

Conformal symmetry: towards the link between the Fermi and the Planck scales

Mikhail Shaposhnikov

based of works with Andrey Shkerin



International conference
Recent Advances in Theoretical Physics
of Fundamental Interactions

Outline

- Introduction and motivations, naturalness?
- Semiclassical relation between the Fermi and Planck scales?
- Conclusions

Introduction and motivations, naturalness?

Triumph of the SM in particle physics

- The Standard Model is now complete: the last particle - Higgs boson, predicted by the SM, has been found

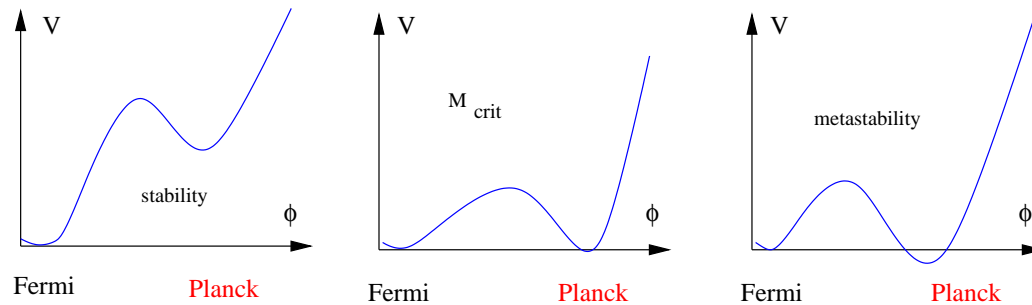
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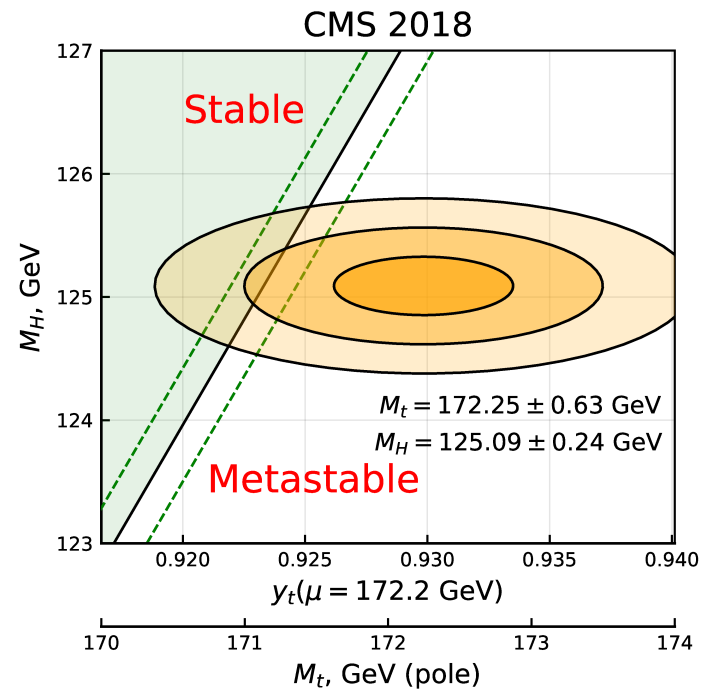
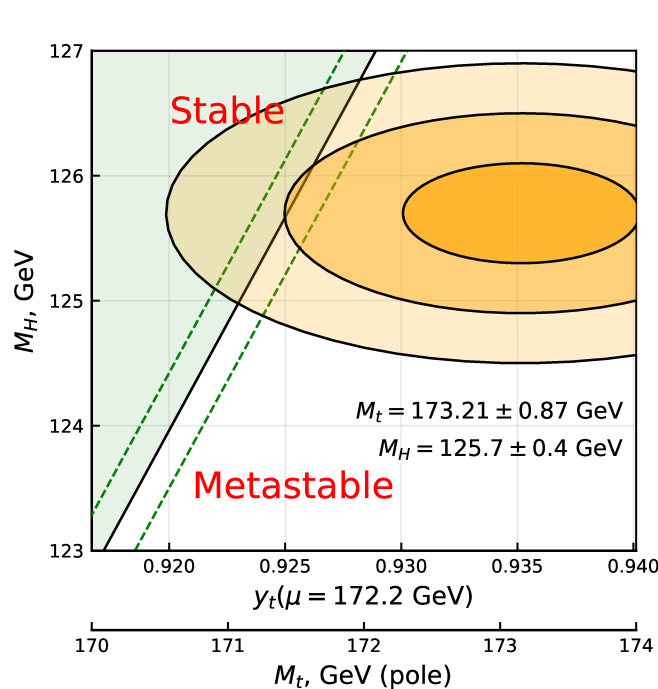
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- The Standard Model is now complete: the last particle - Higgs boson, predicted by the SM, has been found
- No significant deviations from the SM have been observed
- With experimental values of the masses of the top quark and of the Higgs boson the SM is a self-consistent effective field theory all the way up to the quantum gravity Planck scale M_P .

