Conference:

"Recent advances in theoretical physics of fundamental interactions"

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Conformal Fishnet Theory in Any Dimension

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ENS

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Collaborations with

Ö. Gürdogan	arXiv:1512.06704
J. Caetano and Ö. Gürdogan	arXiv:1612.05895
D. Grabner, N. Gromov, G. Korchemsky	arXiv:1711.04786
N. Gromov, G. Korchemsky, S. Negro, G. Sizov	arXiv:1706.04167
D. Chicherin, F. Loebbert, D. Mueller, D. Zhong	arXiv:1704.01967, arXiv:1708.00007
E. Olivucci	arXiv:1801.09844
N. Gromov, G. Korchemsky	arXiv:1808.02688
S. Derkachev, E. Olivucci	arXiv:1811.10623
E. Olivucci, M. Preti	arXiv:1901.00011
G. Karananos, M. Shaposhnikov	arXiv:19***
B. Basso, G. Ferrando, D. Zhong	arXiv:19***

Outline

- CFTs are important for description of many physical phenomena: phase transitions, quantum gravity (via AdS/CFT duality), Regge limit in QCD, etc

- Non-supersymmetric strongly coupled CFTs in 4D are rare species. None of them are known to have a moduli space of vacua (possible key to hierarchy problem)

- We proposed a family of non-supersymmetric, logarithmic 4D CFTs, from double scaling limit of N=4 SYM: strong γ-deformation & weak coupling.

Gurdogan, V.K. '15

- Example: bi-scalar "fishnet" CFT: $\mathcal{L}[\phi_1,\phi_2] = N_c \mathrm{tr}\left(\partial^\mu\phi_1^\dagger\partial_\mu\phi_1 + \partial^\mu\phi_2^\dagger\partial_\mu\phi_2 + \xi^2\,\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\right)$

- Generalization to any dimension V.K , Olivucchi, '18

- Integrable! Inherits integrability of N=4 SYM and sheds some light on its origins.

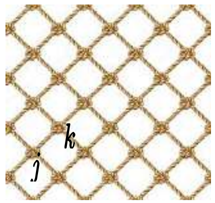
- Analytic insight into non-perturbative structure of CFTs at D>2 (with spontaneous conformal symmetry breaking)

“Fishnet” Feynman graphs

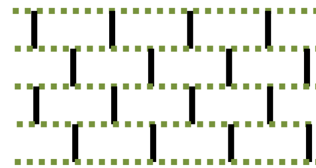
- Fishnet CDF is dominated by specific, integrable (computable!) multi-loop planar diagrams:

“fishnet” graph A. Zamolodchikov 1980
(massless, 4-scalar int.)

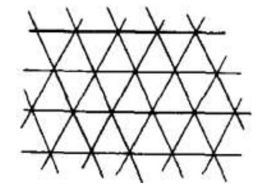
$$\int \prod_i d^D x_i \prod_{<jk>} \frac{1}{|x_j - x_k|^{D/2}}$$



“brick wall” Gurdogan, V.K. 2015
Caetano, Gurdogan, V.K. 2016
(Yukawa ψψϕ interaction)



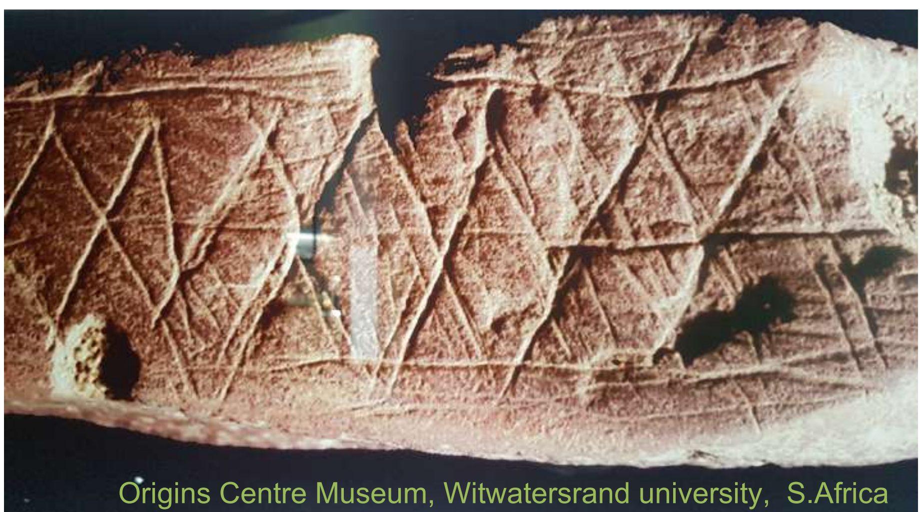
Caetano, Gurdogan, V.K. 2016
3D ABJM in our limit



$$\frac{1}{|x_j - x_k|}$$

A. Zamolodchikov 1980

- Earlier approaches



D=0 case: regular triangular lattice, 75.000 BC

possible author

Approaches and results

- Chiral "fishnet" CFT can be studied by methods inherited from N=4 SYM integrability (Asymptotic Bethe Ansatz (ABA), Y/TBA →Quantum Spectral Curve (QSC), Hexagon approach)

Minahan, Zarembo '02
Beisert, Eden, Staudacher '05

Al. Zamolodchikov '90
Gromov, V.K., Vieira '09
Gromov, V.K. Kozak, Vieira
Bombardelli, Fioravanti, Tateo
Arutyunov, Frolov

Gromov, V.K. ,Leurent, Volin '13

Basso, Caetano, Flory'18
Basso, Komatsu, Vieira '15

- Or by direct map to non-compact SU(2,2) Heisenberg spin chain

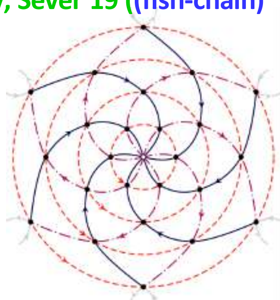
A. Zamolodchikov '80
Chicherin, Derkachev, Isaev '11
Gurdogan, V.K. '15
Gromov, V.K , Korchemsky, Negro, Sizov '17

Gurdogan, V.K. '15
Caetano, Gurdogan, V.K. 2016

dimensions of local operators… … with magnons

Gromov, V.K , Korchemsky, Negro, Sizov '17
Gromov, Sever'19 ((fish-chain)

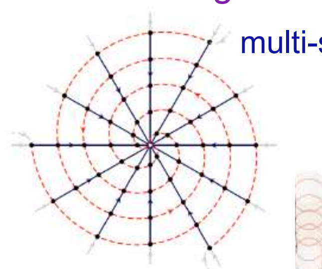
ABJM wheel



wheels

$\mathrm{tr}[\phi_1(x)]^L$

multi-spirals



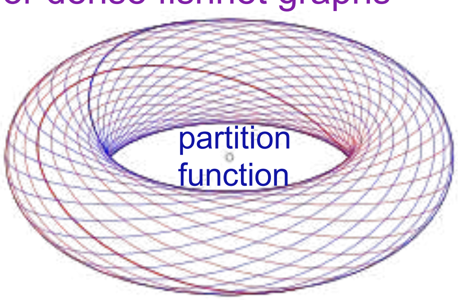
4-point correlators and OPE, annulus amplitude, Chaos in massive fishnet CFT

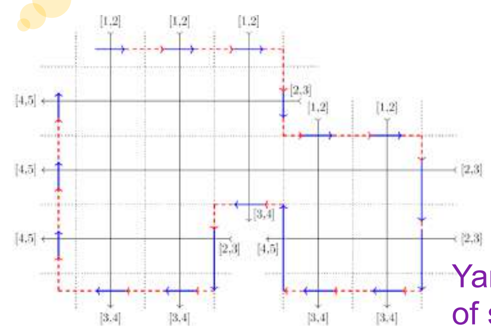
Grabner, Gromov, V.K., Korchemsky '17
V.K , Olivucchi, '18
Gromov, V.K., Korchemsky '18
V.K , Olivucchi, Preti '18
Korchemsky '18
De Mello Koch, LiMing, Van Zyl, Rodrigues '18

Computable physical quantities and related Feynman graphs

A. Zamolodchikov '80
Basso, Zhang '18

AdS$_5$ sigma-model for dense fishnet graphs





Basso, Dixon '17 (4D)
Derkachev, V.K., Olivucchi '18 (2D)

