Gauge/Gravity Duality and the Phenomenology of Hadrons

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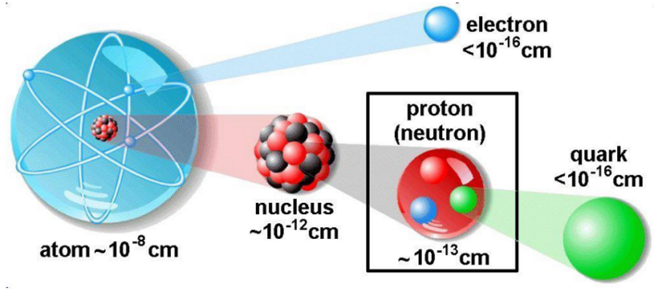
Summary:

Problem: QCD is strongly coupled at low energies → the spectrum of hadrons is not fully understood

Idea: use gauge/gravity duality to derive effective models for the interactions of hadrons and compare the predictions with experimental data

Concrete example: glueball decay, based on my work with Josef Leutgeb, Denis Parganlija, and Anton Rebhan (1501.07906, 1504.05815, 1510.07605, 1610.10034, 1807.10164)



Fundamental constituents of the nucleus: quarks and gluons

Quarks and gluons remained undetected for a long time, until deep inelastic scattering was used to probe the structure of protons and neutrons

Two reasons: quarks and gluons ...

1. ... interact strongly at low energy (*asymptotic freedom*)
2. ... appear only in colour-neutral states (*confinement*)

Most accurate description of quarks and gluons incorporating these principles: quantum chromodynamics (QCD)

QCD: quantum field theory with gauge group $\mathrm{SU}(3)$, incorporating quarks as spinors and gluons as gauge fields

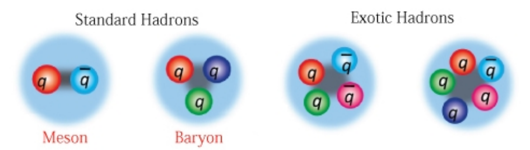
$$\mathcal{L}_{\mathrm{QCD}} = \sum_{j=1}^{N_f} \bar{\psi}_j \left[i\not{D} - m_j \right] \psi_j - \frac{1}{2} \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu}$$

Important side note:

In the absence of quark mass, this Lagrangian is invariant under the global *chiral symmetry* group $\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}$, which is broken spontaneously as

$$\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R} \to \mathrm{SU}(3)_\mathrm{V}$$

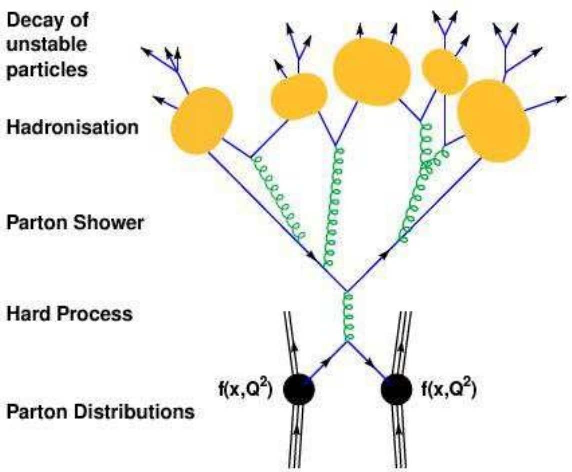
Spontaneous symmetry breaking means that while the theory (Lagrangian) itself is invariant, the ground state solution is not



In collider experiments at low energies (up to at least a few GeV) we do not observe quarks and gluons as dynamical objects, but rather a large number of distinct hadronic states.

Constituent quark model predicts two types of hadrons: mesons (two quarks) and baryons (three quarks)

Additional possibilities: tetraquarks, pentaquarks, *glueballs*, ...

