$$G^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu}R = T^{\mu}_{\nu}$$

$$\Downarrow$$

$$R = -T$$

$$\text{DM} + \text{DE} \rightarrow \underline{\underline{T^{\mu}_{\nu}}} = \underset{\substack{\uparrow \\ \text{particles}}}{\varepsilon u^{\mu} u_{\nu}} + \underset{\substack{\uparrow \\ \text{cosmol. const.}}}{\Lambda \delta^{\mu}_{\nu}}$$

Can we find DM + DE in gravitational equations themselves?

$$R^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu}R + \frac{1}{4}\delta^{\mu}_{\nu}R = T^{\mu}_{\nu} - \frac{1}{4}\delta^{\mu}_{\nu}T$$

(no trace eq.)

$$\underline{\underline{T^{\mu}_{\nu} = 0}}$$

$$R^{\mu}_{\nu} - \frac{1}{4}\delta^{\mu}_{\nu}R = 0$$

Solution:

$$R = \Lambda \leftarrow \underline{\text{constant of integration}}!$$

$$R^{\mu}_{\nu} = \frac{1}{4}\Lambda\delta^{\mu}_{\nu}$$

D$\boxed{\text{E}}$ as constant of integration!!

Mimetic Dark Matter as Constant of integration.

$$G^{\mu}_{\nu} = 0 \Rightarrow \underline{G = 0}$$

$$G^{\mu}_{\nu} - G\,\partial^{\mu}\varphi\,\partial_{\nu}\varphi = 0$$

$$(\underbrace{1 - \partial^{\mu}\varphi\,\partial_{\mu}\varphi}_{=0})\,G = 0 \Rightarrow \underline{G \neq 0}$$

$$\left\{ \begin{array}{l} G^{\mu}_{\nu} = \overset{\varepsilon}{\overset{\shortparallel}{G}}\,\overset{u^{\mu}}{\overset{\shortparallel}{\partial^{\mu}\varphi}}\,\overset{u_{\nu}}{\overset{\shortparallel}{\partial_{\nu}\varphi}} \\ \partial^{\mu}\varphi\,\partial_{\mu}\varphi = 1 \end{array} \right. \qquad \Rightarrow \boxed{p = 0}$$

Dust as a constant of integration.

$$\Downarrow$$

$$G \propto \frac{C(x^{i})}{\sqrt{\gamma}}$$

in synchronous coordinate system.

Action

Dust as integr. const.

$$S = \int \Big[-\frac{1}{2} R + \lambda \left(\partial_\mu \varphi \partial^\mu \varphi - 1 \right)$$

Cosm. const. as integration constan.

$$+ \tilde{\lambda} \left(\partial_\mu V^\mu - 1 \right) + \ldots$$

(Bunster, Henneaux)

+ extensions $V(\varphi)$, $\varphi F(\varphi_{\text{inflaton}})$

$\Downarrow$

mimic any eq. of state $\quad p = \gamma \varepsilon$ but $c_s^2 = 0$

$+ \gamma (\Box \varphi)^2 \rightarrow$ to produce $c_s^2 \neq 0$

NCG

Spectral triple $(\mathcal{A}, \mathcal{H}, D)$

$\mathcal{A} \downarrow$ Algebra (gauge)

$\mathcal{H} \downarrow$ Hilbert space

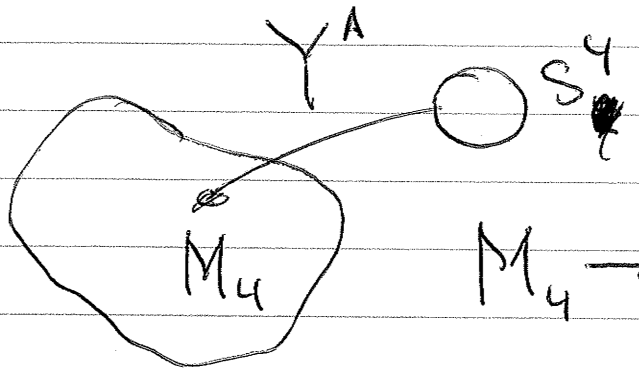
$D \downarrow$ Dirac

$$\gamma = \sum a_0 [D, a_1] \dots [D, a_4]$$

$\gamma \downarrow$ chirality operator

$a_j \in \mathcal{A}$

4d - compact



$M_4 \Rightarrow S^4$

$\langle Y, Y \rangle = 1$ ← unit sphere

$$Y = Y^A \Gamma_A \qquad A = 1 \dots 5$$

$$\{\Gamma_A, \Gamma_B\} = 2\underset{\pm 1}{k}\, \delta_{AB}$$

$$\langle Y, \overset{\gamma \frac{\partial}{\partial x}+\omega}{\overset{\downarrow}{[D,Y]}} \ldots [D,Y] \rangle \equiv \langle Y, [DY]^4 \rangle = \sqrt{k}\,\gamma$$

Quntazation

$$\Downarrow$$

$$\sqrt{g} = \frac{1}{4!} \varepsilon^{\mu\nu\sigma\lambda} \varepsilon_{ABCDE} Y^A \partial_\mu Y^B \partial_\nu Y^C \partial_\sigma Y^D \partial_\lambda Y^E$$

$$\Downarrow$$

$$V = \int \sqrt{g}\, d^4x \quad \propto \text{ integer!}$$

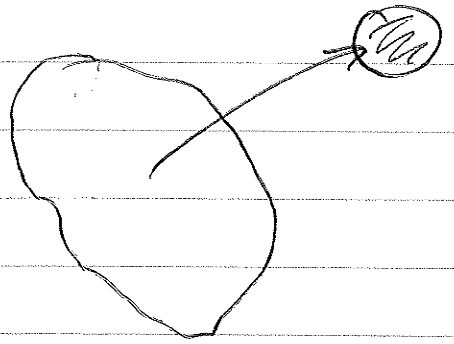
If we add to action.