- Marginal evidence (less than 2σ) for the SM vacuum metastability given uncertainties in relation between Monte-Carlo top mass and the top quark Yukawa coupling



Vacuum is unstable at $\sim 1.5\sigma$



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[uhn-**nach**-er-uh l, -**nach**-ruh l]

Spell Syllables

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adjective

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This is unfair to “unnatural” SM as it describes **the Nature** better than “natural” theories...

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If this does not happen, the theory is called unnatural and fine-tuned

The original source of the naturalness requirement: hierarchy problem in Grand Unified theories

PHYSICAL REVIEW D

VOLUME 14, NUMBER 6

15 SEPTEMBER 1976

Gauge-symmetry hierarchies*

Eldad Gildener

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 15 June 1976)

It is shown that one cannot artificially establish a gauge hierarchy of any desired magnitude by arbitrarily adjusting the scalar-field parameters in the Lagrangian and using the tree approximation to the potential; radiative corrections will set an upper bound on such a hierarchy. If the gauge coupling constant is approximately equal to the electromagnetic coupling constant, the upper bound on the ratio of vector-meson masses is of the order of $\alpha^{-1/2}$, independent of the scalar-field masses and their self-couplings. In particular, the usual assumption that large scalar-field mass ratios in the Lagrangian can induce large vector-meson mass ratios is false. A thus far unsuccessful search for natural gauge hierarchies is briefly discussed. It is shown that if such a hierarchy occurred, it would have an upper bound of the order of $\alpha^{-1/2}$.

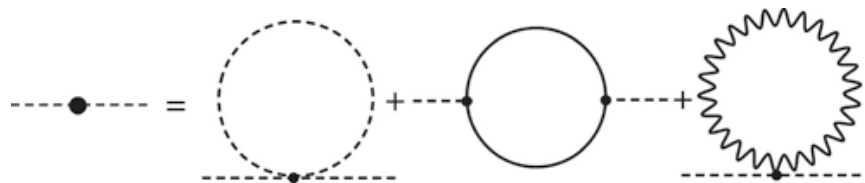
Extra GUT particles beyond the SM – leptoquarks (vector and scalar) must be very heavy, $M_X > 10^{15}$ GeV

- this is required by the gauge coupling unification
- this is needed for stability of matter, proton lifetime $\tau_p > 10^{34}$ years

Hierarchy: $\left(\frac{M_X}{M_W}\right)^2 \simeq 10^{28}$

Two faces of hierarchy

- Ad hoc tuning between the parameters (masses and couplings of different multiplets) at the tree level with an accuracy of **26 orders** of magnitude
- Stability of the Higgs mass against radiative corrections **Gildener, '76**



$$\delta m_H^2 \simeq \alpha_{GUT}^n M_X^2$$

Tuning is needed up to **14th order** of perturbation theory!

Proposed solutions

Stability of EW scale – requirement of “naturalness”: absence of quadratic divergencies in the Higgs mass

- Low energy SUSY: compensation of bosonic loops by fermionic loops
- Composite Higgs boson - new strong interactions
- Large extra dimensions

All require new physics right above the Fermi scale, which was expected to show up at the LHC

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**An attempt towards this direction:
M.S., Shkerin, arXiv:1803.08907 +
arXiv:1804.06376**

Semiclassical relation between the Fermi and Planck scales?

