Renormalizable quantum gravity with anisotropic scaling

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w/ A.Barvinsky, D.Blas, M.Herrero-Valea, C.Steinwachs

1512.02250, 1705.03480, 1706.06809, 1905.03798

Recent Advances in Theoretical Physics of Fundamental Interactions, Jun 11, 2019

Can gravity be formulated as a UV complete Quantum Field Theory ?

(unitary, finite number of d.o.f., under control, ...)

General Relativity

Beautiful **local** gauge theory based on **diff-invariance**

Classical: describes phenomena from $10^{-2}\mathrm{cm}$ to $10^{28}\mathrm{cm}$

fails in black hole / cosmological singularities

Quantum: an effective theory valid up to $M_P \simeq 2 \times 10^{19}\mathrm{GeV}$

requires **UV completion** at energies higher than M_P

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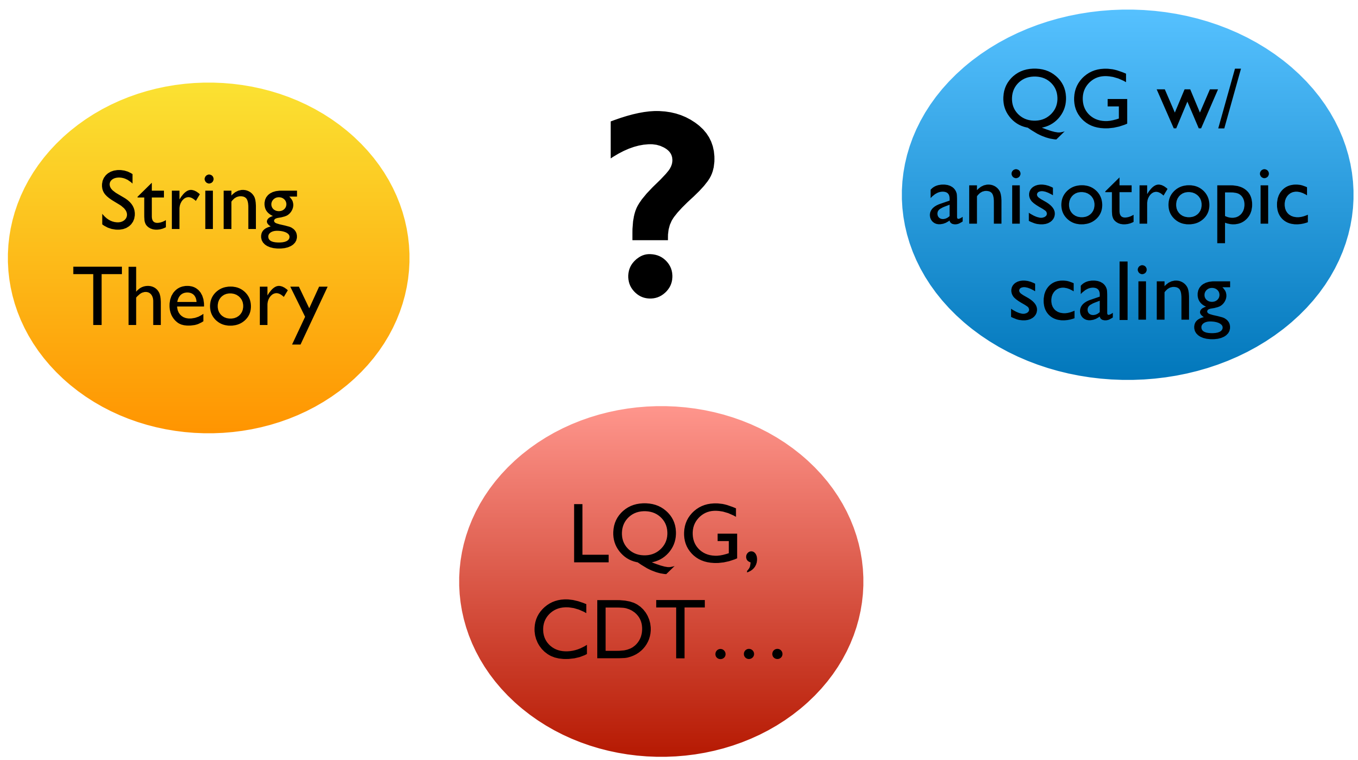

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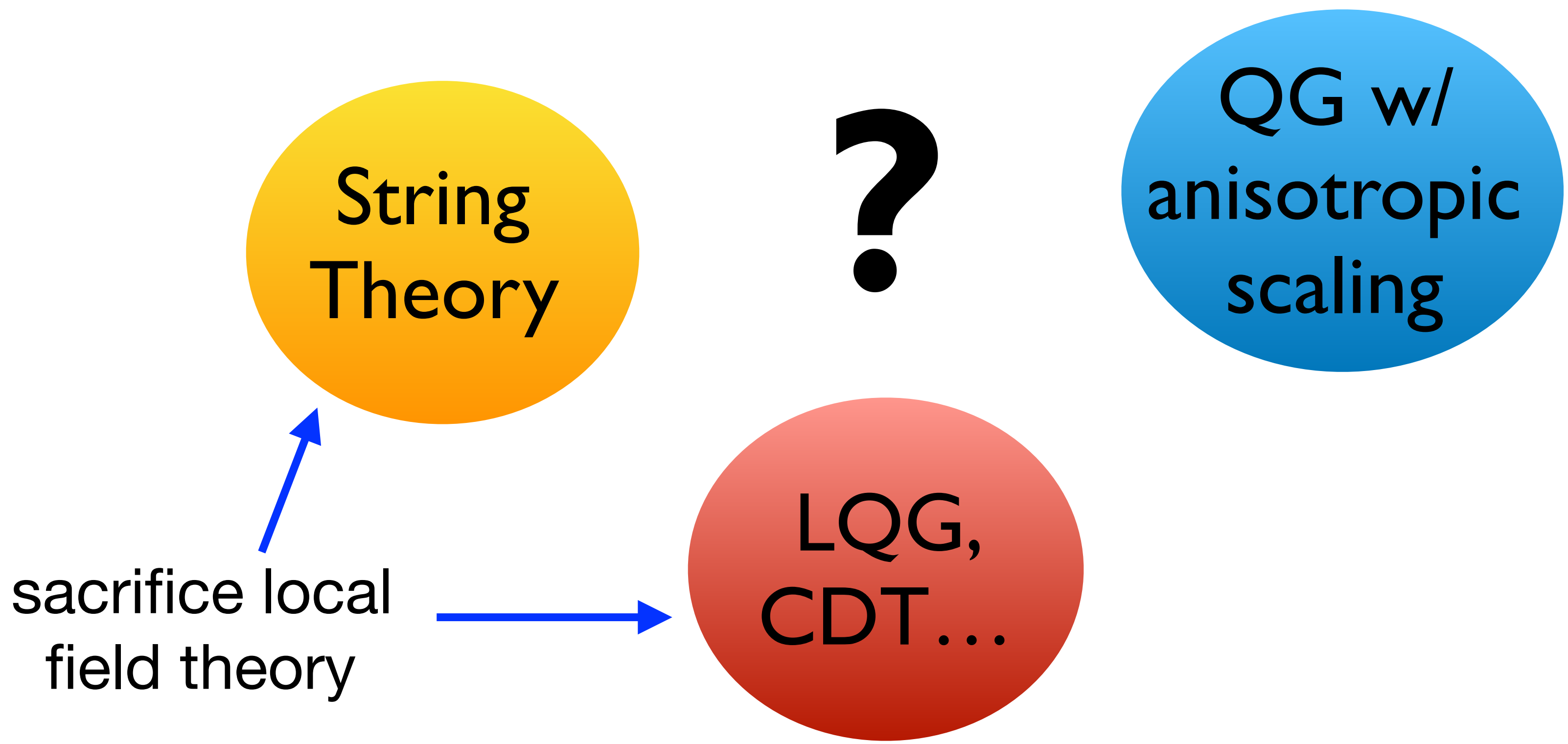

