Infinite Statistics and Integrability on the Confining String Worldsheet

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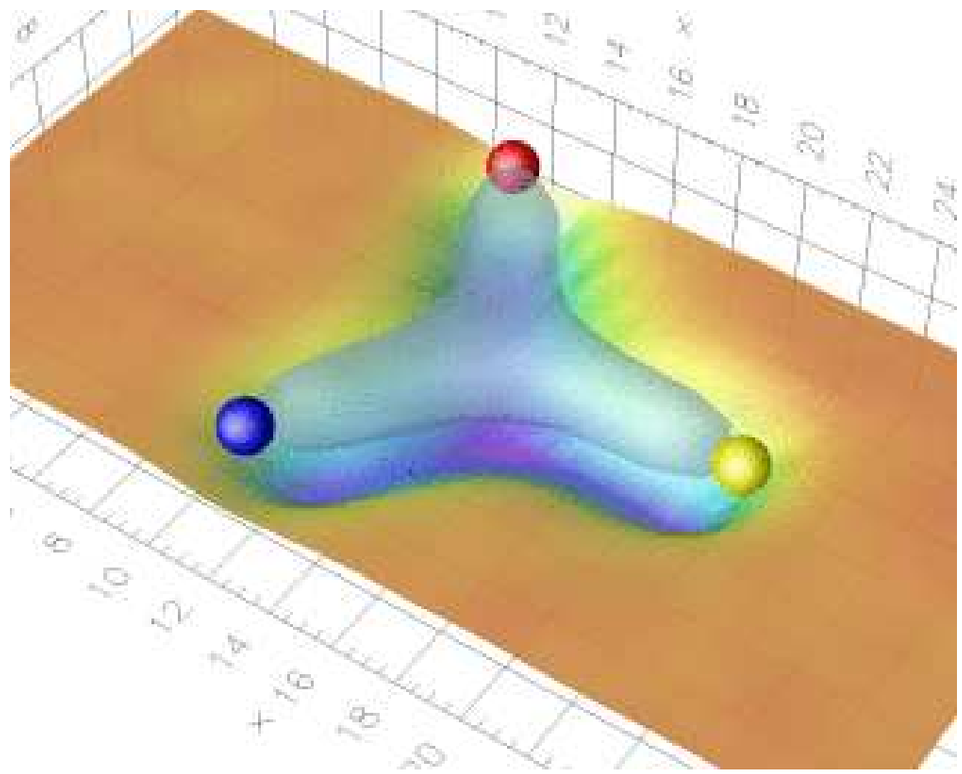
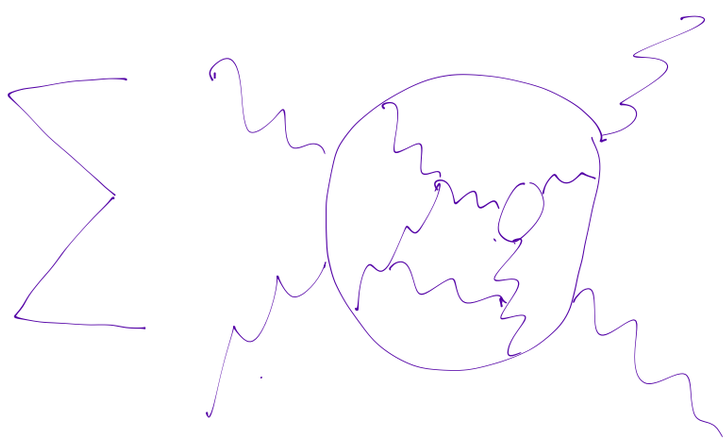
1803.00577, 1807.00254
+ work in progress with
Jack Donahue



Continuation of earlier work with
Raphael Flauger, Victor Gorbenko, Guzman Hernandez-Chifflet, Mehrdad Mirbabayi, Peter Conkey, Patrick Cooper, Ali Mohsen, Stefano Storace, Chang Chen

The underlying goal is to “solve” large N QCD

’t Hooft’74



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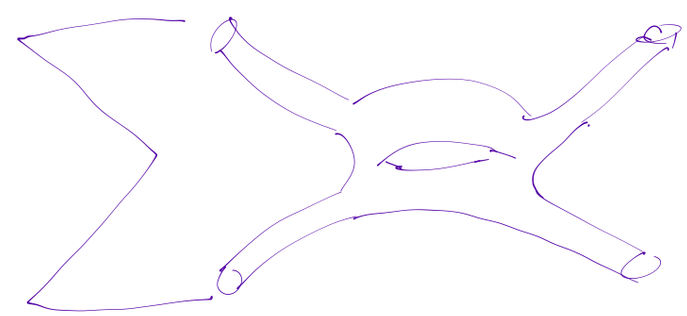


$$= \sum_g N^{2-2g} F_g(\lambda)$$

which reduces to solving a certain free string theory

Has proven to be very hard

WHY?

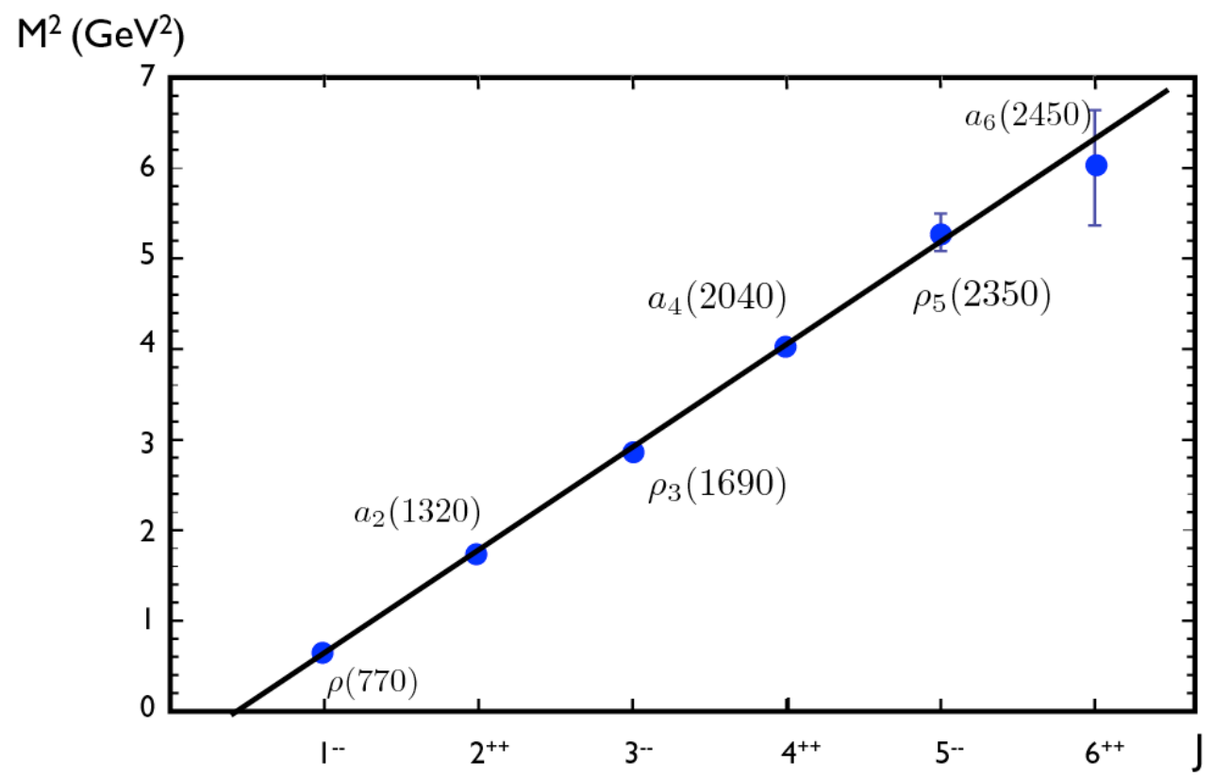
Maybe it is hopelessly hard?

NO: it worked out in N=4 SYM!!!

many people are here

Maybe it is hopelessly misguided?