Chicherin, V.K., Loebbert, Mueller, Zhong '17

Yangian symmetry of scalar amplitudes

γ-twisted N=4 SYM

$$\mathcal{L} = N_c \mathrm{tr}[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D^\mu\phi_i^\dagger D_\mu\phi^i + i\bar\psi_A^{\dot\alpha} D_{\dot\alpha}^{\alpha}\psi_\alpha^A] + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{int}} = N_c g^2 \,\mathrm{tr}\left(\frac{1}{4}\{\phi_i^\dagger, \phi^i\}\{\phi_j^\dagger,\phi^j\} - e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger\phi_j^\dagger\phi^i\phi^j\right) +$$

$$N_c g\,\mathrm{tr}(-e^{-\frac{i}{2}\gamma_j^-}\bar\psi_j\phi^j\bar\psi_4 + e^{+\frac{i}{2}\gamma_j^-}\bar\psi_4\phi^j\bar\psi_j + i\epsilon_{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\bar\psi^k\phi^i\bar\psi^j + \text{conjugate terms}) .$$

$$\gamma_1^\pm = -\frac{\gamma_3 \pm \gamma_2}{2}$$

$$\gamma_2^\pm = -\frac{\gamma_1 \pm \gamma_3}{2}$$

$$\gamma_3^\pm = -\frac{\gamma_2 \pm \gamma_1}{2}$$

- γ-twisted N=4 SYM Lagrangian: product of matrix fields $\rightarrow$ star-product

$$A\,B \to A \star B \equiv q_{A,B}\,A\,B \qquad \text{where} \qquad q_{A,B} = e^{-\frac{i}{2}\epsilon^{mjk}\gamma_m J_j^A J_k^B} = (q_{B,A})^{-1}$$

Leigh, Strassler
Frolov, Tseytlin
Beisert, Roiban
Lunin, Maldacena

$J_1^A,\, J_2^A,\, J_3^A \in SO(6)$ - Cartan charges of R-symmetry $\quad \gamma_1, \gamma_2, \gamma_3$ - twists

- Breaks R-symmetry and all supersymmetry: PSU(2,2|4)→SU(2,2)×U(1)^3

- γ-twisted N=4 SYM is an integrable non-supersymmetric non-unitary CFT

V.K., Leurent, Volin '14
Grabner, Gromov, V.K.,Korchemsky'18

γ-twisted N=4 Super-Yang-Mills

- γ-twisting of N=4 SYM : product of $N_c \times N_c$ matrix fields → star-product

Leigh, Strassler
Frolov, Tseytlin
Beisert, Roiban
Lunin, Maldacena

$$A\,B \to A \star B \equiv q_{A,B}\, A\, B \quad \text{where} \quad q_{A,B} = e^{-\frac{i}{2}\epsilon^{mjk}\gamma_m\, J_j^A\, J_k^B} = (q_{B,A})^{-1}$$

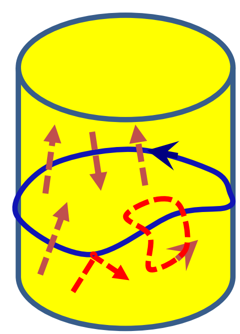
$J_1^A,\ J_2^A,\ J_3^A \in SO(6)$ - Cartan charges of R-symmetry $\gamma_1,\ \gamma_2,\ \gamma_3$ - twists

$$[A\,,B] \to [A,B]_q \equiv q_{AB}\, A\, B - \frac{1}{q_{AB}}\, B\, A$$

- PSU(2,2|4)→SU(2,2) × U(1)³ - breaks R- symmetry and all supersymmetry
- γ-twisted N=4 SYM is a CFT, integrable at $N_c \to \infty$

V.K., Leurent, Volin '14
Grabner, Gromov, V.K.,Korchemsky'18

- γ-twist is a topological factor on a planar graph:
  it produces quasiperiodic b.c. on cylindric graph



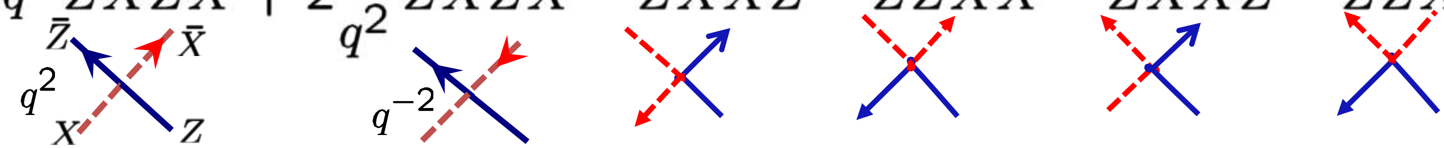
$$\sim\ (q^2)^{\#_\uparrow - \#_\downarrow}$$

4-scalar term

$$q = e^{-i\gamma_m/2}$$

$$\frac{g^2}{16}\mathrm{tr}\left([\Phi^{AB},\Phi^{CD}]_q[\bar\Phi^{AB},\bar\Phi^{CD}]_q\right) =$$

$$= g^2\mathrm{tr}\left(2q^2\, ZX\bar Z\bar X + 2\frac{1}{q^2}\, Z\bar X\bar Z X - Z\bar X X\bar Z - Z\bar Z X\bar X - ZX\bar X\bar Z - Z\bar Z\bar X X\right) + \ldots$$



Double scaling limit

Gurdogan, V.K. '15

- Strong imaginary twist, weak coupling

$$g \to 0, \qquad \gamma \to i\infty, \qquad \xi_j = g\, e^{-i\gamma_j/2} - \text{fixed}, \qquad (j = 1,2,3.)$$

- γ-twisted N=4 SYM becomes in this limit a non-unitary, chiral CFT (χCFT):

$$\mathcal{L} = N_c \mathrm{tr}[-\frac{1}{2}\partial^\mu \bar{\phi}_i \partial_\mu \phi^i + i\bar{\psi}^{\dot\alpha}_A \partial^\alpha_{\dot\alpha} \psi^A_\alpha] + \mathcal{L}_{\rm int}$$

$$\mathcal{L}_{\rm int} = N_c\, \mathrm{tr}[\xi_1^2\, \bar{\phi}_2\bar{\phi}_3\phi_2\phi_3 + \xi_2^2\, \bar{\phi}_3\bar{\phi}_1\phi_3\phi_1 + \xi_3^2\, \bar{\phi}_1\bar{\phi}_2\phi_1\phi_2 +$$

$$+ i\sqrt{\xi_2\xi_3}(\psi^3\phi^1\psi^2 + \bar{\psi}_3\bar{\phi}_1\bar{\psi}_2) + i\sqrt{\xi_1\xi_3}(\psi^1\phi^2\psi^3 + \bar{\psi}_1\bar{\phi}_2\bar{\psi}_3) + i\sqrt{\xi_1\xi_2}(\psi^2\phi^3\psi^1 + \bar{\psi}_2\bar{\phi}_3\bar{\psi}_1)\,].$$

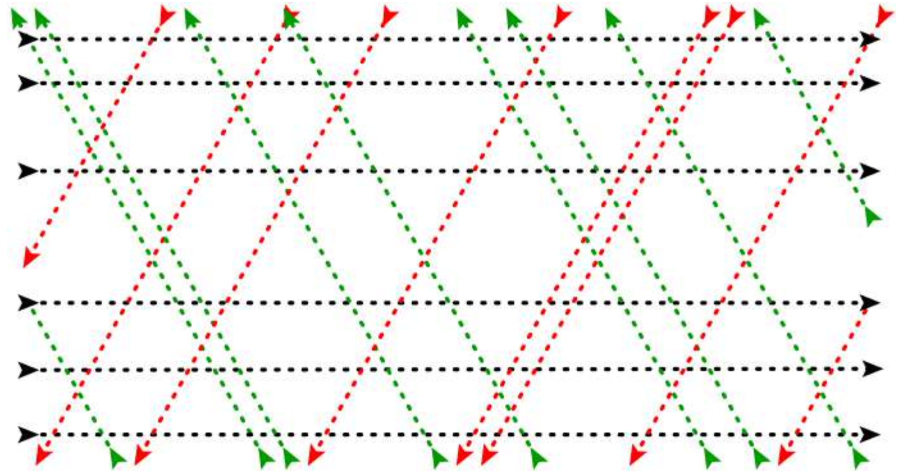
- 3 flavors of bosons and fermions

- Chirality: terms don't have their complex conjugates

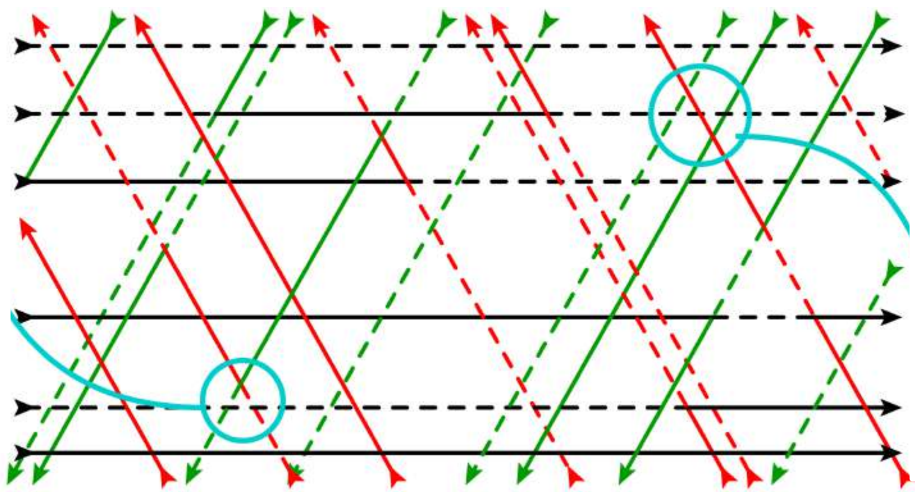
V.K., Olivucci, Preti 2018

Dynamic “fishnet” graphs in full chiral CFT

Basic skeleton graph:
3 systems of parallel lines with different flavors (black, red, green).
Encodes multiple graphs



Dotted lines encode solid (bosonic) and dashed (fermionic) stretches, forming continuous zig-zag lines



Intersecting bosonic lines give quartic vertices

Intersection with fermionic lines should be disentangled into two Yukawa vertices

A challenge: to uncover the underlying integrable spin chain.
A step towards understanding of full N=4 SYM integrability.

