Energy correlations at conformal collider

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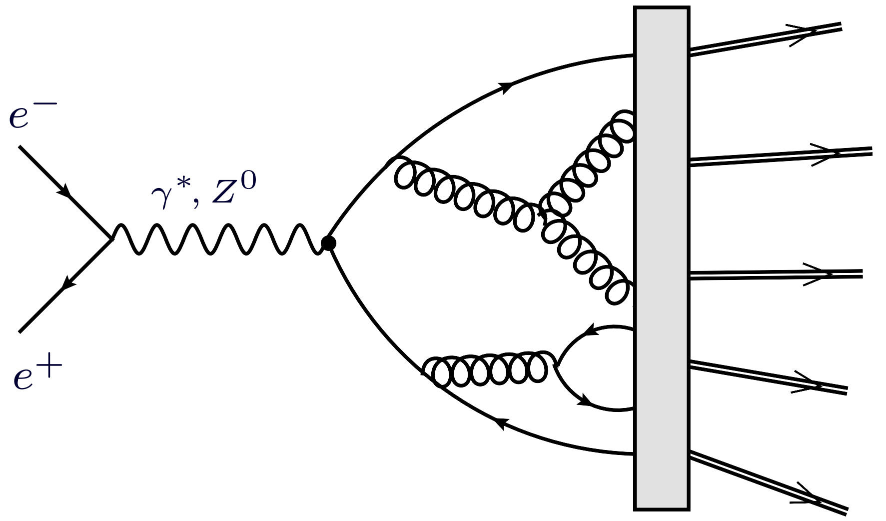
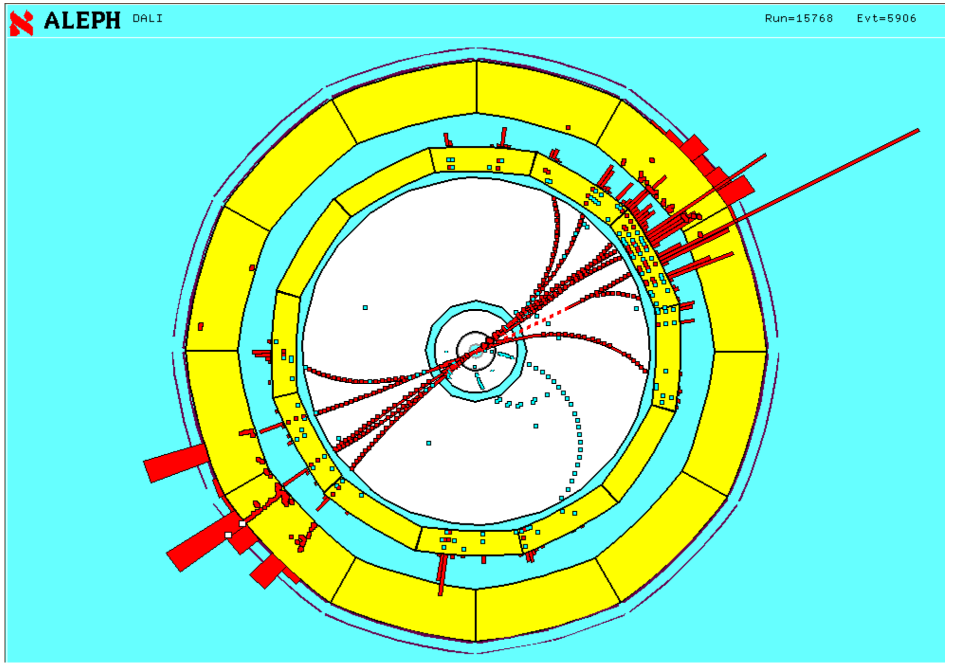
Outline

- ✔ Energy flow correlations in QCD
- ✔ Conformal collider for $\mathcal{N}=4$ SYM
- ✔ Why conventional approach is not efficient
- ✔ Generalized optical theorem
- ✔ Energy flow correlations in $\mathcal{N}=4$ SYM
- ✔ From $\mathcal{N}=4$ SYM to QCD

e^+e^- annihilation in QCD

- PETRA (1978-1986) and LEP (1989-2010)





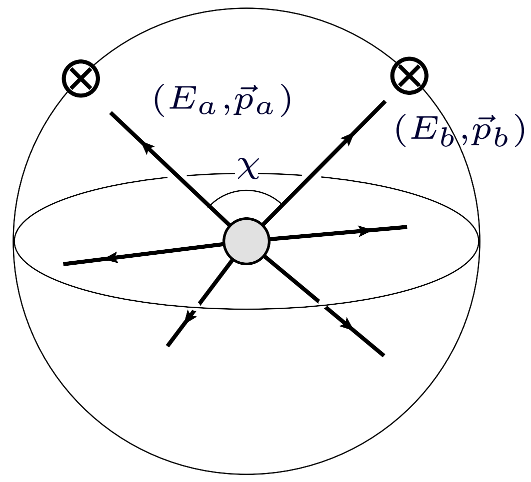
- A virtual photon or Z^0-boson decay into quarks and gluons that undergo a hadronization process into hadrons
- Final states can be described using the class of *infrared finite* observables (event shapes): energy-energy correlations (EEC), thrust, heavy mass, . . .
- Can be computed in perturbative QCD, hadronisation corrections are 'small' at high energy

