The Holographic Fishchain

By Nikolay Gromov

based on 1903.10508 with A. Sever

Aimes

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* First principle derivation of a holographic dual of a large N CFT in 4D (weak/strong dual)

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* Get proper playground for holography, in particular without SUSY
* Develop integrability approach to non-perturbative correlation functions

Slide with citations

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* `t Hooft: string world sheet from diagrams 1974
* Fishnet pioneers: H.B.Nielsen P.Olesen 1970
* Integrability in fishnets: A.Zamolodchikov 1980
* B.Thorn - studied fishnets intensively

Fishnet CFT

Gurdogan, Kazakov 2015

$$\mathcal{L}_{4d} = N\,\mathrm{tr}\left(|\partial\phi_1|^2 + |\partial\phi_2|^2 + (4\pi)^2\xi^2\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\right)\ ,$$

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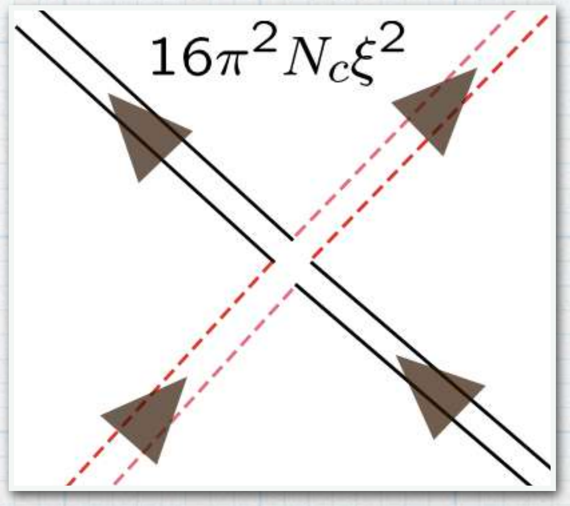

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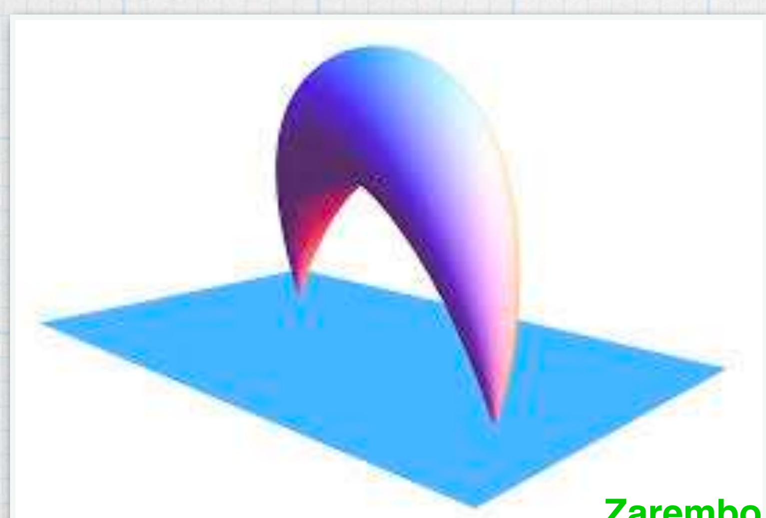

$$\Delta \sim \sqrt{\lambda}$$

Energy

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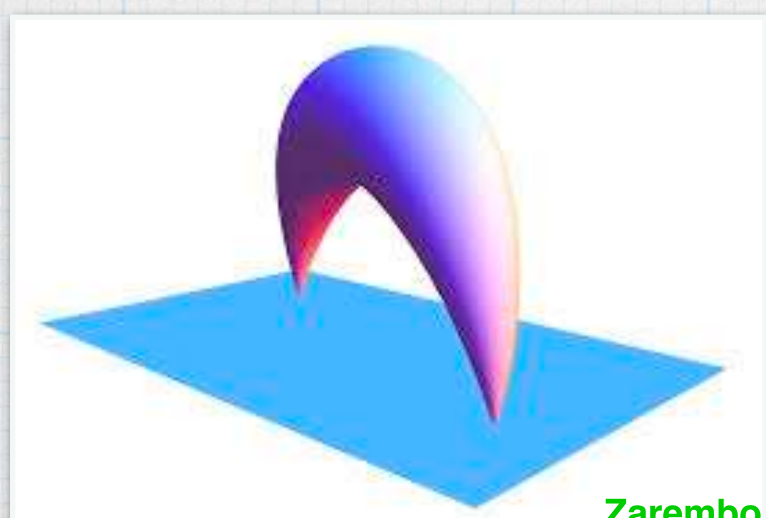

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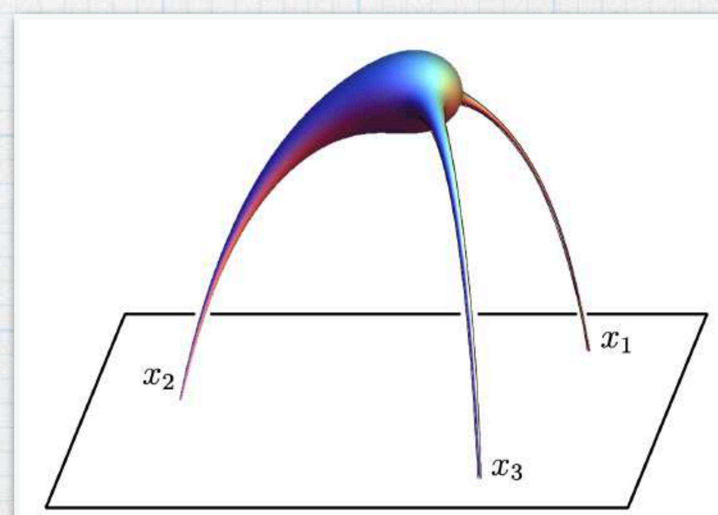

$$\mathcal{G} \sim e^{-\sqrt{\lambda}A}$$

Correlators

Relation to Fishnet CFT

Twisted N=4 interaction:

$$\lambda\ \mathrm{tr}|\phi_1\phi_2 - \phi_2\phi_1|^2 \to \lambda\ \mathrm{tr}|e^{+i\theta}\phi_1\phi_2 - e^{-i\theta}\phi_2\phi_1|^2$$

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Light-cone?

Problem formulation

Given

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Derive weak/strong holographic dual

$$\mathcal{L}_{\mathrm{dual}} = \xi \ldots$$

Evidence of existence

Why do we believe at strong coupling the theory is described by a dual weakly coupled theory?

see: Kazakov, Korchemsky, Sizov, Negro, Grabner, N.G.

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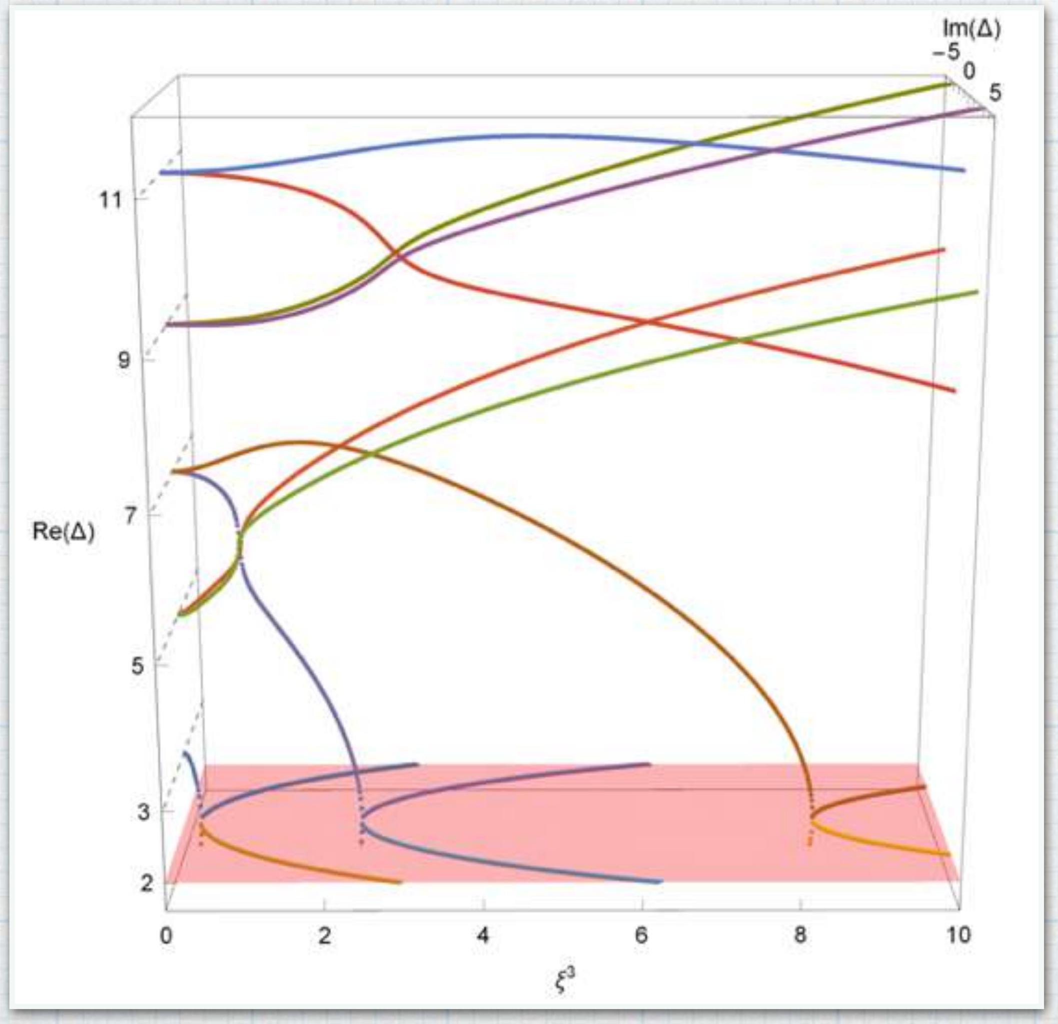
Dimensions scale as