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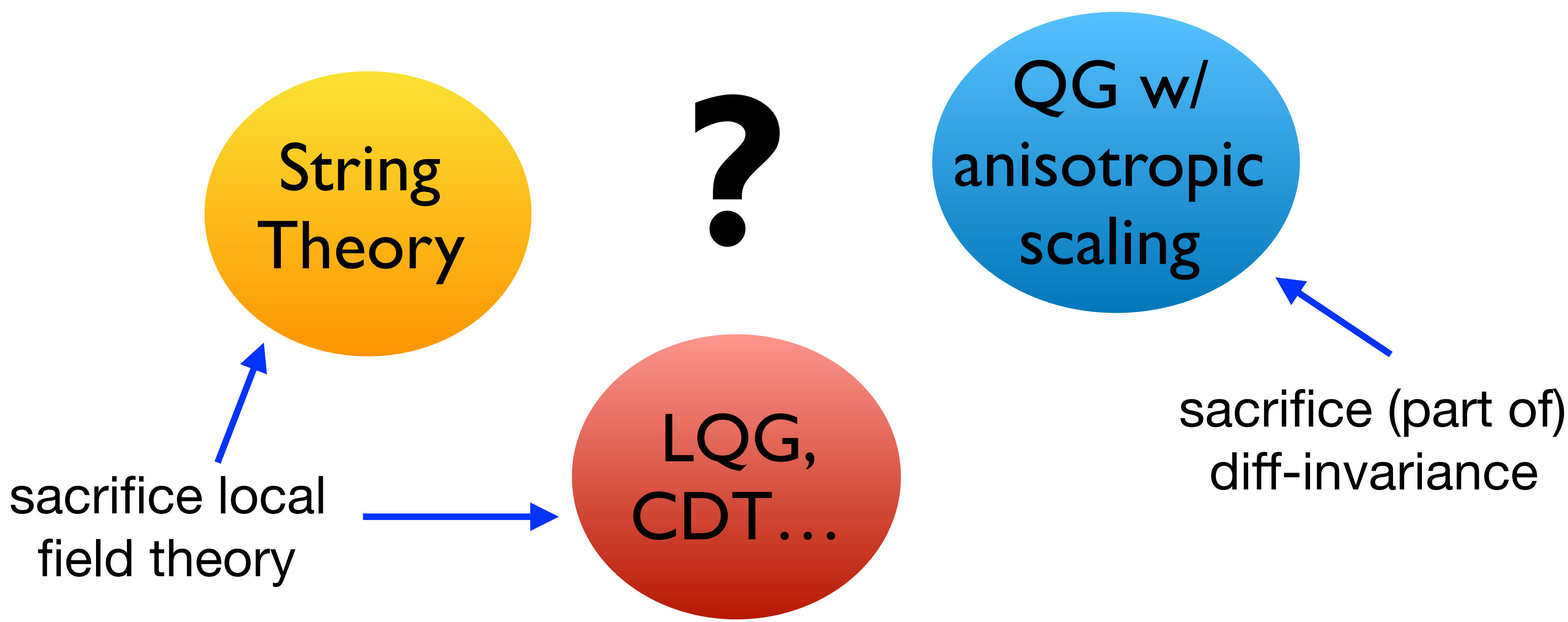

Why GR is not UV complete ?

$$S_{EH} = \frac{M_P^2}{2} \int d^{d+1}x \sqrt{g} R$$

quadratic action sets the amplitude of fluctuations

$$\Rightarrow \frac{M_P^2}{2} \int d^{d+1}x \left(\underbrace{h_{ij} \Box h_{ij}}_{} + \underbrace{h^2 \Box h + \dots}_{} \right)$$

interaction terms

Zoom in on shorter scales: $x^\mu \mapsto b^{-1} x^\mu$

To preserve the quadratic action scale the metric:

$$h_{\mu\nu} \mapsto b^{(d-1)/2} h_{\mu\nu}$$

scaling dimension

Look at the interactions: $S_{int} \mapsto b^{(d-1)/2} S_{int}$

For $d > 1$ the interaction strength grows unboundedly at short distances ($b \to \infty$)

Generation of higher-order operators ➡ loss of predictive power

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A failed attempt

Stelle (1977)

Interactions contain arbitrarily high powers of the metric

Different from Yang-Mills theory, similar to sigma models

➡ If we want to bound the interactions in UV we need to reduce the scaling dimension of h_{ij} to zero

$$\int d^4x \sqrt{g}\left(M_P^2 R + R_{\mu\nu}R^{\mu\nu} + R^2\right)$$

dominates at high energies, determines the scaling dim of the metric in UV

$$\Rightarrow \int d^4x \left(M_P^2 h_{ij}\Box h_{ij} + \underbrace{h_{ij}\Box^2 h_{ij}} + \dots\right)$$

Fast decrease of the graviton propagator $\langle h\, h\rangle \propto 1/k^4$ improves convergence of the loop integrals. The theory is renormalizable and asymptotically free !

Fradkin, Tseytlin (1981)

Avramidi, Barvinsky (1985)

But higher time derivatives give **ghost poles**

➡ no unitary interpretation

$$\langle h\, h\rangle \sim \frac{1}{k^2} - \frac{1}{k^2 + M_P^2}$$

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Never give up

Imagine that spacetime is endowed with a preferred spacelike foliation

General covariance is reduced to foliation-preserving diffeomorphisms

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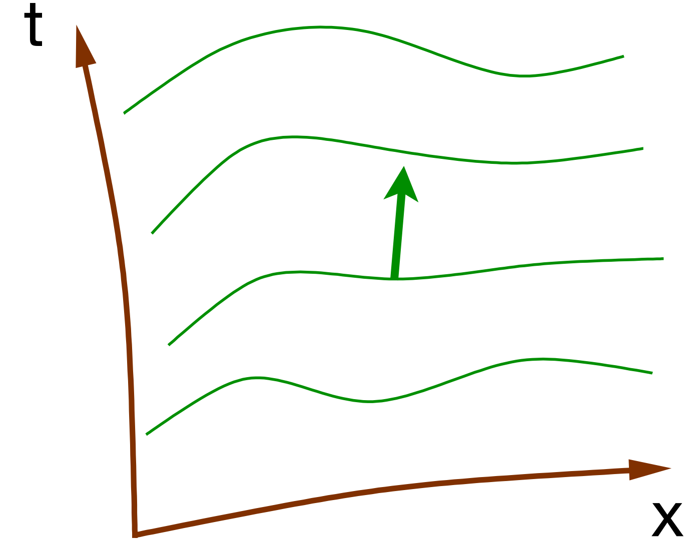