Energy-energy correlation

- ✔ Function of the angle $0 \le \chi \le \pi$ between detected particles
  [Basham,Brown,Ellis,Love'78]

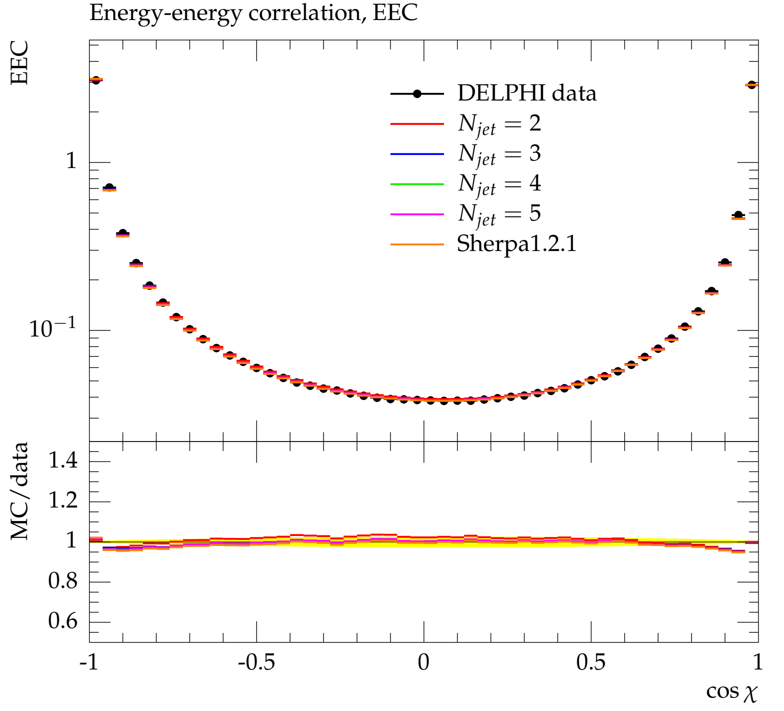
$$\mathrm{EEC}(\chi) = \sum_{a,b} \int d\sigma_{a+b+X} \frac{E_a E_b}{Q^2} \delta(\cos\theta_{ab} - \cos\chi)$$

  Total energy $\sum_a E_a = Q$



- ✔ One of the best studied event shapes
- ✔ The final states are dominated by two-jet events
- ✔ Current status (1978 – today):
  - ✗ Very precise experimental data
  - ✗ Slow progress on the theory side

$$\mathrm{EEC}(\chi) = \underbrace{a_S(Q)A(\chi)}_{\text{Basham et al 1978}} + \underbrace{a_S^2(Q)B(\chi)}_{\text{Dixon et al 2018}} + O(a_S^3)$$



- ✔ *Final goal: develop more efficient method to computing EEC*

Conformal collider

“Mathematics is a part of physics. Physics is an experimental science, a part of natural science. Mathematics is the part of physics where experiments are cheap.”

V. Arnold

From QCD to Maximally supersymmetric Yang-Mills theory

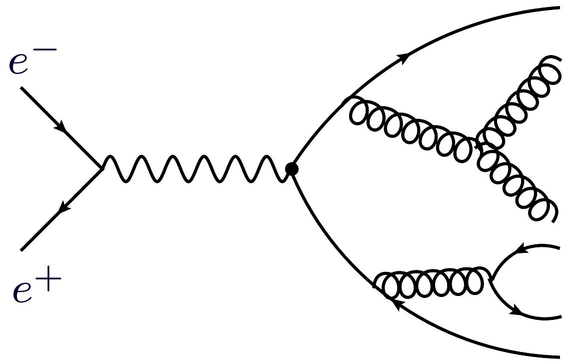
- ✔ Supersymmetric cousin of QCD (extended spectrum of on-shell states)

$$\mathcal{N} = 4 \text{ SYM} = [\text{gluon}] + [4 \text{ gauginos}] + [6 \text{ scalars}]$$

- ✔ Maximally supersymmetric, conformal four-dimensional gauge theory
- ✔ Is believed to be integrable, in the planar limit at least
- ✔ Weak/strong coupling duality (AdS/CFT correspondence)

Use $\mathcal{N} = 4$ SYM for developing new approaches to computing physical observables in QCD

e^+e^- annihilation in $\mathcal{N}=4$ SYM



- Introduce an analog of the QCD electromagnetic current: the stress-energy supermultiplet

$$J \;=\; \Big\{ \; \underbrace{O_{\mathbf{20'}}}_{\text{1/2-BPS operator}} \;,\; \underbrace{\epsilon^{\mu} J_{R,\mu}}_{R-\text{current}} \;,\; \underbrace{\epsilon^{\mu\nu} T_{\mu\nu}}_{\text{stress-energy tensor}} \; \Big\}$$