$$\Delta \propto \xi$$



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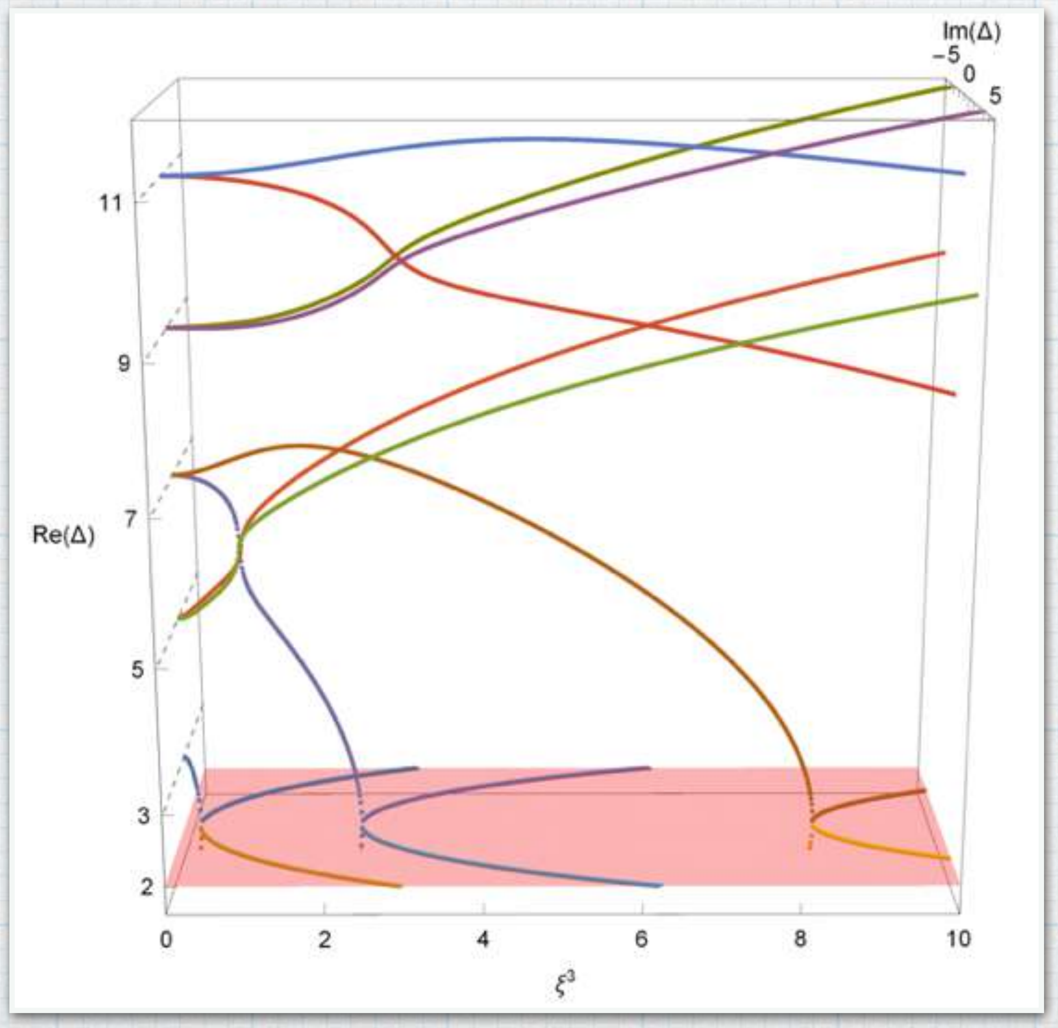
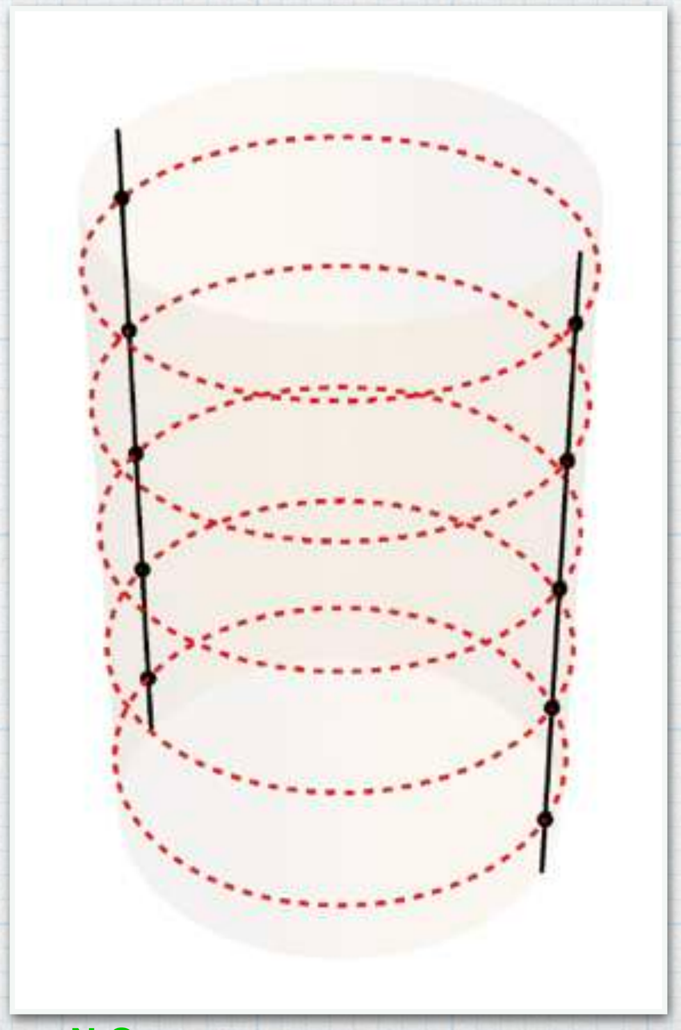


Correlation functions scale as

$$\exp(-\xi A)$$



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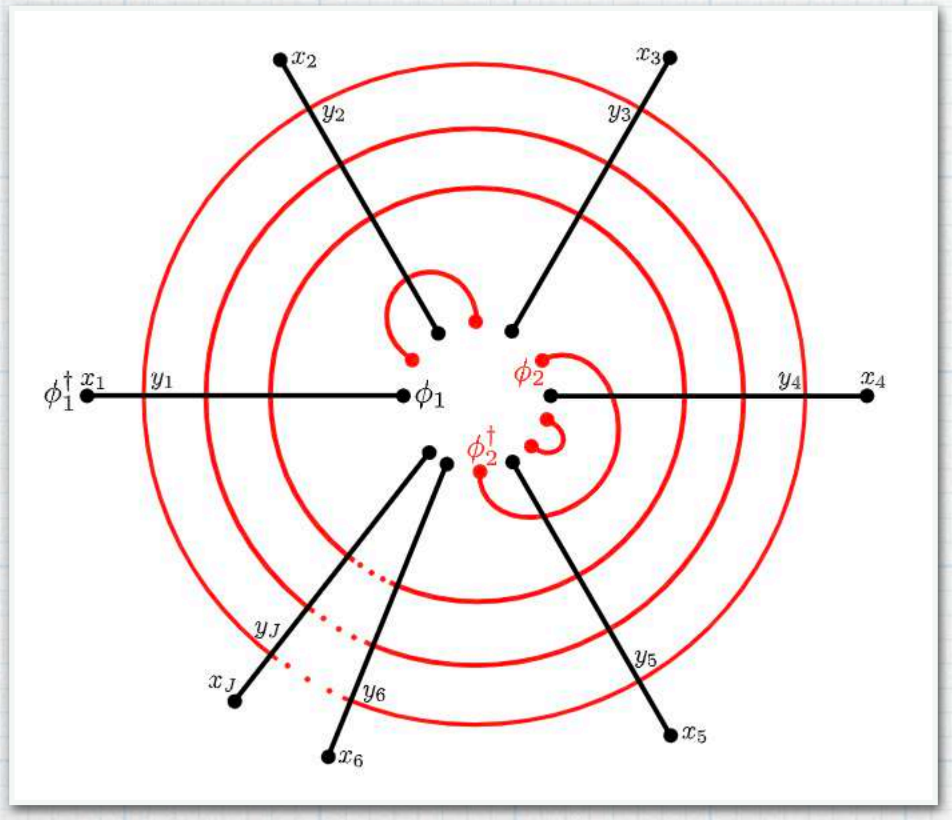
Graph-building operator

$\mathcal{O} = \phi_1^J(\phi_2\phi_2^\dagger)^n$



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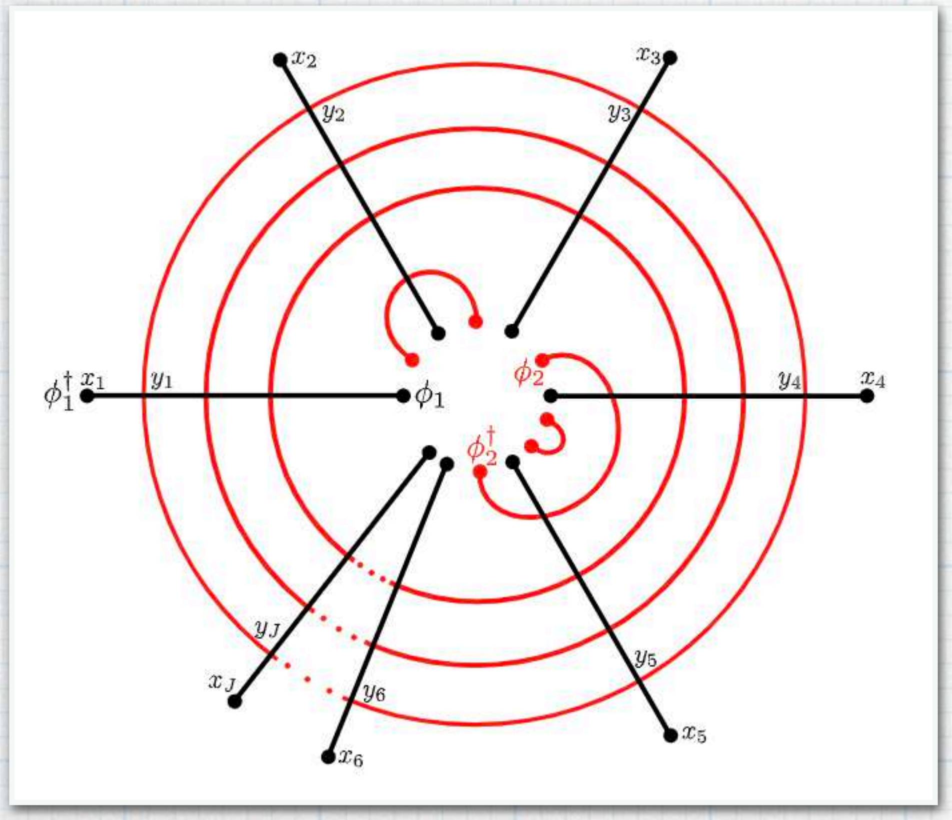