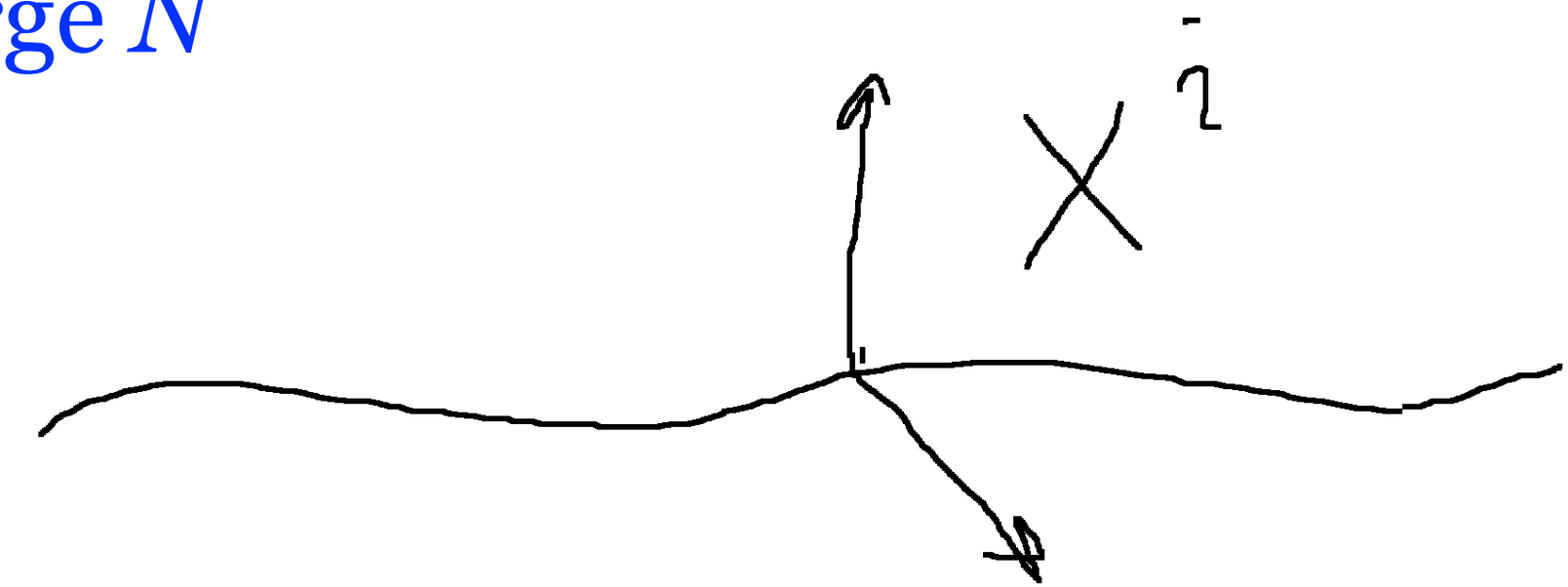
NO: it worked out in N=4 SYM!!!

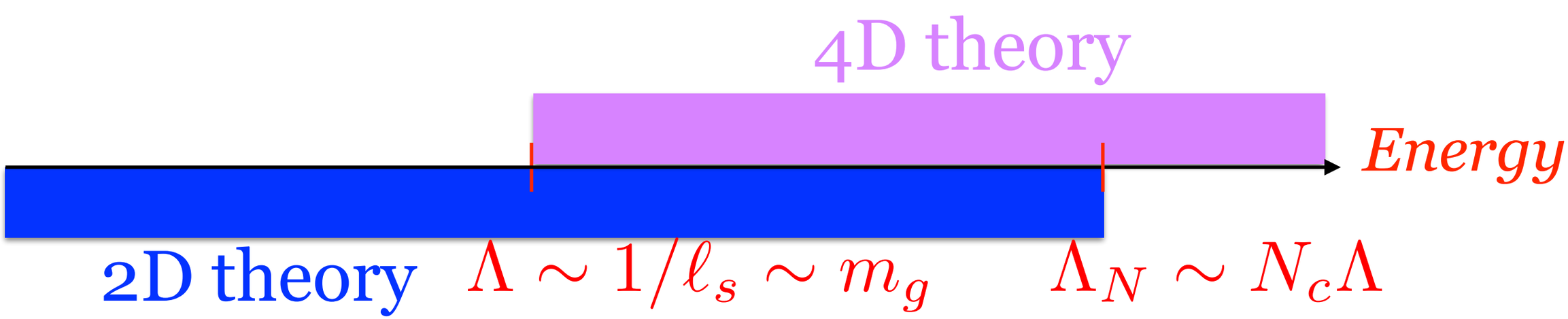


Maybe it is hopelessly ill-posed?

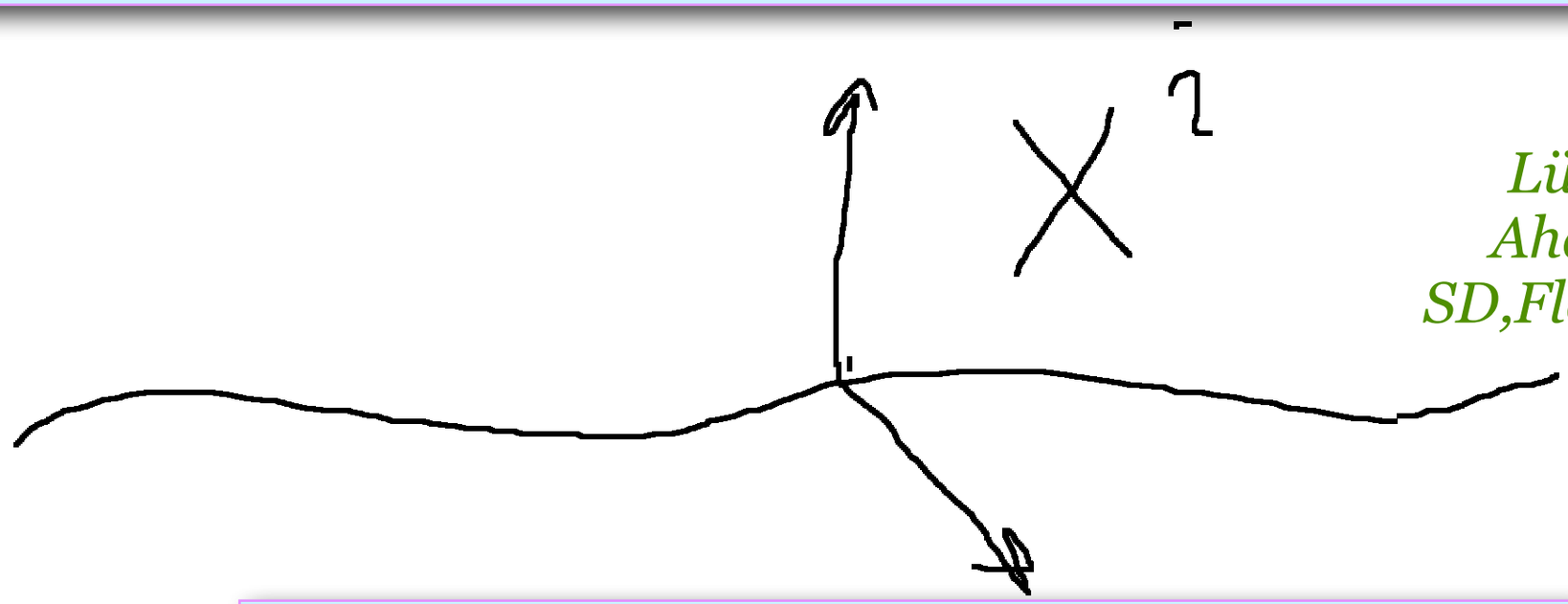
SETUP

- Confining gauge theory with a gap
- Unbroken center symmetry
- Large N





(Long) String as seen by an Effective Field Theorist



Lüscher '81
Lüscher, Weisz '04
Aharony et al '07-11
SD,Flauger, Gorbenko'12

Theory of Goldstone Bosons

$$ISO(1, D-1) \to ISO(1,1) \times SO(D-2)$$

$$\delta_\epsilon^{\alpha i} X^j = -\epsilon(\delta^{ij}\sigma^\alpha + X^i \partial^\alpha X^j)$$

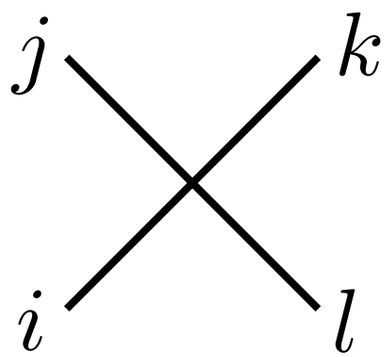
$$S_{string} = -\int d^2\sigma \sqrt{-\det h_{\alpha\beta}} \left(\ell_s^{-2} + \frac{1}{\alpha_0} \left(\cancel{K^i_{\alpha\beta}} \right)^2 + \ldots \right)$$