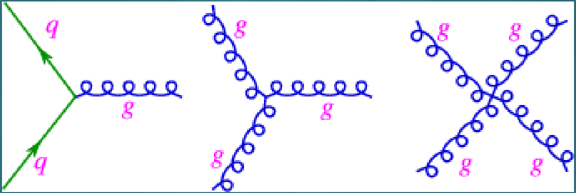

Decay of unstable particles
Hadronisation
Parton Shower
Hard Process
Parton Distributions
f(x,Q^2)
f(x,Q^2)

Experimental status:

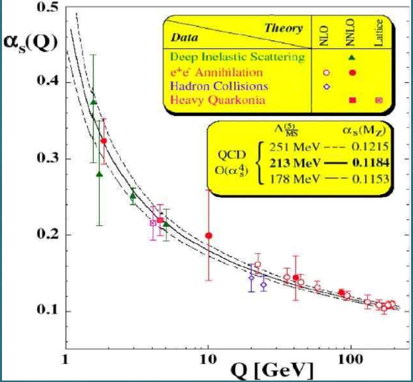
Few stable hadrons: protons and neutrons

Many unstable hadrons (resonances): pions, kaons, η-meson, η'-meson, ρ-meson ...

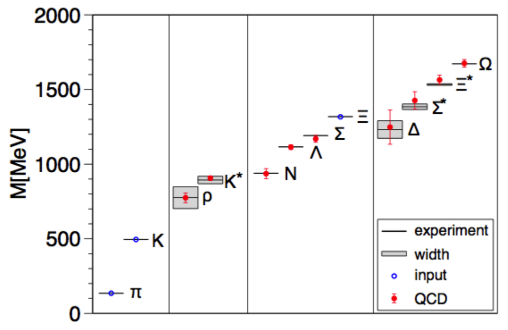
Task: find a full classification of all hadronic resonances and their properties, i.e., determine their *mass*, their *decay rate*, and their *decay channels*



Conventional approach in a quantum field theory: calculate physical observables (like decay rates) by expanding them into a series for small values of the coupling parameter, making use of Feynman diagrams (= graphical representations of terms in the expansion)



Problem: the coupling constant of QCD, α_s, is very large at low energy, so perturbation theory is not applicable.



Part of the problem can be circumvented by placing QCD on a space-time lattice, and determining the mass spectrum of hadrons by numerical simulation.

However, lattice QCD is not able to provide predictions for decay rates or scattering amplitudes.

Because of this, people have constructed effective models, field theories that involve mesons and baryons themselves as degrees of freedom, instead of quarks of gluons.

These theories are not rigorously derived from QCD, but built based on symmetry considerations.

The basic building block for low-energy effective models of QCD is the so-called *chiral Lagrangian*:

$$\mathcal{L}_{\mathrm{chiral}} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(U^{-1}\partial_\mu U\right)^2,$$

with $U \equiv \exp(2i\pi_a T_a)/f_\pi$ and $a = 1 \ldots 8$ for $N_f = 3$.

It encodes the spontaneous symmetry breaking present in QCD:

$$\mathrm{SU(3)_L} \times \mathrm{SU(3)_R} \to \mathrm{SU(3)_V}$$

by describing the dynamics of the corresponding pseudo-goldstone bosons, i.e., pions, kaons, and eta meson

Beyond the chiral Lagrangian, it is very hard to find reliable and unique predictions for the interactions of mesons (and baryons).

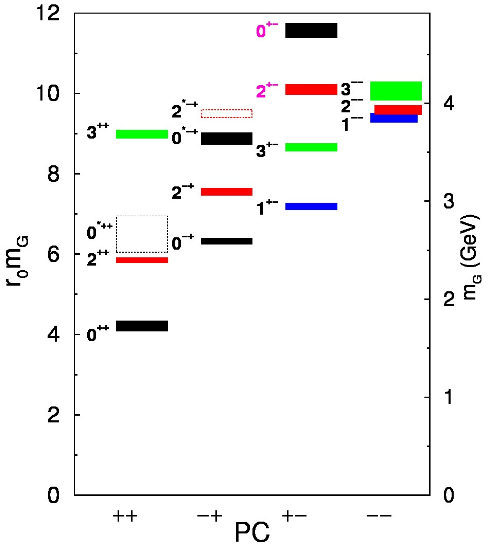
Effective models of low energy QCD often involve many parameters that have to be adjusted to experimental data, and there are many different conflicting predictions in the literature.

Are there alternatives to guessing interactions based on symmetries?