To add an extra layer:

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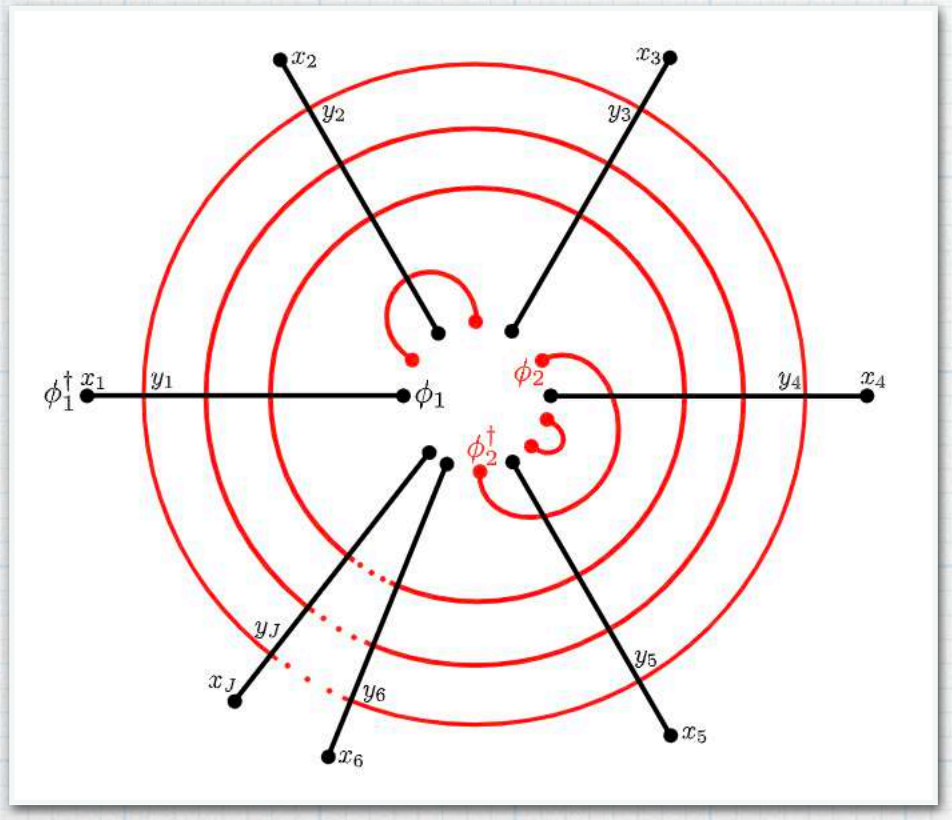

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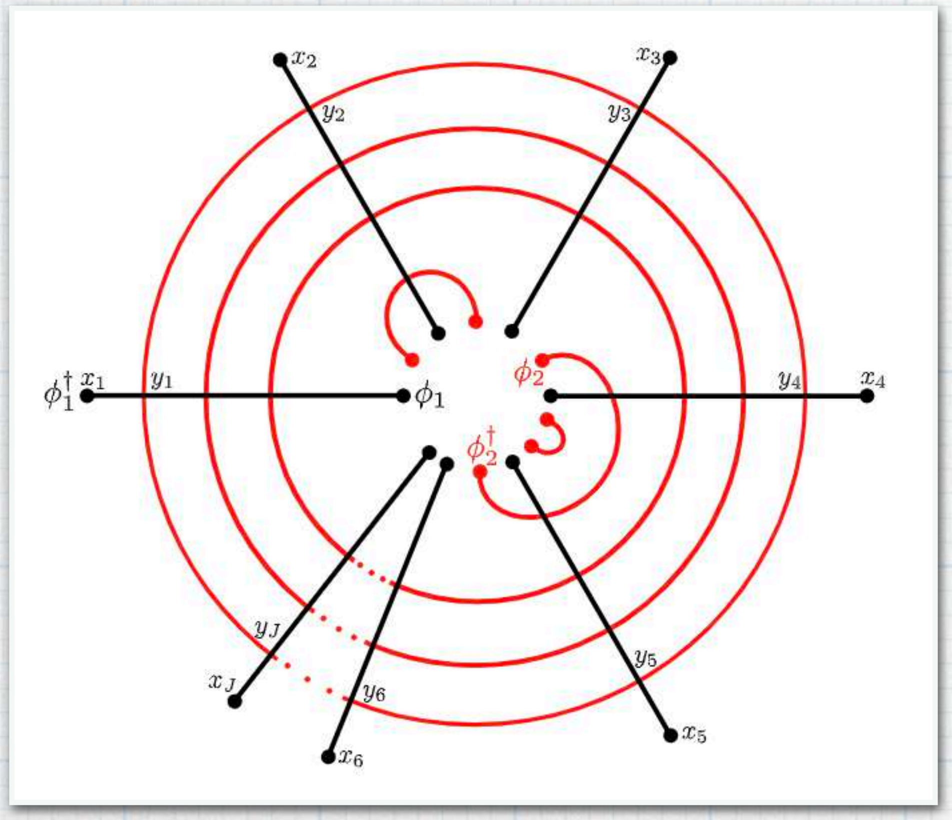

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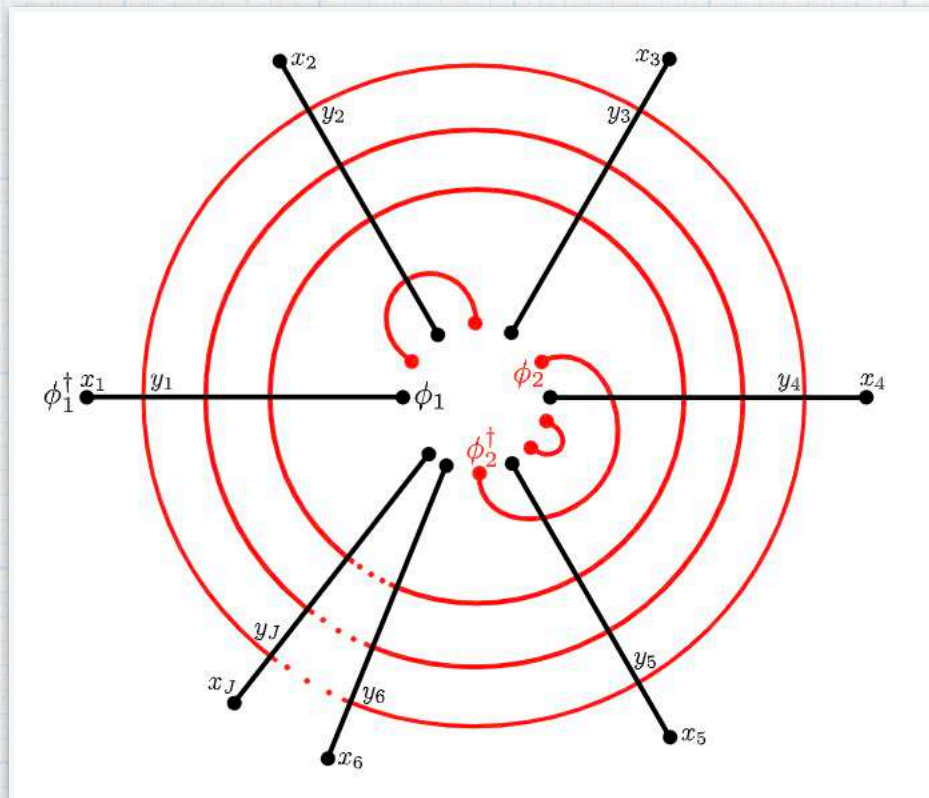

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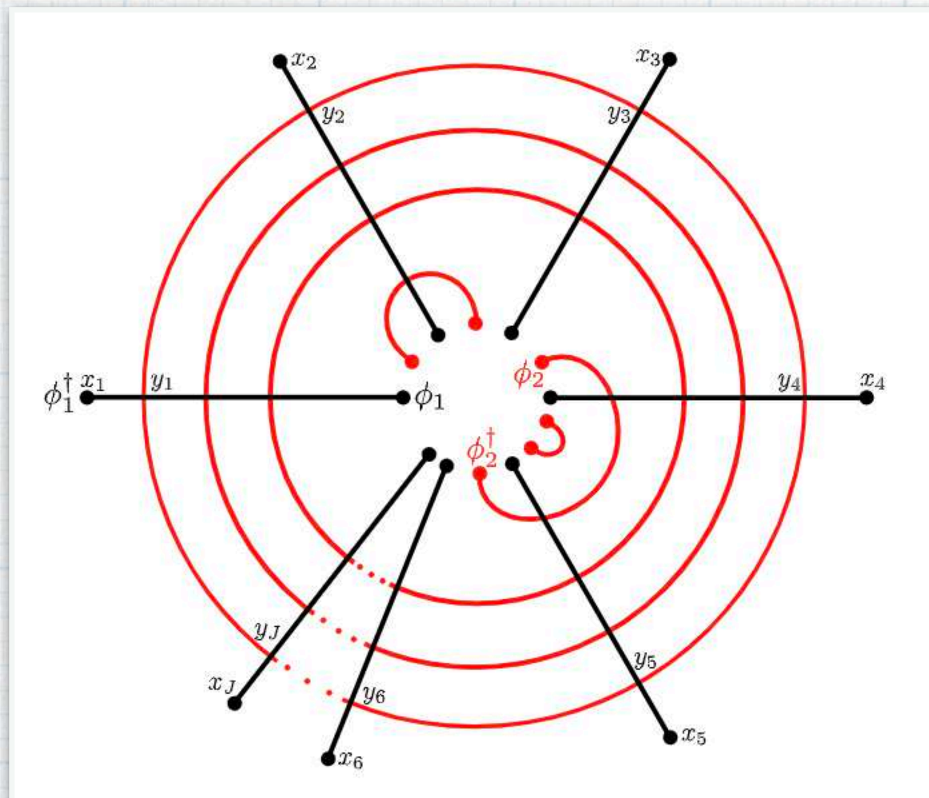

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Diagonalize B?