Nambu-Goto

rigidity
vanishes on-shell

Large N QCD is hard because the worldsheet theory is

- ✳not free

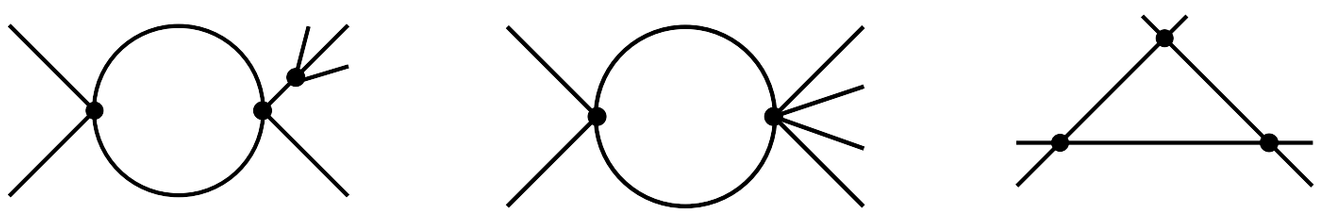


$$= \delta_{ik}\delta_{jl}\frac{i\ell_s^2 s}{4}$$

tree level

- ✳not integrable

$$X^i(p_+ + q_+)X^i(p_- + q_-) \to X^j(p_+)X^j(q_+)X^k(p_-)X^k(q_-)$$



one loop

$$-\ell_s^6\frac{D-26}{24\pi}p_+q_+p_-q_-(p_+ + q_+)^2(p_- + q_-)^2$$

- ✳strongly coupled in the UV: 2d gravity rather than a QFT

The best we can hope for is to find a “close by” integrable model and perturb around it

An alternative complementary approach would be the S-matrix bootstrap: we are after an isolated theory here!

Miro, Guerrieri, Hebbar, Penedones, Vieira’ to appear next week

Examples of similar simple integrable models

D=3 or 26

Ward identities of non-linearly realized Poincare plus integrability determine:

$$e^{2i\delta(s)} = e^{is\ell_s^2/4}$$

shock wave phase shift
aka
$T\bar{T}$ deformation of m=0 free bosons

Finite volume spectrum from TBA

$$E(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell^4} + \frac{4\pi}{\ell^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$$

At D=26 this a critical bosonic string
NO scale invariance in the UV!

A simple example of a close by integrable model at general D

$$S_{string} = -\ell_s^{-2} \int d^2\sigma \sqrt{-det(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i + \partial_\alpha \phi \partial_\beta \phi)} + Q \int d^2\sigma \phi R[X] + \ldots$$

$$Q = \sqrt{\frac{25 - D}{48\pi}}$$

$$e^{2i\delta(s)} = e^{is\ell^2/4}$$

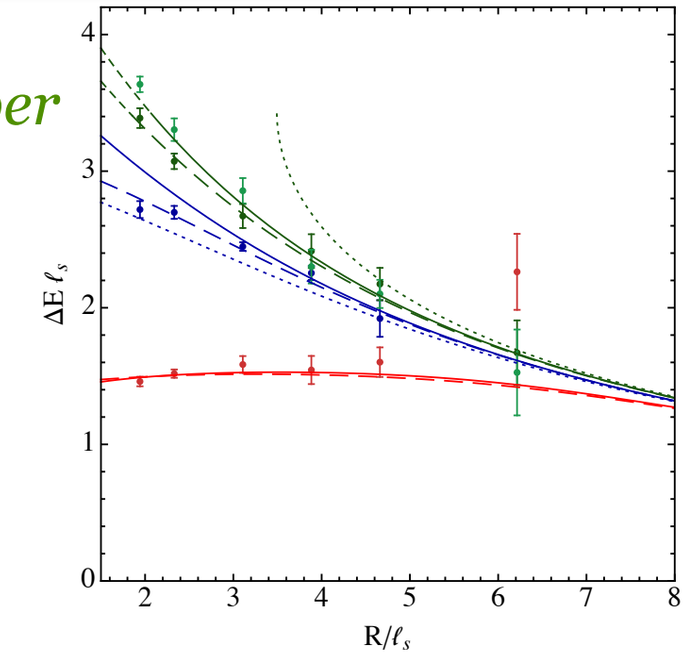
This is also known as a linear dilaton background,

aka Polyakov's non-critical strings

aka holographic QCD

Experimental Summary

based on lattice data by Athenodorou, Bringholtz and Teper



✳D=4

field content $X^1\ ,\ X^2$ ϕ

Goldstones pseudoscalar

"worldsheet axion"

action

$$S = S_{NG}[X^1, X^2] + \int d^2\sigma \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}m^2\phi^2 + Q\phi\epsilon^{\alpha\beta}\epsilon_{ij}K^i_{\alpha\gamma}K^{j\,\gamma}_{\beta} + \ldots$$

$$m\ell_s \approx 1.85^{+0.02}_{-0.03} \qquad Q \approx 0.38 \pm 0.04$$

✳D=3

field content X action $S = S_{NG}[X] + \ldots$

Goldstone

Another simple close by integrable model

$$S_{string} = -\ell_s^{-2} \int d^2\sigma \sqrt{-det(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i + \partial_\alpha \phi \partial_\beta \phi)} + Q \int d^2\sigma \phi K \tilde{K} + \ldots$$

$$Q = \sqrt{\frac{25 - D}{48\pi}} = \sqrt{\frac{7}{16\pi}} \approx 0.373176\ldots$$

??!

$$e^{2i\delta(s)} = e^{is\ell^2/4}$$

Axionic String Ansatz

Both at D=3,4 the worldsheet theory is a deformation of an integrable theory with

$$e^{2i\delta(s)} = e^{is\ell_s^2/4}$$

Correct matter content+suggestive numerology

Why on earth one would expect integrability at high energies?

Well, the underlying gauge theory is asymptotically free...

Worldsheet theory lives in d=2 independently of D

Good chances to draw useful lessons from analytically tractable lower dimensional models (D=2)

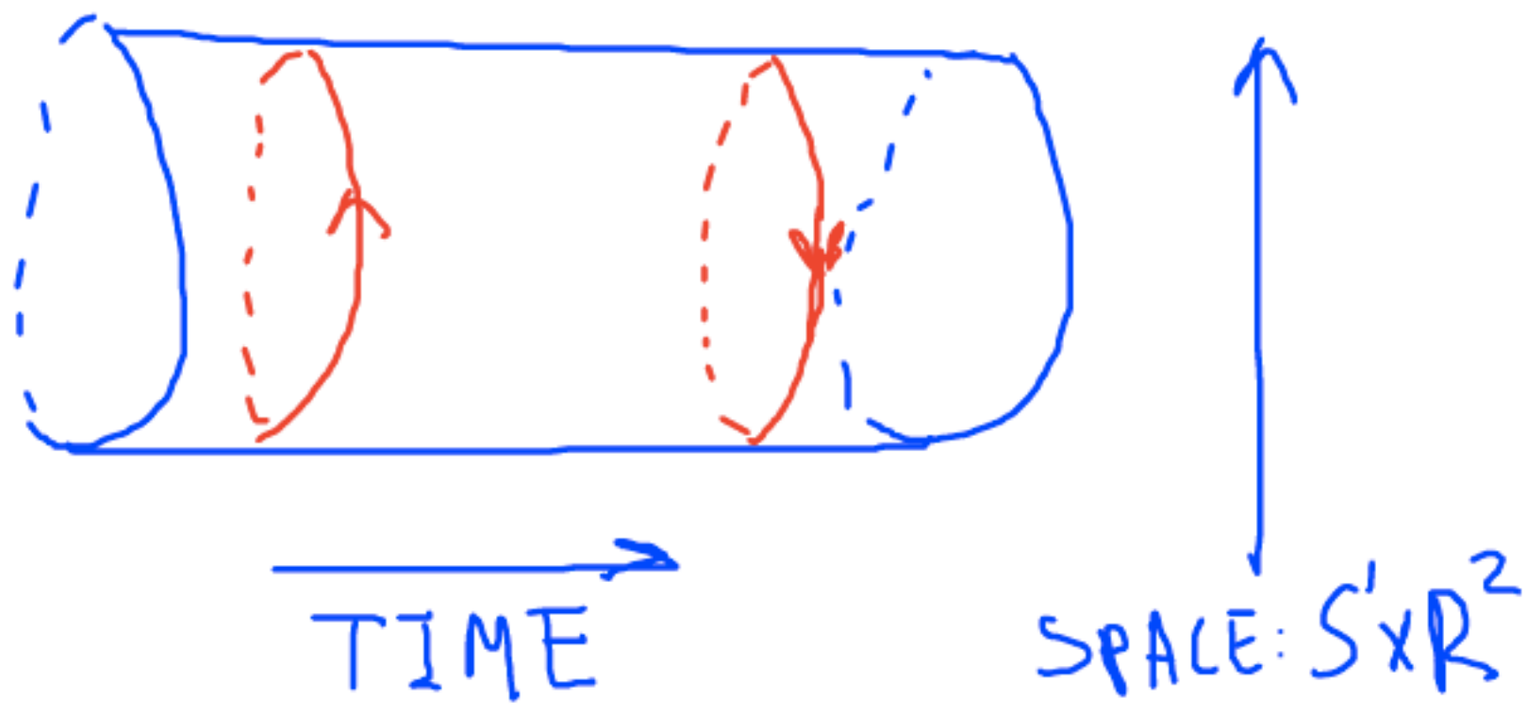
$$S = \int d^2\sigma \mathrm{Tr}\left\{-\frac{1}{2g^2}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\gamma^\mu \nabla_\mu^{(ad)} - m)\psi\right\}$$