- The final state contains an arbitrary number of scalars (s), gauginos (q) and gauge fields (g)

$$\int d^4x \, \mathrm{e}^{iQx} \, J(x)|0\rangle = |ss\rangle + |ssg\rangle + |sqq\rangle + \ldots$$

- Energy-energy correlation in $\mathcal{N}=4$ SYM

$$\mathrm{EEC}(\chi) = \sum_{a,b=s,q,g} \int d\sigma_{a+b+X} \frac{E_a E_b}{Q^2} \delta(\cos\theta_{ab} - \cos\chi)$$

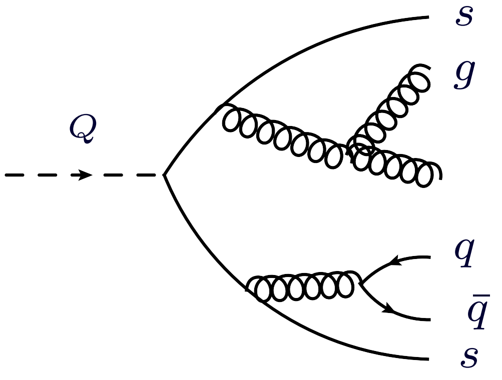
Conventional approach

- EEC as a weighted cross-section

$$\mathrm{EEC}(\chi) = \sum_{a,b,X} \int \mathrm{dLIPS}\, \left|\mathcal{A}_{a+b+X}\right|^2 \frac{E_a E_b}{Q^2} \delta(\cos\chi - \cos\theta_{ab})$$

The amplitude of creation of the final state $|a, b, X = \mathrm{everything}\rangle$

$$\mathcal{A}_{a+b+X} = \int d^4x\, \mathrm{e}^{iQx} \langle a, b, X|J(x)|0\rangle$$



- Main disadvantages:
  - ✗ presence of infrared divergences in transition amplitudes $\mathcal{A}_{a+b+X}$
  - ✗ integration over the Lorentz invariant phase space of the final states $d\mathrm{LIPS}$
  - ✗ necessity for summation over all final states $\sum_X$
  - ✗ no analytical results beyond one loop
- New approach: EEC can be computed from *correlation functions of energy flow operators*

EEC from correlation functions

- Total cross section from the optical theorem

$$\sigma_{\rm tot}(q) = \sum_X (2\pi)^4 \delta^{(4)}(Q - p_X) |\mathcal{A}_{J\to X}|^2$$

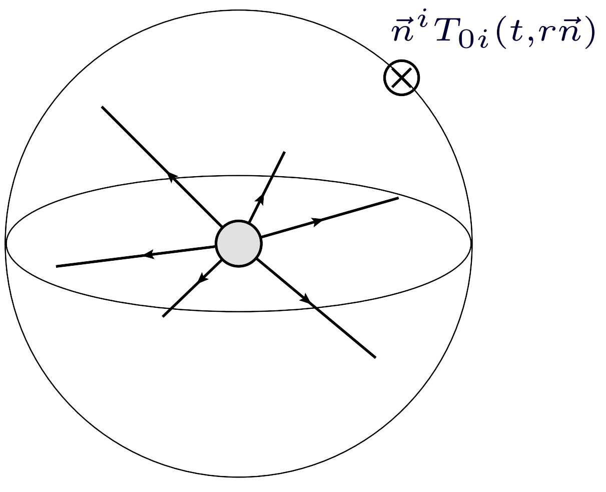
$$= \int d^4x \, \mathrm{e}^{iQx} \sum_X \langle 0|J^\dagger(0)|X\rangle \, \mathrm{e}^{-ixp_X} \, \langle X|J(0)|0\rangle$$

$$= \int d^4x \, \mathrm{e}^{iQx} \underbrace{\langle 0|J^\dagger(x)J(0)|0\rangle}_{\text{Wightman correlation function}} = \frac{1}{16\pi}(N^2-1)\theta(Q^0)\theta(Q^2)$$

- Generalization to EEC

$$\mathrm{EEC} \sim \sum_X \langle 0|J^\dagger(x)|X\rangle \underbrace{w(X)}_{\text{EEC weight factor}} \langle X|J(0)|0\rangle = \langle 0|J^\dagger(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)J(0)|0\rangle$$

- *Energy flow operator* [Sveshnikov,Tkachov],[GK,Oderda,Sterman]