$$\int \ldots + \lambda \left(\sqrt{g} - \frac{1}{4!} \varepsilon^{\cdots} \varepsilon_{\cdots} Y \partial Y \partial Y \partial Y \partial Y \right) + \tilde{\lambda} \left(Y^A Y^A - 1 \right)$$

$$\Downarrow$$

cosmologial constant as constant of integration!

Boot.



Jacobian vanishes $\Rightarrow$ Manifold will be topologically discon. unit spheres!

How to fix?

$$\langle Z, [DZ]^4 \rangle = \gamma$$

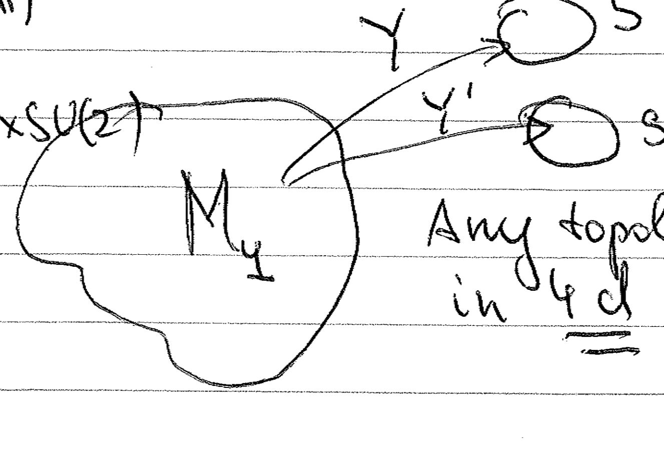
$$Z = \frac{1}{4}(1+Y)(1+Y')$$

$$Y = Y^A \Gamma_A, \quad Y' = Y'^A \Gamma'_A$$

$M_4(\mathbb{C})$
$\downarrow$
$SU(4)$

$$\{\Gamma_A, \Gamma_B\} = +\delta_{AB} \qquad \{\Gamma'_A, \Gamma'_B\} = -\delta_{AB}$$

$M_2(\mathbb{H})$
$\downarrow$
$SU(2)\times SU(2)$



Any topology in 4d

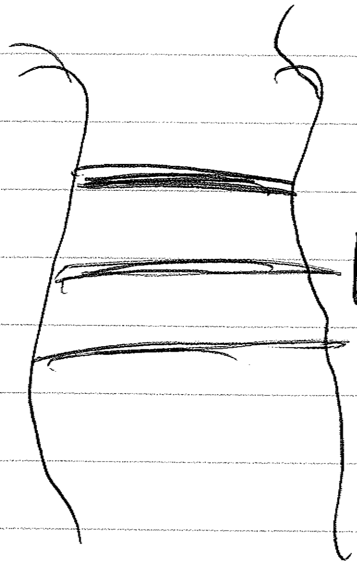
$$\sqrt{g} \propto \varepsilon\varepsilon\, Y\partial Y \partial Y \partial Y \partial Y + \varepsilon\varepsilon\, Y'\partial Y'\partial Y'\partial Y'\partial Y'$$

$\Downarrow$

Volume $\propto$ integer in 2,4 dim. But not in 5,6

Modified constraint $\Rightarrow$
Λ as constant of integration
$\Rightarrow$ DE

Dark matter from NCG.



M_3 are quantized.

$$M_4 = \underline{S_3} \times R$$

Wick rotation + $Y^5 = \eta X$

$$x^4 = \eta t$$

$$0$$

$$Y^A Y^A = 1 \Rightarrow Y^a Y^a = 1 \quad a = 1,2,3$$

$$\sqrt{g} = \lim_{\eta \to 0} \left(\frac{1}{4!} \varepsilon \varepsilon Y^A \partial Y \partial Y \partial Y \partial Y \right) \to$$

$$\to \frac{1}{3!} \varepsilon^{\mu\nu\sigma\lambda} \varepsilon_{abcd} \partial_\mu X \, Y^a \partial_\nu Y^b \partial_\sigma Y^c \partial_\lambda Y^d$$

$$+ \quad \underline{\underline{\partial_\mu X \partial^\mu X = 1}}$$

$$\left(\underset{\substack{\uparrow \\ \text{lapse}}}{N} \underset{\substack{\uparrow \\ \text{det} \\ \text{of 3d} \\ \text{metric}}}{\sqrt{h}} \right)_{\underset{\substack{\uparrow \\ t = \text{const}}}{\Sigma}} = \frac{1}{3!} N \varepsilon^{ijk} \varepsilon_{abcd} Y^a \partial_i Y \partial_j Y \partial_k Y$$

$$\Downarrow$$

$$\int \sqrt{h} \, d^3x \propto \text{integer.}$$

<u>Action</u> $\int \ldots \tilde{\lambda} (\partial_\mu X \partial^\mu X - 1) + \ldots$

$$\downarrow$$

DM !

Conclusion!

Quant. of 4d → DE

Quantiz. of 3d → DM