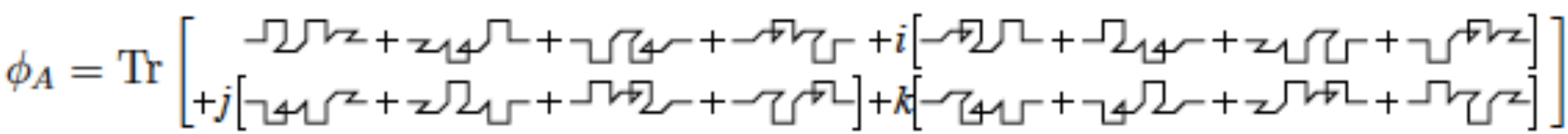
perturbative in heavy mass regime $m^2 \gg g^2 N$

But is there a worldsheet theory to talk about at D=2?

Worldsheet theory at D=4
(as seen on a lattice)

$$\mathcal{O}_P = Tr P e^{i \oint A}$$

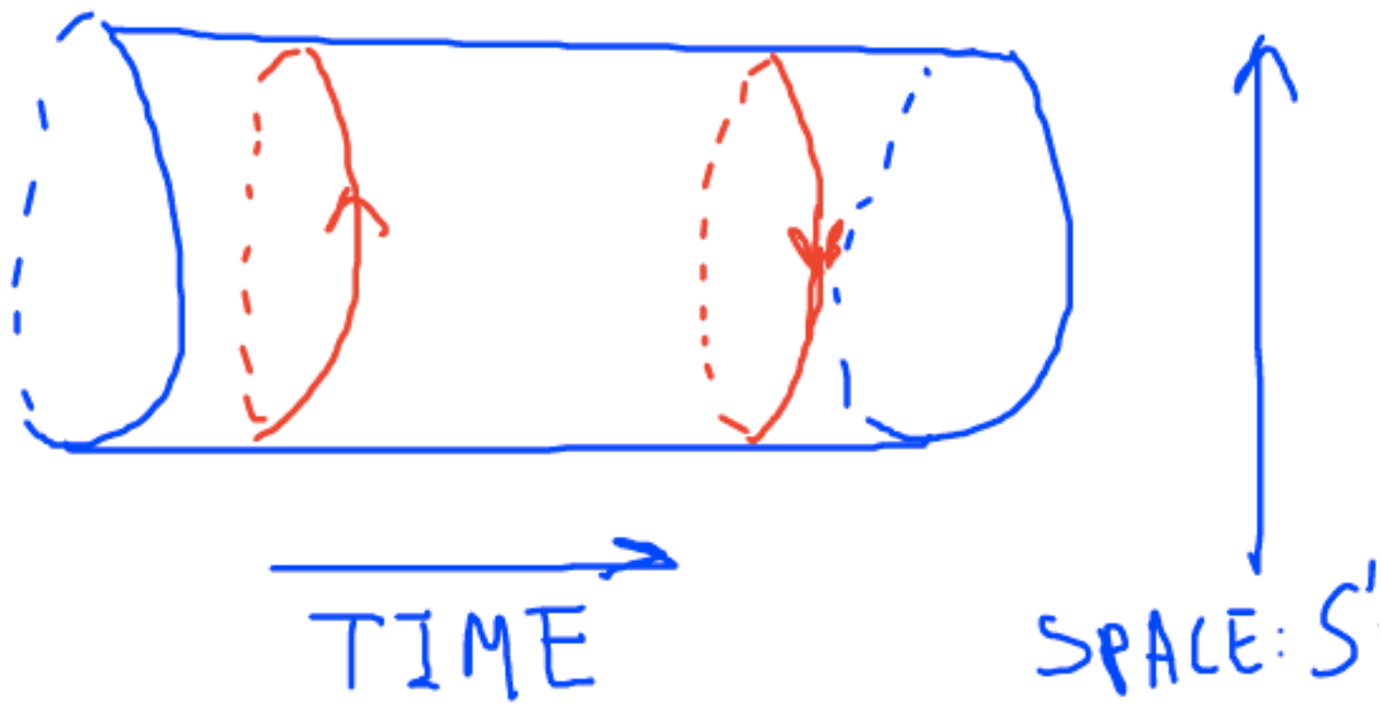




$$\int \mathcal{D}A e^{-S_{YM}} \mathcal{O}(0)\mathcal{O}^\dagger(t) \to e^{-E_{\mathcal{O}} t} + \ldots$$

Worldsheet Theory at D=2

$$\mathcal{O}_P = Tr P e^{i \oint A}$$



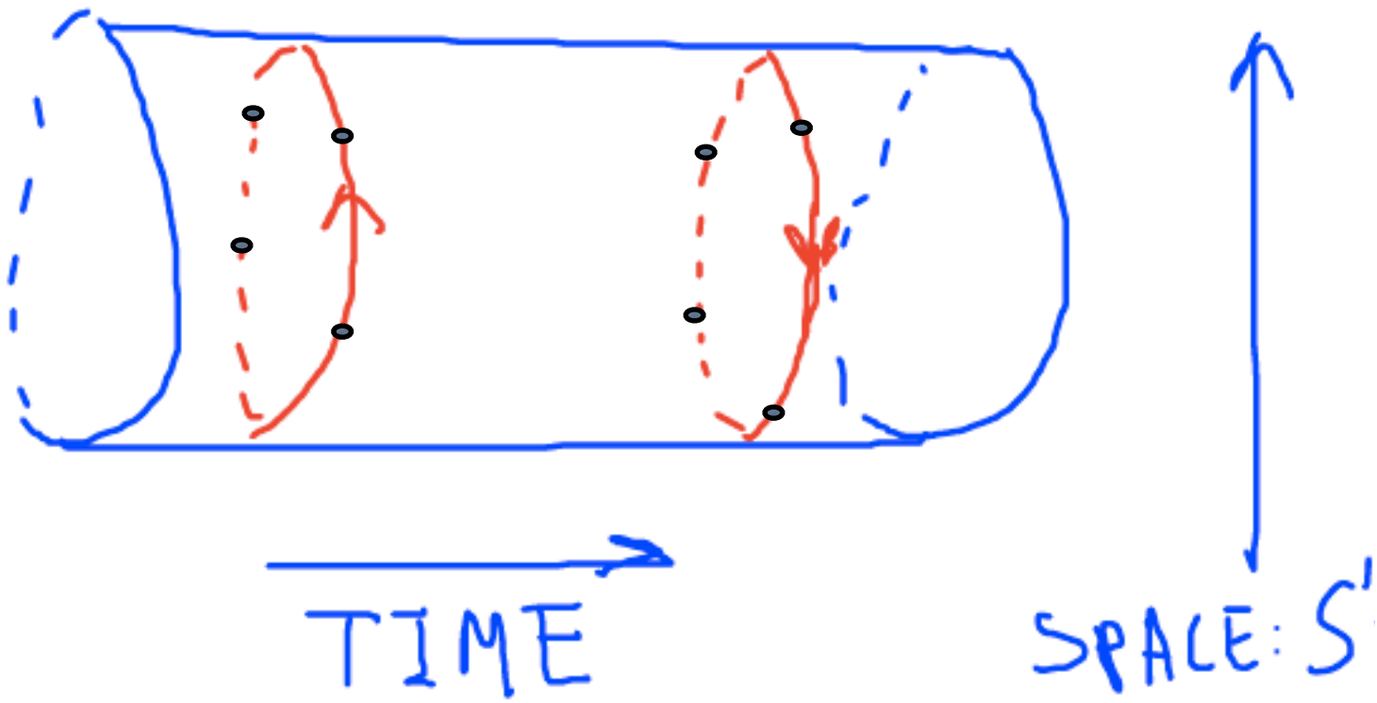
- pure YM is an interesting theory of non-critical strings solved at all N

Migdal'75, Kazakov, Kostov'80, Rusakov'90, Witten'91, Gross,Taylor'93,...