Baxter lattice (ancient version)

Gurdogan, V.K. 2015

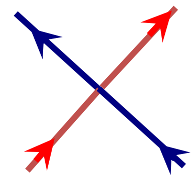
Special case of bi-scalar, “fishnet” CFT_4

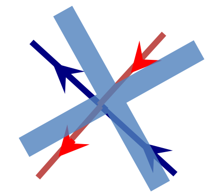
- Single coupling: $\xi := \xi_3 \; - \text{fixed}, \qquad \xi_1 = \xi_2 = 0$

$$\mathcal{L}[\phi_1,\phi_2] = \frac{N_c}{2}\mathrm{tr}\,\left(\partial^\mu\bar{\phi}_1\partial_\mu\phi_1 + \partial^\mu\bar{\phi}_2\partial_\mu\phi_2 + 2\xi^2\,\bar{\phi}_1\bar{\phi}_2\phi_1\phi_2\right).$$

- Propagators

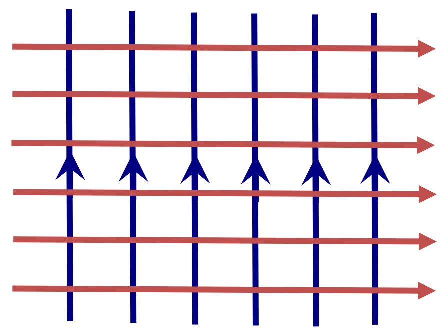
$\sim \frac{1}{(x-y)^2}$



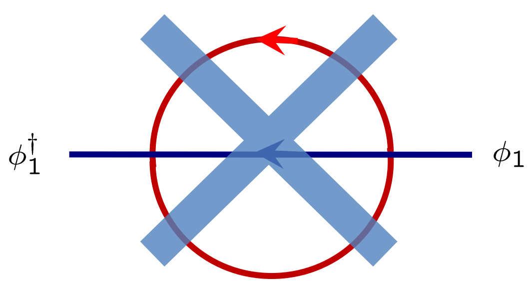


Missing
“anti-chiral” vertex

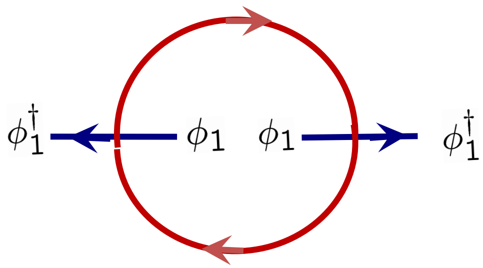
- Bulk structure of
  planar graphs - fishnet



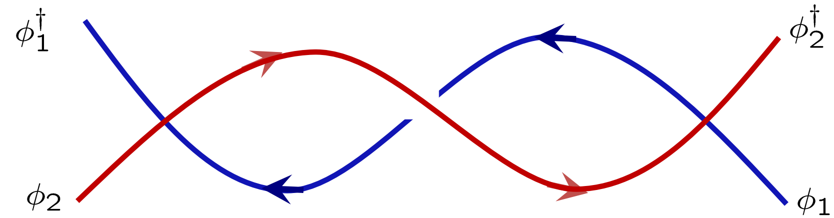
- Very limited number of planar graphs. No mass or vertex renormalization!
  The coupling ξ does not run!



Double trace terms



$\mathrm{tr}(\phi_1\phi_1)\mathrm{tr}(\bar\phi_1\bar\phi_1)$



$\mathrm{tr}(\bar\phi_1\phi_2)\mathrm{tr}(\bar\phi_2\phi_1)$

Tseytlin, Zarembo
Dymarsky, Klebanov, Roiban
Witten
Sieg, Wilhelm, Fokken

- ξ doesn't run in planar limit. Double-trace counter-terms do run:

$$\mathcal{L}_{\mathrm{dt}} = \alpha_1^2 \sum_{i=1}^{2} \mathrm{tr}(\phi_i\phi_i)\,\mathrm{tr}(\bar\phi_i\bar\phi_i) - \alpha_2^2\,\mathrm{tr}(\phi_1\phi_2)\mathrm{tr}(\bar\phi_2\bar\phi_1) - \alpha_2^2\mathrm{tr}(\phi_1\bar\phi_2)\mathrm{tr}(\phi_2\bar\phi_1)\,,$$

Pomoni, Rastelli 2009

- Beta-function quadratic in double-trace coupling (agrees with general arguments)

Grabner, Gromov, V.K., Korchemsky '17

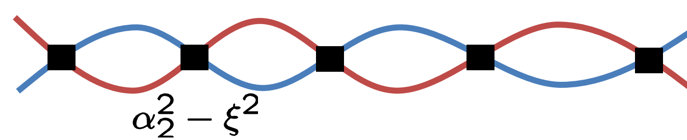
$$\beta_{\alpha_1^2} = a(\xi) + b(\xi)\alpha_1^2 + c(\xi)\,\alpha_1^4 \qquad \text{where} \qquad \frac{1}{2}\sqrt{b^2-4ac} = \gamma_{\mathrm{tr}\phi_1^2}$$

- At critical (trace)² couplings - non-unitary CFT at any ξ (same for γ-def. N=4 SYM)

$$\alpha_{1,\pm}^2 = \pm\frac{i\xi^2}{2} - \frac{\xi^4}{2} \mp \frac{3i\xi^6}{4} + \xi^8 \pm \frac{65i\xi^{10}}{48} - \frac{19\xi^{12}}{10} + O\left(\xi^{14}\right)$$

Sieg, Wilhelm 2016
Grabner, Gromov, V.K., Korchemsky '17

- The other double-trace coupling also tuned to the critical value $\alpha_2^2(\xi) = \xi^2$ to renormalize the only type of divergent graphs:



Karananos, V.K., Shaposhnikov (to appear)

Flat Vacua (moduli)

- The Fishnet CFT does obey a quantum moduli space!

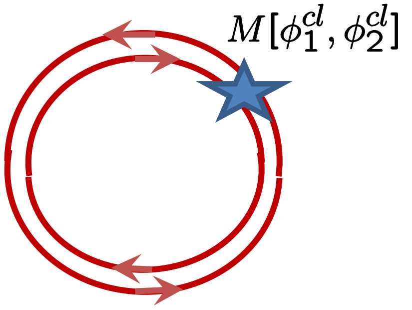
- We expand fields around non-zero constant values:

$$\phi_j(x) \to \phi_j^{cl} + \delta\phi_j(x)\,, \qquad j = 1,2$$

and compute eff. action $\mathcal{L}_{eff}[\phi_1^{cl},\phi_2^{cl}] = \mathcal{L}[\phi_1^{cl},\phi_2^{cl}] + \mathrm{tr}\left(M^4 \log\frac{M^2}{\mu^2}\right)[\phi_1^{cl},\phi_2^{cl}]$

where $\mathcal{L}[\phi_1,\phi_2] = \mathcal{L}_{single-tr}[\phi_1,\phi_2] + \mathcal{L}_{double-tr}[\phi_1,\phi_2]$

$M[\phi_1^{cl},\phi_2^{cl}]$ - "mass-matrix" of quadratic fluctuations in new vacuum



- No multi-loop corrections (no such planar graphs)!
  One-loop Coleman-Weinberg potential appears to be exact

- We found indeed diagonal flat vacua obeying

$$\frac{\partial}{\partial\phi_j^{cl}}\mathcal{L}_{eff}[\phi_1^{cl},\phi_2^{cl}] = 0 \qquad j = 1,2.$$

- It seems that the supersymmetry mechanism of stabilization of flat vacua is replaced by the absence of dangerous Feynman graphs which could lift the flatness property via Coleman-Weinberg potential.

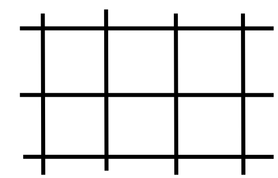
Bi-scalar fishnet CFT at any D

V.K., Olivucci 2018

- Bi-scalar CFT can be defined at any D by a non-local action

$$\mathcal{L}_\phi = N_c \,\mathrm{tr}\left[-\phi_1^\dagger \,\Delta^\gamma\, \phi_1 - \phi_2^\dagger\, \Delta^{\frac{D}{2}-\gamma}\, \phi_2 + \xi^2\, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2\right]$$

- D-dimensional "fishnet" graphs generalize 4D "fishnets"
- Described by integrable conformal SO(1,D) spin chain !