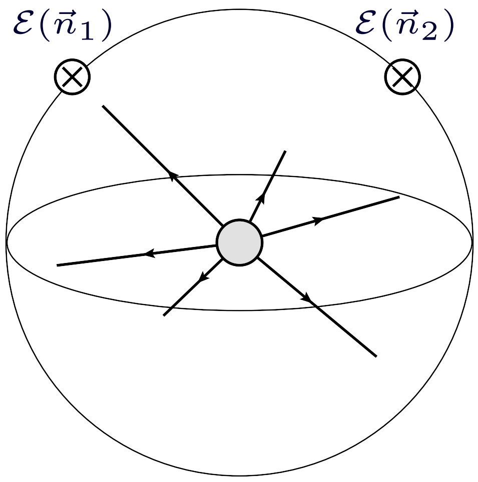
$$\mathcal{E}(\vec{n})|X\rangle = \sum_a E_a \, \delta^{(2)}(\Omega_{\vec{p}_a} - \Omega_{\vec{n}})|X\rangle$$

Relation to the energy-momentum tensor in $\mathcal{N}=4$ SYM

$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 \, \vec{n}^i T_{0i}(t, r\vec{n})$$

EEC from correlation functions II

- ✔ Energy flow correlations [GK,Sterman],[Belitsky,GK,Sterman],[Hofman,Maldacena]



$$\mathrm{EEC}(\chi) = \int d^4x\ \mathrm{e}^{iQx} \langle 0| J^\dagger(x)\, \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\, J(0)|0\rangle$$

Energy flow in the direction of $\vec{n}_1$ and $\vec{n}_2$ with the relative angle χ

- ✔ Multi-fold integral of *Wightman* 4pt function

$$\mathrm{EEC} \sim \underbrace{\int d^4x\, \mathrm{e}^{iQx}}_{\text{Fourier}} \underbrace{\int_0^\infty dt_1 dt_2 \lim_{r_i\to\infty} r_1^2 r_2^2}_{\text{Detector limit}} \underbrace{\langle 0| J^\dagger(x)\, T_{0\vec{n}_1}(x_1) T_{0\vec{n}_2}(x_2)\, J(0)|0\rangle}_{\text{Wightman corr. function}} \Big|_{x_i = (t, r\vec{n}_i)}$$

  - ✗ Compute corr. function $\langle J^\dagger(x) T(x_1) T(x_2) J(0)\rangle$ in Euclid
  - ✗ Continue to Minkowski with Wightman prescription
  - ✗ Take detector limit + perform Fourier

Correlation functions in $\mathcal{N} = 4$ SYM

✔ Correlation functions of $J = \left\{ O_{\mathbf{20'}} \, , J_{R,\mu} \, , T_{\mu\nu} \right\}$ in the stress-energy multiplet are determined by the *same* scalar function

$$\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle_E = \frac{1}{x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2} \Phi(u, v; a)$$

$$\langle J(x_1)T(x_2)T(x_3)J(x_4)\rangle_E = \frac{1}{(x_{12}^2 x_{23}^2 x_{34}^2)^2} \underbrace{P(\partial_u, \partial_v)}_{\text{diff. operator}} \Phi(u, v; a)$$

Cross-ratios $\quad u = x_{12}^2 x_{34}^2/(x_{13}^2 x_{24}^2) \, , \qquad v = x_{23}^2 x_{41}^2/(x_{13}^2 x_{24}^2)$

✔ Universal function at weak coupling $a = g_{\rm YM}^2 N_c/(4\pi^2)$ [Eden,Schubert,Sokatchev],[Bianchi et al]

$$\Phi(u, v) = a\, \Phi^{(1)}(u, v) + a^2 \left(\frac{1}{2}(1 + u + v) \left[\Phi^{(1)}(u, v)\right]^2 + 2 \left[\Phi^{(2)}(u, v) + \frac{1}{u}\Phi^{(2)}(v/u, 1/u) + \frac{1}{v}\Phi^{(2)}(1/v, u/v)\right] \right) + O(a^3)$$

$\Phi^{(1)}(u, v)$ ‘box’ integral, $\Phi^{(2)}(u, v)$ ‘double’ box integral

✔ AdS/CFT correspondence predicts $\Phi(u, v)$ at strong coupling [Arutyunov, Frolov]

All-loop prediction for EEC

Master formula

$$\mathrm{EEC}(\chi) = \frac{1}{4z^2(1-z)} \int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} \underbrace{M(j_1,j_2;a)}_{\text{corr.function}} \underbrace{K(j_1,j_2)}_{\text{detector}} \underbrace{\left(\frac{1-z}{z}\right)^{j_1+j_2}}_{\text{angular dependence}}$$

- ✔ The dependence on the angle χ enters through

$$z = (1-\cos\chi)/2\,, \qquad\qquad 0 < z < 1$$

- ✔ Detector function is independent on the coupling

$$K(j_1,j_2) = \frac{2\,\Gamma(1-j_1-j_2)}{\Gamma(j_1+j_2)[\Gamma(1-j_1)\Gamma(1-j_2)]^2}$$

- ✔ The dependence on the coupling constant resides in the Mellin amplitude

$$\Phi(u,v;a) = \int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} M(j_1,j_2;a)\, u^{j_1} v^{j_2}$$