- confirms that there is a path to non-critical strings without a Liouville mode
- a bit too minimalistic—no local d.o.f. whatsoever

Worldsheet Theory at D=2

$$\mathcal{O}_P = Tr P e^{i \oint A} \psi_1 \psi_2 \ldots \psi_n$$



- local d.o.f. appear in the presence of adjoint matter
- interesting results on the spectrum

Daley, Klebanov'92, Kutasov'93, Katz et al'13,...

- not much on the worldsheet dynamics

technical analysis is a linear superposition of

More about the Massive Schwinger Model*

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Received April 2, 1976

+

θ-Vacua in Two-Dimensional Quantum Chromodynamics (*).

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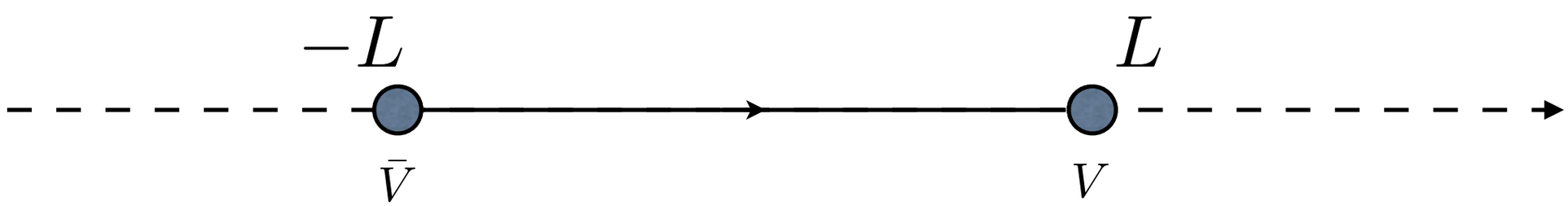
(ricevuto il 23 Febbraio 1979)

Hamiltonian Formalism on the Worldsheet

✳Introduce heavy "end-of-the-world" fundamental quarks



✳Fix $A_x = 0$ gauge, and solve for A_0 from the Gauss' law

$$H = \int d\sigma \left(\frac{1}{2g^2}(E^a)^2 + \frac{1}{2}\bar{\psi}^a(-i\gamma^1\partial_\sigma + m)\psi^a \right)$$

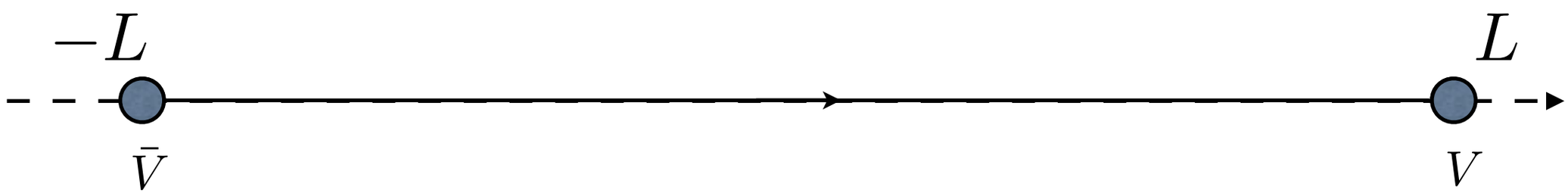
$$\int d\sigma \frac{(E^a)^2}{2g^2} = \frac{g^2 N L}{2} - \frac{g^2}{4}\int d\sigma d\sigma' |\sigma - \sigma'| \rho^a(\sigma)\rho^a(\sigma') + \frac{g^2}{2}(\bar{T}^a - T^a)\int d\sigma \sigma \rho^a(\sigma)$$

string tension

Coulomb force

interaction with the background electric field

quantum mechanical operators



$$H = \int d\sigma \left(\frac{1}{2g^2}(E^a)^2 + \frac{1}{2}\bar{\psi}^a(-i\gamma^1\partial_\sigma + m)\psi^a \right)$$

$$\int d\sigma \frac{(E^a)^2}{2g^2} = \frac{g^2 N L}{2} - \frac{g^2}{4}\int d\sigma d\sigma' |\sigma - \sigma'| \rho^a(\sigma)\rho^a(\sigma') + \frac{g^2}{2}(\bar{T}^a - T^a)\int d\sigma \sigma \rho^a(\sigma)$$

Extended Hilbert space

$$\mathcal{H}_{ext} = \mathcal{H}_{free} \otimes \bar{V} \otimes V$$

operators in V

Physical states

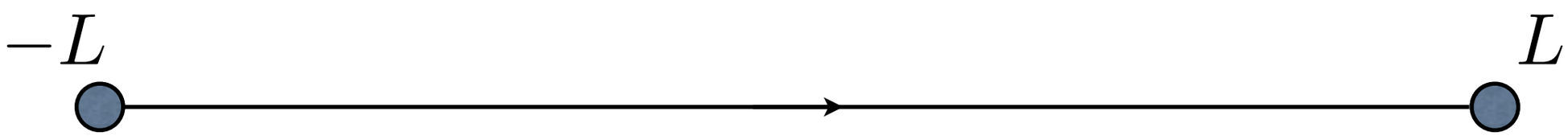
$$Q^a|ph\rangle = \left(T^a + \bar{T}^a + \int \rho^a \right)|ph\rangle = 0$$

- required for the energy to be finite
- ensures translational invariance

At $m^2 \gg g^2 N$ states can be characterized by a number of fermions ("partons")

✳ Vacuum

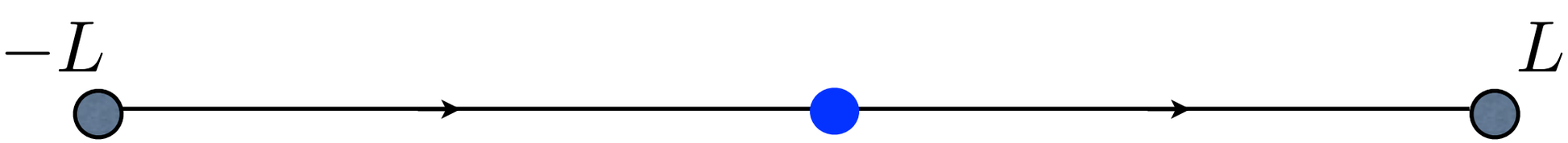
$$|0\rangle = |0\rangle_f \otimes \bar{e}_i \otimes e^i \equiv |0\rangle_f \otimes \mathbf{1}$$



$$E_0 \approx \frac{g^2 N}{2} L \equiv 2L/\ell_s^2$$

✳ One-particle state ("free quark")

$$|k\rangle = |k, a\rangle \otimes T^a$$

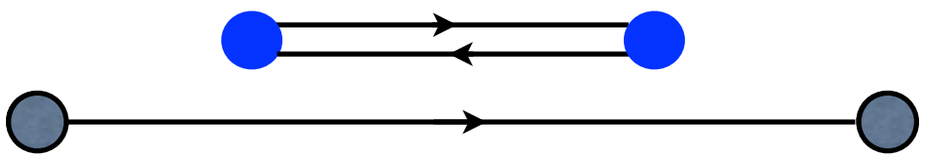


$$\text{mass} \approx m$$

✳ Two-particle states

mesons

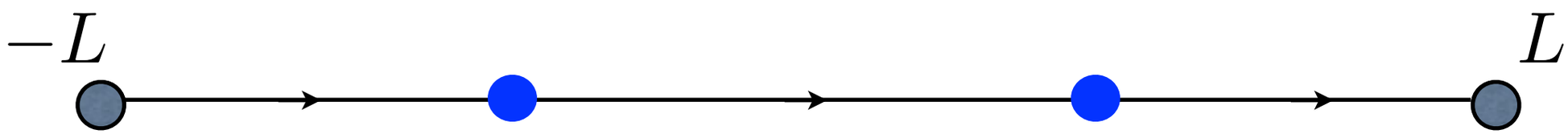
$$|k_1, k_2\rangle_{mes} = \sum_a |k_1, k_2, a, a\rangle \otimes \mathbf{1}$$



decouple from the worldsheet
in the planar limit

✳ Two-particle states

scattering states on the worldsheet



$$|k_1, k_2\rangle_{sc} = \sum_{a_1, a_2} |k_1, k_2, a_1, a_2\rangle \otimes T^{a_1} T^{a_2}$$

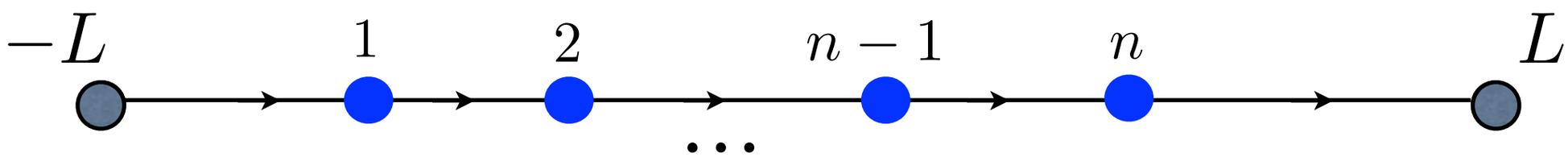
Too many states for a pair of identical particles ?!

