N=2* holography

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J. Russo, E. Widén, K.Z., 1901.02835

“Recent Advances in Theoretical Physics of Fundamental Interactions”, ITMP, Moscow, 12.06.19

AdS/CFT correspondence

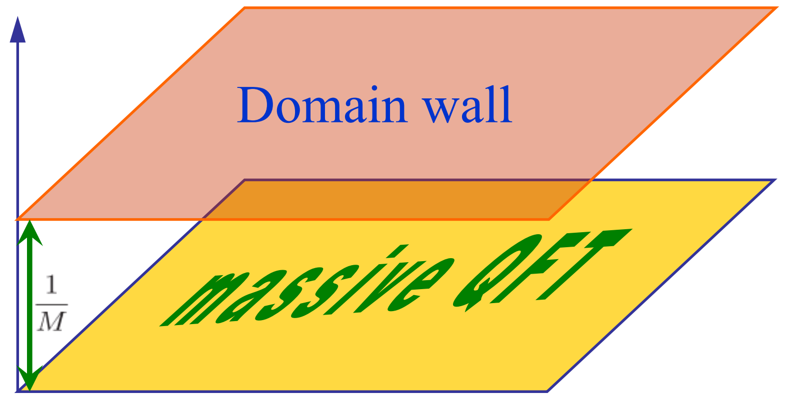
$$ds^2 = \frac{dx^\mu dx_\mu + dz^2}{z^2}$$

AdS/CFT correspondence

Maldacena'97
Gubser,Klebanov,Polyakov'98
Witten'98

$\mathcal{N}=4$ SYM	Strings on $AdS_5 \times S^5$
't Hooft coupling: $\lambda = g_{YM}^2 N$	String tension: $T = \frac{\sqrt{\lambda}}{2\pi}$
Number of colors: N	String coupling: $g_s = \frac{\lambda}{4\pi N}$
Large-N limit	Free strings
Strong coupling	Classical strings
Local operators Scaling dimension: Δ	String states Energy: E

Beyond CFT



Massless

Massive

N=2* theory

$$\mathcal{L} = \frac{1}{g^2}\,\mathrm{tr}\left\{-\frac{1}{2}F_{\mu\nu}^2 + |D_\mu\Phi|^2 + D_\mu Z_i^\dagger D^\mu Z^i + \ldots\right.$$

← N=4 SYM

$$\left. -M^2 Z_i^\dagger Z^i - M\,\mathrm{Im}\,\Phi\varepsilon^{ij}Z_iZ_j + \mathrm{c.c.} + \ldots\right\}$$

hypermultiplet

Z^1

Ψ^1 Ψ^2 $+$ conj.

Z^2

mass $= \pm M$

vector multiplet

A_μ

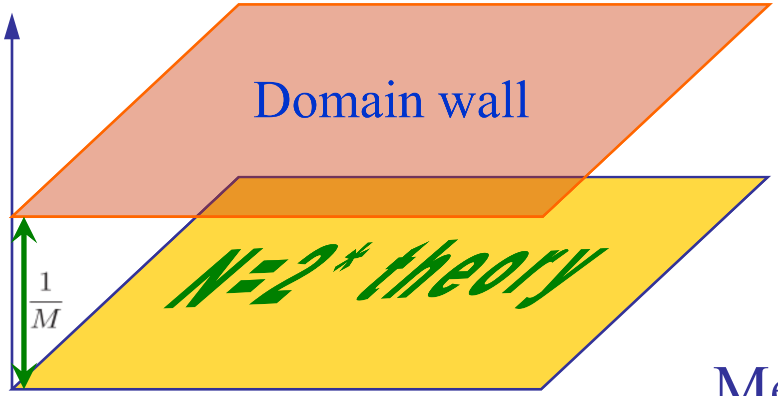
ψ λ

Φ

mass $= 0$

Holographic dual of N=2*

Pilch,Warner'00
Buchel,Peet,Polchinski'00



Metric known explicitly:

$$ds^2 = \frac{A}{c^2-1}\,M^2 dx_\mu^2 + \frac{1}{A\,(c^2-1)^2}\,dc^2$$

$$A = c + \frac{c^2-1}{2}\,\ln\frac{c-1}{c+1} \qquad c = \cosh Mz$$

$Mz \ll 1:$

$$ds^2 \simeq \frac{dx_\mu^2 + dz^2}{z^2}$$

AdS_5

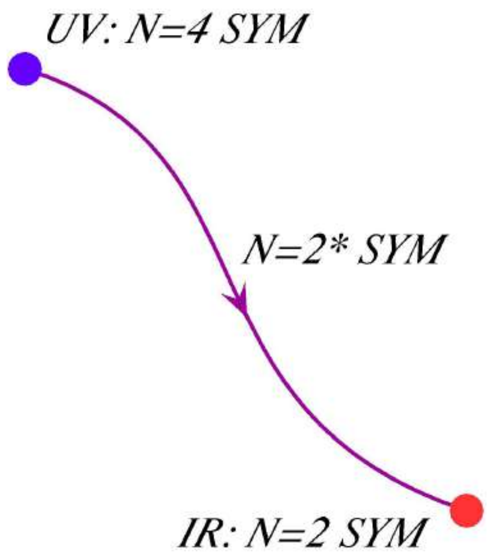
$Mz \gg 1:$

$$ds^2 \approx \frac{\frac{2}{3}\,dx_\mu^2 + \frac{3}{2}\,dc^2}{c^3}$$

AdS_5 with dilaton

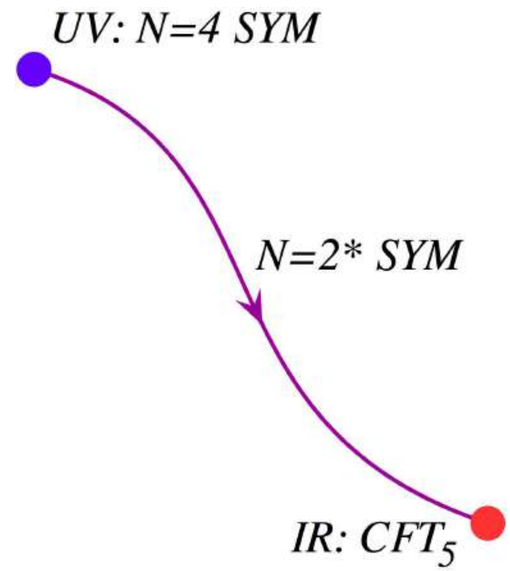
- UV regularization of pure N=2 SYM

Weak coupling



$$\Lambda = M\, e^{-\frac{4\pi^2}{\lambda}}$$

Strong coupling



Hoyos'10

Eguchi-Kawai mechanism?