- ✔ The Mellin amplitude $M(j_1,j_2;a)$ is known in $\mathcal{N}=4$ SYM at weak and at strong coupling

EEC at weak coupling

$$\mathrm{EEC}_{\mathcal{N}=4} = \frac{1}{4z^2(1-z)}\Big\{aF_1(z) + a^2 F_2(z) + a^3 F_3(z) + O(a^4)\Big\}, \qquad z = \tfrac{1}{2}(1-\cos\chi)$$

- ✔ Leading order $F_1(z) = -\ln(1-z)$
- ✔ Next-to-leading order [Belitsky, Hohenegger,GK,Sokatchev,Zhiboedov'2013]

$$F_2(z) = (1-z)(4\sqrt{z}\Big[\mathrm{Li}_2(-\sqrt{z}) - \mathrm{Li}_2(\sqrt{z}) + \tfrac{1}{2}\ln z \ln\Big(\frac{1+\sqrt{z}}{1-\sqrt{z}}\Big)\Big] + (1+z)\big[2\mathrm{Li}_2(z) + \ln^2(1-z)\big] + 2\ln(1-z)\ln\Big(\frac{z}{1-z}\Big) + z\frac{\pi^2}{3})$$

$$+ (1-z)(1+2z)\Big[\ln^2\Big(\frac{1+\sqrt{z}}{1-\sqrt{z}}\Big)\ln\Big(\frac{1-z}{z}\Big) - 8\mathrm{Li}_3\Big(\frac{\sqrt{z}}{\sqrt{z}-1}\Big) - 8\mathrm{Li}_3\Big(\frac{\sqrt{z}}{\sqrt{z}+1}\Big)\Big] - 4(z-4)\mathrm{Li}_3(z) + 6(3+3z-4z^2)\mathrm{Li}_3\Big(\frac{z}{z-1}\Big)$$

$$-2z(1+4z)\zeta_3 + 2\Big[(3-4z)z\ln z + 2(2z^2-z-2)\ln(1-z)\Big]\mathrm{Li}_2(z) + \tfrac{1}{3}\ln^2(1-z)\Big[4(3z^2-2z-1)\ln(1-z) + 3(3-4z)z\ln z\Big]$$

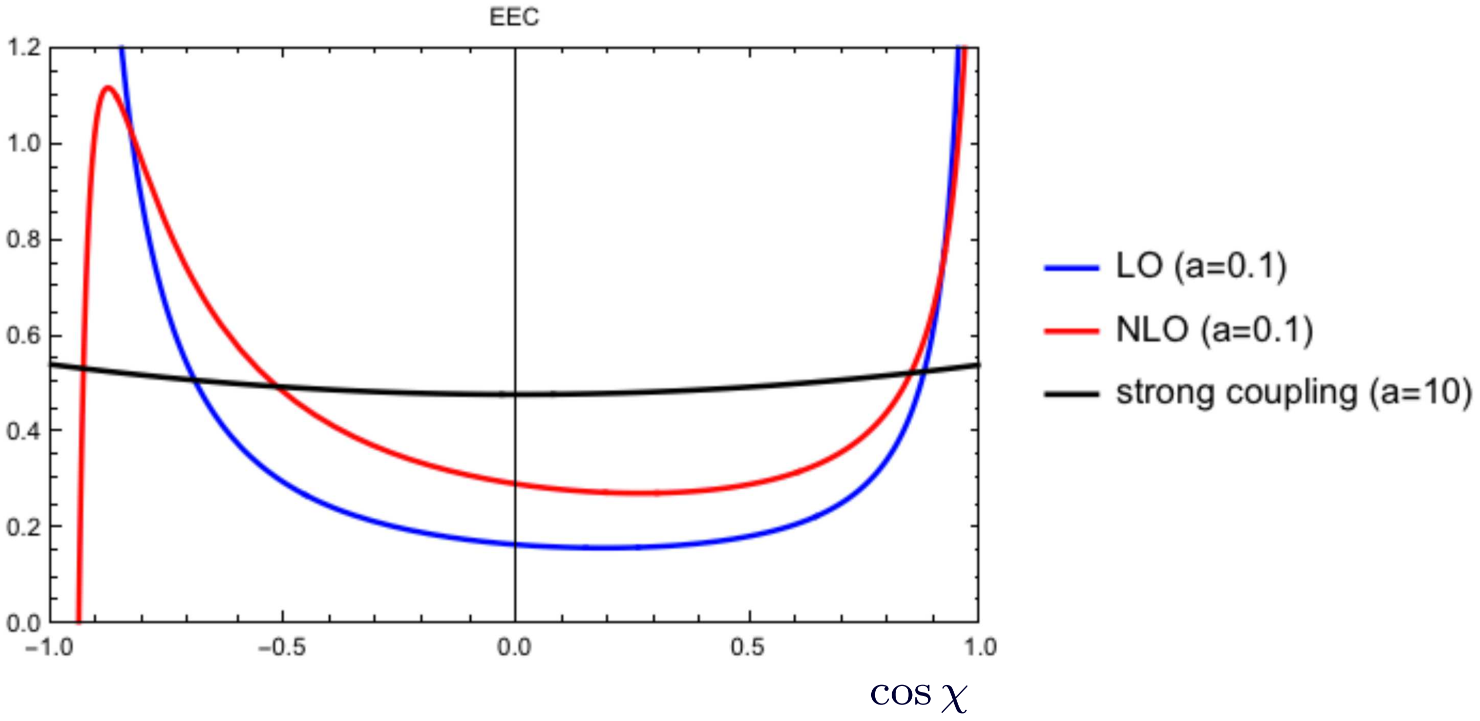
$$+\frac{\pi^2}{3}\big[2z^2\ln z - (2z^2+z-2)\ln(1-z)\big]$$