J=2 example

Conformal symmetry alone fixes the eigenvectors of B

$$\Psi_{\Delta,S}(x_1,x_2) = \frac{1}{x_1^{\Delta-S} x_2^{\Delta-S} x_{12}^{S-\Delta+2}} \left(\text{tensor}\right)^S$$

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Note that Δ is an arbitrary parameter so far

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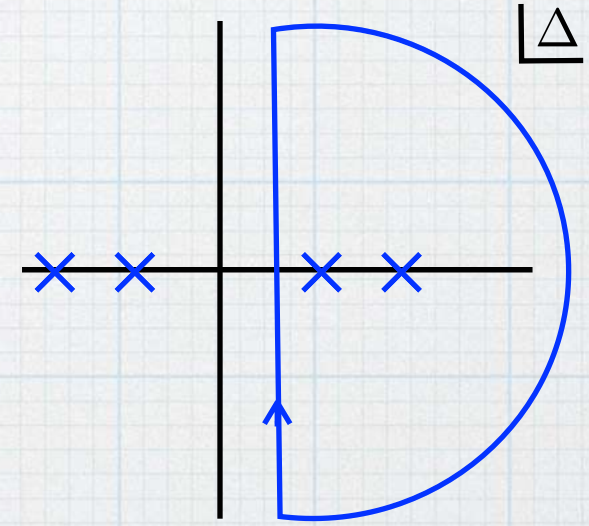

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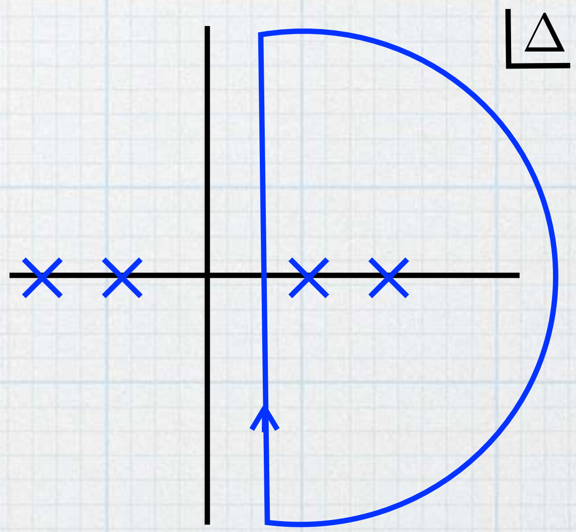

Residue gives the structure function

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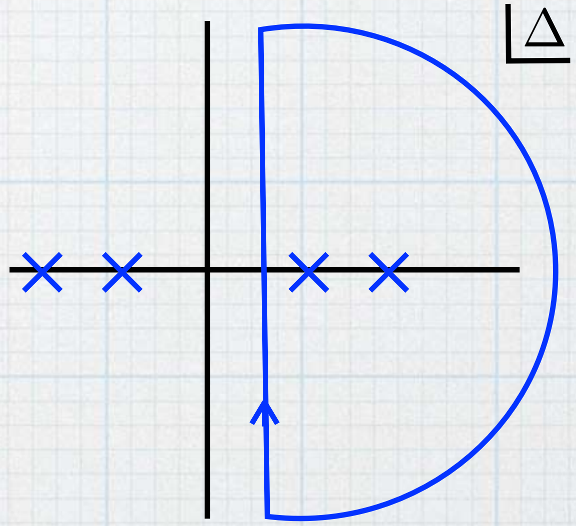

Compute integration by poles. Poles at

$E_{\Delta,S}(\xi)=1$ Gives spectrum $\Delta(S,\xi)$

Residue gives the structure function

J=2 we get:

$$\mathcal{G}(x_1,x_2|y_1,y_2)=\sum_{S,\Delta}C_{\Delta,S}\,g_{\Delta,S}(u,v)$$

J=2 example

N.G., Kazakov, Korchemsky

$\langle \mathrm{tr}(\phi_1\phi_2)\mathrm{tr}(\phi_1\phi_2^\dagger)\mathrm{tr}(\phi_1^\dagger\phi_2)\mathrm{tr}(\phi_1^\dagger\phi_2^\dagger)\rangle$

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Exact structure constant:

$$C_{\Delta,S} = \frac{(S+1)\Gamma(S-\Delta+4)\Gamma\left(\frac{1}{2}(S+\Delta-2)\right)\Gamma\left(\frac{1}{2}(S+\Delta)\right)}{((4-\Delta)\Delta + S(S+2)-2)\,\Gamma^2\left(\frac{1}{2}(S-\Delta+4)\right)\Gamma(S+\Delta-2)}$$

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At weak coupling $\xi \to 0$:

$$\Delta_2(S) = 2 + S - \frac{2\xi^4}{S(S+1)} + \frac{2\xi^8((S-1)S-1)}{S^3(S+1)^3} + \mathcal{O}\left(\xi^{12}\right),$$

$$\Delta_4(S) = 4 + S + \frac{2\xi^4}{(S+1)(S+2)} - \frac{2\xi^8(S(S+5)+5)}{(S+1)^3(S+2)^3} + \mathcal{O}\left(\xi^{12}\right)$$

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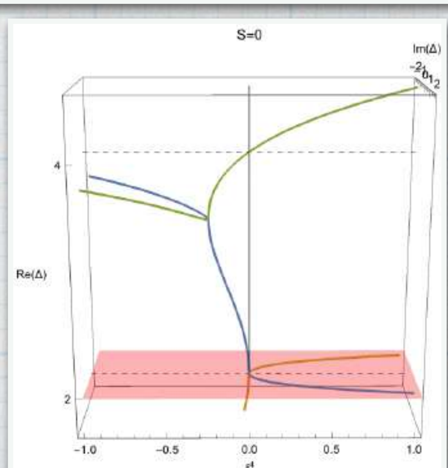

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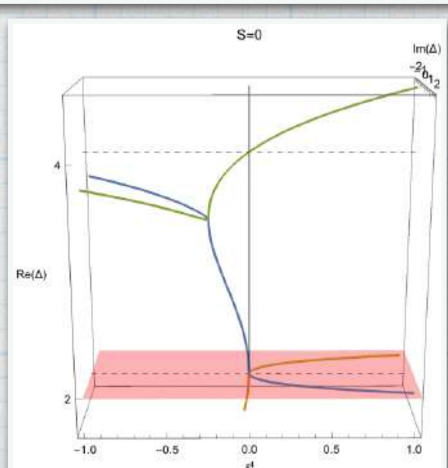

$$\mathcal{G}_0^{(0)}=z-\bar z$$

$$\mathcal{G}_0^{(1)}=H_{1,0}-\bar H_{1,0}+H_1\bar H_0-H_0\bar H_1+\bar H_{0,1}-H_{0,1}$$

$$\begin{aligned}\mathcal{G}_0^{(2)}=&\,H_1\bar H_{0,0}-\bar H_1 H_{0,0}+\bar H_0 H_{1,0}-H_0\bar H_{1,0}+\bar H_{1,0}-H_{1,0}-\bar H_{1,0,0}+H_{1,0,0}\\&-H_1\bar H_0+H_0\bar H_1-\bar H_{0,1}+\bar H_{0,0,1}+H_{0,1}-H_{0,0,1}\,,\end{aligned}$$

Harmonic Poly-Logarithms

J=2 example

N.G., Kazakov, Korchemsky

$\langle \mathrm{tr}(\phi_1\phi_2)\mathrm{tr}(\phi_1\phi_2^\dagger)\mathrm{tr}(\phi_1^\dagger\phi_2)\mathrm{tr}(\phi_1^\dagger\phi_2^\dagger)\rangle$

$$\mathcal{G}(x_1,x_2|y_1,y_2) = \sum_{S,\Delta} C_{\Delta,S}\, g_{\Delta,S}(u,v)$$

Exact spectrum:

$$\Delta_2(S) = 2 + \sqrt{(S+1)^2 + 1 - 2\sqrt{(S+1)^2 + 4\xi^4}}$$
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$$C_{\Delta,S} = \frac{(S+1)\Gamma(S-\Delta+4)\Gamma\left(\frac{1}{2}(S+\Delta-2)\right)\Gamma\left(\frac{1}{2}(S+\Delta)\right)}{((4-\Delta)\Delta + S(S+2) - 2)\Gamma^2\left(\frac{1}{2}(S-\Delta+4)\right)\Gamma(S+\Delta-2)}$$