- Isotropic fishnet $\gamma = \frac{D}{4}$

$$\int \prod_i d^D x_i \prod_{<jk>} \frac{1}{|x_j - x_k|^{D/2}} \xrightarrow{D=2} \int \prod_i d^2 z_i \prod_{<jk>} \frac{1}{|z_j - z_k|}$$

- Anisotropic "fishnet" at D=2, γ=2s -1, where s is the "on-site" SL(2,C) spin

- D=2, $\gamma \to 1$ -- Lipatov's model of reggeized gluons (BFKL)

Gurdogan, V.K , '15
Caetano, Gurdogan, V.K , '16

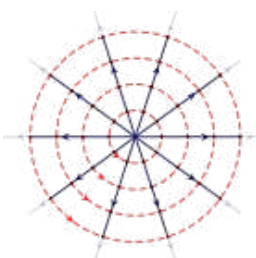
Operators, correlators, graphs…

- Local operators

$$\mathcal{O}(x) = C^{\mu_1\cdots\mu_n}\,\mathrm{tr}\left[\partial_{\mu_1}\ldots\partial_{\mu_n}(\phi_1)^K(\phi_2)^L(\phi_1^\dagger)^M(\phi_2^\dagger)^N\right](x) + \text{permutations}$$

- Correlators $\langle \mathcal{O}(x)\ \mathcal{O}(0)\rangle \sim |x|^{-2\Delta_0 - 2\gamma(\xi)}$ anomalous dimension

- BMN "vacuum" operator $\mathrm{tr}[\phi_1(x)]^L$ renormalized by "wheel" graphs:

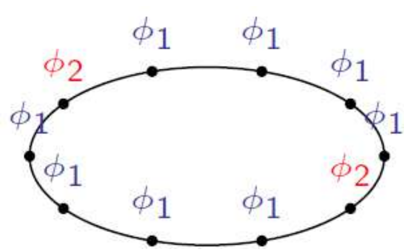


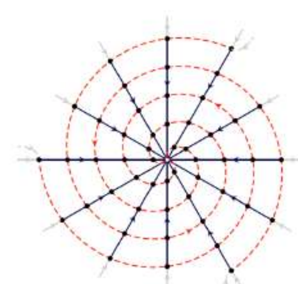
$\epsilon = 4 - D$

$$= \xi^{2LM}\left(\frac{C_M^{(L,M)}}{\epsilon^M} + \frac{C_{M-1}^{(L,M)}}{\epsilon^{M-1}} + \ldots + \frac{C_1^{(L,M)}}{\epsilon} + \text{finite}\right) \Rightarrow \gamma(\xi) = \sum_{M=1}^{\infty}\gamma_M\,\xi^{2LM}$$

wheel with M frames and L spokes

- Multi-magnon operators renormalized by multi-spiral (spider-web) graphs. Typical integrable "fishnet" bulk structure.





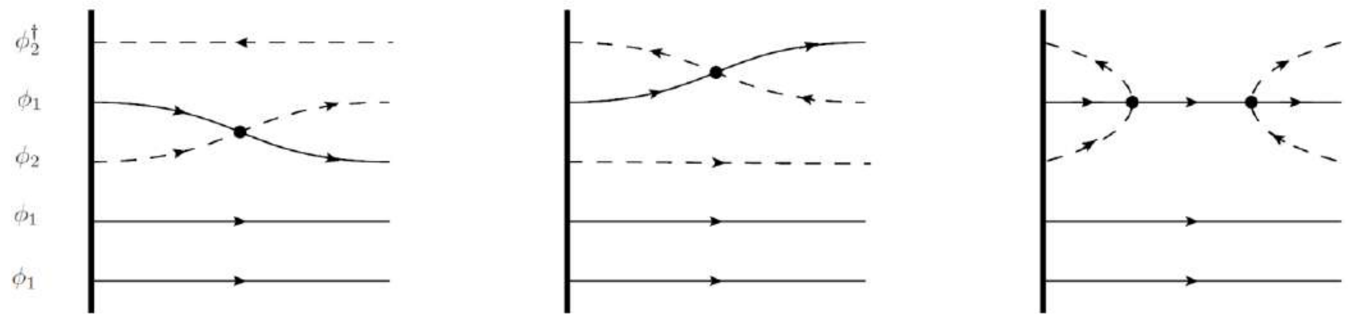
Logarithmic multiplets

Caetano
Gromov, V.K , Negro, Korchemsky, Sizov

- Mixing in multiplet of operators of length L=5:

protected

$$O_1 = \mathrm{tr}\,(\phi_1^3\phi_2\bar\phi_2)\,,\qquad O_2 = \mathrm{tr}\,(\phi_1^2\phi_2\phi_1\bar\phi_2)\,,\qquad O_3 = \mathrm{tr}\,(\phi_1\phi_2\phi_1^2\bar\phi_2)\,,\qquad O_4 = \mathrm{tr}\,(\phi_2\phi_1^3\bar\phi_2)\,.$$



- Non-unitary mixing matrix. “Diagonalisation” means bringing to Jordan form

$$\mu\frac{d}{d\mu}O_i(x) = -V_{ij}O_j(x),\qquad V = \frac{1}{4\pi^2}\begin{bmatrix} 0 & \xi^2 & 0 & 0\\ 0 & 0 & \xi^2 & 0\\ 0 & -\xi^4 & 0 & \xi^2\\ 0 & 0 & 0 & 0\end{bmatrix} = U\cdot\begin{pmatrix} \boxed{\begin{matrix}0 & 1\\ 0 & 0\end{matrix}} & \begin{matrix}0 & 0\\ 0 & 0\end{matrix}\\ \begin{matrix}0 & 0\\ 0 & 0\end{matrix} & \boxed{\begin{matrix}-i\xi^3 & 0\\ 0 & i\xi^3\end{matrix}}\end{pmatrix}\cdot U^{-1}$$

- Lower block contains usual operators with dimensions $\Delta_{\pm}^{L=5} = 5 \pm 2i\left(\frac{\xi}{4\pi}\right)^3 + \dots$

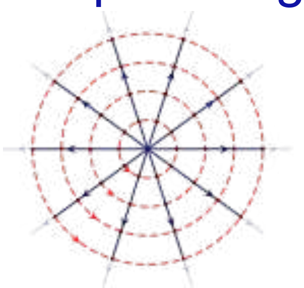
- Upper block is Jordan cell, it gives log-conformal correlators:

Gurarie 1993
Caetano 2016

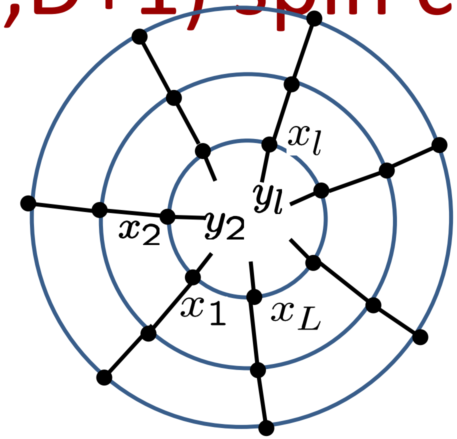
$$\langle \tilde{O}^\dagger_\alpha(x)\tilde{O}_\beta(0)\rangle_{\mathrm{ren}} = \frac{1}{x^{10}}\begin{bmatrix} 0 & 1\\ 1 & \log x^2\mu^2\end{bmatrix}_{\alpha\beta}$$

Wheel graph as integrable SO(1,D+1) spin chain

- Operator generating a wheel (fishnet) graph



$$\hat{t}_L = \xi^{2L} \prod_{i=1}^{L} \frac{1}{(x_i - x_{i+1})^{D/4}\,(y_i - x_i)^{D/4}}$$

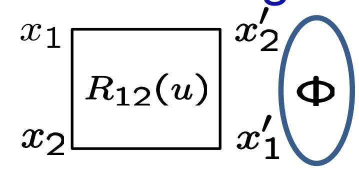


$x_l \in \mathbf{R}^D$

$\sim (\hat{t}_L)^3$

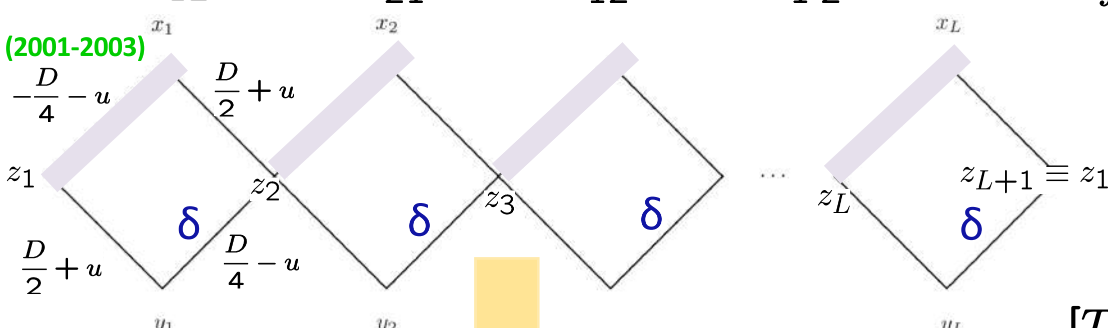
- Integrability: graph-generating operator emerges at special value of spectral parameter of Transfer-matrix in physical irrep on 4D conformal group

$$[R_{12}(u) \star \Phi](x_1, x_2) = \int \frac{d^D x_{1'} d^D x_{2'}\, \Phi(x_{1'}, x_{2'})}{(x_{12}^2)^{-\frac{D}{4}-u}(x_{21'}^2)^{\frac{D}{2}+u}(x_{12'}^2)^{\frac{D}{2}+u}(x_{1'2'}^2)^{-u+\frac{D}{4}}}$$