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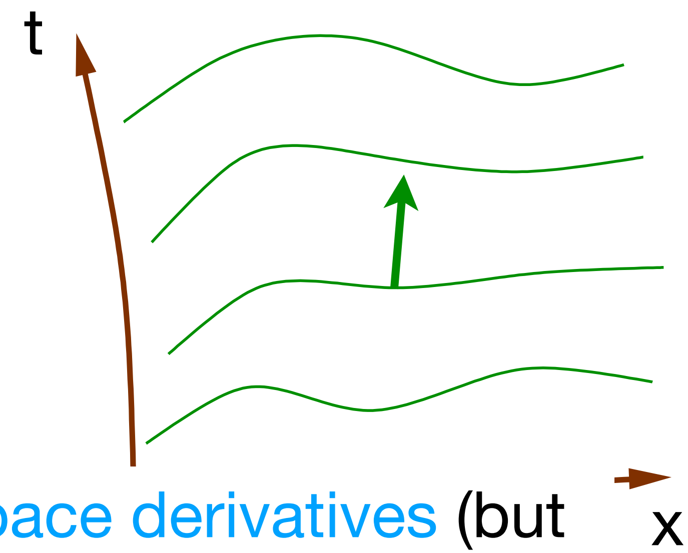

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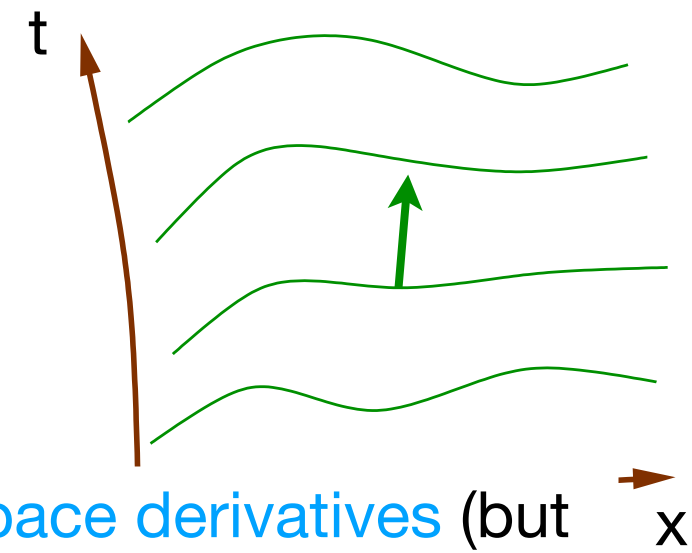

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$$\int \underbrace{dt\, d^d x}_{\propto\, b^{-(z+d)}} \left(\dot{h}_{ij}\dot{h}_{ij} - h_{ij}(-\Delta)^z h_{ij} + \dots \right)$$

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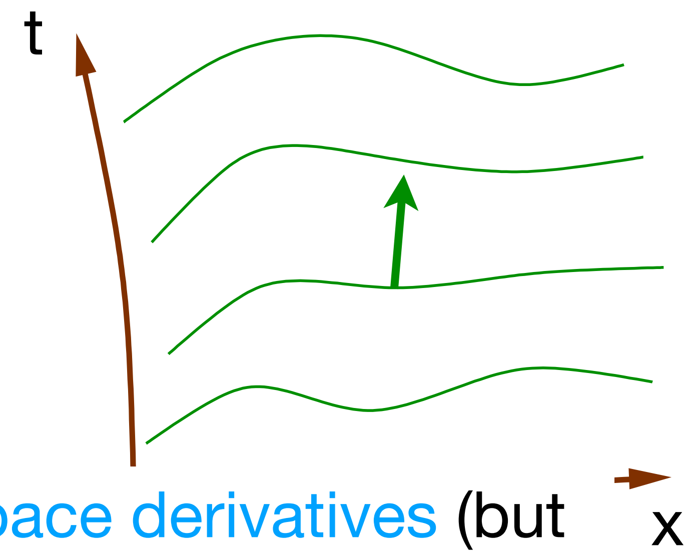

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critical theory in $z = d$

"LET'S SEE IF WE COULD PUT A SPIN ON IT AND GET THE PUBLIC INTERESTED."

Field content and low-E limit

We want to preserve as many symmetries, as possible

foliation-preserving diffeos

$$\begin{cases} x^i \mapsto \tilde{x}^i(\mathbf{x}, t) & \longrightarrow \quad \gamma_{ij}, \quad N^i \\ t \mapsto \tilde{t}(t) & \longrightarrow \quad N \end{cases}$$

$$\mathcal{L} = M_P^2 \sqrt{\gamma} N (K_{ij} K^{ij} - \lambda K^2 - \mathcal{V}[\gamma_{ij}, N])$$

$$\text{dim}\gamma_{ij} = \text{dim}N = 0$$

$$\text{dim}N^i = d - 1$$

$$K_{ij} = \frac{\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i}{2N}$$

contains terms with up to $2d$ spatial derivatives

Reduces to a scalar-tensor gravity at low energies

$$\longrightarrow \quad \mathcal{L} = M_P^2 \sqrt{g} R + \mathcal{L}_\chi [g_{\mu\nu}, \chi]$$

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Blas, Pujolas, S.S. (2009, 2010)

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What about renormalizability
or why power-counting is not enough ?

A naive “proof”:

$$\mathcal{Z} = \int [dh] e^{-S}$$

$$= \sum \frac{(-1)^n}{n!} \int [dh] e^{-S_0} \underbrace{\int dx_1 \ldots dx_n \mathcal{L}_{int}(x_1) \ldots \mathcal{L}_{int}(x_n)}_{dim \leq 0}$$

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this is not guaranteed because of gauge invariance

Toy model: d=2 “projectable”

“projectability condition” $N = N(t)$ → set $N = 1$ by gauge-fixing time and forget

$\mathrm{dim}\gamma_{ij} = 0 \quad \mathrm{dim}N^i = 2$

$$\mathcal{L} = \frac{1}{2G}\left(K_{ij}K^{ij} - \lambda K^2 - \mu R_{\mathrm{sp}}^2\right)$$

- is fully parameterized by 3 couplings
- unlike GR in 3d, has propagating d.o.f., a single scalar
- is well-behaved for $G, \mu > 0$ and $\lambda < 1/2$ or $\lambda > 1$

Gauge fixing

We need to fix spatial diffeos

linear combination of the fields

$$\mathcal{L}_{gf} = \frac{\sigma}{2G} F^i \mathcal{O}_{ij} F^j$$

must have dim=4 to preserve power-counting

invertible operator

Local $\mathcal{O}_{ij}$ → $F^i = N^i$, $F^i = \partial_j h_{ij} + \sigma' \partial_i h$ → $\langle N^i N^j \rangle \ni \delta_{ij} \frac{G}{k^2}$

no dependence on energy

$$\langle N^i(t,x) N^j(0) \rangle \ni \frac{\delta_{ij}}{4\pi} \delta(t) \log |\mathbf{x}|$$

the singularity is **non-local** (in space)
Hard to keep track of divergences

Similar to the Coulomb gauge in YM. What is the analog of covariant gauges ?