At weak coupling $\xi \to 0$:

$$\Delta_2(S) = 2 + S - \frac{2\xi^4}{S(S+1)} + \frac{2\xi^8((S-1)S-1)}{S^3(S+1)^3} + \mathcal{O}\left(\xi^{12}\right),$$
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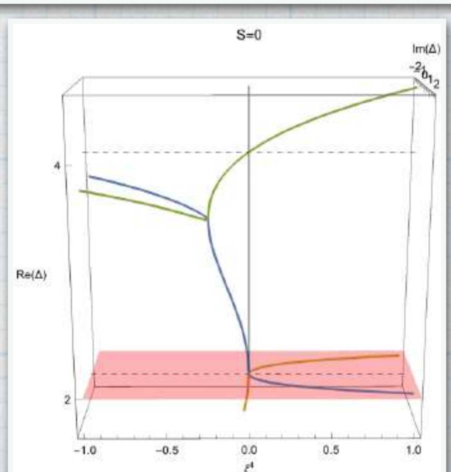

$$\begin{aligned}
\mathcal{G}_0^{(1)} &= -(\mathcal{L}_{01} - \mathcal{L}_{10}) \\
\mathcal{G}_0^{(2)} &= (\mathcal{L}_{01} - \mathcal{L}_{10}) - \mathcal{L}_{001} + \mathcal{L}_{100} \\
\mathcal{G}_0^{(3)} &= \frac{3}{2}(\mathcal{L}_{01} - \mathcal{L}_{10}) - \mathcal{L}_{0001} + \mathcal{L}_{0010} - \mathcal{L}_{0100} - \mathcal{L}_{0101} + \mathcal{L}_{1000} + \mathcal{L}_{1010} + 4\zeta_3\mathcal{L}_1 \\
\mathcal{G}_0^{(4)} &= (3 - 4\zeta_3)(\mathcal{L}_{01} - \mathcal{L}_{10}) - \mathcal{L}_{00001} + \mathcal{L}_{00010} - \mathcal{L}_{01000} - \mathcal{L}_{01001} + \mathcal{L}_{10000} + \mathcal{L}_{10010} \\
\mathcal{G}_0^{(5)} &= \frac{49}{8}(\mathcal{L}_{01} - \mathcal{L}_{10}) + \frac{1}{2}(\mathcal{L}_{0001} - \mathcal{L}_{0010} + \mathcal{L}_{0100} + \mathcal{L}_{0101} - \mathcal{L}_{1000} - \mathcal{L}_{1010}) - 2(\zeta_3 - 6\zeta_5)\mathcal{L}_1 \\
&\quad + 4\zeta_3(\mathcal{L}_{001} - \mathcal{L}_{010} + \mathcal{L}_{100} + \mathcal{L}_{101}) - \mathcal{L}_{000001} + \mathcal{L}_{000010} - \mathcal{L}_{000100} - \mathcal{L}_{000101} + \mathcal{L}_{001000} \\
&\quad + \mathcal{L}_{001010} - \mathcal{L}_{010000} - \mathcal{L}_{010001} - \mathcal{L}_{010100} - \mathcal{L}_{010101} + \mathcal{L}_{100000} + \mathcal{L}_{100010} + \mathcal{L}_{101000} + \mathcal{L}_{101010}
\end{aligned}$$

Single-valued Harmonic Poly-Logarithms

J=2 example

$$\mathcal{G}(x_1, x_2|y_1, y_2) = \sum_{S,\Delta} C_{\Delta,S}\, g_{\Delta,S}(u,v)$$

Exact spectrum:

$$\Delta_2(S) = 2 + \sqrt{(S+1)^2 + 1 - 2\sqrt{(S+1)^2 + 4\xi^4}}$$

$$\Delta_4(S) = 2 + \sqrt{(S+1)^2 + 1 + 2\sqrt{(S+1)^2 + 4\xi^4}}$$

Exact structure constant:

$$C_{\Delta,S} = \frac{(S+1)\Gamma(S-\Delta+4)\Gamma\left(\frac{1}{2}(S+\Delta-2)\right)\Gamma\left(\frac{1}{2}(S+\Delta)\right)}{((4-\Delta)\Delta + S(S+2) - 2)\,\Gamma^2\left(\frac{1}{2}(S-\Delta+4)\right)\Gamma(S+\Delta-2)}$$

At strong coupling $\xi \to \infty$:

$$\Delta = \sqrt{S^2 \mp 4\xi^2}$$

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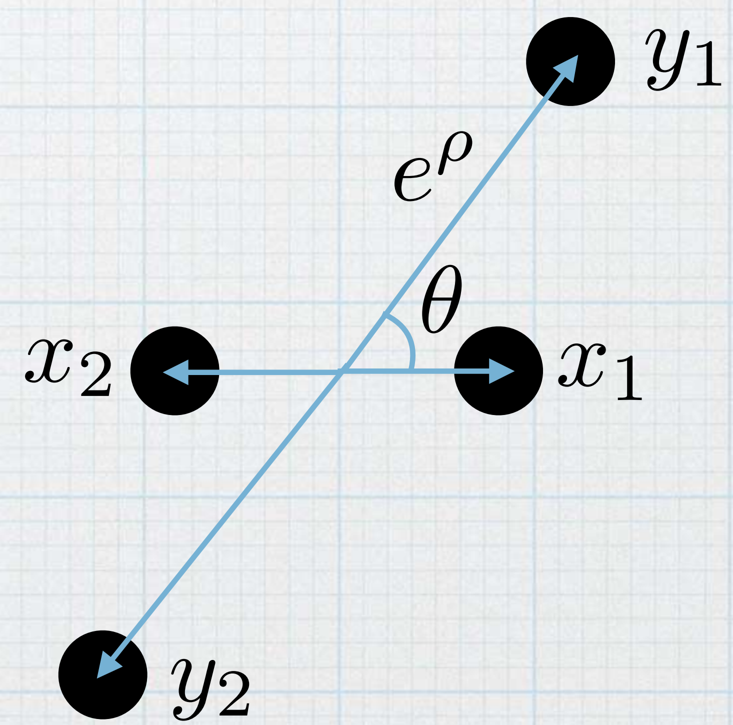
In COM kinematics



Resembles classical dual string @ $\xi \to \infty$

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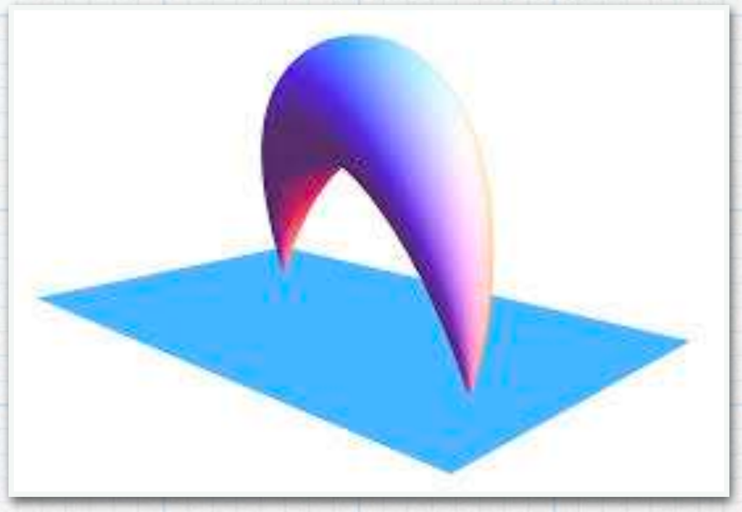
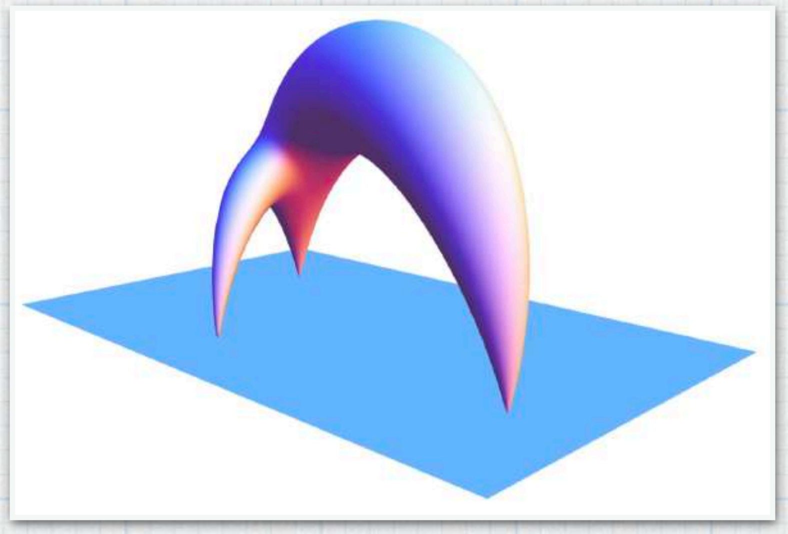
Energy $\Delta \sim \xi$



Correlation functions $\langle \mathcal{O}_1 \mathcal{O}_2 \dots \rangle \sim e^{-\xi A}$



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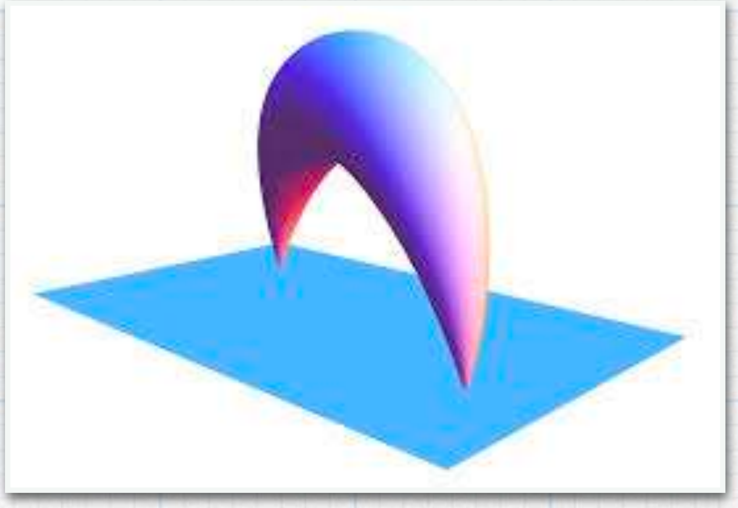