- ✔ Next-to-next-to-leading order [Henn,Sokatchev, Yan,Zhiboedov'2019]

$F_3(z) =$ a sum of harmonic polylogarithms + a two-fold elliptic integral

- ✔ Large logarithmically enhanced corrections for $z \to 0$ (small angle) and $z \to 1$ (back-to-back region)

From weak to strong coupling



- ✔ At weak coupling $\mathrm{EEC}_{\mathcal{N}=4}$ has a shape which is remarkably similar to the one in QCD
- ✔ Going from one to two loops, EEC flattens
- ✔ This agrees with strong coupling prediction for EEC in planar $\mathcal{N}=4$ SYM [Hofman,Maldacena]

$$\mathrm{EEC}_{\mathcal{N}=4} \overset{a\to\infty}{\sim} \frac{1}{2}\left[1 + a^{-1}\left(1-6z(1-z)\right) + O(a^{-3/2})\right]$$

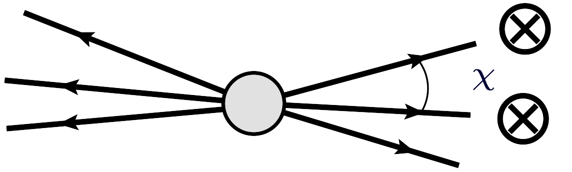
No jets at strong coupling!

What is a manifestation of integrability of $\mathcal{N}=4$ SYM?

End-point asymptotics

✔ Small angle correlations $\chi \to 0$ (or $z \sim \chi^2 \to 0$): calorimeters measure nearly collinear particles

$$\text{EEC} \overset{z\to 0}{\sim} \frac{a}{4z}\left[1 + a\left(\ln z - \tfrac{1}{2}\zeta_3 + \zeta_2 - 3\right)\right]$$



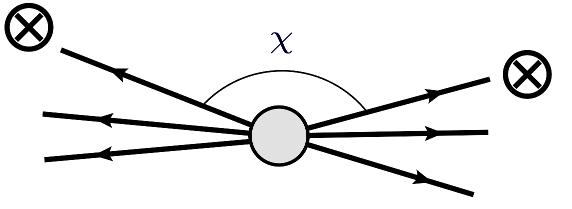
✗ Resummation of leading log's $a(a\ln z)^k$ using the "jet calculus" [Konishi,Ukawa,Veneziano]

$$\text{EEC} \overset{z\to 0}{\sim} \frac{a}{4z}\int_0^1 dx\, x^2 \underbrace{D(x, Q^2 z)}_{\text{fragmentation function}} = \frac{a}{4} z^{-1+\gamma(3)/2}$$

$\gamma(3) = 2a + O(a^2)$ – twist-2 anom. dimension of spin $S = 3$

✔ EEC in the back-to-back kinematics $\chi \to \pi$ (or $y \equiv 1 - z \sim (\pi - \chi)^2 \to 0$)

$$\text{EEC} \overset{z\to 1}{\sim} \frac{1}{4y}\left\{a\ln(1/y) - \frac{a^2}{2}\left[\ln^3(1/y) + \frac{\pi^2}{2}\ln(1/y)\right]\right\}$$

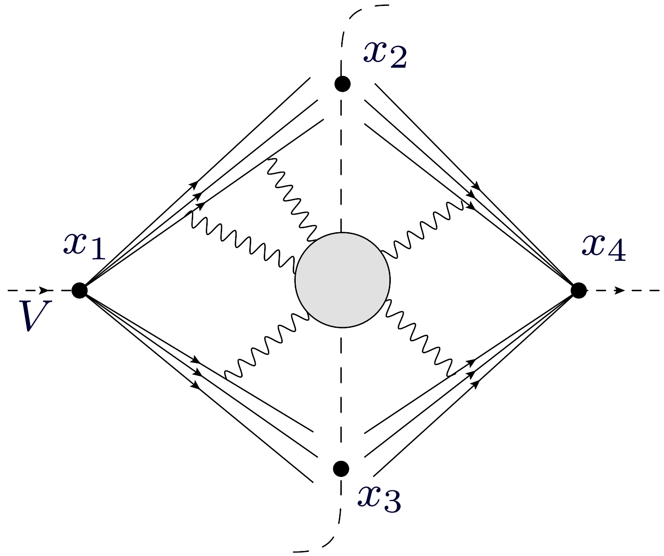


✗ Large (Sudakov) corrections $a^k(\ln y)^n$ come from the emission of soft and collinear particles