Young,Z.'14

Moduli space

$$SU(N+1) \to SU(N) \times U(1)$$

$$\Phi_i = \begin{pmatrix} a_i & \\ & \mathbf{0} \end{pmatrix}$$

N=4

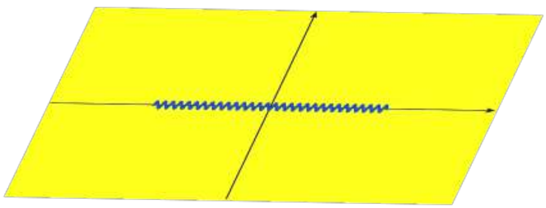
$$a_i - \text{any} \in \mathbb{R}^6$$

Flat, 6 dim

N=2

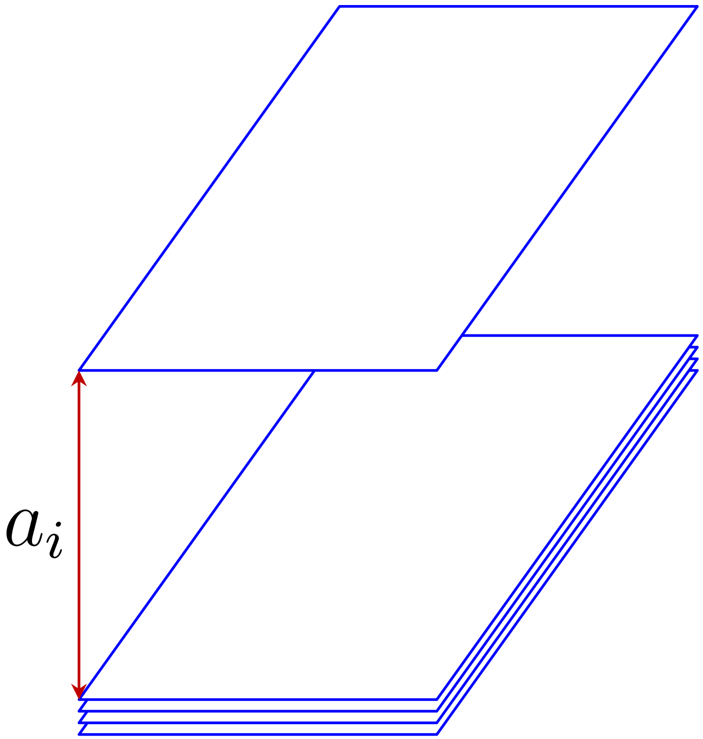
$$a, \bar{a} \in \text{vect mult}$$

$$a = a(u)$$



2 dim complex: SW curve

Moduli space: holography



Space-filling D3-brane

$$AdS_5 \times S^5$$

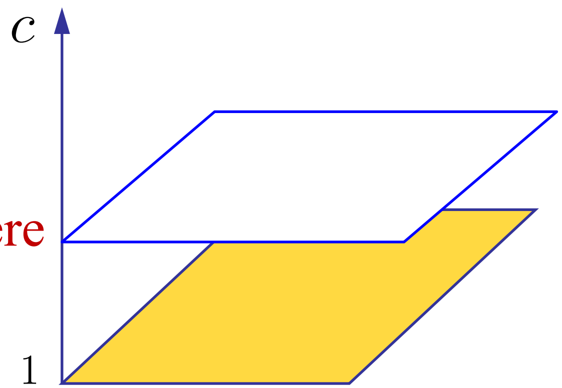
x_0	x_1	x_2	x_3	z	S^5
+	+	+	+	•	•

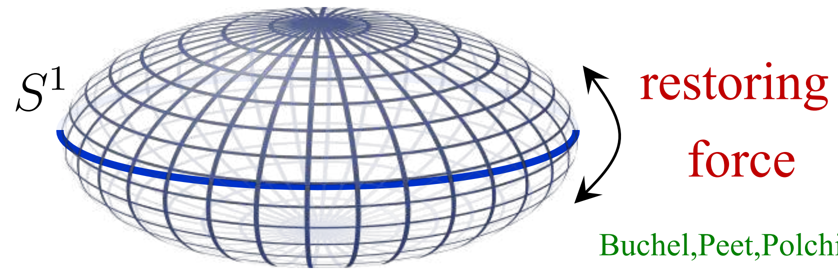
any point

Moduli space: $[0, \infty) \times S^5 = \mathbb{R}^6$

Pilch-Warner background

$$''AdS_5'' \quad \times \quad ''S^5''$$





Buchel,Peet,Polchinski'00
Evans, Johnson, Petrini'00

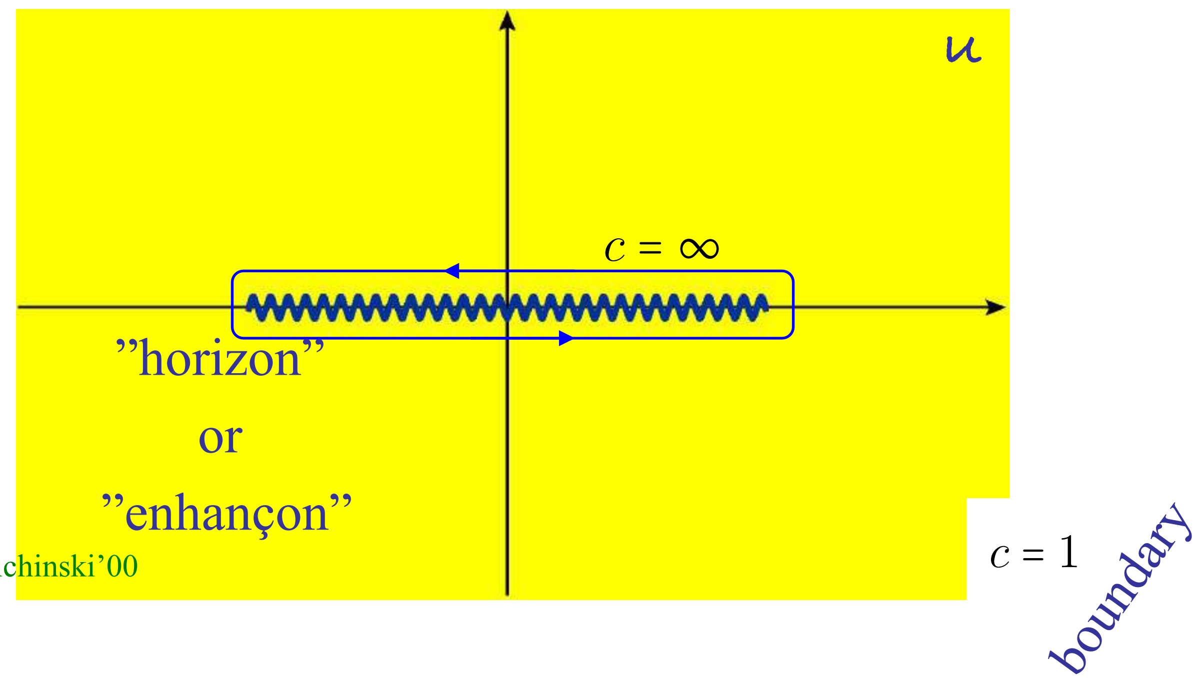
Moduli space: $\quad [1,\infty) \times S^1 = \mathbb{C} \setminus D^2$

u-plane

$$\mathcal{L} = e^{-\Phi(u)}\left(F^2 + \partial u\, \partial \bar{u} + \ldots\right)$$

- unambiguously determines u,ū

$$z = e^{-i\varphi}\sqrt{\frac{c+1}{c-1}} \qquad u = \frac{\mu}{2}\left(z + \frac{1}{z}\right)$$ (Zhukovsky map)