Higgs-Planck hierarchy: the ratio of the two scales is exponentially small,

$$\frac{m_H}{M_P} \simeq 10^{-16} \simeq e^{-S}, \quad S \simeq 36$$

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Requirements to the theory

- Perturbatively $m_H = 0$, symmetry protection (?)
- Existence of (semi) classical configurations leading to $\langle H \rangle \neq 0$, $\langle H \rangle \ll M_P$

If the mass of the Higgs boson is put to **zero** in the SM, the classical Lagrangian has a wider symmetry: it is scale and conformally invariant:

Dilatations - global scale transformations ($\sigma = \text{const}$)

$$\Psi(x) \rightarrow \sigma^n \Psi(\sigma x) ,$$

$n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

It is tempting to use this symmetry for solution of the hierarchy problem

Bardeen '95: why the Higgs boson mass is so small in comparison with the Planck scale?

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But what about quantum corrections?

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No go theorem?

Conformal anomaly

The statement "any cutoff procedure necessarily involves a large mass" is not true. Counter-example: dimensional regularisation of t'Hooft and Veltman does not involve any large mass, just a normalisation point μ which is not sent to infinity. Still, the scale invariance is anomalous due to "dimensional transmutation": the renormalisation-group running of the parameters leads to a non-vanishing trace of the energy-momentum tensor, which enters the divergence of the scale current. However, the physical quantities depend on the renormalisation scale only **logarithmically**. Any quadratically divergent contributions to the Higgs boson mass are purely technical and are introduced by explicitly breaking the conformal invariance by regulators.

Radiative symmetry breaking

Lagrangian is invariant at the classical level, and scale symmetry is broken by quantum corrections (conformal anomaly) a'la

Coleman-Weinberg: Linde '76; Weinberg '76; Buchmuller, Dragon '88; Hempfling '96; Meissner, Nicolai '06; Foot et al '07, '11; Iso, et al '09; Boyle et al '11; Wetterich '11, Salvio, Strumia '14; Lindner et al, '14, '15, '17

Does not work for the SM:

- If the top quark mass $m_t \lesssim 172$ GeV, then the minimum of the effective potential is generated at $\langle H \rangle \simeq 100$ MeV due to chiral symmetry breaking in QCD
- If the top quark mass $m_t \gtrsim 172$ GeV, then an extra minimum of the effective potential is generated at $\langle H \rangle \gtrsim M_P$ due to top quark loops

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Gravity + conformally invariant SM is an ideal playground for looking at non-perturbative generation of the weak scale

Toy Model

Scalar theory plus gravity with non-minimal coupling, $\xi > 0$:

$$\frac{\mathcal{L}_{\varphi,g}}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi\varphi^2)R + \frac{1}{2}(\partial\varphi)^2 + V(\varphi)$$

Scale invariant “matter” with

$$V(\varphi) = \frac{\lambda}{4}\varphi^4$$

We want to compute the Higgs vev:

$$\langle\varphi\rangle \sim \int \mathcal{D}\varphi \mathcal{D}g_{\mu\nu} \varphi e^{-S_E} .$$

S_E is the **euclidean** action of the model.

Remarks:

- Euclidean path integral for gravity may not be well defined due to the problem with the conformal factor of the metric
- We will ignore this problem and follow the crowd: Hawking; Coleman, de Luccia; Veneziano; ..., Isidori, Rychkov, Strumia, Tetradis; ... Branchina, Messina, Sher;...

Small $\varphi \ll M_P$ - gravity is irrelevant – no contribution to the vev of the Higgs from scalar loops.

Challenge: account for contributions with $\varphi \gg M_P$.

Theory for large h :

$$\mathcal{L} = -\frac{1}{2}\xi\varphi^2 R + \frac{1}{2}(\partial\varphi)^2 + \frac{\lambda}{4}\varphi^4$$

- Scale-invariant
- Planck scale is dynamical, $\propto \sqrt{\xi}\varphi$

Conjecture: contribution of large Higgs fields $\varphi > M_P/\sqrt{\xi}$ to path integral is better to be found in the Einstein frame. Conformal transformation:

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} , \quad \Omega^2 = 1 + \frac{\xi\varphi^2}{M_P^2}$$

Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\chi}{d\varphi} = \sqrt{\frac{\Omega^2 + 6\xi\varphi^2/M_P^2}{\Omega^4}} \implies \begin{cases} \varphi \simeq \chi & \text{for } \varphi < M_P/\xi \\ \varphi \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6}M_P}\right) & \text{for } \varphi > M_P/\sqrt{\xi} \end{cases}$$

Resulting kinetic part of the action

$$S_{kin} = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2} \hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} \right\}$$

Most important:

$$\langle \varphi(x) \rangle \sim \int \mathcal{D}A \mathcal{D}\varphi(x) \mathcal{D}g_{\mu\nu} \varphi(x) e^{-S_E} \implies \int \mathcal{D}A \mathcal{D}\chi \mathcal{D}\hat{g}_{\mu\nu} e^{\frac{\chi(x)}{\sqrt{6}M_P} - S_E}$$

Modification of the action and equations of motion!