- Momenta take values on a plane, rather than on a half-plane
- Exchange term is missing in the inner product

$$\langle k_1', k_2' | k_1, k_2\rangle = \delta(k_1 - k_1')\delta(k_2 - k_2')$$

✳ n-particle states

scattering states on the worldsheet



$$|k_1, \ldots, k_n\rangle = \sum_{a_i} |k_1, a_1; \ldots; k_n, a_n\rangle T_1^a \ldots T_n^a$$

$$\langle k_1, \ldots, k_n | k_1', \ldots, k_n'\rangle = \delta(k_1 - k_1') \ldots \delta(k_n - k_n')$$

Infinite (quantum Boltzman) Statistics

Need to learn how to do perturbation theory in this space

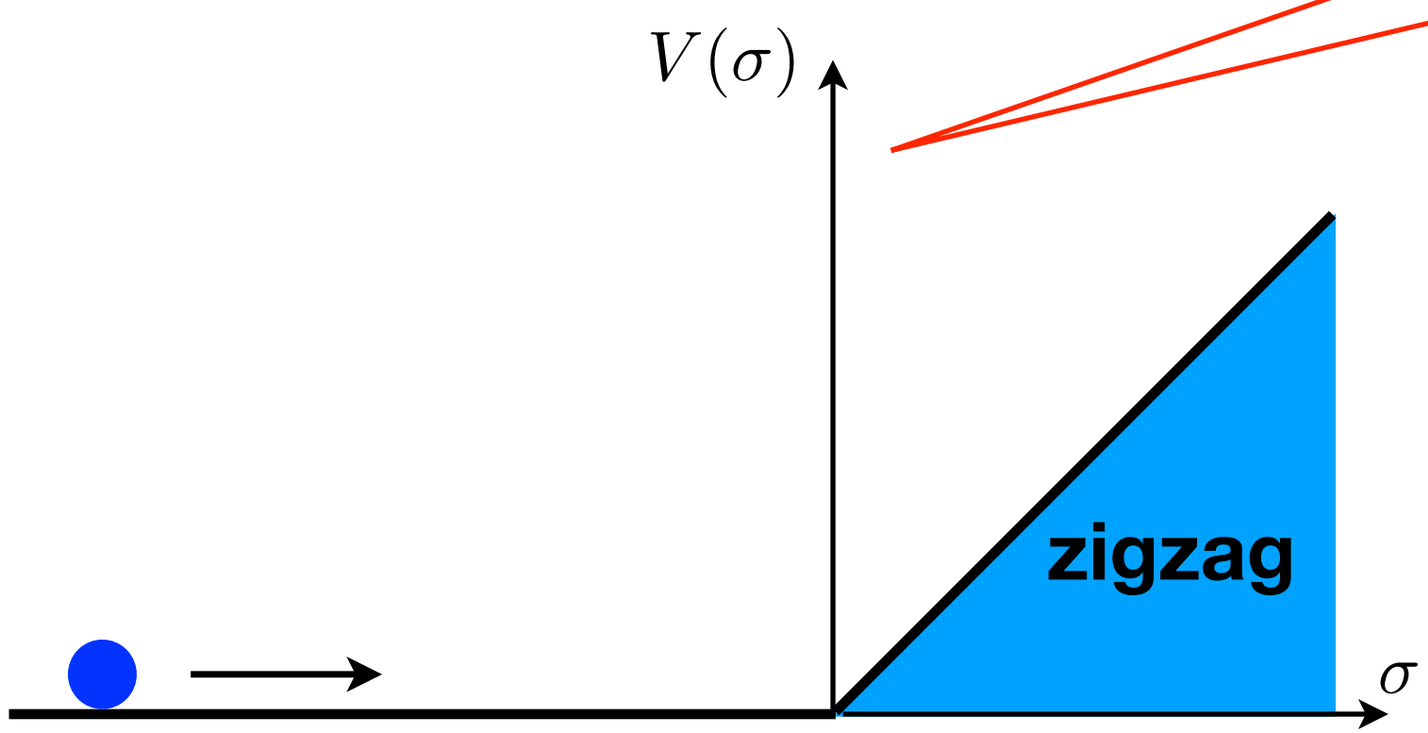
Dynamics

Let's calculate the effective Hamiltonian

$$\langle k_1', k_2'|H|k,-k\rangle = \delta(k_1'+k_2')\langle k'|H_{eff}|k\rangle$$

One finds

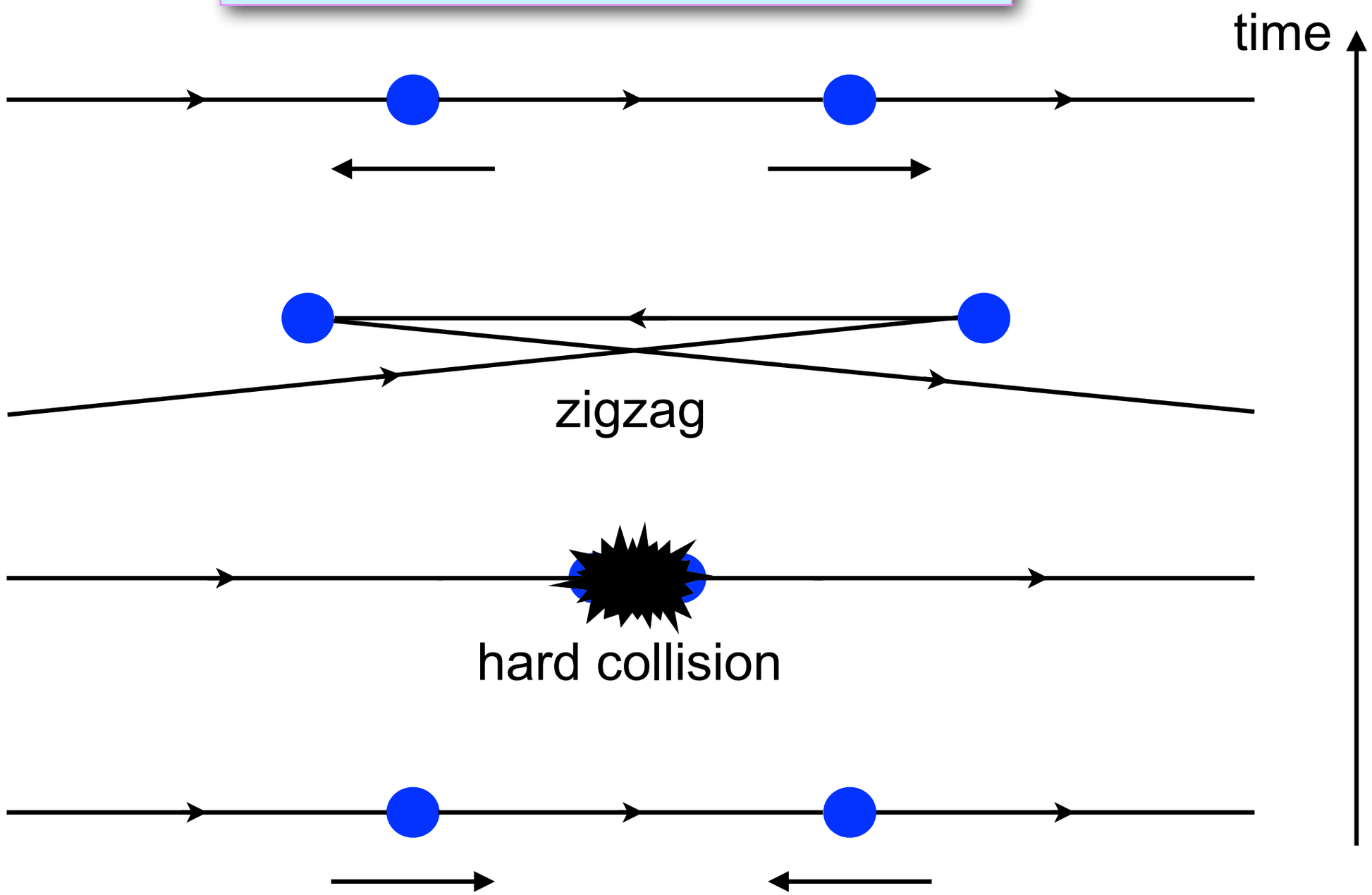
$$\langle k'|H_{eff}^{L(R)}|k\rangle = \delta(k-k')2\omega_k - \frac{g^2N}{4\pi}\left(\mathcal{U}(k,k')\frac{\mathcal{P}}{(k-k')^2} \pm i\pi\delta'(k-k')\right)$$



conventional statistics is restored on-shell

long time delay without a resonance!