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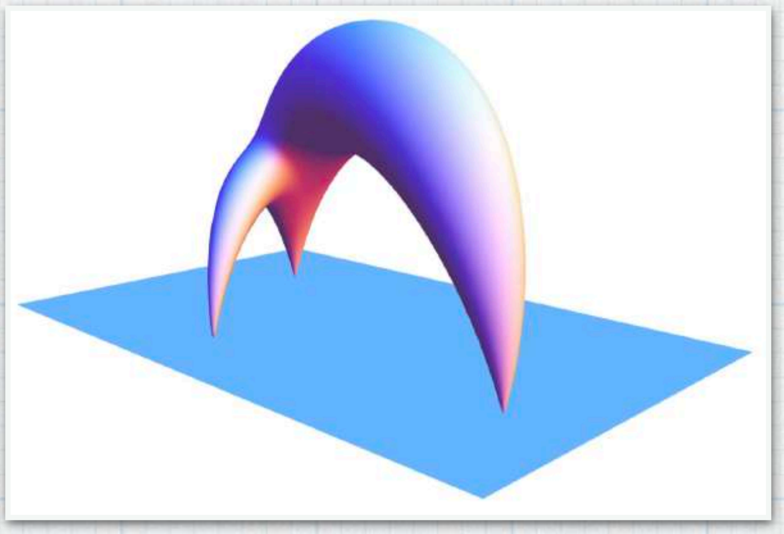

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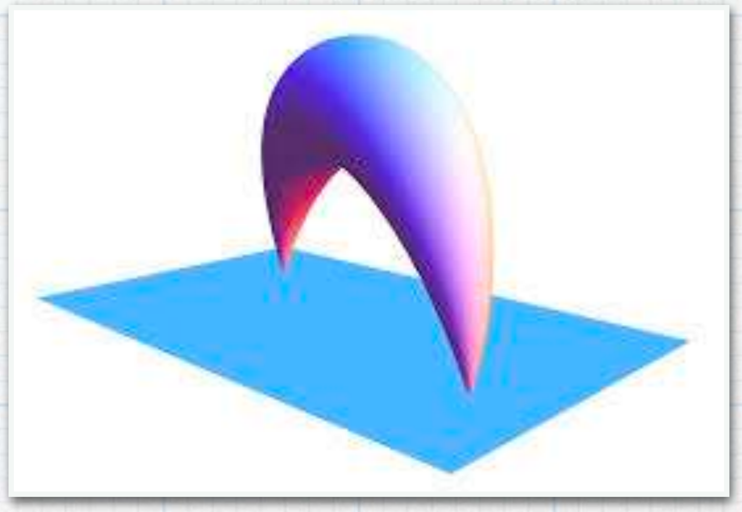

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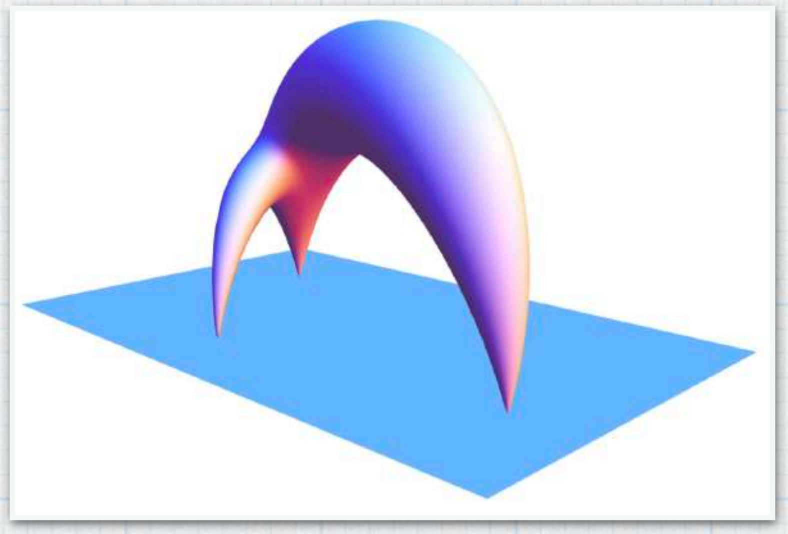

Derivation of the dual

Wave functions, corresponding to the dilatation operator spectrum are at the residues

$$(\hat{B}-1)\Psi = 0 \qquad B(\{y\},\{x\}) = \prod_{i=1}^{J} \frac{\xi^2/\pi^2}{(y_i - y_{i+1})^2 (x_i - y_i)^2}$$

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Bring to a nicer form by inverting x-y propagator

$$H \circ \Psi(\{x_i\}) = 0\ , \quad H = \prod_{i=1}^{J} \vec{p}_i^{\,2} - \prod_{i=1}^{J} \frac{4\xi^2}{(\vec{x}_i - \vec{x}_{i+1})^2}\ , \quad p_i = -i\partial_{x^i}$$

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$$L = \frac{2J-1}{2^{\frac{2J}{2J-1}}} \left(\frac{1}{\gamma} \prod_{i=1}^{J} \dot{\vec{x}}_i^{\,2} \right)^{\frac{1}{2J-1}} + \gamma \prod_{i=1}^{J} \frac{4\xi^2}{(\vec{x}_i - \vec{x}_{i+1})^2}$$

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Symmetries:

Derivation of the dual

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* Time reparameterization

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$\hbar = 1/\xi$ classical at strong coupling!

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$\hbar = 1/\xi$ classical at strong coupling!

Symmetries:

- Time reparameterization
- Rotations, Translations

Derivation of the dual

$$L = \frac{2J-1}{2^{\frac{2J}{2J-1}}} \left(\frac{1}{\gamma} \prod_{i=1}^{J} \dot{\vec{x}}_i^2 \right)^{\frac{1}{2J-1}} + \gamma \prod_{i=1}^{J} \frac{4\xi^2}{(\vec{x}_i - \vec{x}_{i+1})^2}$$

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$\hbar = 1/\xi$ classical at strong coupling!

Symmetries:

* Time reparameterization
* Rotations, Translations
* Special conformal

Derivation of the dual

$$L = \frac{2J-1}{2^{\frac{2J}{2J-1}}} \left(\frac{1}{\gamma} \prod_{i=1}^{J} \dot{\vec{x}}_i^2 \right)^{\frac{1}{2J-1}} + \gamma \prod_{i=1}^{J} \frac{4\xi^2}{(\vec{x}_i - \vec{x}_{i+1})^2}$$

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Next:

try to make symmetries more manifest...

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$$L = -\sum_i\left(\frac{\alpha_i\dot{X}_i^2}{2} + \prod_k\left(-\alpha_k X_k . X_{k+1}\right)^{-\frac{1}{J}}\right) + \eta_i X_i^2 \ ,$$

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Finally get the dual action:

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Reproduced correct spectrum!

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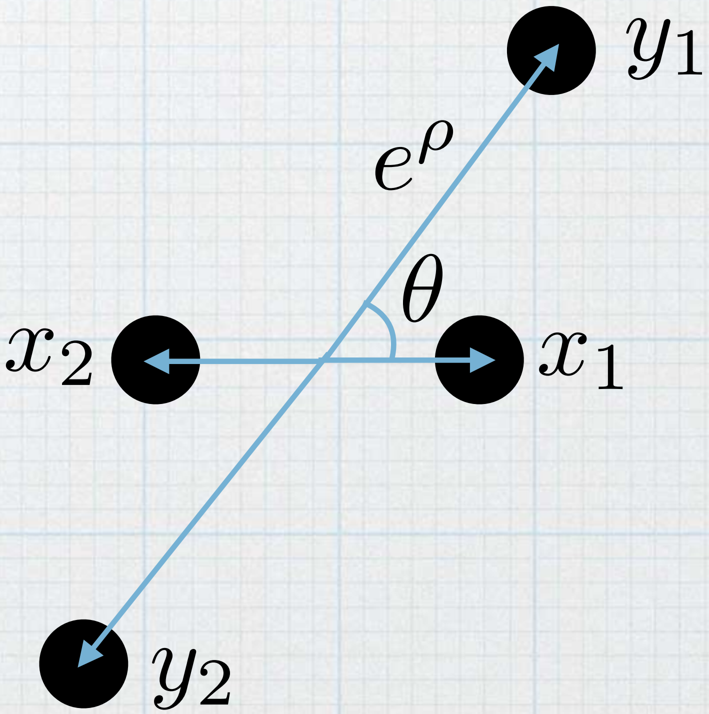

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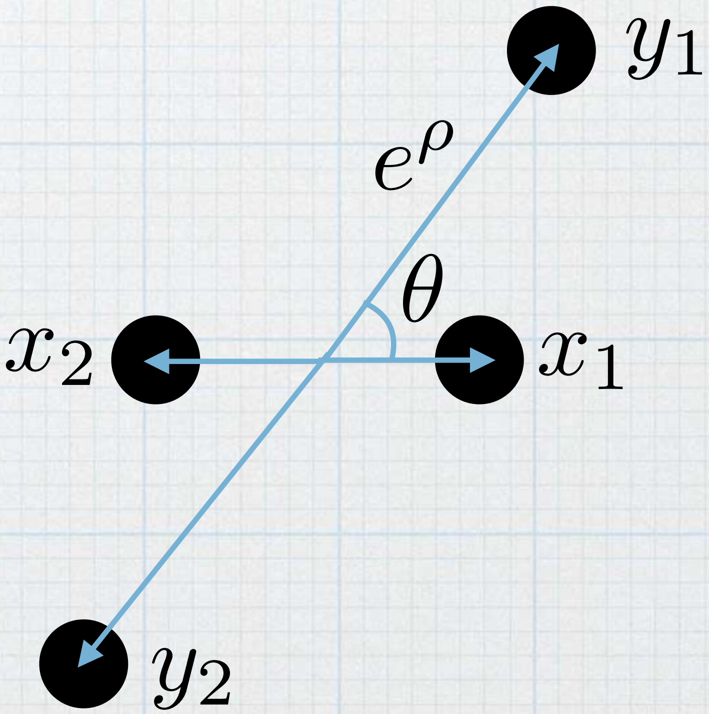