Zamolodchikov 1980
Derkachev, Manashov (2001-…)
Derkachev, Korchemsky, Manashev (2001-2003)
Chicherin, Derkachev, Isaev 2012

$T_L(u; \mathbf{y}|\mathbf{x}) =$



$[T_L(u), T_L(u')] = 0$

Gurdogan, V.K. 2015
Gromov, V.K , Korchemsky, Negro, Sizov
V.K., Olivucci

$u = -D/4 + \epsilon$

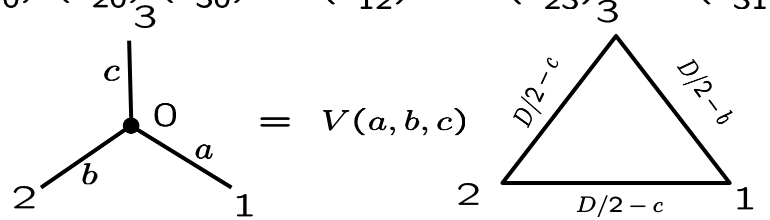
$\sim \frac{1}{\epsilon^L} \hat{t}_L$

- Hence integrability of bi-scalar model is demonstrated explicitly in all orders!

Integrability: Star-Triangle relations and Yang-Baxter

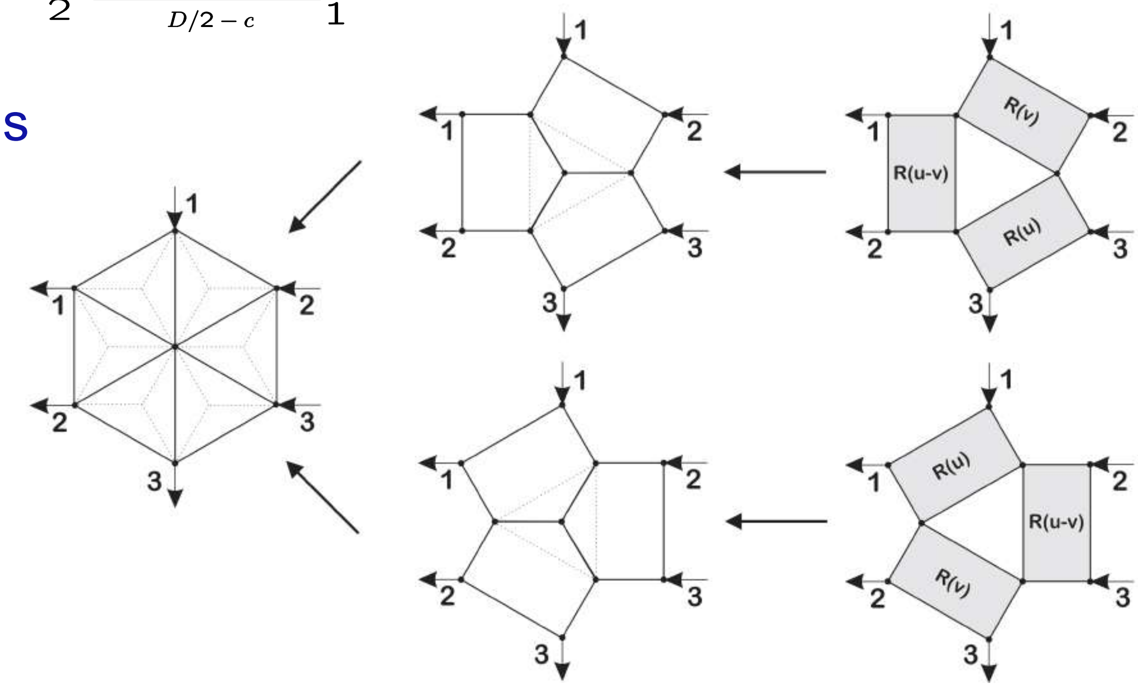
- Star-triangle relations

$$\int \frac{d^D x_0}{(x_{10}^2)^a (x_{20}^2)^b (x_{30}^2)^c} = \frac{V(a,b,c)}{(x_{12}^2)^{D/2-c}(x_{23}^2)^{D/2-a}(x_{31}^2)^{D/2-b}}, \qquad (a+b+c=D\,, \quad x_{ij} := x_i - x_j)$$



$$V(a,b,c) = \pi^{D/2}\frac{\Gamma(\frac{D}{2}-a)\Gamma(\frac{D}{2}-b)\Gamma(\frac{D}{2}-c)}{\Gamma(a)\Gamma(a)\Gamma(c)}$$

- Yang-Baxter relations



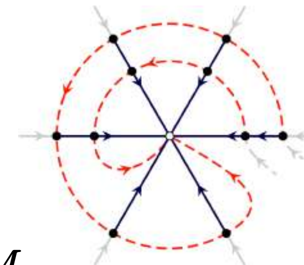
- Integrability allows to compute many complicated physical quantities in bi-scalar CFT

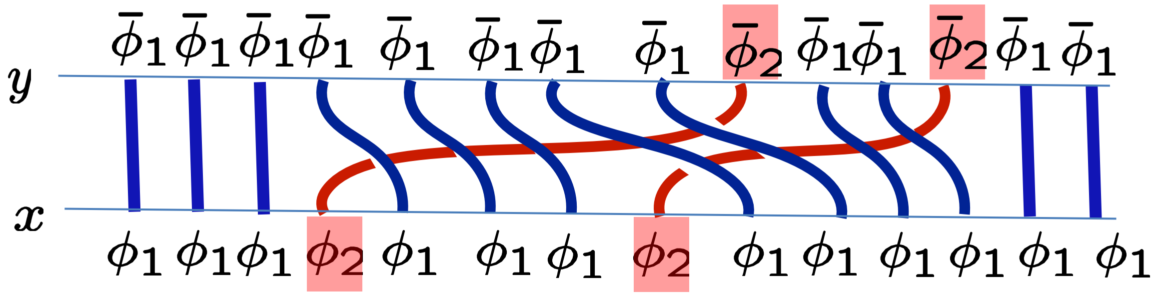
Caetano, Gurdogan, V.K 2016 (D=4)
Basso, Ferrando, V.K., Zhong (any D, to appear)

Dimensions of long operators from $\mathrm{ABA_D}$

Beisert, Eden, Staudacher, 2005

- Asymptotic Bethe ansatz describes magnon scattering in $U(1)^2$ subsector of effective spin chain made of $\phi_1,\ \phi_2$





$$1 = e^{iLp(u_j)} \prod_{k(\neq j)}^{M} S_D(u_j,u_k)$$

- We obtained S-matrix of two-magnon scattering for any D. E.g., for D=4

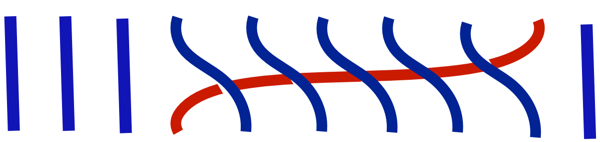
$$S_4(u,v) = -\frac{f(u)}{f(v)}\frac{\Gamma(1+i(u-v))\Gamma(2+i(u-v))}{\Gamma(1-i(u-v))\Gamma(2-i(u-v))}, \qquad f(u)=\frac{\Gamma\left(\frac{1}{2}-iu\right)\Gamma\left(\frac{3}{2}-iu\right)}{\Gamma\left(\frac{1}{2}+iu\right)\Gamma\left(\frac{3}{2}+iu\right)}$$

- Momentum of magnon

$$e^{ip(u)} = \xi^2\frac{\Gamma\left(\frac{D}{8}-iu\right)\Gamma\left(\frac{D}{8}+iu\right)}{\Gamma\left(\frac{3D}{8}-iu\right)\Gamma\left(\frac{3D}{8}+iu\right)} \underset{D=4}{\rightarrow} \frac{\xi^2}{u^2+1/4}$$

- The anomalous dimension $\gamma = \sum_{j=1}^{M}\epsilon(u_j)$ $\qquad \epsilon(u) = 2iu + \frac{D}{4}$

- Single magnon energy γ_1 and momentum 2iLog(ξ) related by dispersion:

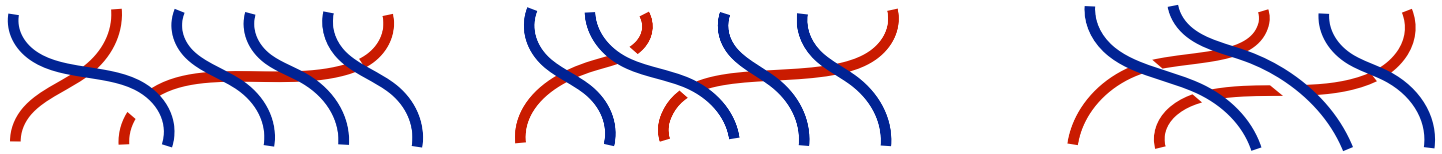


$$\frac{\Gamma\left(\frac{D}{2}-\frac{\gamma_1}{2}\right)\Gamma\left(\frac{D}{4}+\frac{\gamma_1}{2}\right)}{\Gamma\left(\frac{D}{4}-\frac{\gamma_1}{2}\right)\Gamma\left(\frac{\gamma_1}{2}\right)} = \xi^2 \qquad \underset{D=4}{\rightarrow} \quad \gamma_1 = 1-\sqrt{1-4\xi^2}$$

Feynman graphs from ABA

- Helps to compute unwrapped magnon graphs entering the mixing matrix:

Caetano, Gurdogan, V.K 2016



I_a I_b I_c

$$I_b\big|_{1/\epsilon} = -I_a\big|_{1/\epsilon} - \frac{160\zeta(3)}{9} + \frac{53\pi^4}{72} - \frac{187}{5} - \frac{25\pi^2}{12} \qquad I_c\big|_{1/\epsilon} = I_a\big|_{1/\epsilon} + \frac{418\zeta(3)}{45} + \frac{121\pi^4}{360} + \frac{2\pi^2}{9} - \frac{112}{5}$$

- "Minimally connected" Feynman graph $I_a\big|_{1/\epsilon}$ is computed directly

Georgoudis, Goncalves, Pereira

- By "particle-hole" duality, this ABA is equivalent to O(2+D) sigma model with unusual, non-relativistic dual dispersion relation: O(2+D) $\rightarrow$ AdS_{D+1}

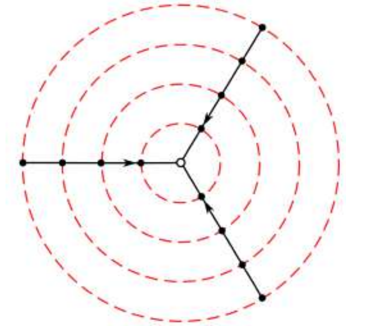
Basso, Zhang '18
Basso, Ferrando, V.K., Zhang, to appear

- TBA, Y-system, quantum spectral curve (QSC) for fishnet model

Gromov, V.K., Korchemsky, Negro, Sizov 2017

Solution for L=3 "vacuum" operator

$\mathrm{tr}(\phi_1)^3$



- Solution of the problem is obtained using Quantum Spectral Curve (QSC) of twisted N=4 SYM

Gromov, V.K. ,Leurent, Volin
V.K. ,Leurent, Volin

- In double scaling limit, QSC gives 2nd order Baxter eq.

$$\left(\frac{(\Delta-1)(\Delta-3)}{4u^2}-\frac{i\,\xi^3}{u^3}-2\right)q(u)+q(u+i)+q(u-i)=0$$

- Two solutions $q_2(u,\xi)$, $q_4(u,\xi)$ satisfy the following quantization condition

$$q_2(0,\xi)\,q_4(0,-\xi)+q_2(0,-\xi)\,q_4(0,\xi)=0$$

- These q-functions should obey "pure" large u asymptotics

$$q_2(u,\xi)\sim u^{\Delta/2-1/2}(1+\frac{\alpha_1}{u}+\frac{\alpha_2}{u^2}+\cdots),\qquad q_4(u,\xi)\sim u^{-\Delta/2+3/2}(1+\frac{\beta_1}{u}+\frac{\beta_2}{u^2}+\cdots)$$

- Solved numerically for anomalous dimensions with virtually arbitrary precision at all relevant couplings

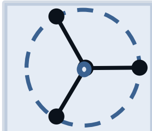
- Perturbative solution in ξ gives the exact periods of "wheel" graphs

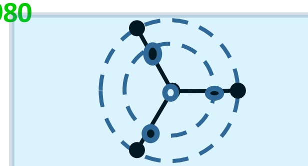
Gromov, V.K , Korchemsky, Negro, Sizov (2017)

Periods of wheel graphs from Quantum Spectral Curve

$\mathrm{tr}(\phi_1)^3$

Broadherst 1980



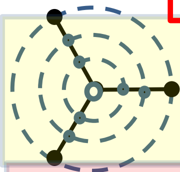


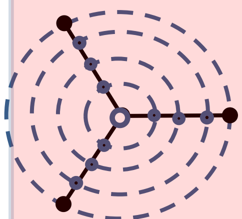
Ahn, Bajnok, Bombardelli, Nepomechie 2013

E.Panzer, 2015

Gurdogan, V.K. '15 (any number of spokes)

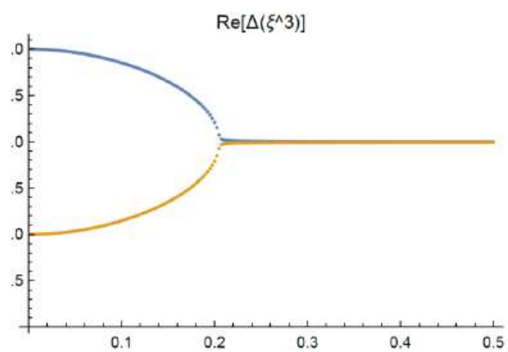
In terms of Riemann (multi)-zeta numbers

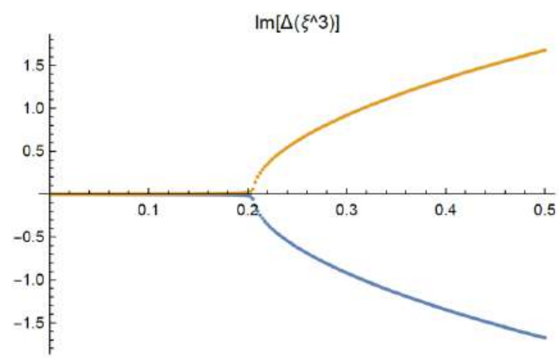




$$\boxed{\Delta - 3 =} -12\xi^6\zeta(3) + \xi^{12}\left(189\zeta(7) - 144\zeta(3)^2\right)$$
$$+\xi^{18}\left(-1944\zeta(8,2,1) - 3024\zeta(3)^3 - 3024\zeta(5)\zeta(3)^2 + \frac{198\pi^8\zeta(3)}{175} + 6804\zeta(7)\zeta(3) + \frac{612\pi^6\zeta(5)}{35} + 270\pi^4\zeta(7) + 5994\pi^2\zeta(9) - \frac{925911\zeta(11)}{8}\right) +$$
$$\xi^{24}\left(\frac{10368}{5}\pi^4\zeta(8,2,1) + 5184\pi^2\zeta(9,3,1) + 51840\pi^2\zeta(10,2,1) - 148716\zeta(11,3,1) - 1061910\zeta(12,2,1) + 62208\zeta(10,2,1,1,1) - 93312\zeta(3)\zeta(8,2,1) - 288\zeta(3)^5 + 72\gamma\pi^2\zeta(3)^4 - 77760\zeta(3)^4 - \frac{80756\pi^6\zeta(3)^3}{945} - 145152\zeta(5)\zeta(3)^3 - \frac{29}{270}\gamma\pi^8\zeta(3)^2 + \frac{9504\pi^8\zeta(3)^2}{175} - 879\pi^4\zeta(5)\zeta(3)^2 - 2025\pi^2\zeta(7)\zeta(3)^2 + 244944\zeta(7)\zeta(3)^2 + 186588\zeta(9)\zeta(3)^2 + \frac{2910394\pi^{12}\zeta(3)}{2627625} - 2592\pi^2\zeta(5)^2\zeta(3) + \frac{29376}{35}\pi^6\zeta(5)\zeta(3) + 12960\pi^4\zeta(7)\zeta(3) + 298404\zeta(5)\zeta(7)\zeta(3) + 287712\pi^2\zeta(9)\zeta(3) - 5555466\zeta(11)\zeta(3) + 57672\zeta(5)^3 - 71442\zeta(7)^2 + \frac{13953\pi^{10}\zeta(5)}{1925} + \frac{7293\pi^8\zeta(7)}{175} - \frac{19959\pi^6\zeta(9)}{5} + \frac{119979\pi^4\zeta(11)}{2} + \frac{10738413\pi^2\zeta(13)}{2} - \frac{4607294013\zeta(15)}{80}\right) + O\left(\xi^{25}\right)$$

- Numerics of high precision: Gromov, V.K , Korchemsky, Negro, Sizov (2017)

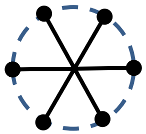


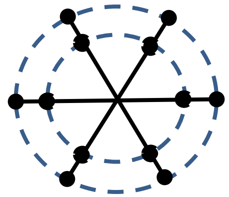


- Generalization to any number of spokes&magnons is in work Grabner, Gromov, V.K , Korchemsky