✗ Can be resummed to all orders in the coupling [Collins, Soper]

Back-to-back region

Unitary diagram describing a two-jet cross-section



EEC $\sim \langle J_{\mu_1}(x_1) T_{\mu_2\nu_2}(x_2) T_{\mu_3\nu_3}(x_3) J_{\mu_4}(x_4)\rangle$

The 'source operators' are at the points $x_1,\ x_4$, the 'calorimeter operators' are at $x_2\ ,\ x_3$

The four operators are light-like separated $x_{12}^2, x_{13}^2, x_{24}^2, x_{34}^2 \to 0$

EEC in the back-to-back region $\quad=\quad$ Light-like limit of the correlation function

$$\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle = \frac{1}{x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2}\Phi(u,v;a)$$

Cross-ratios $u,\ v \to 0$

$\Phi(u,v;a)$ receives large double-log corrections $(a \ln u \ln v)^\ell$

Light-like limit of the correlation function

OPE expansion $(x_i - x_{i+1})^2 \to 0$

$$O(x_i)O(x_{i+1}) \sim \sum_{\Delta,S} \frac{C_S}{(x_{i,i+1}^2)^{2-(\Delta-S)/2}} O_S(x_i)$$

The leading contribution in all channels comes from the twist-2 operators with large spin $S \gg 1$

$$\Delta - S = 2 + \gamma_S(a), \qquad \gamma_S = 2\Gamma_{\text{cusp}}(a)\ln S + \Gamma(a)$$

$$C_S(a) = H(a)\, \mathrm{e}^{-\Gamma(a)\ln S}\, 2^{-\gamma_S(a)}\Gamma^2\left(1 - \tfrac{1}{2}\gamma_S(a)\right)$$

$\Gamma_{\text{cusp}}(a)$ and $\Gamma(a)$ are known for any coupling from integrability; $H(a)$ is known at three loops

$$\Phi(u,v) \sim \sum_{S\gg 1} C_S\, u^{\gamma_S/2} \underbrace{g_S(v)}_{\text{conformal block}}$$

Leading asymptotics of the correlation function for $u, v \to 0$ [Alday,Eden,GK,Maldacena,Sokatchev]

$$\Phi(u,v) = H(a)\int_0^\infty \frac{dy_1}{y_1}\int_0^\infty \frac{dy_2}{y_2}\, \mathrm{e}^{-\frac{1}{2}\Gamma_{\text{cusp}}(a)\ln(u/y_1)\ln(v/y_2) + \frac{1}{2}\Gamma(a)\ln(uv/(y_1y_2))} f(y_1)f(y_2)$$

The function $f(y) = 2yK_0(2\sqrt{y})$ describes the large spin limit of the conformal block

EEC in the back-to-back region

Prediction for the EEC in the back-to-back region $\delta = (\pi - \chi)^2 \to 0$

$$\mathrm{EEC}(\chi) = \frac{H(a)}{8\delta} \int_0^\infty dy\, J_0(\sqrt{y}) \exp\left[-\frac{1}{2}\Gamma_{\mathrm{cusp}}(a) \ln^2(y/\delta) - \Gamma(a) \ln(y/\delta)\right]$$

Weak-coupling expansion (with $L = \ln(1/\delta) \gg 1$)

$$\begin{aligned}
\text{EEC} = \frac{1}{4\delta}\Big\{ & aL + a^2\left(-\frac{L^3}{2} - \frac{\pi^2 L}{4} + \frac{\zeta_3}{2}\right) + a^3\left(\frac{L^5}{8} + \frac{\pi^2 L^3}{6} - \frac{11\zeta_3 L^2}{4} + \frac{61\pi^4 L}{720} - \frac{\pi^2\zeta_3}{3} - \frac{7\zeta_5}{2}\right) \\
& + a^4\Big[-\frac{L^7}{48} - \frac{5\pi^2 L^5}{96} + \frac{95\zeta_3 L^4}{48} - \frac{29\pi^4 L^3}{480} + \left(\frac{67\pi^2\zeta_3}{48} + \frac{69\zeta_5}{4}\right) L^2 \\
& - \left(\frac{97\zeta_3^2}{8} + \frac{367\pi^6}{12096}\right) L + \frac{187\pi^4\zeta_3}{1440} + \frac{95\pi^2\zeta_5}{48} + \frac{785\zeta_7}{32}\Big] + O(a^5)\Big\}
\end{aligned}$$

Homogenous weight property: $w(L) = 1,\, w(\pi) = 1,\, w(\zeta_n) = n \quad \mapsto \quad w(\mathrm{EEC}\big|_{a^\ell}) = 2\ell + 1$

