

# Quantizing solitonic strings

(or what to do with strings made of flesh and blood)

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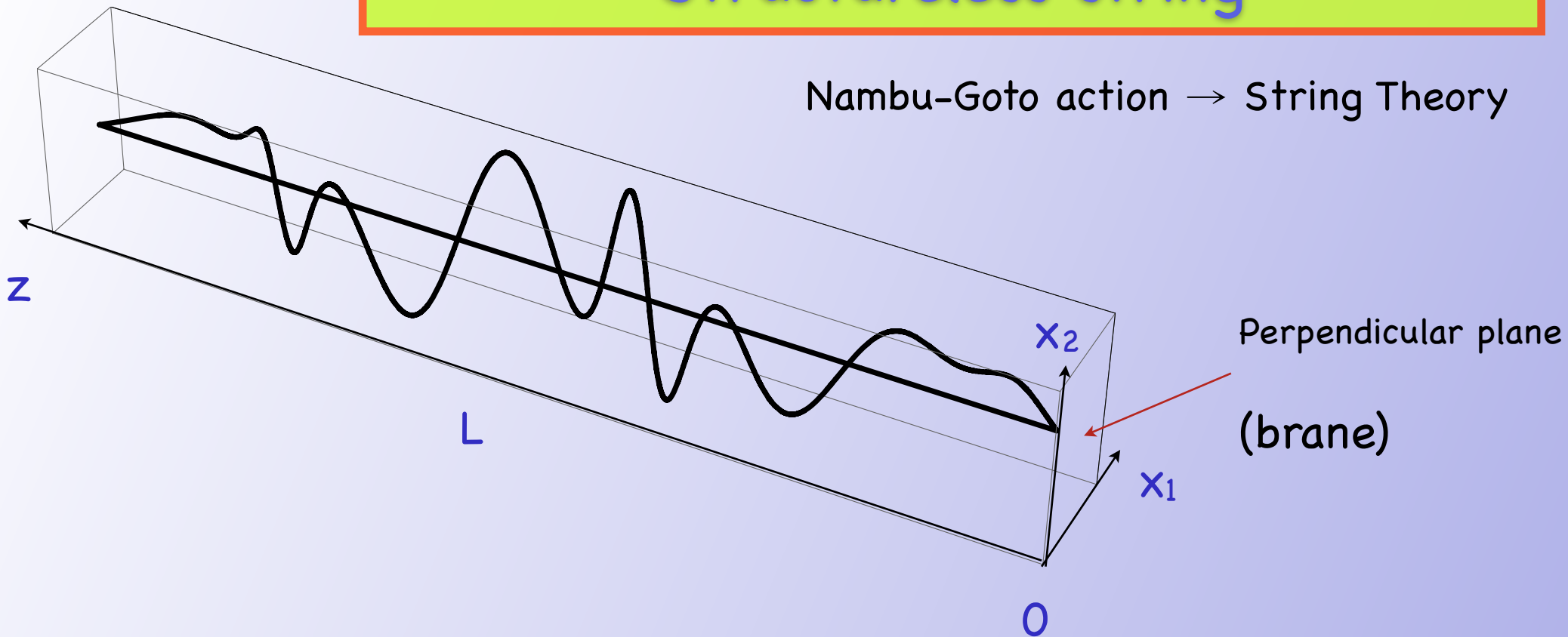
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# Structureless string

Nambu-Goto action  $\rightarrow$  String Theory



Infinitely thin string, fully **defined** by its coordinates

Quantization possible in 26 (10) dimensions (Or else, Liouville field)

$\uparrow$   
SUSY

Problem of quantization is the UV problem. Critical string = world sheet conformal

Concept: start from 4D Yang-Mills theory (8 supercharges) which support non-trivial vortex string with 4 supercharges.

Study moduli on world sheet (other than translational)

There should be 6 extra moduli ( $6+4=10$ )

Determine sigma model on the world sheet, its target space

Try to find a UV completion of the above sigma model making it conformal in the UV

Mission completed?

2003: Hanany, Tong  
Auzzi, Yung, et al.