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Regular propagators

$[\Phi_1] = r_1 \; , \quad [\Phi_2] = r_2$ under Lifshitz scaling with $z = d$

$$\langle \Phi_1 \Phi_2 \rangle = \sum \frac{P(\omega, k)}{D(\omega, k)}$$

$$D = \prod_{m=1}^{M} \left(A_m \omega^2 + B_m k^{2d} + \ldots \right) , \quad A_m, B_m > 0$$

P polynomial of degree $r_1 + r_2 + 2(M-1)d$ (to ensure the correct scaling at short distances)

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Regular propagators have local singularities in position space

Regular gauges

We have to allow for non-local gf. Lagrangian. Good choice:

$$\mathcal{O}_{ij} = -\big(\delta_{ij}\Delta + \xi\partial_i\partial_j\big)^{-1}$$

$$F^i = \dot{N}^i + \frac{1}{2\sigma}\mathcal{O}^{-1}_{ij}\partial_k h_{jk} - \frac{\lambda}{2\sigma}\mathcal{O}^{-1}_{ij}\partial_j h$$

- disentangle h_{ij} from N^i in the quadratic action
- regular propagators for all fields (including Faddeev-Popov ghosts)
- two free gf. parameters $\sigma,\ \xi$
- straightforward generalization to $d>2$, e.g.

$$\mathcal{O}^{d=3}_{ij} = \Delta^{-1}\big(\delta_{ij}\Delta + \xi\partial_i\partial_j\big)^{-1}$$

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Diagrammatics in brief

- induction in the number of loops
- subdivergences are cancelled by counterterms introduced at the previous steps *Anselmi, Halat (2007)*
- introduce the degree of divergence $\mathcal{D}$ defined as the scaling of the diagram under stretching the loop momenta and frequencies $k_{loop} \mapsto b\, k_{loop}$, $\omega_{loop} \mapsto b^d\, \omega_{loop}$
- if all propagators are regular, diagrams with $\mathcal{D} < 0$ converge
- diags. with $\mathcal{D} > 0$ require **local** counterterms of scaling dimension at most $2d$

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Comments

- Straightforward generalization to projectable HL gravity in any dimensions

- Does not work for non-projectable: additional variable $N = 1 + \phi$

$$\langle \phi\phi \rangle = \text{regular} + \frac{1}{k^{2d}}$$

present even in $\sigma\xi$ - gauges
physical: shows up in the interaction of local sources

Cancellation of non-local divergence due to
time-reparameterization ???

Gauge invariance ?

GI is explicitly broken by the gauge-fixing. Instead, we have to rely on the BRST symmetry (Slavnov-Taylor identities)

Non-linearity of BRST → deformation by quantum corrections. To restore original BRST, the gauge fields must be redefined at every loop order

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Everything from scratch !

Renormalization in the background-field method

Decompose the fields in the “background” and “quantum fluctuations”

$$\gamma_{ij} = \bar{\gamma}_{ij} + h_{ij} \; , \quad N^i = \bar{N}^i + n^i$$

Doubles the number of GI:

- BRST transformations acting on fluctuations
- **Backgroung GI** acting on both. Acts linearly !

⇒ one-loop counterterms are manifestly gauge-invariant

at higher loops BGI helps to explicitly separate redefinition of quantum fields from renormalization of couplings

General result: BRST structure is preserved in any non-anomalous gauge theory admitting sensible BF formulation

- YM, GR and their higher-derivative extensions
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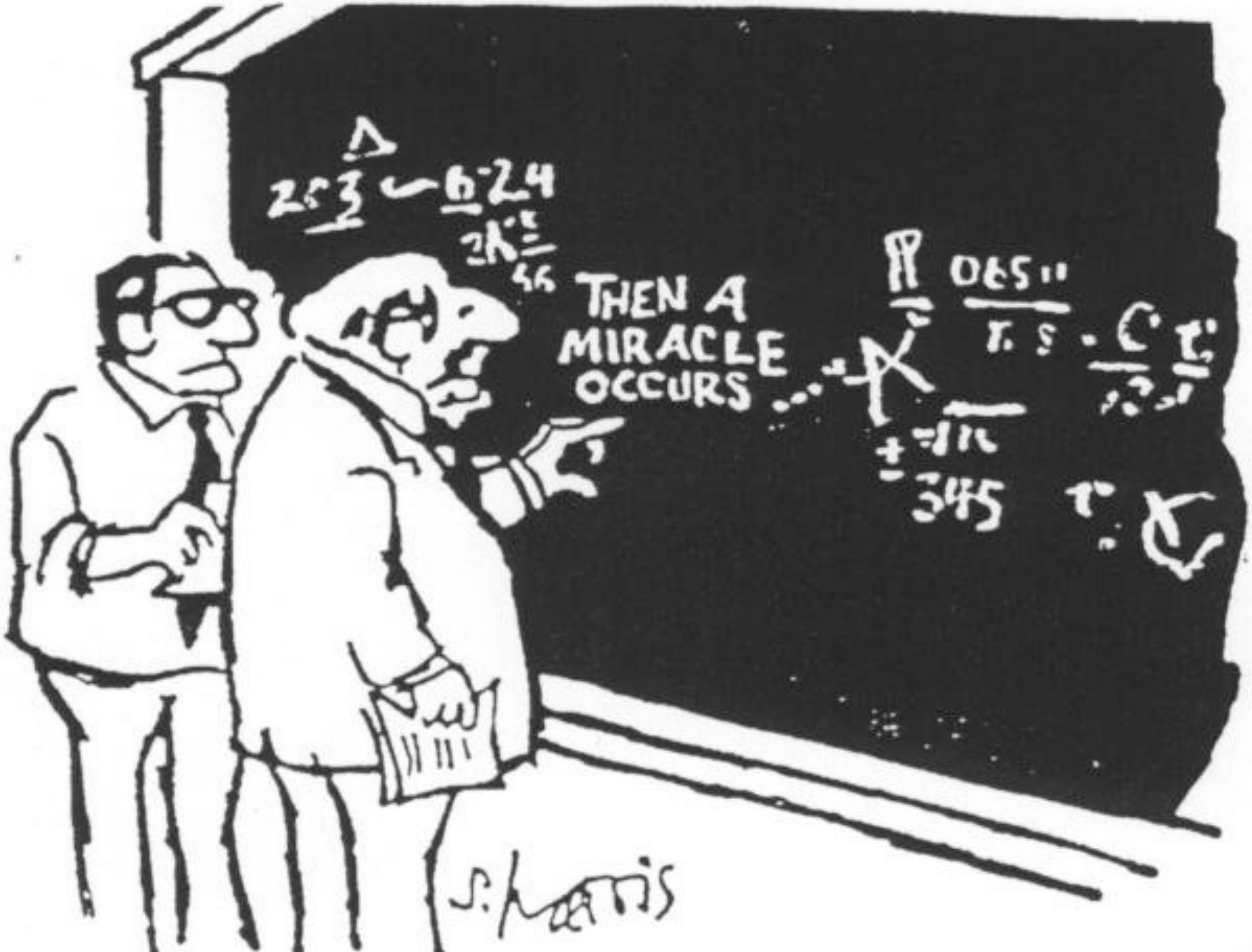

I think you should be a little
more specific, here in Step 2

Renormalization group

$$\mathcal{L} = \frac{1}{2G}\left(K_{ij}K^{ij} - \lambda K^2 - \mu R_{\rm sp}^2\right)$$