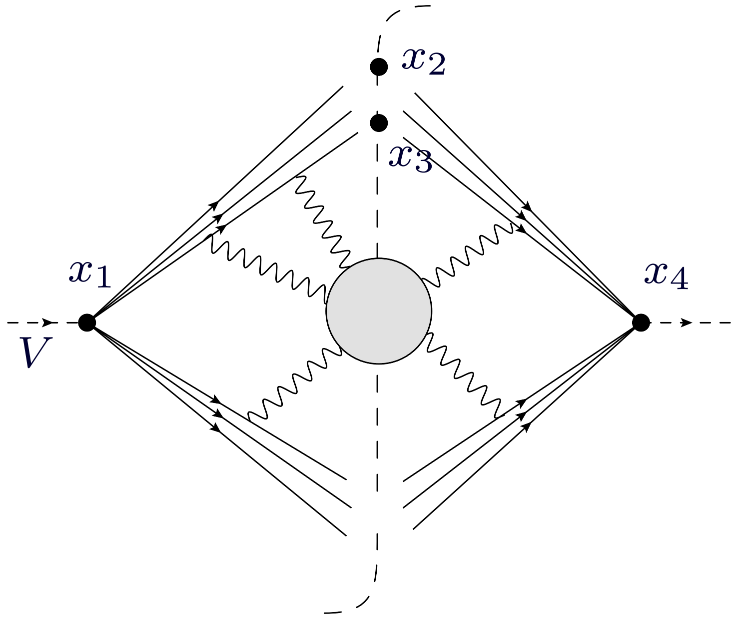
Relation to QCD

$$\mathrm{EEC}_{\mathrm{QCD}}(\chi \to \pi) = \mathrm{EEC}_{\mathcal{N}=4}(\chi \to \pi) + \text{lower weight terms}$$

$\mathcal{N} = 4$ captures the most complicated part of the QCD result

EEC at small angles

Correlation between the particles within the same jet



EEC $\sim \langle J_{\mu_1}(x_1) T_{\mu_2\nu_2}(x_2) T_{\mu_3\nu_3}(x_3) J_{\mu_4}(x_4)\rangle$

Small angle limit $(x_2 - x_3)^2 \to 0$:

$$u' = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = 1/v \to 0\,, \qquad v' = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = u/v \to 1$$

The leading contribution comes from the twist-six operator

Belongs to the twist-two supermultiplet, its conformal data are related to those of the twist-two operator : $\Delta_{\mathrm{tw}-6}(S) = 6 + S + \gamma_{S+2}$ and $C_{\mathrm{tw}-6}(S) = C_{S+2}$

Conformal partial wave expansion

$$\Phi(u,v) = \sum_{S=0,2,4,\dots} C_{S+2}(a)(u')^{3+\gamma_{S+2}(a)/2} g_S(v') + \dots$$

EEC at small angles II

$$\text{EEC} = 2\int_{-\infty}^{\infty} \frac{dj}{2\pi} \sum_{S=0,2,4,\dots} z^{\tau/2-4} A_{S,\tau} \underbrace{P_{S,\tau}(j)}_{\text{Mellin amplitude of the conformal block}}$$

Sum over the exchanged operators with twist $\tau = 6 + \gamma_{S+2}$

$$A_{S,\tau} = \frac{\Gamma(2S+\tau)\Gamma(\tau/2-1)}{4\left[\Gamma(\tau/2)\Gamma\left(S+\tau/2\right)\right]^2 \Gamma(2-\tau/2)} C_{S+2}(a)$$

$P_{S,\tau}(x)$ is the continuous Hahn polynomial $\Longrightarrow$ the sum collapses to a single term with $S=-1$

$$\text{EEC} = z^{\tau/2-4} A_{S=-1,\tau} \frac{(\tau/2-1)^2}{\tau-3}$$

EEC at small angle $z = \chi^2/4 \to 0$ [Kologlu,Kravchuk,Simmons-Duffin,Zhiboedov]

$$\text{EEC} = z^{-1+\gamma_1(a)/2} \frac{C_1(a)\,\Gamma(3+\gamma_1(a))}{4\Gamma^3\left(2+\gamma_1(a)/2\right)\Gamma(-1-\gamma_1(a)/2)}$$

Depends on the twist-2 conformal data C_S and γ_S for *small* spin $S=1$ [Dixon,Mould,Zhu] [GK]

$$\mathrm{EEC}_{\mathrm{QCD}}(\chi\to 0) = \mathrm{EEC}_{\mathcal{N}=4}(\chi\to 0) + \text{lower weight terms}$$

Conclusions and open questions

- ✔ Energy correlations are good/nontrivial physical observables in $\mathcal{N} = 4$ SYM
- ✔ Relation to energy-energy correlations in QCD (most complicated part)?
- ✔ Interpolation between weak and strong coupling? what is the manifestation of integrability?
- ✔ Other proposals for ‘good’ observables?

Many thanks to the Organizers

for setting up this wonderful Conference!