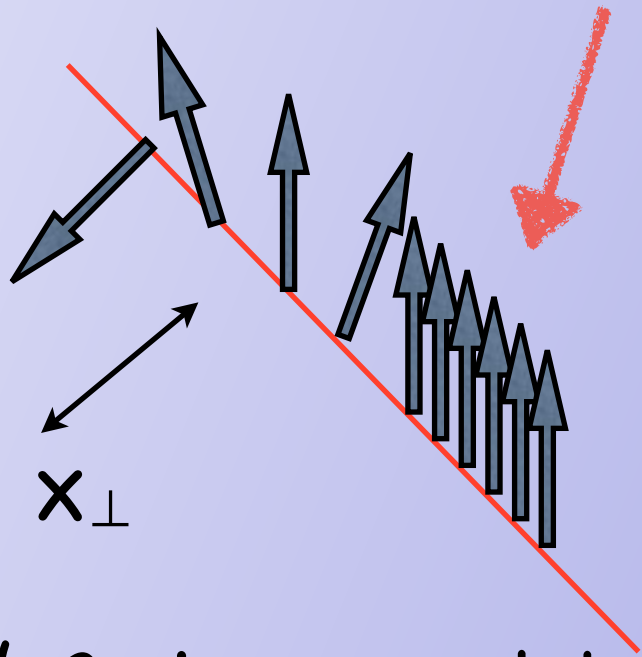
Yung + M.S.

# Vortex strings from flesh and blood

4D bulk,  $N = 2$  Yang-Mills  
with  $N$  flavors &  $N$  colors

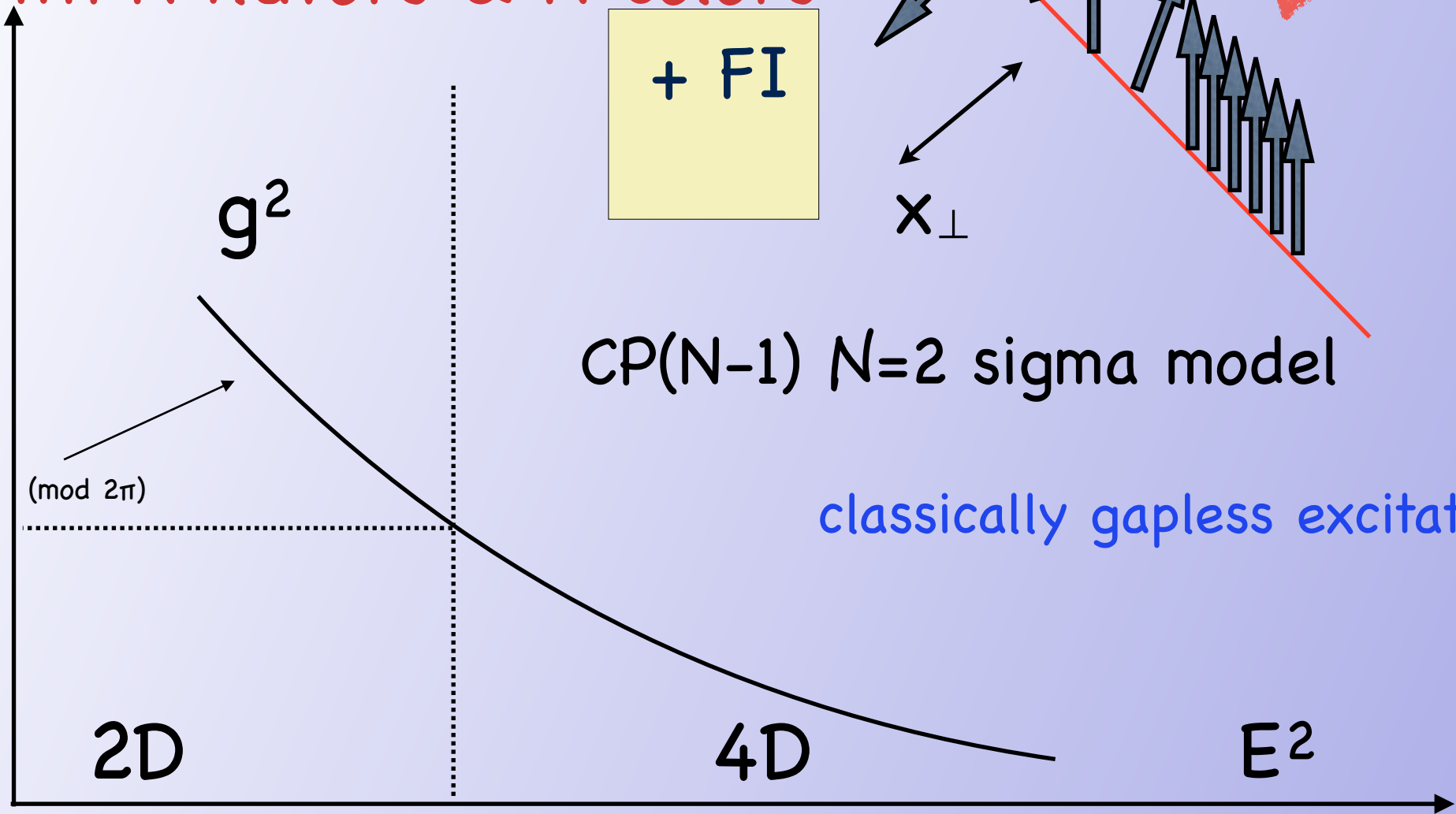
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Non-Abelian internal d.o.f