β -functions are **not** separately gauge invariant

Background effective action gets contributions proportional to eom's when the gauge is changed

$$\Gamma \mapsto \Gamma + \alpha \int dt\, d^2x \left(\bar{K}_{ij}\bar{K}^{ij} - \lambda \bar{K}^2 + \mu \bar{R}^2\right)$$

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invariant combinations: λ, $\mathcal{G} = \frac{G}{\sqrt{\mu}}$

Sit down and calculate

- fix the background gauge

  3 gauge choices: 2 regular + conformal $h_{ij} = \bar{\gamma}_{ij} e^{2\zeta}$

- expand background: $\bar{\gamma}_{ij} = \delta_{ij} + H_{ij}$
- integrate out fluctuations

- extract divergent parts of coefficients in front of

$$\dot{H}_{ij}\dot{H}_{ij} \quad , \quad (\dot{H}_{ii})^2 \quad , \quad (\partial_i \partial_j H_{ij})^2$$

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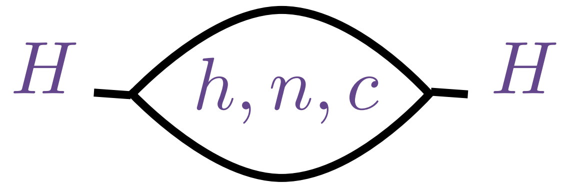

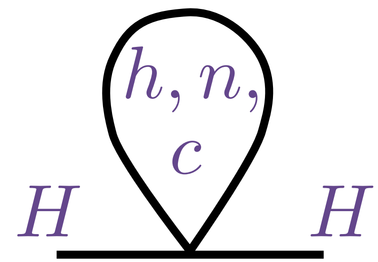

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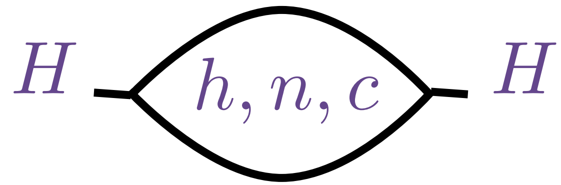

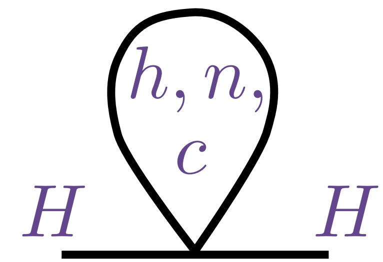

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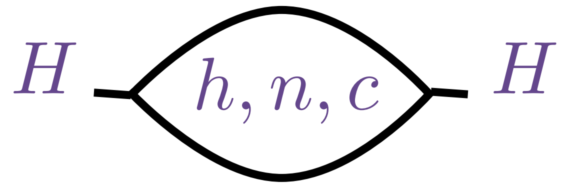

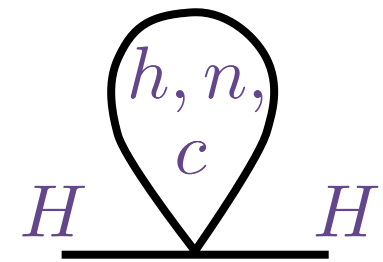

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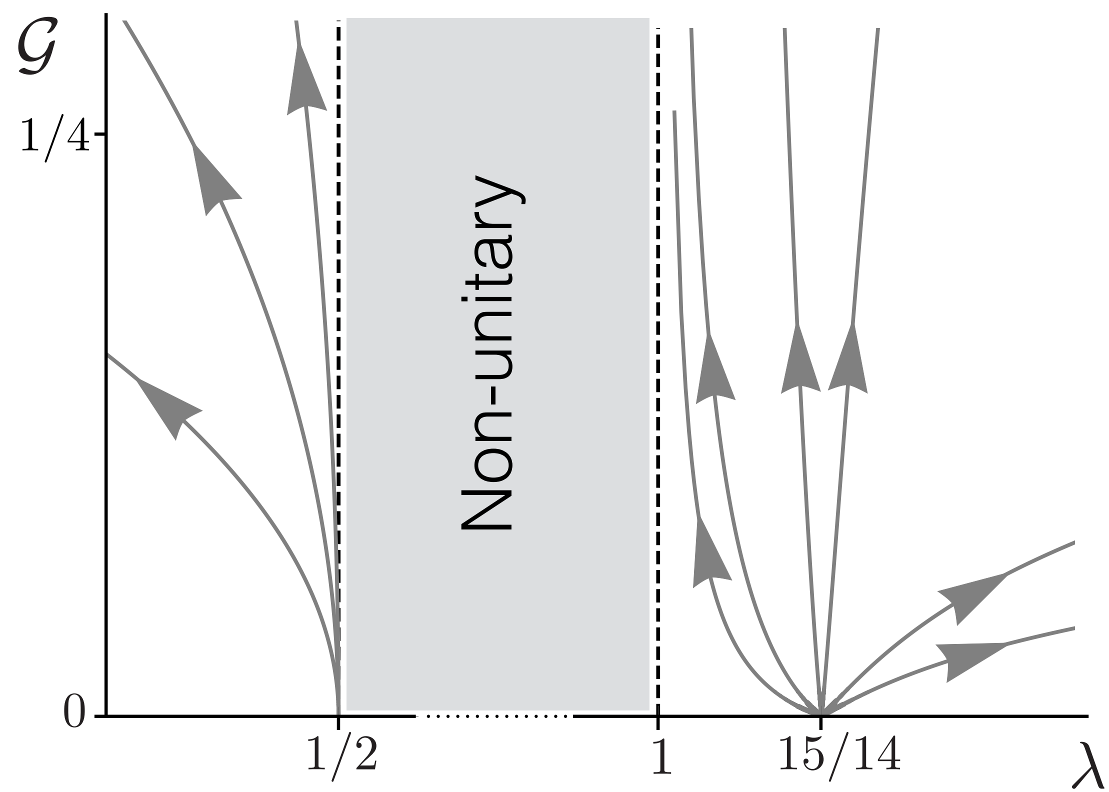
$$\frac{d\lambda}{d\log\Lambda} = \frac{15 - 14\lambda}{64\pi}\sqrt{\frac{1-2\lambda}{1-\lambda}}\,\mathcal{G} \qquad \frac{d\mathcal{G}}{d\log\Lambda} = -\frac{(16 - 33\lambda + 18\lambda^2)}{64\pi(1-\lambda)^2}\sqrt{\frac{1-\lambda}{1-2\lambda}}\,\mathcal{G}^2$$

RG portrait of Horava-Lifshitz gravity in (2+1)d

RG flow of essential couplings:



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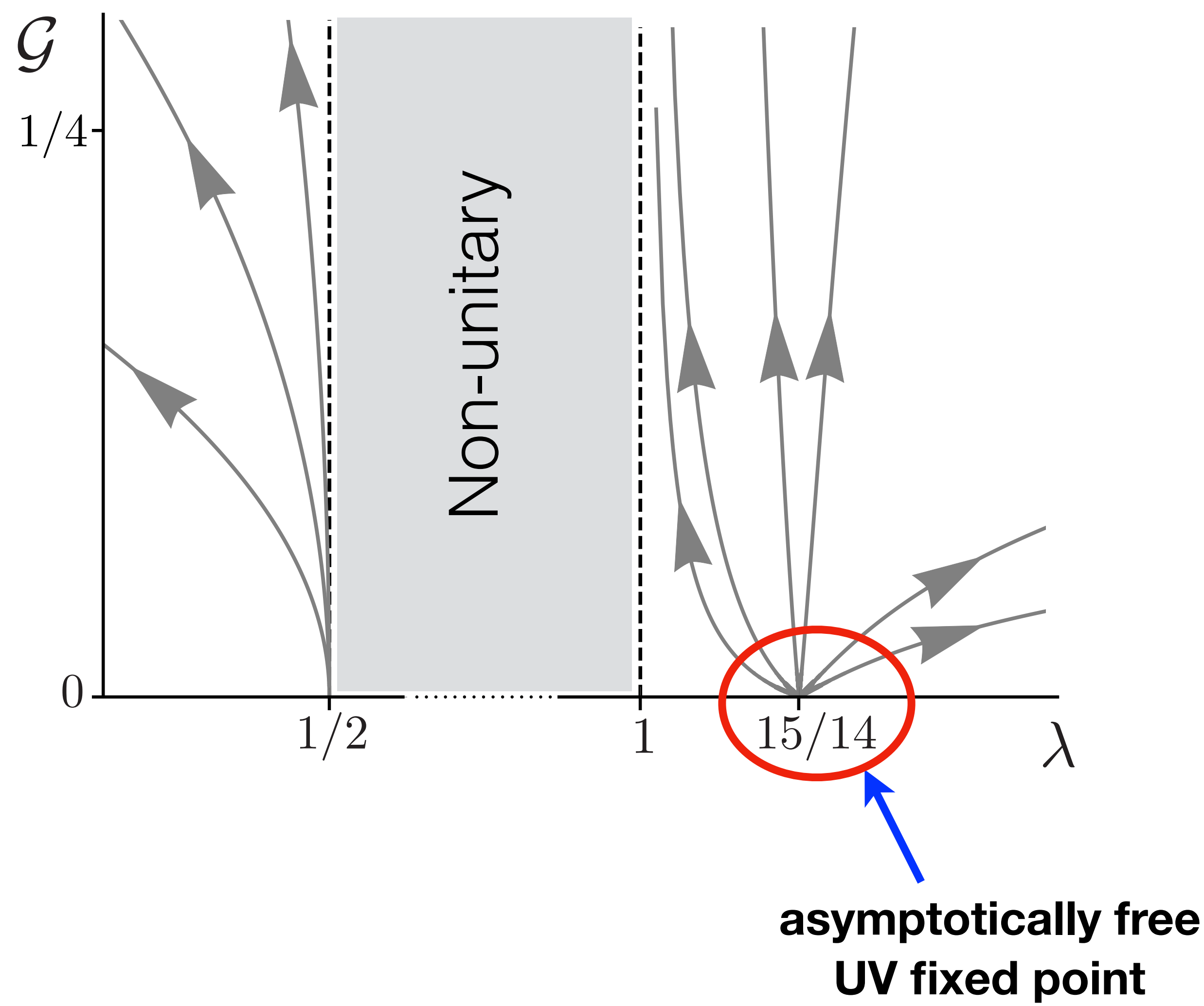

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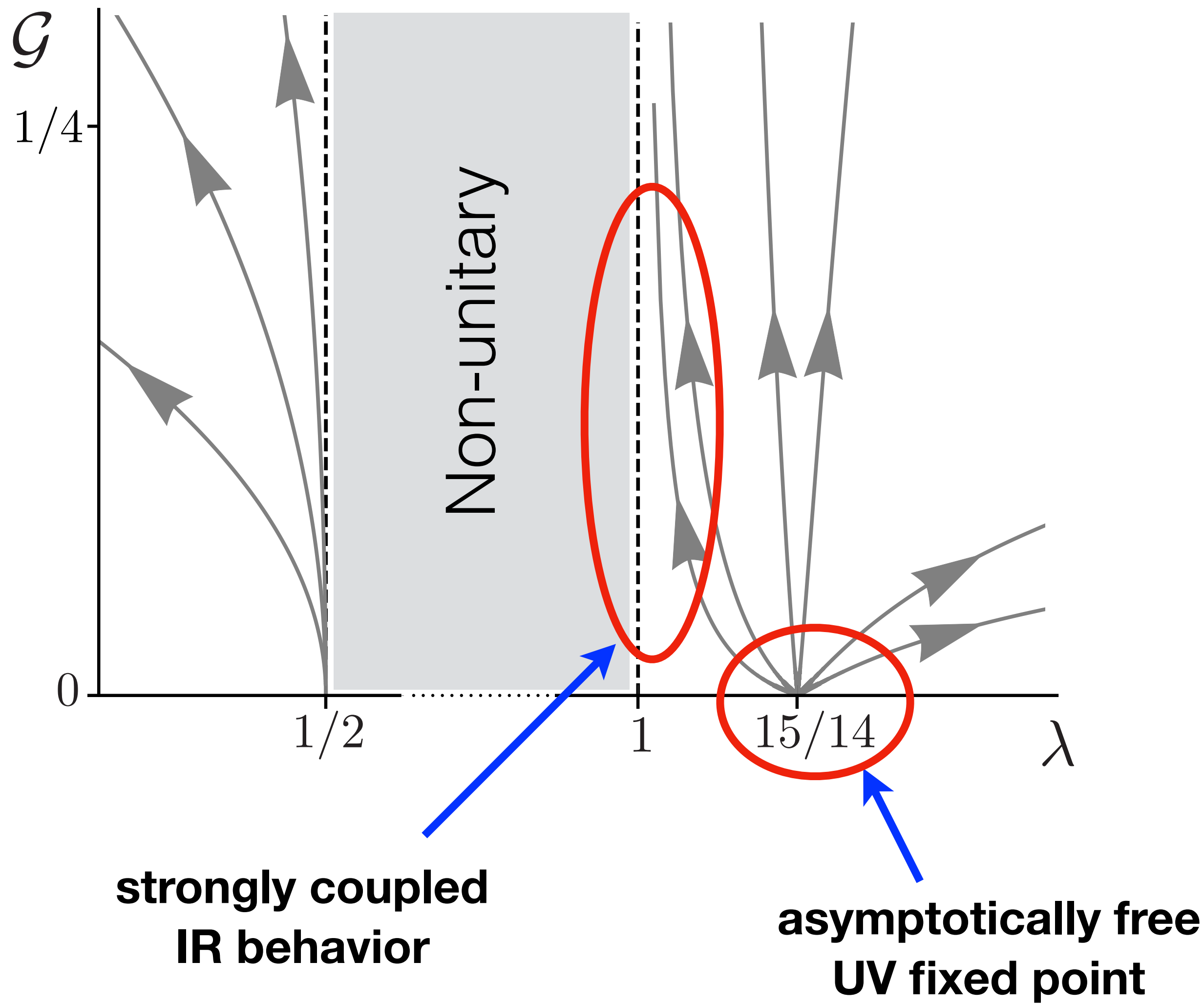