Worldsheet Scattering

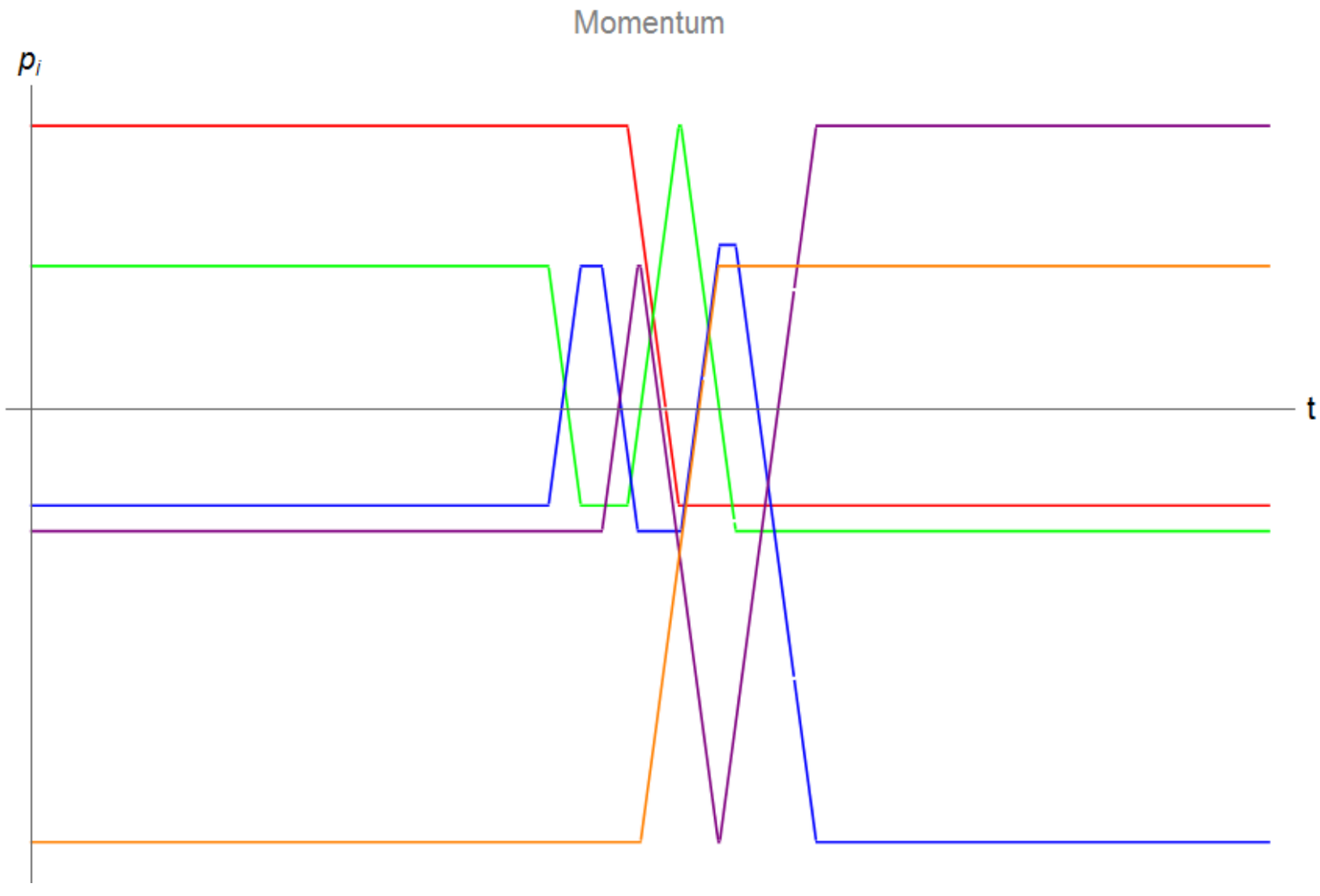


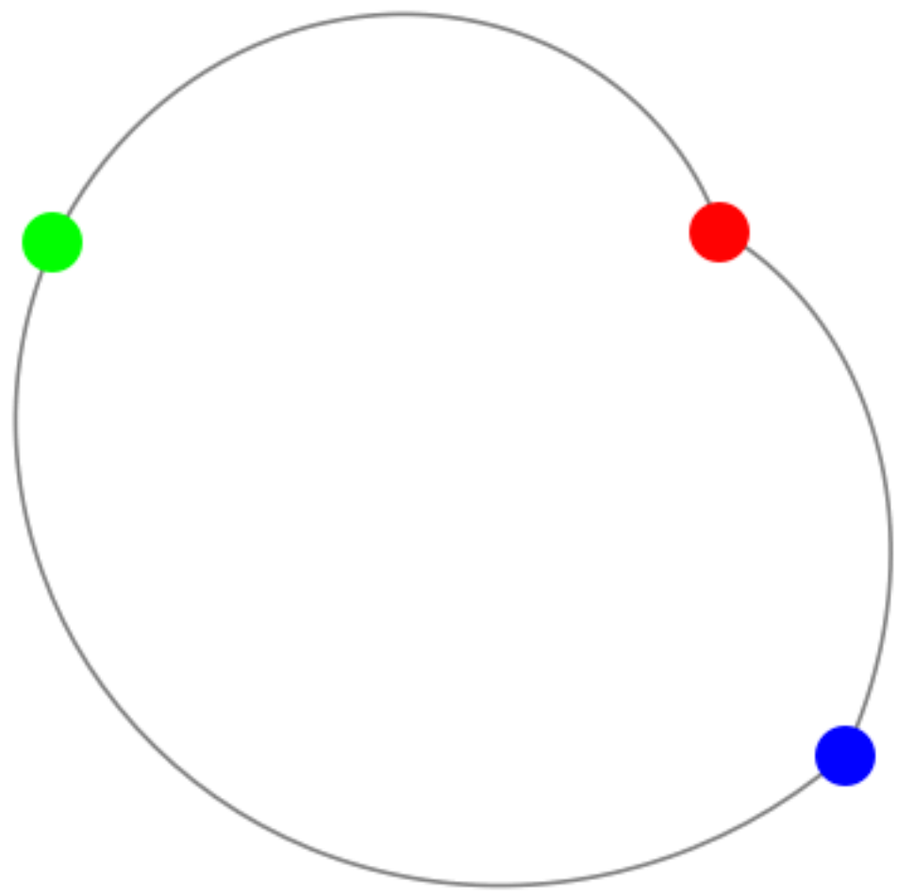
Zigzags restore much of the D>2 stringy dynamics