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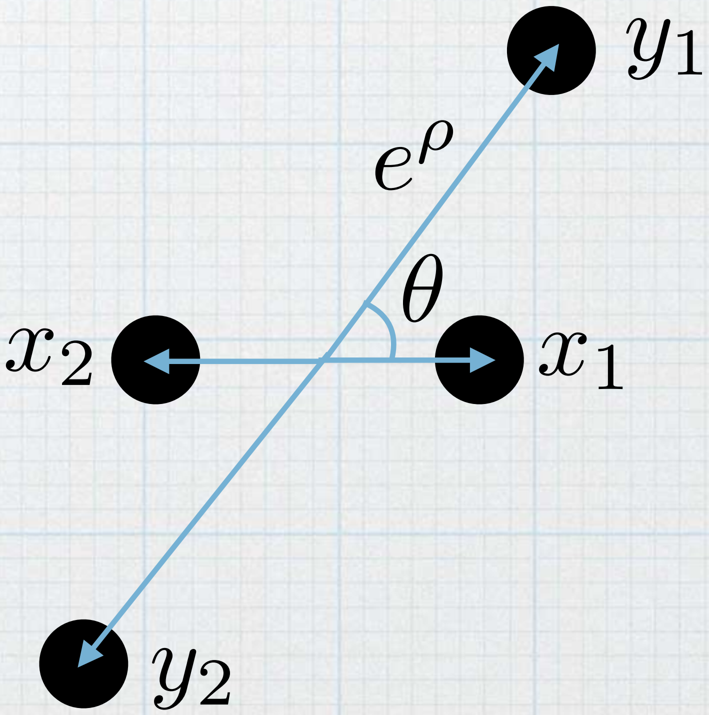

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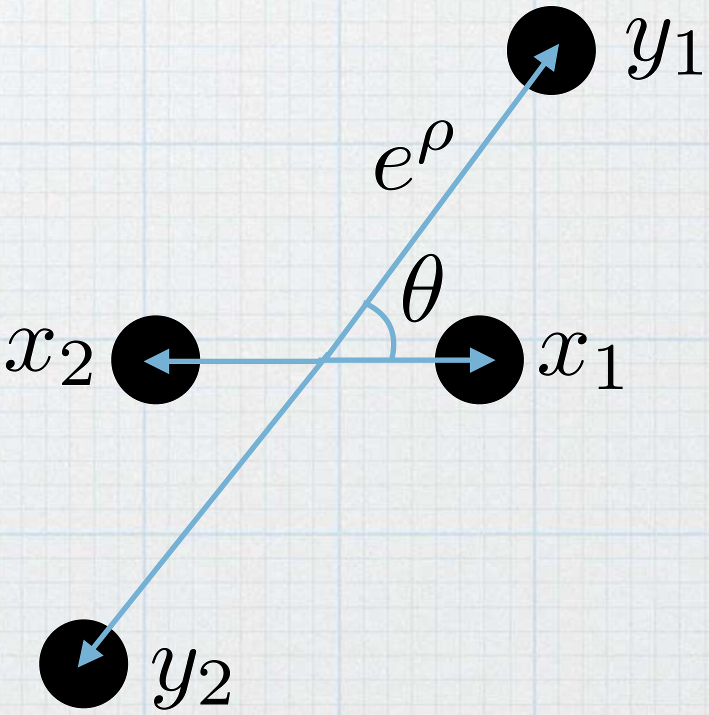
Reproduced correctly 4-point correlator!



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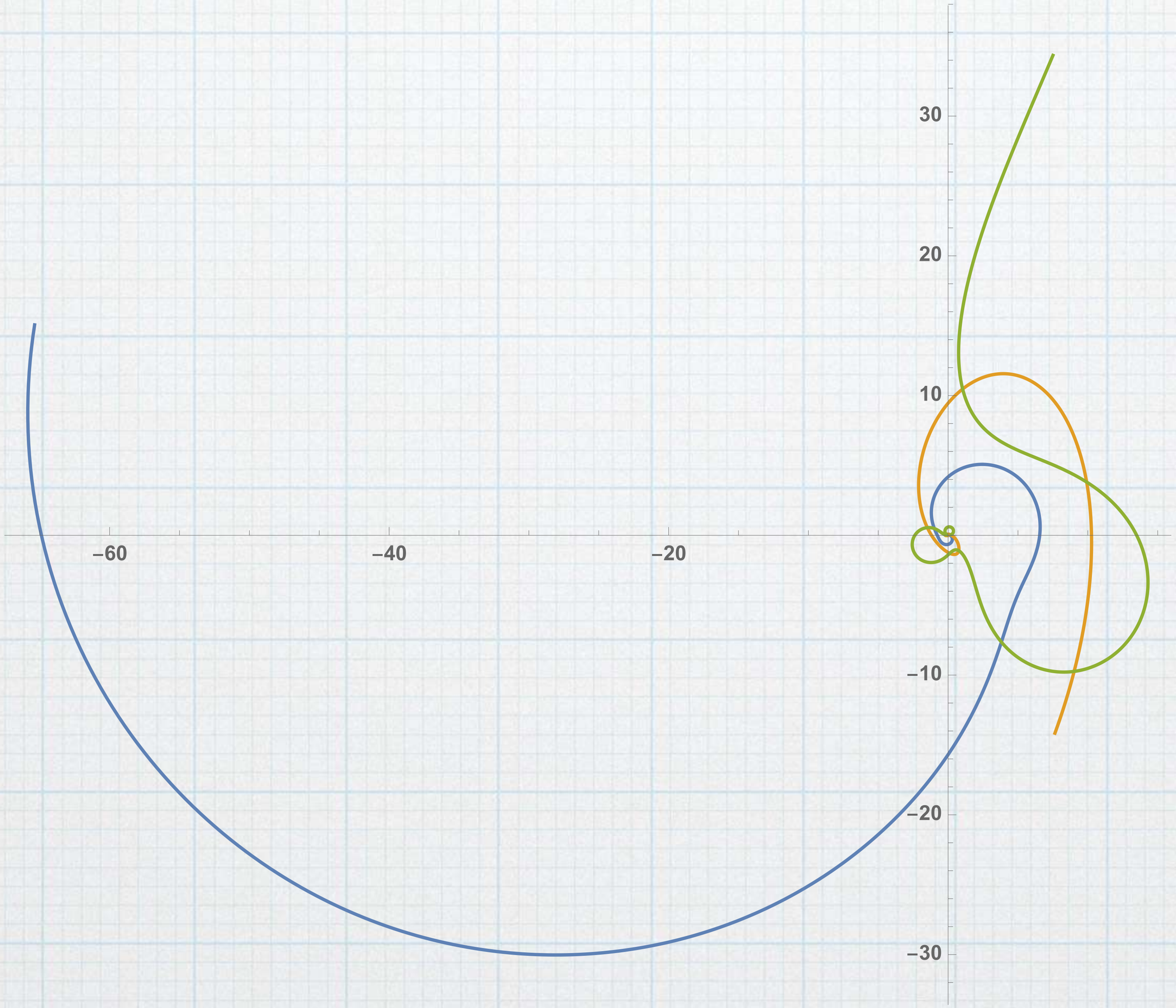

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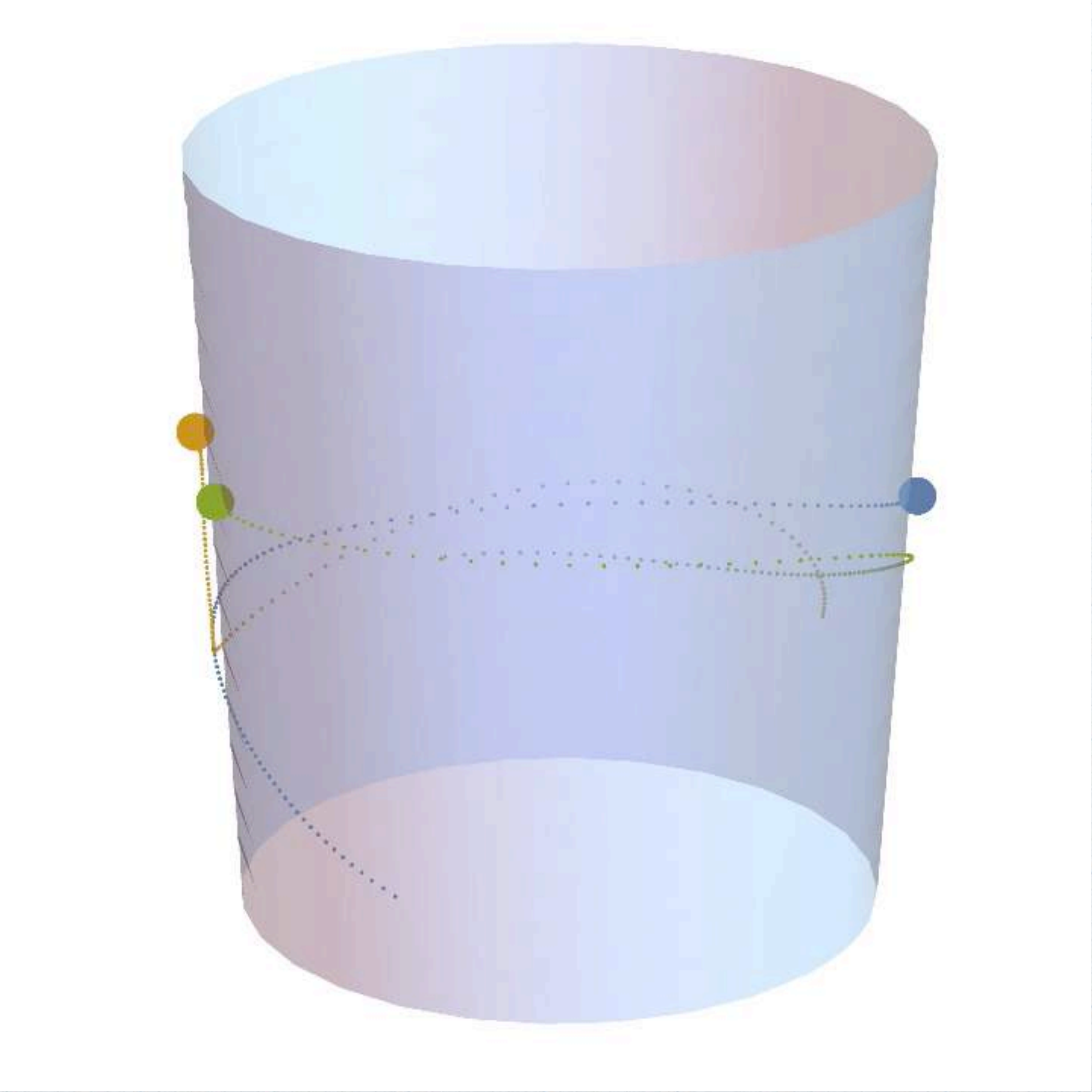