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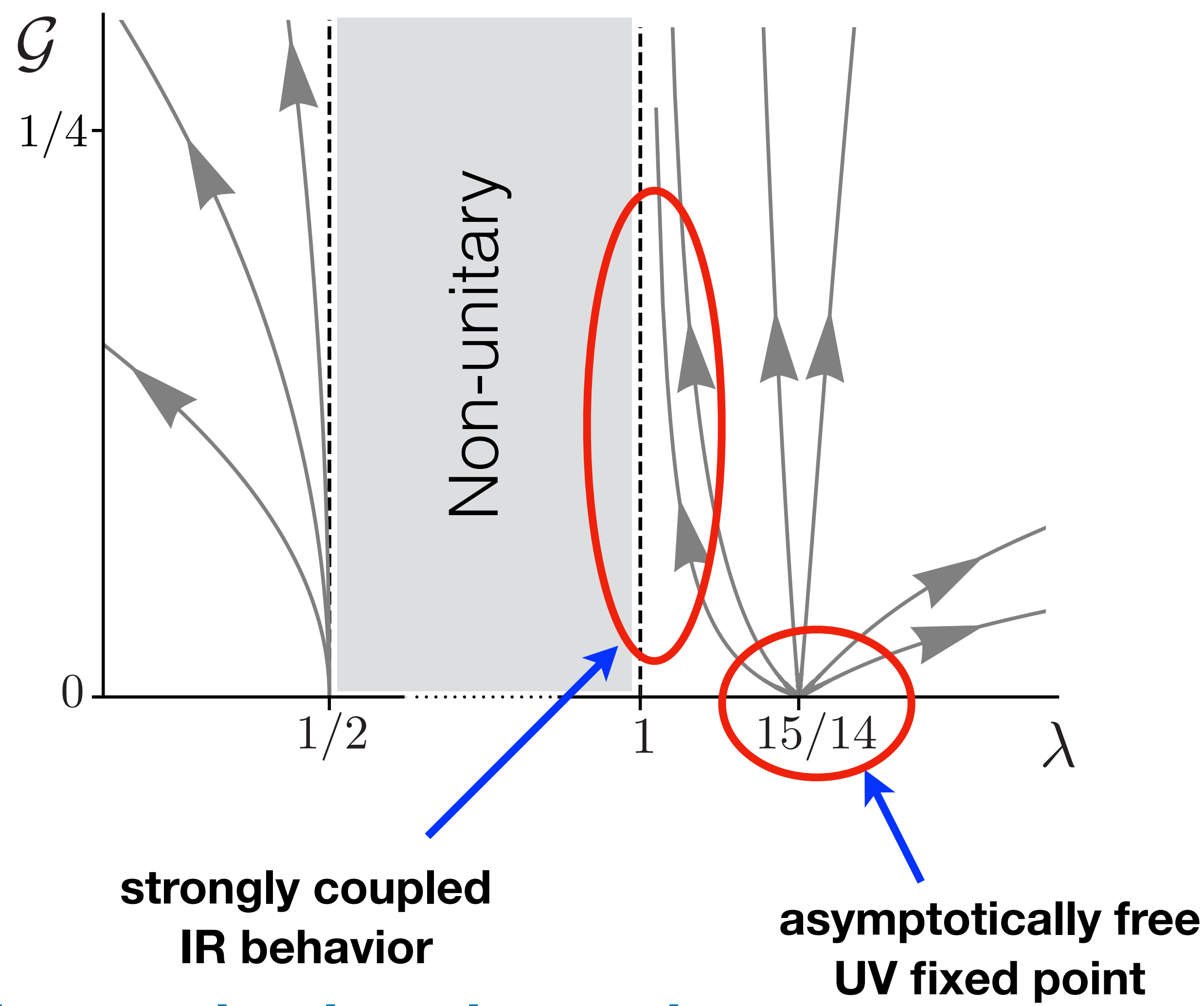

Smth interesting is going on here

Towards RG flows in (3+1)d

Contains a dynamical graviton (transverse-traceless tensor mode)

7 couplings, 6 essential

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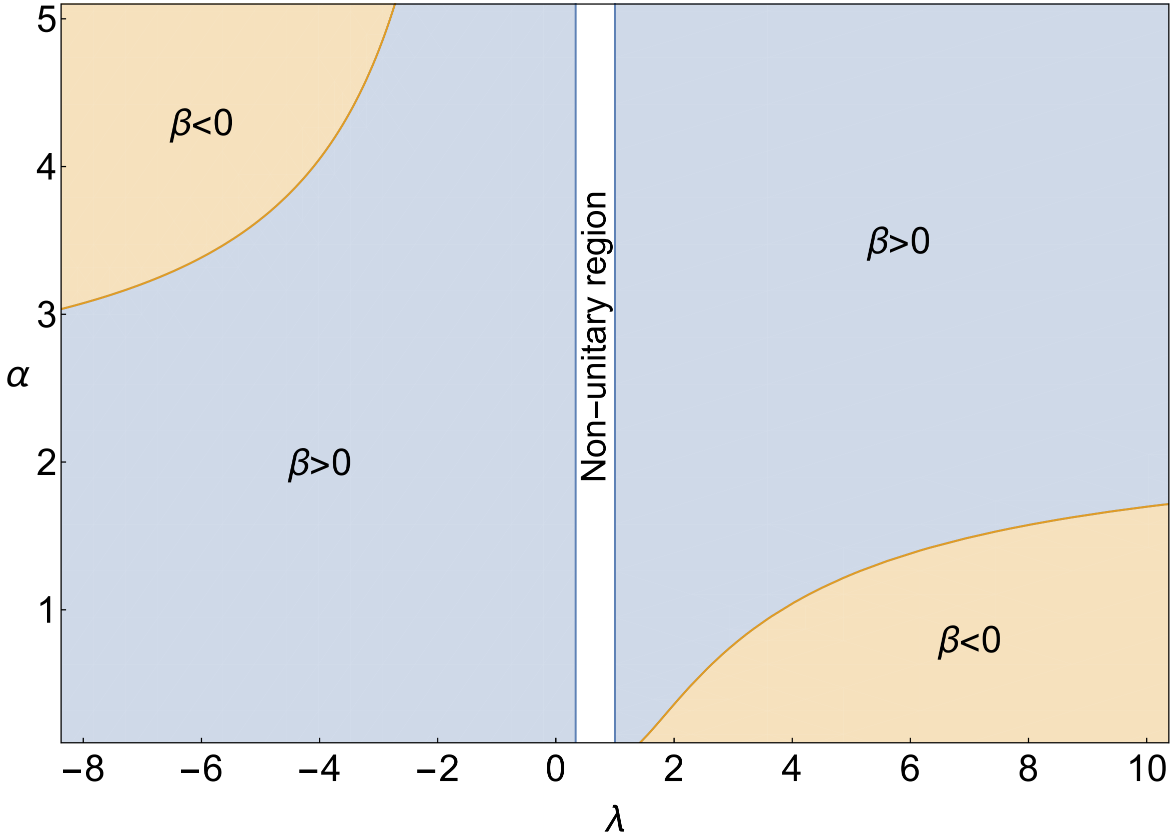