CP(N-1)  $N=2$  sigma model

classically gapless excitations



In 4D bulk we have only two discrete free parameters:  $N_f$  and  $N$   
 WE WANT CONFORMAL THEORY ON THE STRING. How?

$$N_f = 2 \times N_c, \quad N_c = 2$$


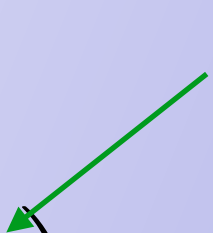
In this case the vortex string is semilocal;  
 rotational moduli  $n$  are supplemented by scale  
 moduli  $\rho$  

WCP(N,N)

$$S = \int d^2\sigma \sqrt{h} h^{ab} \left\{ \sum_{i=1}^N [(\partial_a - iA_a) n^i] \dagger [(\partial_a - iA_a) n^i] \right. \\
 \left. + \sum_{i=1}^N [(\partial_a + iA_a) \rho^i] \dagger [(\partial_a + iA_a) \rho^i] \right. \\
 \left. + \frac{e^2}{2} [ |n|^2 - |\rho|^2 - 2\beta ]^2 \right\}$$

Verification:

Virasoro central charge (including ghosts) = 0


$$c_{\text{Vir}} = \frac{3}{2} \left( D + \frac{2}{3} c_{\text{WCP}} - 10 \right),$$