Buchel,Peet,Polchinski’00

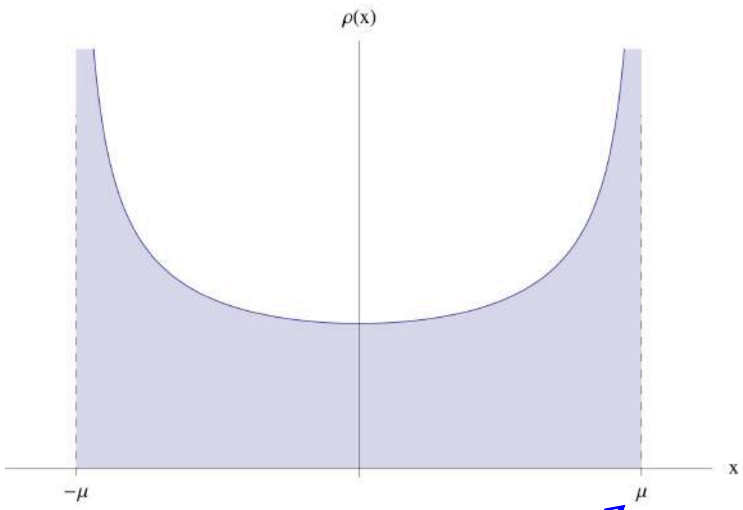
Master field

$$SU(N) \to U(1)^{N-1}$$

$$\langle \Phi \rangle = \mathrm{diag}\,(a_1, \ldots, a_N)$$

Large-N master field:

$$\rho(x) = \frac{1}{N} \sum_{i=1}^{N} \delta\,(x - a_i)$$



largest eigenvalue

Holomorphic coupling:

$$\frac{1}{\lambda_{\text{eff}}} + \frac{\theta}{8\pi^2 i N} \equiv \tau(u) = \frac{1}{\lambda} + \frac{M^2}{8\pi^2} \int_{-\mu}^{+\mu} \frac{dx\, \rho(x)}{(u-x)^2}$$

From supergravity:

$$\tau(u) = \frac{1}{\lambda} \frac{u}{\sqrt{u^2 - \mu^2}}$$

$$\boxed{\rho(x) = \frac{2}{\pi\mu^2}\sqrt{\mu^2 - x^2}} \qquad \mu = \frac{\sqrt{\lambda}\, M}{2\pi}$$

Buchel,Peet,Polchinski'00

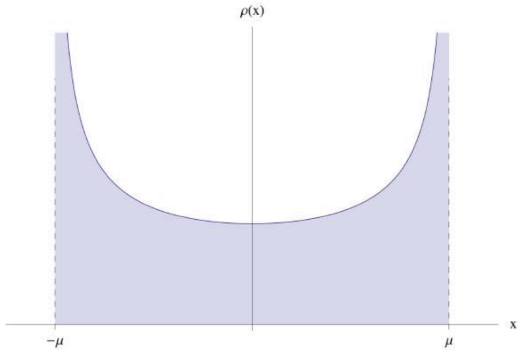
- Wigner distribution. Gaussian random matrices?

Carlisle, Johnson'03

Differs from large-N solution of pure N=2 SYM (without matter):

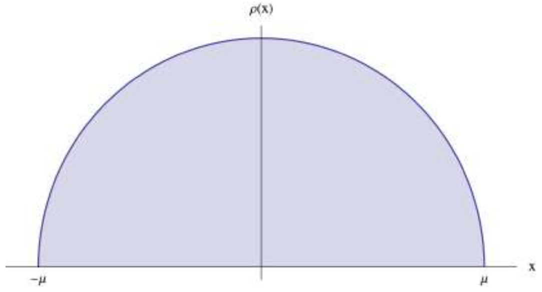
$$\rho(x) = \frac{1}{\pi}\,\frac{1}{\sqrt{\mu^2 - x^2}}$$

Douglas, Shenker'95



In N=2* theory:

$$\rho(x) = \frac{2}{\pi\mu^2}\sqrt{\mu^2 - x^2}$$

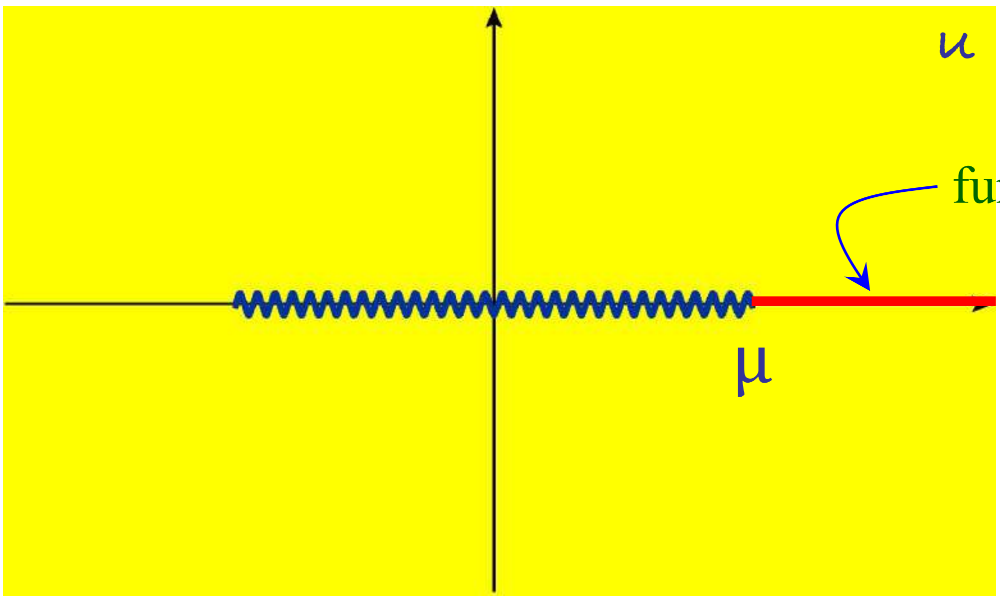


Wilson loop

$$W(C)=\left\langle \frac{1}{N}\,\mathrm{tr}\,\mathrm{P}\exp\oint_C ds\;\left(i\dot{x}^{\mu}A_{\mu}+|\dot{x}|\Phi\right)\right\rangle$$

Holographic description:

$$W(C)=\int_{\partial X=C}\mathcal{D}X\;\mathrm{e}^{-\int(\partial X)^2}\simeq\mathrm{e}^{-A_{\min}(C)}$$



fundamental string

$$A=\int_{\mu}^{\infty}\int_{0}^{L}du\,dx=\infty-\mu L$$

Perimeter law:

$$W(C_L)=\mathrm{e}^{\mu L}$$

$$\mu=\frac{\sqrt{\lambda}\,M}{2\pi}$$

Buchel, Russo, Z.'13

Summary

Eigenvalue density:

$$\rho(x) = \frac{2}{\pi\mu^2}\sqrt{\mu^2 - x^2} \qquad \mu = \frac{\sqrt{\lambda}\,M}{2\pi}$$

- points towards some matrix model description
- differs from large-N solution of N=2 SYM $\quad \rho(x) = \frac{1}{\pi}\,\frac{1}{\sqrt{\mu^2 - x^2}}$

Wilson loop expectation value:

$$W(C_L) = \mathrm{e}^{\,\mu L}$$

Strong coupling from field theory: localization

Pestun'07

Semiclassics is exact:

path int → sum over cr pts

$$\int \mathcal{D}\varphi\mathcal{D}\psi\, \mathrm{e}^{-S[\varphi,\psi]} = \sum_{\varphi_*} \mathrm{Sdet}^{-1/2} K\, \mathrm{e}^{-S[\varphi_*]}$$

Localization locus in N=2* theory:

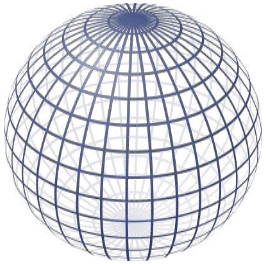
- constant scalar mode on S^4

$$\Phi = \mathrm{diag}\left(a_1, \ldots, a_N\right)$$

$$Z=\int d^{N-1}a\prod_{i<j}\frac{\left(a_i-a_j\right)^2H^2\left(a_i-a_j\right)}{H\left(a_i-a_j+M\right)H\left(a_i-a_j-M\right)}\,\mathrm{e}^{-\frac{8\pi^2N}{\lambda}\sum_i a_i^2}$$

Pestun'07

$$H(x)=\prod_{n=1}^{\infty}\left(1+\frac{x^2}{n^2}\right)^n\mathrm{e}^{-\frac{x^2}{n}}$$



- Matrix model (not Gaussian!): saddle-point at large N

Saddle-point equations

$$\fint_{-\mu}^{\mu} dy\, \rho(y) \left(\frac{2}{x-y} - \frac{1}{x-y+M} - \frac{1}{x-y-M} \right) = 0$$

Russo, Z.'13

+ two normalization conditions:

$$\int_{-\mu}^{+\mu} dx\, \rho(x) \ln\left(\frac{M^2}{x^2} - 1 \right)^2 = \frac{16\pi^2}{\lambda}$$

$$\int_{-\mu}^{\mu} dx\, \rho(x) = 1$$

RG log

$$W(C_L) = \mathrm{e}^{\mu L}$$

Buchel, Russo, Z.'13

Weak coupling

$M \gg \mu$

$$\fint_{-\mu}^{\mu} \frac{dy\, \rho(y)}{x-y} = 0$$

Solution:

$$\rho(x) = \frac{1}{\pi} \frac{1}{\sqrt{\mu^2 - x^2}}$$

Russo, Z.'12 agrees with Douglas, Shenker'95

Normalization condition:

$$\ln \frac{M^4}{\mu^4} = \frac{16\pi^2}{\lambda} \quad \Longrightarrow \quad \mu = M\, \mathrm{e}^{-\frac{4\pi^2}{\lambda}}$$

Finite coupling

Resolvent:

$$r(u) = \frac{i}{2}\int_{-\mu}^{\mu} dy\,\rho(y)\left(\frac{2}{u-y} - \frac{1}{u-y+M} - \frac{1}{u-y-M}\right)$$