Equations of motion for χ contain a source term $\delta(x) \implies$ new classical solutions.

Similar to

- computation of $\int dx x^N e^{-x^2}$ for large N ,
- to computation of multi-particle production,
- to proof of confinement in 3D Georgi-Glashow model, $\langle \exp(\int A_\mu dv^\mu) \rangle$ in Polyakov '76.

Schematically, modification of the right-hand side for scalar field equation:

$$\square\chi + \dots = \delta(x) / \sqrt{6}M_P$$

Path integral:

$$\int_{\varphi \gtrsim M_P / \sqrt{\xi}} \mathcal{D}\varphi \varphi e^{-S_E} \rightarrow M_P \int_{\chi \gtrsim M_P \log(1/\sqrt{\xi})} \mathcal{D}\chi J e^{-W},$$

where $W = \chi(0) / \sqrt{6} M_P + S_E$ and J is the corresponding Jacobian.

Conjecture: Higgs vev

$$\langle \varphi \rangle \approx M_P e^{-\bar{W}},$$

is much smaller than M_P because the action \bar{W} on the saddle point

$$\bar{W} \gg 1.$$

Condensed matter analogue: exponentially small mass gap in superconductors.

Computation in toy model

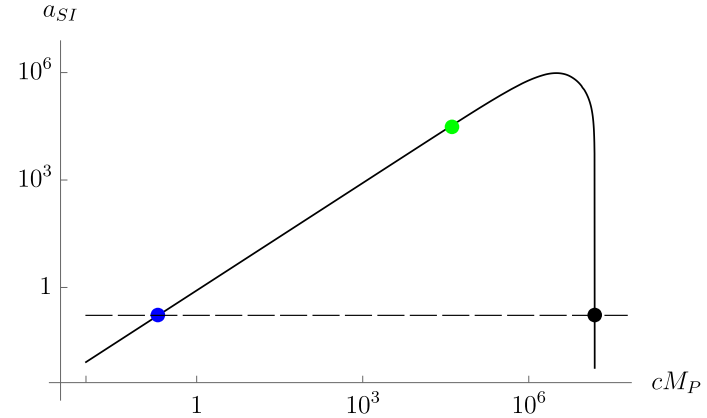
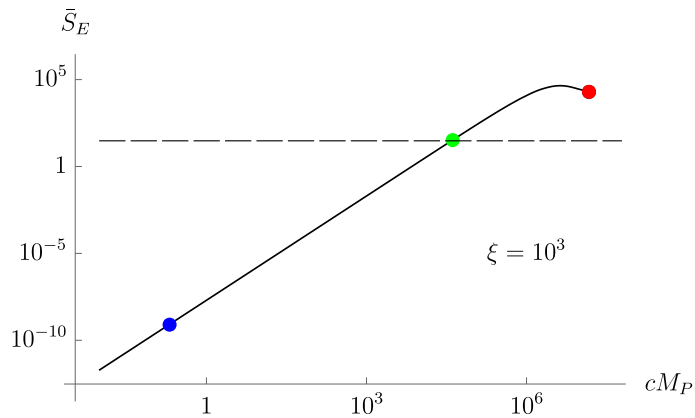
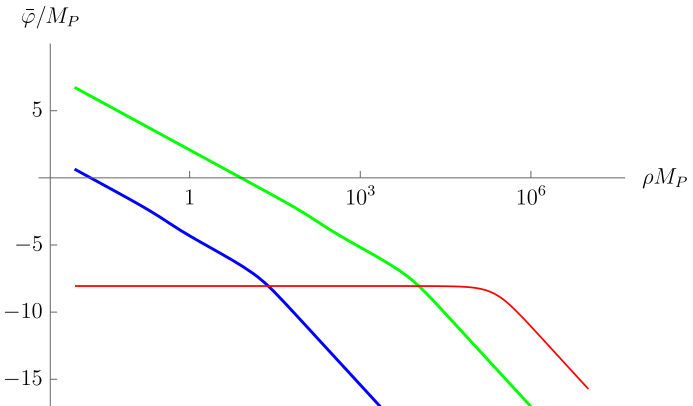
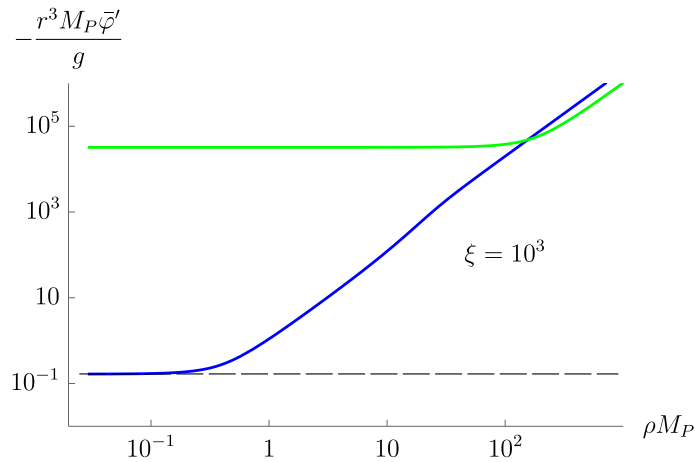
Field equations in the Einstein frame for maximally $O(4)$ symmetric metric $d\tilde{s}^2 = g^2(\rho)d\rho^2 + \rho^2 d\Omega_3^2$ in the large χ regime:

$$\partial_\rho \left(\frac{\rho^3 \bar{\chi}'}{g a_{SI}} \right) = -\frac{1}{M_P} \delta(\rho), \quad g^2 = 1 - \frac{\rho^2 \bar{\chi}'^2}{6 a_{SI} M_P^2}, \quad a_{SI} = \frac{1}{1/\xi + 6}$$

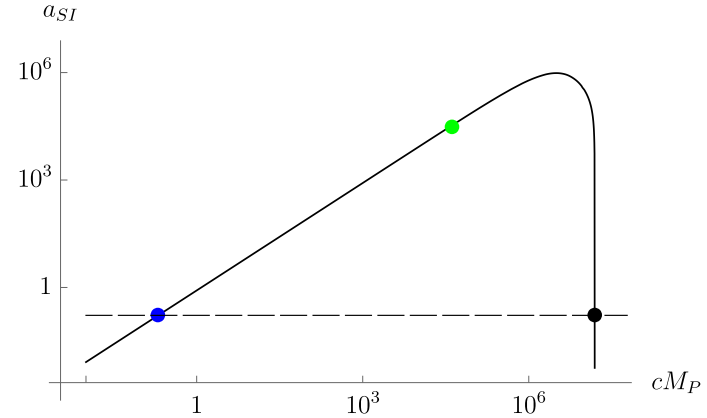
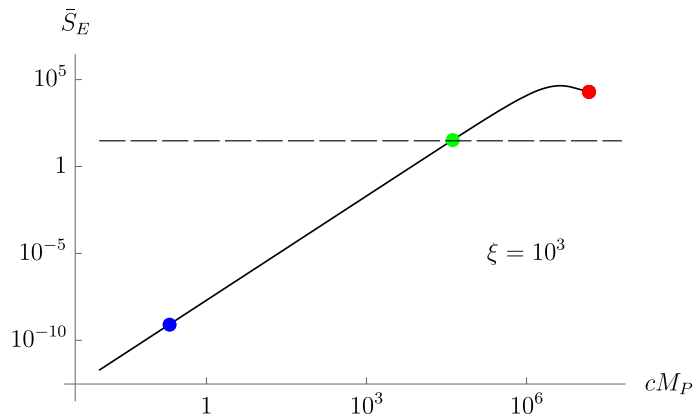
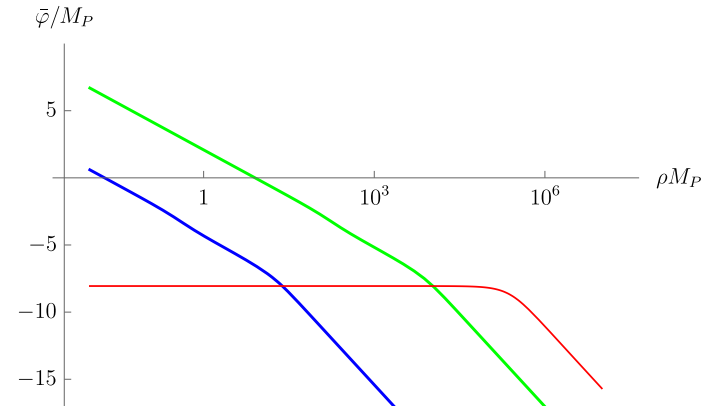
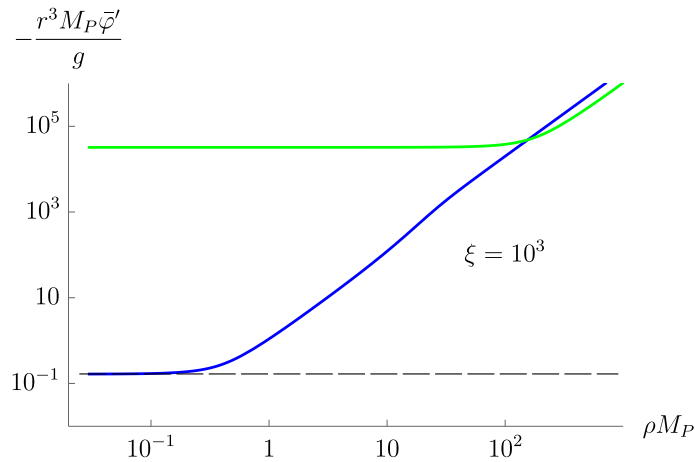
Vacuum boundary conditions at infinity $\rho \rightarrow \infty$:

$g^2(\rho) \rightarrow 1$, $h(\rho) \rightarrow 0$. The asymptotic behaviour of the scalar field χ at $\rho \rightarrow 0$: $\chi = -M_P \sqrt{6 a_{SI}} \log \rho M_P + c$ with c a constant used to match with the asymptotics at large ρ .

Coincides with Hawking-Turok instanton, but the constant in front of \log is fixed by the source term.



blue: configuration obeying the boundary condition imposed by the source. **green** is the one with the large euclidean action $\bar{W} = 40$. **red:** the bounce. **c:** fall-off at infinity, $\chi = c\rho^{-2}$, $\rho \rightarrow \infty$.



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The action is too small! Semiclassical approximation does not work, and $\langle \varphi \rangle \sim M_P$ is expected.

The appearance of a scale small compared with M_P is not generic and requires a specific theory in UV, well along with our conjecture.

Modifications of the theory in the UV:

- Include higher dimensional operators, for example

$$\mathcal{O}_4 = \sqrt{g} \delta \frac{(\partial\varphi)^4}{(M_P\Omega)^4}, \quad \Omega^2 = 1 + \frac{\xi\varphi^2}{M_P^2}$$

Ω^2 is needed to keep the scale symmetry at large h . Leads to finite values of the Higgs field in the centre of the instanton!

- Action of the instanton depends on a_{SI} , $S \propto \sqrt{a_{SI}}$. The parameter a_{SI} is a combination of a non-minimal coupling to gravity and scalar kinetic term. They can be changed for large h to make the action large, without affecting low energy physics