Wheel graphs at any L: 1 and 2 wrappings

$\mathrm{tr}[\phi_1(x)]^L$





$$\gamma(L) = \gamma^{(1)}(L)\,\zeta_{2L-3}\,\xi^{2L} + \gamma^{(2)}(L)\,\xi^{4L} + \mathcal{O}(\xi^{6L}),$$

$$\gamma^{(1)}_{\mathrm{vac}}(L) = -2\binom{2L-2}{L-1}\zeta_{2L-3}$$

Broadherst 1980

Ahn, Bajnok, Bombardelli, Nepomechie 2011

$$\gamma^{(2)}(L) = \frac{2^3}{\Gamma^2(L)}\Bigg\{-\sum_{j_1=0}^{L-1}\frac{(2L-2)!}{L-j_1}\binom{L+j_1-1}{j_1}\zeta_{2L-3}\zeta_{\substack{L+j_1-2,\\ L-j_1}} + \frac{\Gamma^2(L)}{8}\binom{4L-2}{2L-1}\zeta_{4L-5}$$

Gurdogan, V.K. 2015

$$+\sum_{\substack{j_1,j_2>0\\ j_1+j_2<2L-3}}\frac{\Gamma(2L-j_1-j_2-1)\Gamma(L+j_1)\Gamma(L+j_2)}{\Gamma(j_1+1)\Gamma(j_2+1)\Gamma(L-j_1)\Gamma(L-j_2)}\Bigg[\sum_{k=1}^{L+j_1-2}\Bigg(\binom{L+k+j_2-4}{k-1}\Big(2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-3,\\ 2L-j_1-j_2-1}} + \zeta_{\substack{L+j_1-k-1,\\ 3L-j_1+k-4}}\Big)$$

$$-\binom{L+k+j_2-2}{k}\Big(2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-2,\\ 2L-j_1-j_2-2}} - 2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-1,\\ 2L-j_1-j_2-3}}\Big)\Bigg) + 2\zeta_{\substack{2L+j_1+j_2-3,\\ 2L-j_1-j_2-2}} + 2\zeta_{\substack{2L+j_2-1,\\ L+j_1-1,\\ 2L-j_1-j_2-3}}\ \zeta_{\substack{L+j_1-2,\\ 3L-j_1-3}} - \zeta_{\substack{2L+j_1+j_2-2,\\ 2L-j_1-j_2-3}}\Bigg]$$

$$+\frac{\Gamma^2(2L-1)}{2\Gamma^2(L)}\Bigg[\sum_{k=1}^{2L-4}\Bigg(\binom{2L+k-4}{k}\Big(3\zeta_{\substack{2L-k-3,\\ 2L+k-2}} + 2\zeta_{\substack{2L-k-3,\\ 2L+k-4,\\ 2}}\Big) - 2\Bigg(\binom{2L+k-4}{k-1} + \binom{2L+k-4}{k+1}\Bigg)\zeta_{\substack{2L-k-3,\\ 2L+k-3,\\ 1}}\Bigg)$$

$$+2\zeta_{\substack{2L-3,\\ 2L-4,\\ 2}} - 4(L-3)\zeta_{\substack{2L-3,\\ 2L-3,\\ 1}} + \zeta_{\substack{2L-4,\\ 2L-1}} - 4\zeta_{\substack{2L-2,\\ 2L-3}} + 4\zeta_{\substack{4L-6,\\ 1}} + 2\zeta_{\substack{3L-4,\\ 2L-2,\\ 1}} + \frac{2L+1}{L}\zeta_{2L-3}\zeta_{2L-2} - 5\zeta_{4L-5} - \zeta^2_{2L-3}\Bigg]\Bigg\}$$

Grabner, Gromov, V.K., Korchemsky 2017
V.K., Olivucci 2018

4-point correlator and exact OPE data

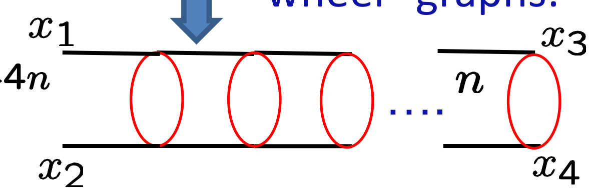
cross-ratios
$u = x_{12}^2 x_{34}^2/(x_{13}^2 x_{24}^2)$
$v = x_{14}^2 x_{23}^2/(x_{13}^2 x_{24}^2)$

- Exact all-loop 4-point correlator

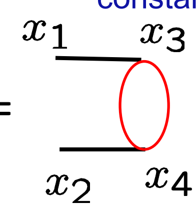
$$\langle \mathrm{tr}[\phi_1(x_1)\phi_1(x_2)]\ \mathrm{tr}[\phi_1^\dagger(x_3)\phi_1^\dagger(x_4)]\rangle = \frac{1}{(x_{12}^2 x_{34}^2)^{\frac{D}{2}}} \sum_{\Delta} \sum_{S/2\in\mathbb{Z}_+} C_{\Delta,S}\ u^{(\Delta-S)/2} g_{\Delta,S}(u,v)$$

structure constant — conformal block

"wheel" graphs:

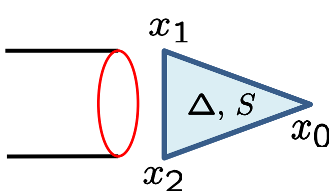
$$\sum_{n=1}^{\infty} \xi^{4n}$$


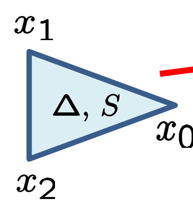
$$= \frac{\xi^4 \mathcal{H}}{1-\xi^4\mathcal{H}}$$

where $\mathcal{H} =$


$$= \frac{1}{(x_{13}^2)^{\frac{D}{4}} (x_{24}^2)^{\frac{D}{4}} (x_{34}^2)^{\frac{D}{2}}}$$

- Solution by Bethe-Salpeter method: we diagonalize the graph-builing kernel



$= h_{\Delta,S}^{-1}$


$$\langle\, \mathrm{tr}[\phi_1(x_1)\phi_1(x_2)]\ O_{\Delta,S,n}(x_0)\,\rangle$$

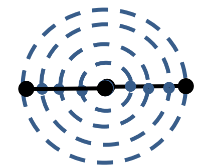
- Dimensions of "exchange" operators $\quad O_{\Delta,S,n}(x_0) = \mathrm{tr}[\phi_1 \partial_+^{\mathrm{S}} \phi_1 (\phi_2^\dagger \phi_2)^{\mathrm{n}}]$

$$1 - \xi^4/h_{\Delta,S} = 0$$

$$h_{\Delta,S} = \frac{\Gamma\left(\frac{3D}{4} - \frac{\Delta-S}{2}\right)\Gamma\left(\frac{D}{4} + \frac{\Delta+S}{2}\right)}{\Gamma\left(\frac{D}{4} - \frac{\Delta-S}{2}\right)\Gamma\left(-\frac{D}{4} + \frac{\Delta+S}{2}\right)} = \xi^4$$

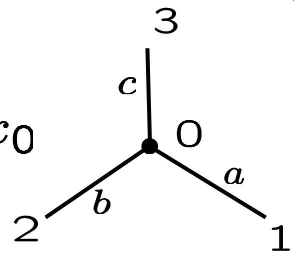
Grabner, Gromov, V.K., Korchemsky 2017
V.K., Olivucci 2018

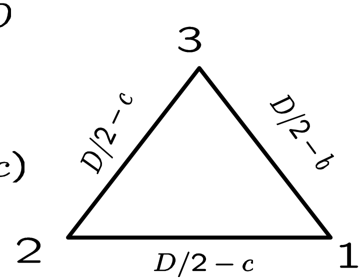
Example: anomalous dimension at L=2, Spin=0

- Length=2 wheel operator: $\mathrm{tr}(\phi_1)^2$ ⇒ 

- Use star-triangle relation:

$$a+b+c=D$$

$$\int d^D x_0$$ 

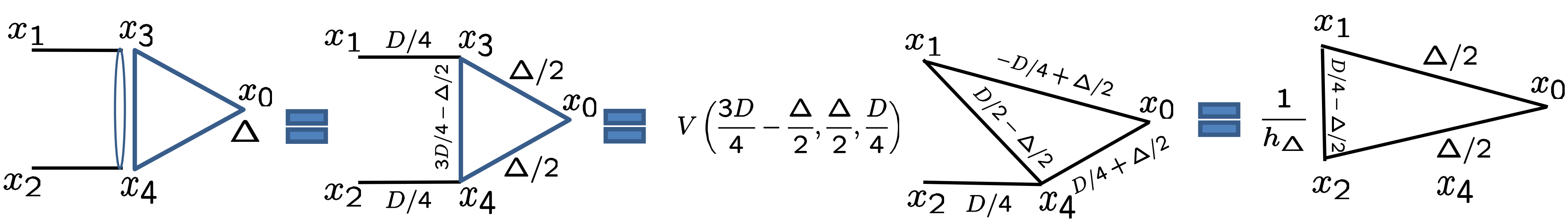
$$= V(a,b,c)$$ 

$$V(a,b,c) = \pi^{D/2} \frac{\Gamma(\frac{D}{2}-a)\Gamma(\frac{D}{2}-b)\Gamma(\frac{D}{2}-c)}{\Gamma(a)\Gamma(b)\Gamma(c)}$$

$$i \overset{a}{\text{————}} j = \frac{1}{(x_i - x_j)^{2a}}$$

- We apply the graph generating Hamiltonian to conformal eigenfunction at S=0

$$\langle \Delta \,|\, x_{10}, x_{20} \rangle = \frac{1}{\left(x_{12}^2\right)^{\frac{D}{4}-\frac{\Delta}{2}} \left(x_{10}^2 x_{20}^2\right)^{\frac{\Delta}{2}}}$$