(by analogy with supergarvity)

$$r(u) = \tau'(u)$$

$$\mathrm{Cont}\, r(x) = \fint_{-\mu}^{\mu} dy\,\rho(y)\left(\frac{2}{x-y} - \frac{1}{x-y+M} - \frac{1}{x-y-M}\right) = 0$$

Discontinuity:

$$2\rho(x) - \rho(x-M) - \rho(x+M) = 2r(x)$$

Exact eigenvalue density

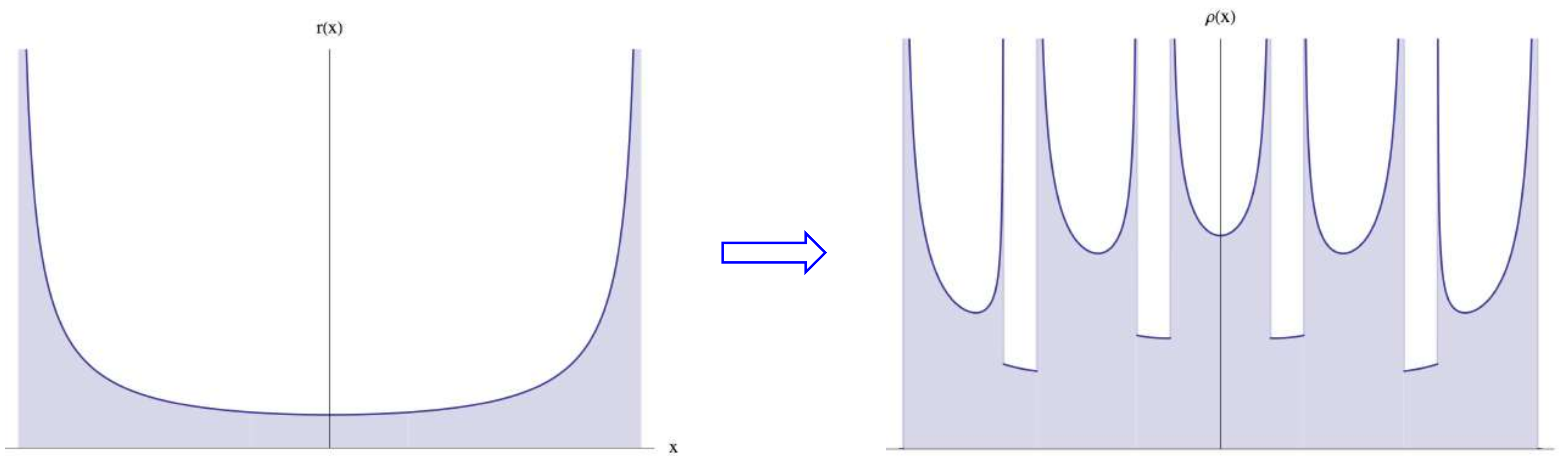
$$2\rho(x) - \rho(x-M) - \rho(x+M) = 2r(x)$$

Inverting discrete Laplace:

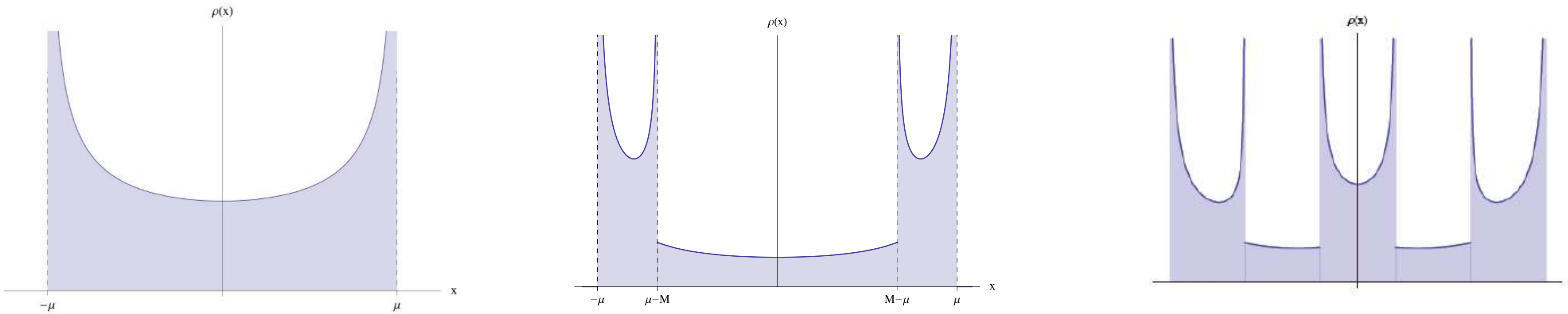
$$\rho(x) = \frac{2k}{n+a}\sum_{l=0}^{n+a-k-l}(n+a-k-l)r(x+lM) + \frac{2(n+a-k)}{n+a}\sum_{l=1}^{k-1}(k-l)r(x-lM)$$

$$k = \left[\frac{\mu+x}{M}\right]+1, \qquad a = \left[\frac{\mu-x}{M}\right]-n+k$$