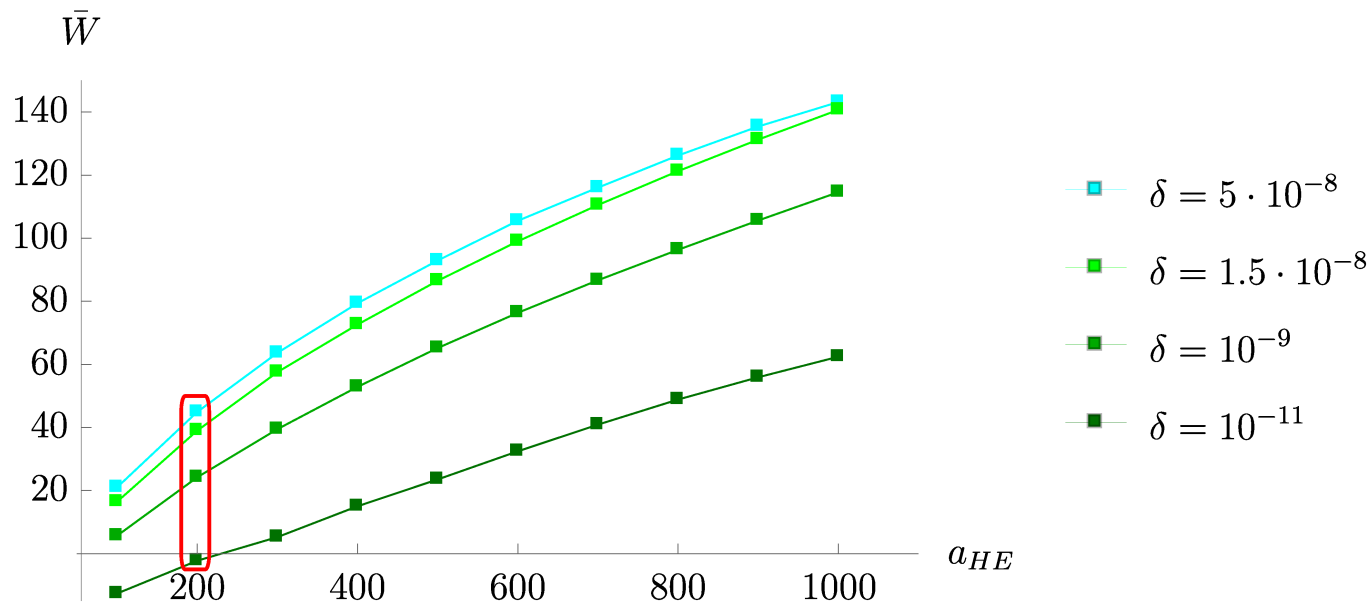
A theory that works

$$\frac{\mathcal{L}_{h,g}}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi h^2)R + \frac{1}{2}G(h/M_P)(\partial h)^2 + \delta\xi^2 \frac{(\partial h)^4}{(M_P\Omega)^4} + \frac{\lambda}{4}h^4,$$

where $G(0) = 1$, $G(\infty) = \kappa = \text{const}$. Then the asymptotic value of a_{SI} in the large- χ regime modifies to

$$a_{SI} \rightarrow a_{HE} = \frac{1}{\kappa/\xi + 6}, \quad \rho \rightarrow 0, \quad h \gtrsim M_P,$$

Must have $\kappa > -\frac{6}{\xi}$ (absence of ghosts). Taking $\kappa = -\frac{6}{\xi} + \epsilon$, $\epsilon > 0$, $\epsilon \ll 1$ can make the action large.



The instanton value of the functional W plotted against the coefficient α_{HE} and with the different choices of the parameter δ .

Intriguing fact: large α_{HE} and small δ correspond to an approximate **Weyl** symmetry.

Conclusions: the mechanism may be viable.

Remark: other known classical solutions

- Bounce: an indication of the vacuum instability Coleman, de Luccia
- Hawking-Moss instanton: dominates transitions between vacua at high temperature
- Gravitational instantons (Taub-NUT, Eguchi-Hanson, Gibbons-Hawking): pure gravity, no relation to scalar field
- Giddings-Strominger instanton: gravity + axion field: wormholes
- Hawking-Turok instanton: creation of Universe from nothing ?

None works for the Higgs vev generation

Conclusions

- Very small m_H/M_P ratio is (perhaps) telling us that
 - There are no new particles with masses between the Fermi and Planck scale
 - The smallness of the Fermi scale is a semiclassical non-perturbative UV effect associated with gravity and new type of instantons
 - The asymptotic theory of the SM at large scalar fields is nearly Weyl invariant
- Open problems
 - Unfortunately, we can make no prediction of the ratio m_H/M_P , as this depends on details of UV theory.
 - We cannot estimate the contribution of the effects other than perturbative and semiclassical.

Remark

Everything can be generalised to a completely scale-invariant theory with spontaneous breaking of scale invariance.

Gravity part:

$$\mathcal{L}_G = - (\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger \varphi) \frac{R}{2},$$

The vev of extra field – dilaton – gives rise to the Planck scale.

For analysis and results see [M.S., Shkerin, arXiv:1804.06376](#).