- Genralization: Dimension for any dimension and spin S:

$$h_{\Delta,S} = \frac{\Gamma\left(\frac{3D}{4}-\frac{\Delta-S}{2}\right)}{\Gamma\left(\frac{D}{4}-\frac{\Delta-S}{2}\right)} \frac{\Gamma\left(\frac{D}{4}+\frac{\Delta+S}{2}\right)}{\Gamma\left(-\frac{D}{4}+\frac{\Delta+S}{2}\right)} = \xi^4$$

Olivucci, V.K. 2018

Gromov, V.K., Korchemsky 2018

Generalizations of 4-point correlators

0-magnon correlator

$$\Delta^{(D=4)} = 2 \pm \sqrt{2+S(2+S) \pm 2\sqrt{(1+S)^2+16\xi^4}}$$

$$C_{\Delta,S} = \frac{(-1)^{-S}}{(2\pi)^3}\frac{\Gamma(S+2)\Gamma\left(\frac{1}{2}(S+\Delta-1)\right)^2\Gamma(S-\Delta+4)}{\Gamma(S+1)\Gamma\left(\frac{1}{2}(S-\Delta+5)\right)^2\Gamma(S+\Delta-1)}$$

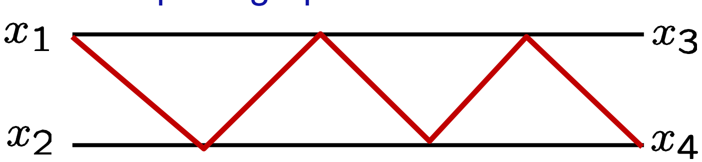
$S=0$

$$\gamma_{\mathrm{tr}(\phi_1^2)} = \sqrt{2}\sqrt{1-\sqrt{1+4\xi^4}} = 2i\xi^2 - i\xi^6 + \ldots$$

1-magnon correlator

$$\langle \mathrm{tr}(X(x_1)Z(x_1)X(x_2))\mathrm{tr}(X^\dagger(x_3)X^\dagger(x_4)Z^\dagger(x_4))\rangle$$

spiral" graphs



Exchange operators: $\mathrm{tr}(XZ\partial_+^S X) + \ldots$

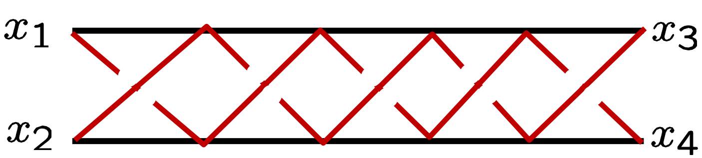
$$\Delta_{1m} = 2 + \sqrt{(S+1)^2 - 4(-1)^S\xi^2}$$

$$C_{\Delta,S}^{(1m)} = (S+1)\frac{\Gamma^2\left(\frac{1}{2}(S+\Delta-1)\right)\Gamma(S-\Delta+4)}{\Gamma^2\left(\frac{1}{2}(S-\Delta+5)\right)\Gamma(S+\Delta-1)}$$

2-magnon correlator

$$\langle \mathrm{tr}(XZ)(x_1)\ \mathrm{tr}(XZ^\dagger)(x_2)\ \mathrm{tr}(X^\dagger Z)(x_3)\ \mathrm{tr}(X^\dagger Z^\dagger)(x_4)\rangle$$

double-spiral graphs



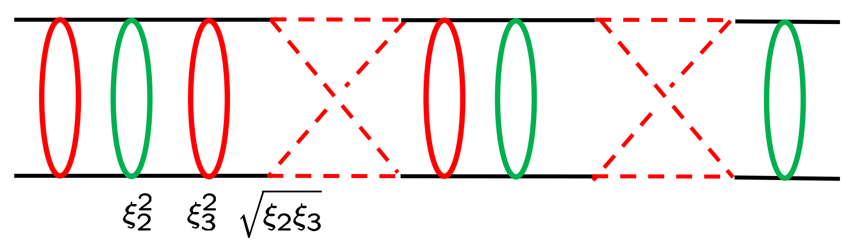
Exchange operators (Konishi-like) $\mathrm{tr}(XZ\partial_+^S XZ) + \ldots$

Dimensions:

$$\xi^{-4} = \frac{\psi'\left(\frac{1}{4}(S-\Delta+4)\right) - \psi'\left(\frac{1}{4}(S-\Delta+6)\right) - \psi'\left(\frac{1}{4}(S+\Delta)\right) + \psi'\left(\frac{1}{4}(S+\Delta+2)\right)}{(4\pi)^4(\Delta-2)(S+1)}$$

- **Full chiral "dynamical fishnet" CFT also computed**

V.K., Olivucci, Preti 2018



Derkachev, V.K., Olivucci 2018

Basso-Dixon-type 4-point correlator in 2D

$$\frac{1}{|x|^{2\gamma}}$$

- Using Sklianin's separation of variables we found explicit $N \times N$ determinant representation for this $L \times N$ fishnet graph with anisotropy parameter γ

$$\frac{1}{|x|^{2-2\gamma}}$$

$$B^{(\gamma)}_{L,N}(\eta,\bar\eta) = \underset{1\le j,k\le N}{\mathrm{Det}} \left[(\eta\partial_\eta)^{i-1}(\bar\eta\partial_{\bar\eta})^{k-1} I^{(\gamma)}_{N+L}(\eta,\bar\eta)\right]$$

conformal ratio

$$\eta = \frac{z_0 - w_1}{w_1 - w_0}\frac{z_1 - w_0}{z_0 - z_1}$$

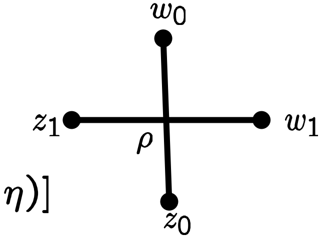
where

hypergeometric function

$$I^{(\gamma)}_M(\eta,\bar\eta) = \partial_\varepsilon^{M-1}\Big|_{\varepsilon=0}\left[\frac{\Gamma^M(1-\gamma-\varepsilon)}{\Gamma^M(\gamma+\varepsilon)}\frac{\Gamma^M(1+\varepsilon)}{\Gamma^M(1-\varepsilon)}[\eta]^{-\varepsilon-\frac{\gamma-1}{2}}\left|{}_{M+1}F_M\left(\begin{matrix}1-\gamma-\varepsilon & \cdots & 1-\gamma-\varepsilon & 1\\ 1-\varepsilon & \cdots & 1-\varepsilon & \end{matrix}\middle|\,\eta\right)\right|^2\right]$$

is the "ladder" integral.

- The homogeneous fishnet BS formula is recovered at $\gamma = \frac{1}{2}$
- Example: 2D cross in terms of elliptic functions



$$\int \frac{d^2\rho}{|z_0-\rho||w_0-\rho||z_1-\rho||w_1-\rho|} = \frac{4\,|1-\eta|}{|w_1-z_1||w_0-z_0|}\left[K(\eta)K(1-\bar\eta) + K(\bar\eta)K(1-\eta)\right]$$

elliptic modulus: $K(x) = \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-x\,t^2)}}.$

- Elliptic polylogarithms for higher BD-integrals? Beilinson, Levin 94'
- Might be interesting applications to BFKL limit $\gamma \to 1$

Conclusions and Prospects

- Fishnet CFT a unique opportunity to study non-supersymmetric quantum conformal world at any coupling. A window into origins of AdS/CFT integrability

- Wheel and spiral graphs of any size and systematic solution for spectra of non-compact integrable spin chains

  Grabner, Gromov, V.K., Korchemsky (in work)

- More complicated 3- and 4-point functions at any D,

  possibly by Sklyanin SoV for O(2,D) spin chain

  A-la Harmark, Wilhelm 17'

- Finite T, chaos, Hagedorn transition, "black hole" regimes for fishnet CFT

- Spontaneous symmetry breaking in "fishnet" CFT

  Karananas, V.K., Shaposhnikov (in work)

- AdS/CFishT: fish-chain, SO(1,1+D) sigma model

  Gromov, Sever '19

  Basso, Zhong '18

  Basso, Ferrando V.K., Zhong (in work)

wheel (from Wits Origins Centre Museum)