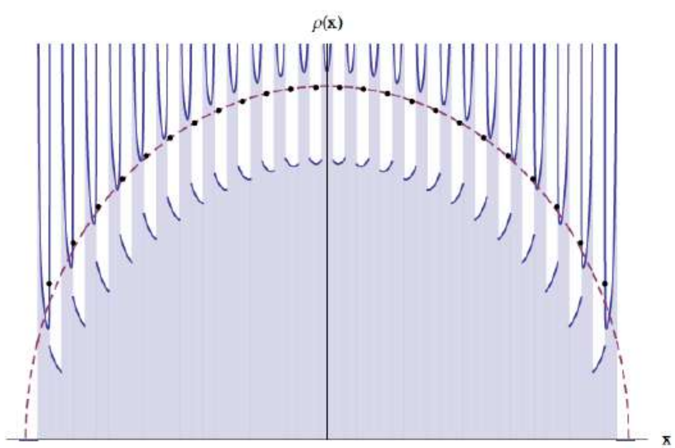
frac. part



Exact density at finite coupling



At $\lambda \to \infty$



Chen-Lin, Gordon, Z.'14
Z.'14

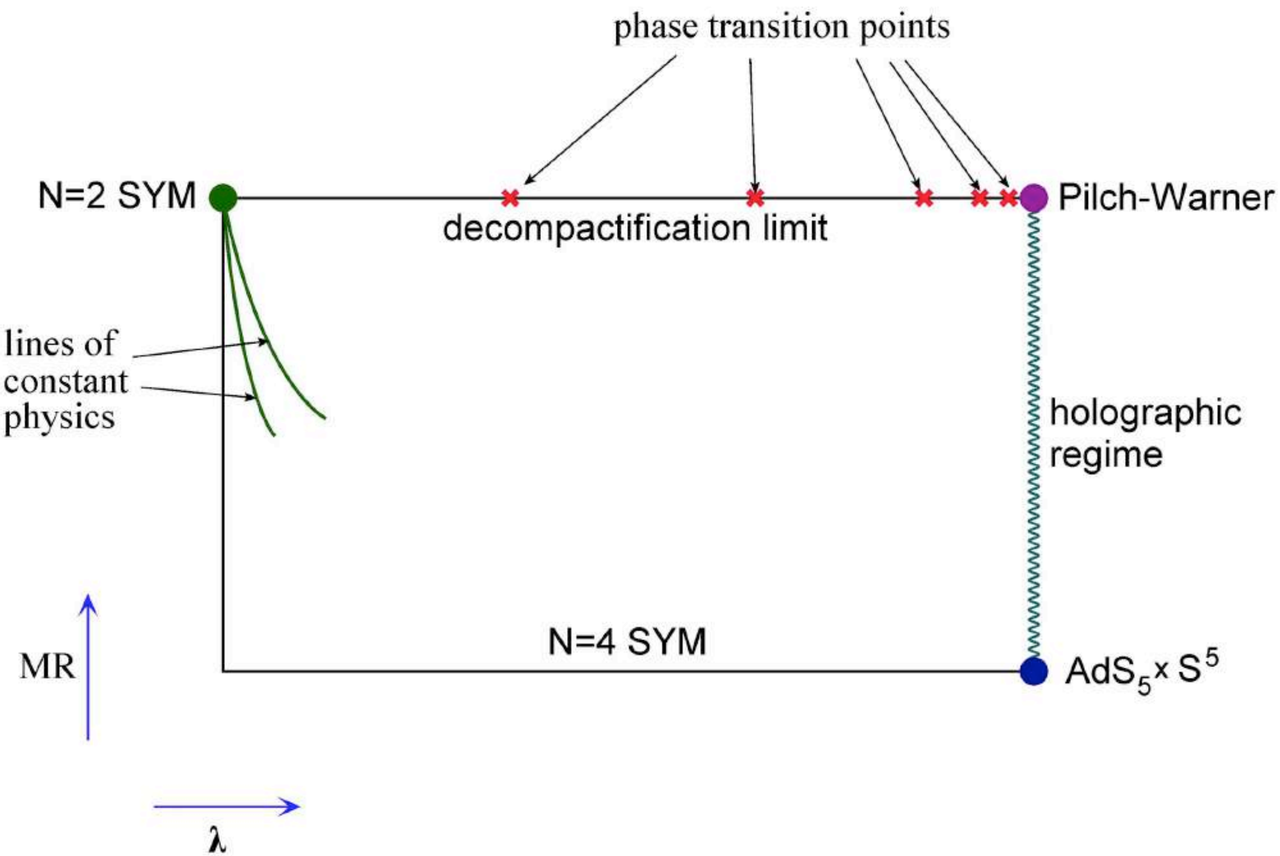
Average density:

$$\bar{\rho}(x) = \frac{2}{\pi\mu^2}\sqrt{\mu^2 - x^2} \qquad \mu = \frac{\sqrt{\lambda}M}{2\pi}$$

Buchel, Russo, Z.'13

Agrees with holography!