$$\alpha = \nu_s / \nu_{tt}$$

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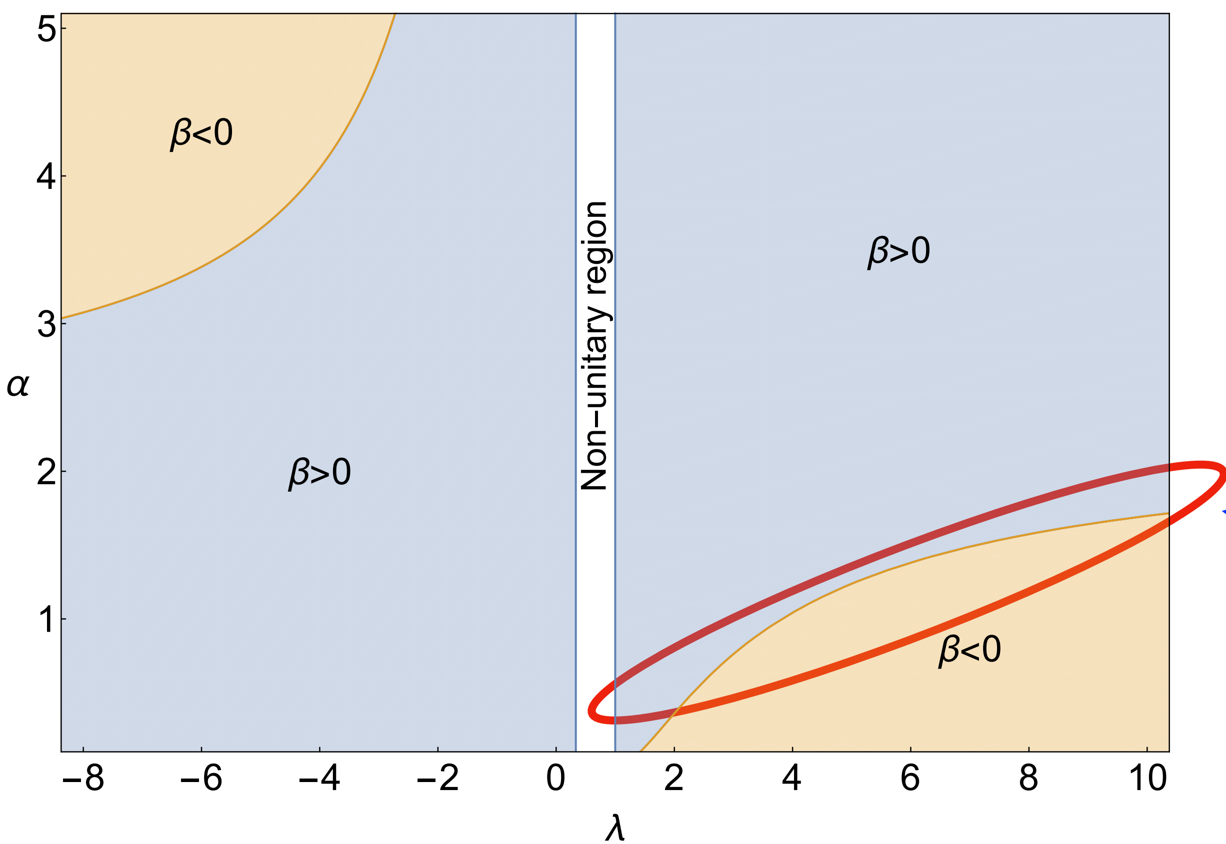

candidate UV fixed points

NB. Non-trivial: AF could have failed

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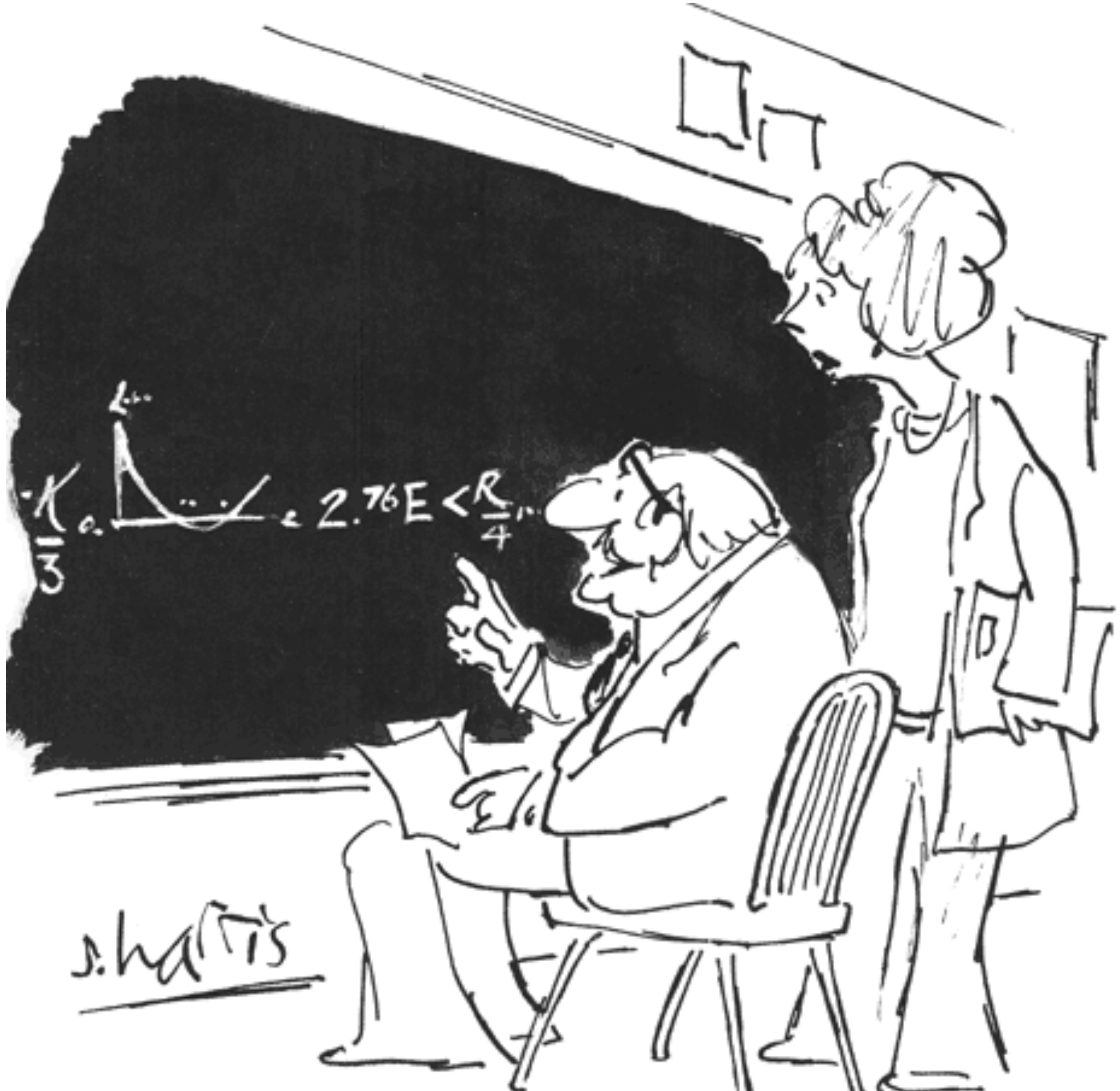
But satisfying $|c_g - c_\gamma| < 10^{-15}$ requires extreme fine-tuning

Gümrükçüoglu, Saravani, Sotiriou (2017)

from GW170817 / GRB170817A



"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."

Outlook

- Use HL as a toy model to address puzzles of GR
  - Characterization of observables
  - Resolution of singularities
  - Information paradox (?)

- Emergence of Lorentz through strong coupling ?