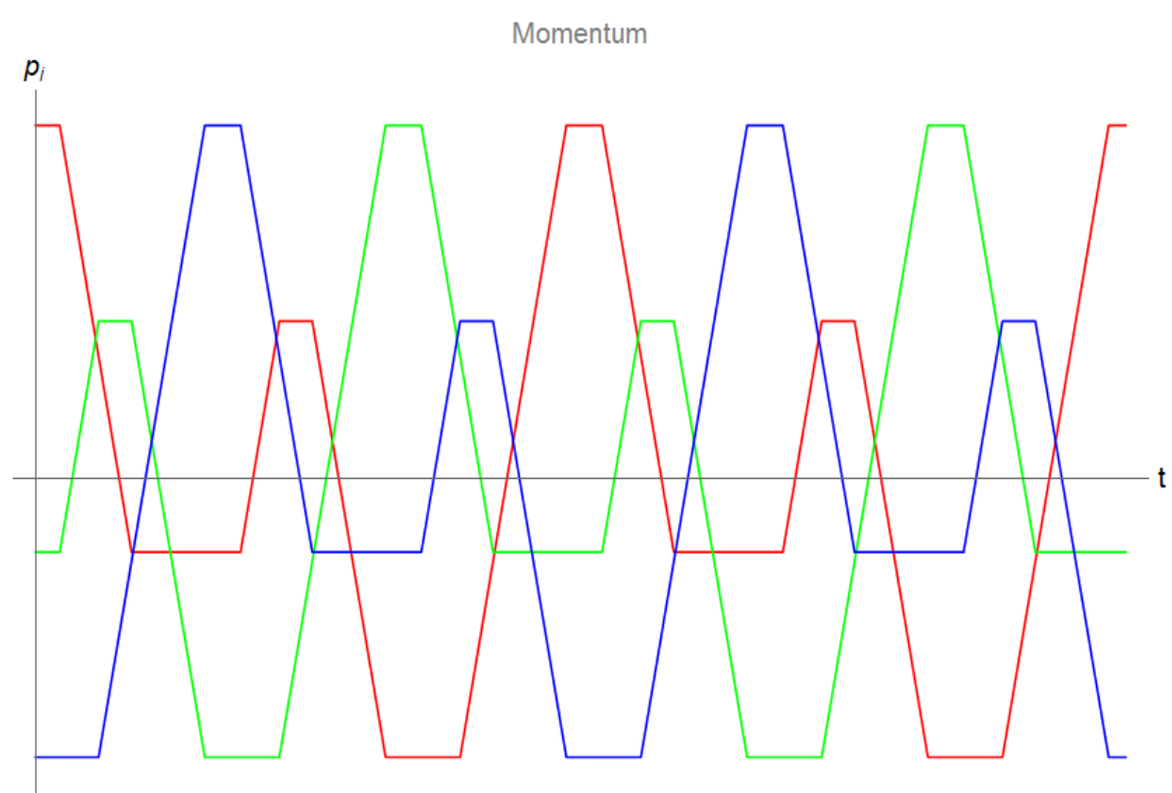
"Leading" N-particle Hamiltonian

$$H_N = \sum_{i=1}^{n} \sqrt{p_i^2 + m^2} + \sum_{i=1}^{N-1} (q_i - q_{i+1} - |q_i - q_{i+1}|)$$

- ✳Poincare invariant at the level of classical Poison brackets
- ✳No integrability at $m \neq 0$
- ✳Integrable in the massless limit!
- ✳Phase shift is the same as for $T\bar{T}$ deformed massless fermion


Momentum
p_i
t




Momentum
p_i
t

Algebra of charges at N=2

$$H = |p_1| + |p_2| + q_1 - q_2 + |q_1 - q_2| \qquad P = p_1 + p_2$$

$$J = p_1 q_1 + p_2 q_2 + \frac{q_1 + q_2}{2}(q_1 - q_2 + |q_1 - q_2|)$$

$$\{J, H\} = P \qquad \{J, P\} = H$$

$$\tilde{P} = |p_1| - |p_2| + 2(q_1 + q_2)$$

$$\tilde{H} = p_1 - p_2 - (\mathrm{sign}(p_1)q_1 + \mathrm{sign}(p_2)q_2)(1 - \mathrm{sign}(q_1 - q_2))$$

$$\{J, \tilde{H}\} = \tilde{P} \qquad \{J, \tilde{P}\} = \tilde{H}$$

$$\{H, \tilde{H}\} = 4 \qquad \{P, \tilde{P}\} = 4$$

The best we can hope for is to find a "close by" integrable model and perturb around it

Can one do better at D=2?

$$m^2 = \frac{g^2 N}{\pi}$$

is an especially interesting point—the model enjoys accidental SUSY.

Kutasov, hep-th/9306013

SUSY is spontaneously broken on the worldsheet. "Free quark" is a Goldstino. Interestingly, a $T\bar{T}$ deformed massless fermion exhibits non-linearly realized SUSY. Conversely, Goldstino is integrable at tree level.

Moreover, there is actually a larger family of models

$$S = \mathrm{Tr}\left(\epsilon^{\alpha\beta} F_{\alpha\beta}\Phi - V(\Phi) + \bar{\psi} i\gamma^{\mu}\nabla_{\mu}\psi + \sum_{n,m} y_{n,m}\Phi^{n}\bar{\psi}\Phi^{m}\psi\right)$$

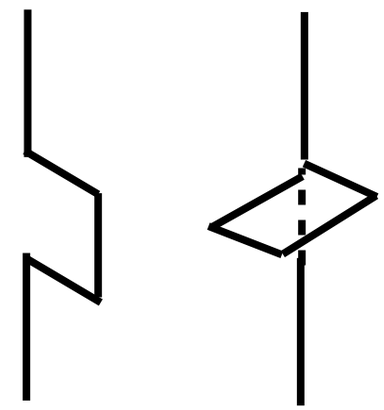
Can some of them be integrable?

Back to D=3,4...

This analysis suggests that there is a direct map between bulk fields and worldsheet excitations.

✳in D=4 YM one expects to find

$$X^i \sim Tr P e^{i \oint_z A} F_{zi} \qquad a \sim Tr P e^{i \oint_z A} F_{xy}$$

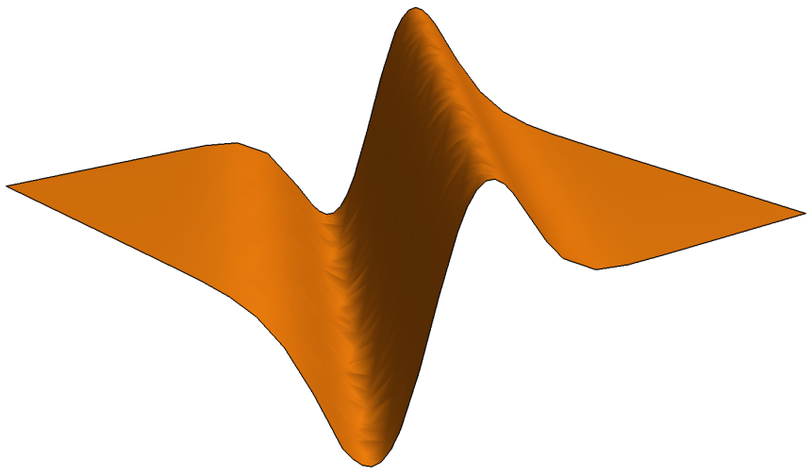


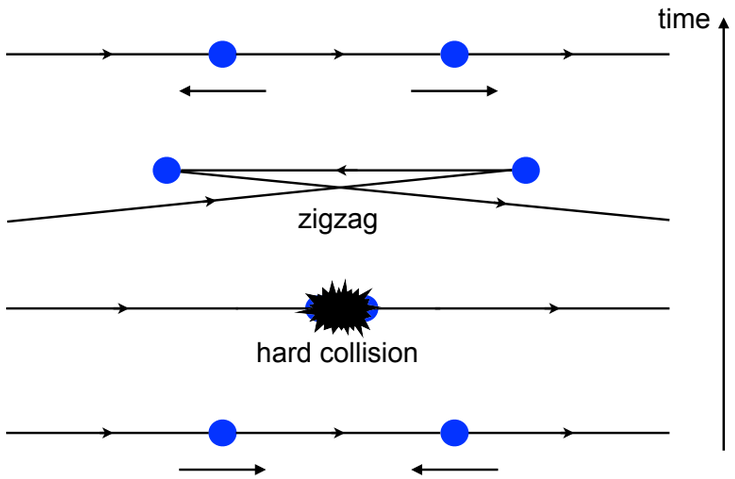
✳in D=3 YM one expects to find

$$X \sim Tr P e^{i \oint_z A} F_{zx}$$

Exactly the ASA matter content!

Goldstone/Gluon Duality





string worldsheet ⟷ heavy quark meson

Goldstones
and other string modes ⟷ pQCD partons

high energy
worldsheet scattering ⟷ 1)perturbative gluon scattering
2)zigzag formation
3)soft gluon production

$T\bar{T}$ deformation + ? ⟷ factorization

The next step seems to be to establish the exact link between pQCD and worldsheet observables

Leading log violation of scale invariance in pQCD

$$\beta(\alpha_s) = -\frac{22 - n_{sc} - 4n_f}{24\pi} C_A \alpha_s^2$$

Leading particle production in Goldstone scattering

$$A_{ann} = \frac{26 - D}{24\pi} \frac{\ell_s^4 s^2}{16}$$

$$26 - 4 = 22$$

c.f. Fradkin, Tseytlin'83, Metsaev, Tseytlin'88

The agreement persists in the presence of adjoint (s)quarks