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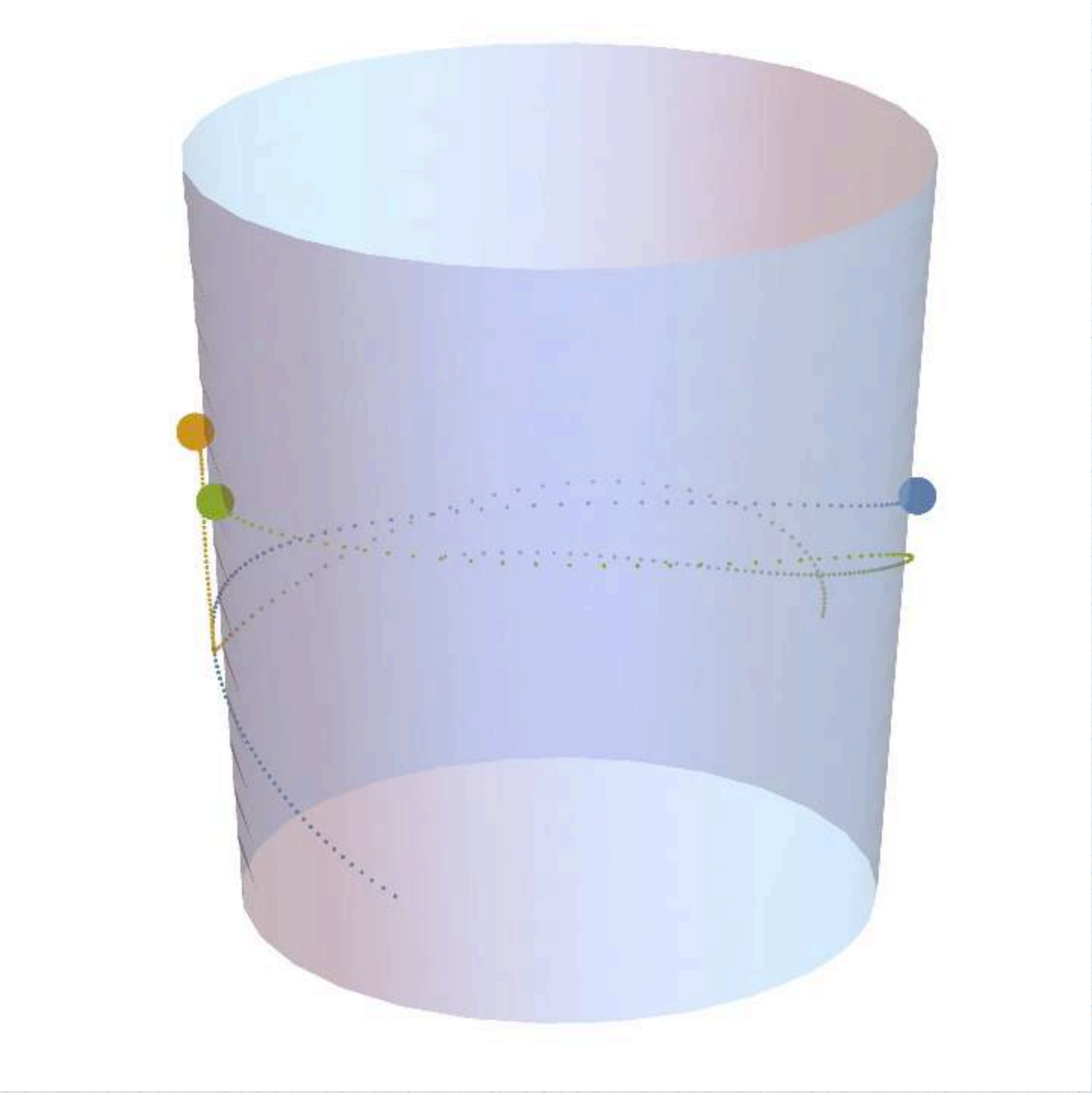

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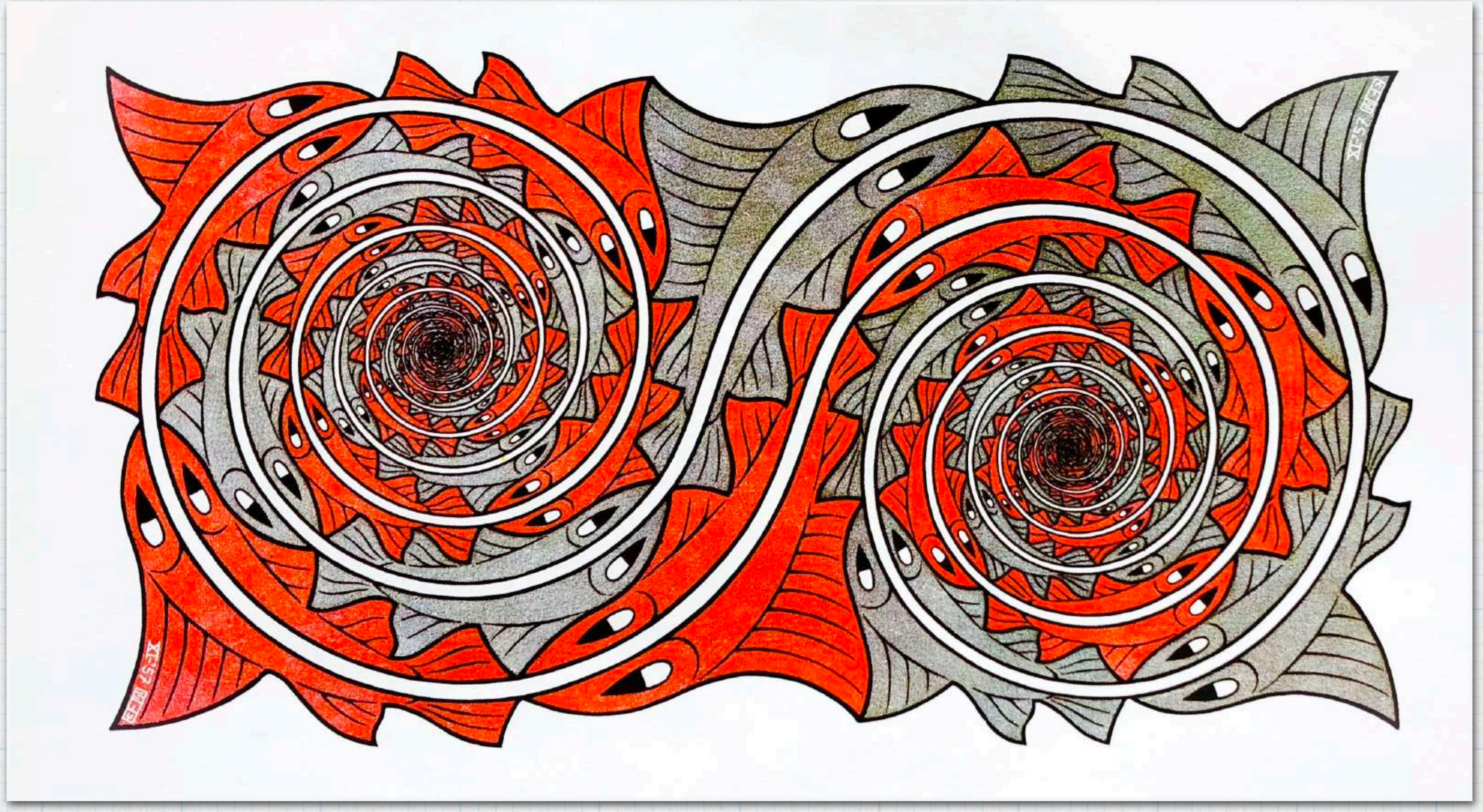
$$q_i^{NM} = \dot{X}_i^N X_i^M - \dot{X}_i^M X_i^N\ , \quad \dot{q}_i = \frac{1}{2}(j_{i+1} - j_i)\ , \quad j_i^{MN} = \frac{X_{i-1}^M X_i^N - X_{i-1}^N X_i^M}{X_{i-1}.X_i}$$

Future directions

* Test at the quantum level, "world-sheet" loops (Amit)
* 1/N corrections? Seems doable
* SOV program - should be very simple
* Match with Ben's "sigma-model" (Amit)
* Derive dual of 3D and 6D fishnets
* Dual of SUSY fishnets
* Incorporate corrections away from the fishnet - should start seeing AdS emerging classically
* Fishchain for SYK?
* Hints about N=4: integrability, AdS/CFT
* ...



thank you!