$$c(\text{WCP}(N, N)) = 3(N + N - 1)$$



N=2

# Spectra of closed string states (no massless gravitons, $\beta = \beta_{\text{crit}}$ )

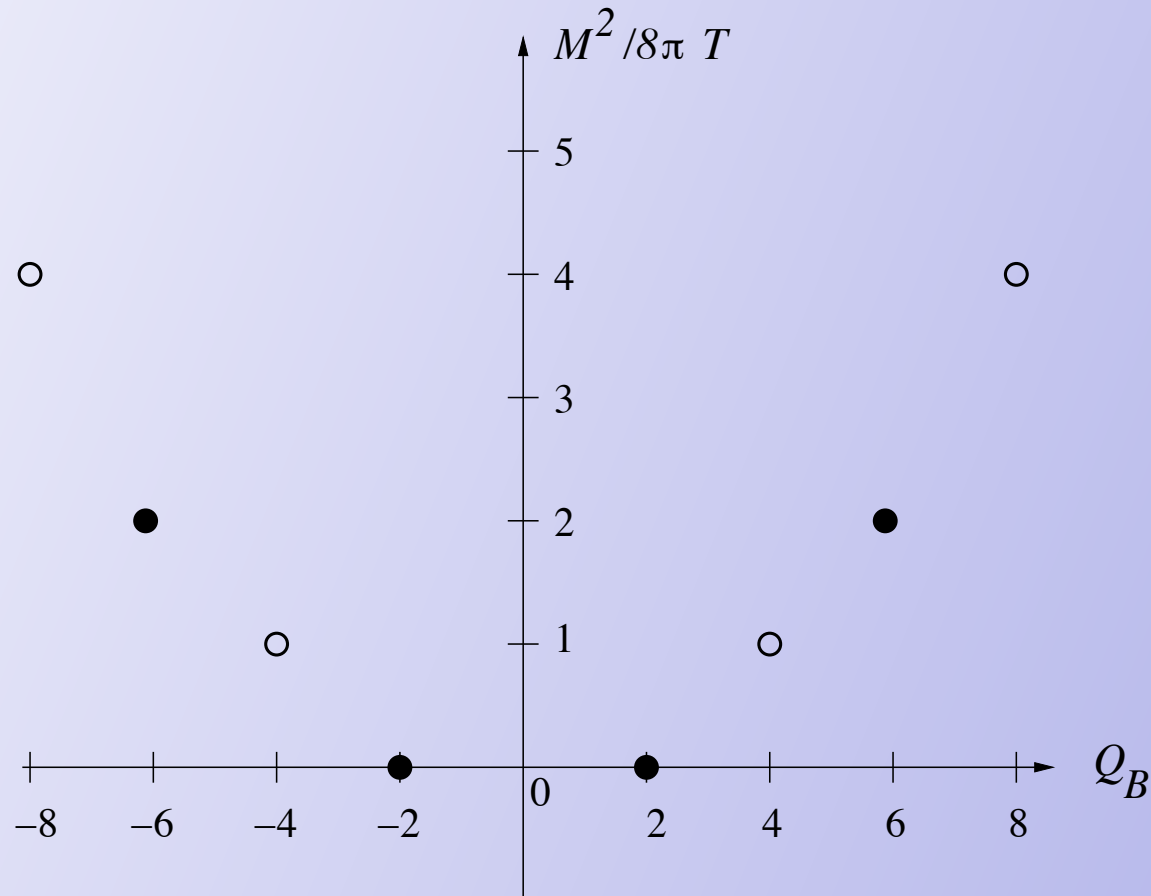


Figure 2: Spectrum of spin-0 and spin-2 states as a function of the baryonic charge. Closed and open circles denote spin-0 and spin-2 states, respectively.

If  $N=2$ , we have two  $n$ 's and two  $\rho$ 's  $\rightarrow$

8 r.dof - one g.c. - one constrains = 6 real dof

+ 4 translational = **10 dof**

Type IIA

\* FULL TARGET SPACE =  $R_4 \times WCP(N,N)$

**Noncompact Calabi-Yau**



The global symmetry of the bulk & world-sheet models is

$SU(2) \times SU(2) \times U(1)$

**WE WANT NON-CONFORMAL THEORY in IR flowing  
to the previous conformal in UV**

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$$\mathbf{N_f = 3, \quad N = 2, \quad \tilde{N} = 1}$$

$$\begin{aligned}
 S_{\text{complete}} = & \int d^2\sigma \sqrt{h} \left\{ h^{\alpha\beta} \left( \tilde{\nabla}_\alpha \bar{n}_P \nabla_\beta n^P + \nabla_\alpha \bar{\rho}_K \tilde{\nabla}_\beta \rho^K + \nabla_\alpha^H \bar{\rho}_H \tilde{\nabla}_\beta^H \rho_H \right) \right. \\
 & + \frac{1}{4e_B^2} B_{\alpha\beta} B^{\alpha\beta} + \frac{1}{e_B^2} \partial_\alpha \bar{\sigma}_B \partial^\alpha \sigma_B - i\sqrt{2} \mathcal{W}_{\sigma_B}^{\text{tree}} B_{01} \\
 & + 2|\sigma|^2 |n^P|^2 + 2|\sigma|^2 |\rho^K|^2 + 2|\sigma_B - (N - \tilde{N})\sigma|^2 |\rho_H|^2 \\
 & + \frac{e^2}{2} \left( |n^P|^2 - |\rho^K|^2 - (N - \tilde{N})|\rho_H|^2 - \beta_1 \right)^2 \\
 & \left. + \frac{e_B^2}{2} \left( |\rho_H|^2 + \sqrt{2} \mathcal{W}_{\sigma_B}^{\text{tree}} - \beta_2 \right)^2 \right\}, \tag{4.1}
 \end{aligned}$$

$$\mathbf{P=1,2; \quad K=1;}$$

$$\mathbf{\sigma=\Lambda}$$

## Conclusions:

10D string made of “flesh and blood” is a BPS soliton in 4D  $N=2$  super-Yang-Mills with  $U(2)$  gauge and three quark flavors.

It's UV completion (conformal!) has one extra  $\rho$  field which is made heavy by design. It describes fluctuations of the string core size.

QCD 10D string from 4D QFT?

# 10D string theory from 4D QFT

## Why this is good?

10D string reincarnation of a BPS soliton in 4D  $N=2$  super-Yang-Mills with  $U(2)_{\text{gauge}}$  and four quark flavors is great because:

- \* Soliton's interpretation and its moduli are absolutely transparent
- \* Reverse holography  $4 \rightarrow 10$  transparent \*
- \* Details & assumptions in strings  $\Leftrightarrow$  details & assumptions in QFT