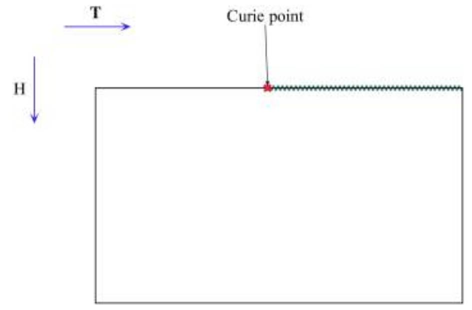
Phase diagram



Russo, Z.'13

R: radius of S^4

Ising model:



Critical indices

String tension:

$$\mu = \frac{nM}{2} + C\left(\lambda - \lambda_c^{(n)}\right)^{\beta_n}$$

$$\beta_1 = \frac{3}{2} \qquad \beta_n = 1 + \frac{2}{\pi n} + \dots$$

Russo, Z.'13
Russo, Widén, Z.' 19

Susceptibility:

$$\frac{\partial}{\partial\lambda} \ln Z \propto \left\langle x^2 \right\rangle = \text{analytic} + C\left(\lambda - \lambda_c^{(n)}\right)^{\gamma_n}$$

$$\gamma_1 = 3 \qquad \gamma_n = 1 + \frac{2}{\pi n} + \dots$$

Russo, Widén, Z.' 19

Edge behavior at criticality:

$$\rho_c(x) \sim \frac{C}{(\mu - x)^{\alpha_n}} \qquad \alpha_n = \frac{1}{2} + \frac{1}{\pi} \arcsin \frac{1}{n+1}$$

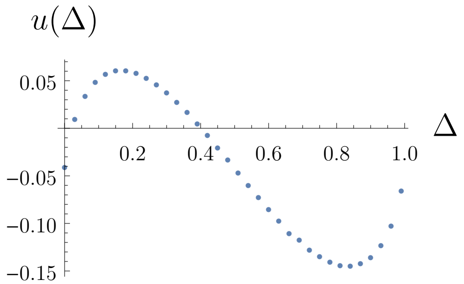
Z.'14

Comparison to string theory

Effective string tension: $W(C) \simeq \mathrm{e}^{\mu L(C)}$

$$\mu = \frac{\sqrt{\lambda}\, M}{2\pi}\left[1 - \frac{\pi}{\sqrt{\lambda}} + \left(\frac{1}{8} - u(\Delta)\right)\frac{\pi^2}{\lambda} + \ldots\right]$$ from localization

function of $\Delta = \left\{\frac{2\mu}{M}\right\} \simeq \left\{\frac{\sqrt{\lambda}}{\pi}\right\}$



Russo, Widén, Z.' 19

Non-analytic at $\Delta \to 0$ or 1

$$u(\Delta) \simeq \frac{2}{\pi}\epsilon \ln \epsilon, \qquad \epsilon = \Delta \text{ or } 1 - \Delta$$

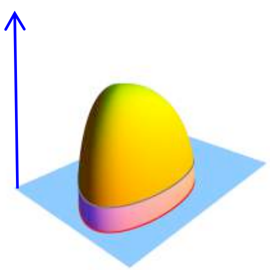
✓ ✓ ?

$$\mu = \frac{\sqrt{\lambda}\,M}{2\pi}\left[1 - \frac{\pi}{\sqrt{\lambda}} + \left(\frac{1}{8} - u(\Delta)\right)\frac{\pi^2}{\lambda} + \ldots\right]$$

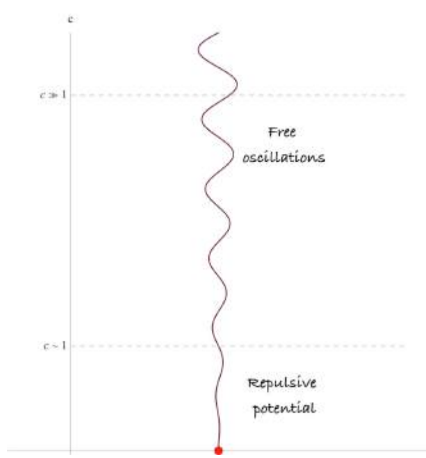
Area law

String fluctuations

Two loops on the worldsheet?



Buchel, Russo, Z.'13



Chen-Lin, Medina-Rincon, Z.'17

Conclusions

- N=2* theory is an ideal playground for non-trivial tests of holography beyond CFT
- highly non-trivial phase structure at large-N
- direct link to matrix models, via localization
- what are the implications of the phase transitions for (non-)AdS/(non-)CFT?
- phase transitions on the string worldsheet?