Before we answer this question, focus on a concrete subset of hadron spectroscopy:

Lattice QCD predicts a discrete spectrum of bound states of gluons, known as glueballs.

Glueballs come with a variety of distinct quantum numbers J^{PC}, the lightest being a scalar (0^{++}) glueball with a mass below 2 GeV.


12
10
8
6
4
2
0
$r_0 m_G$
4
3
2
1
0
m_G (GeV)
++
−+
+−
−−
PC
0^{+-}
2^{+-}
3^{--}
2^{--}
1^{--}
2^{*-+}
3^{++}
0^{*-+}
3^{+-}
2^{-+}
1^{+-}
0^{*++}
0^{-+}
2^{++}
0^{++}

Below 2 GeV, there are several states with the same quantum numbers that might be glueballs, most notably the $f_0(1500)$ and the $f_0(1710)$ resonances.

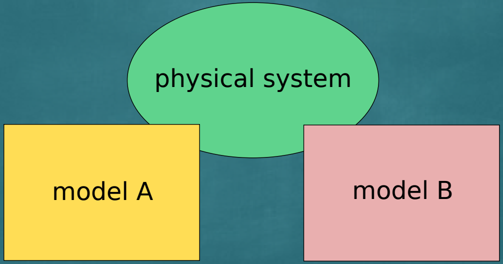
These glueball candidates have similar mass, but distinct decay patterns: their decay products are different. It is not known whether these states are quark-antiquark, glueball, or mixed states.

A reliable theoretical prediction for the decay patterns of a glueball would solve the problem of identifying it within the hadronic spectrum; unfortunately such a prediction does not exist within the framework of conventional effective field theories.

What would a reliable prediction look like?

It would come from a theory known to reproduce all physical observables of QCD in the relevant energy region, even if it is formulated in terms of different degrees of freedom, e.g. mesons and baryons instead of quarks and gluons.

If two theories agree on all their observables, they are said to be *dual* to each other.



If model A and model B agree on all physical observables in the region of interest, there is a duality between the models.

Many instances of duality in quantum field theory are known, especially in the field of supersymmetric gauge theories

Of particular interest to the problem of hadron spectroscopy in general, and glueballs in particular, is *gauge/gravity duality* (also known as holography, and a generalization of the AdS/CFT correspondence)

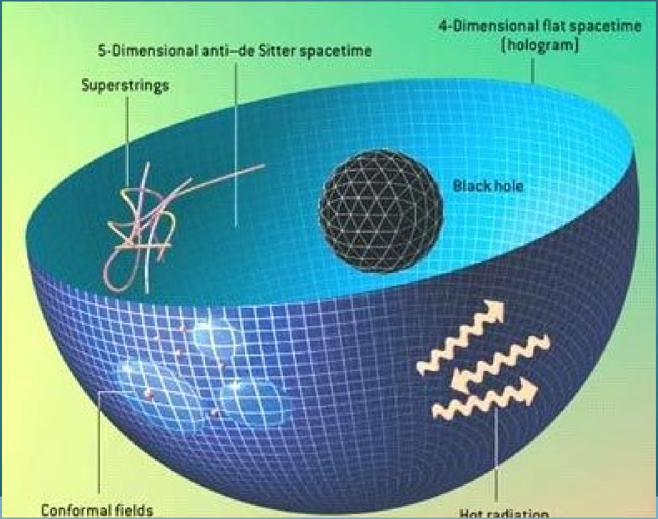
Gauge/gravity duality:

A duality between a gauge theory without gravity in d dimensions and a theory with gravity in at least $d + 1$ dimensions.

A realization of the holographic principle in quantum gravity, i.e., the idea that all physical information of a system (e.g., a black hole) is encoded in its surface.

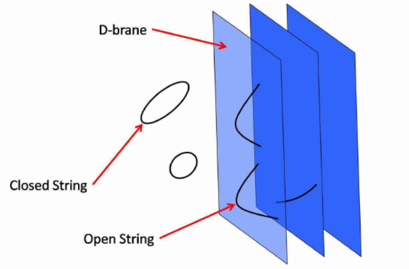
First concrete realization of gauge/gravity duality (due to Maldacena):

The *AdS/CFT correspondence* states that the low energy limit of type IIB string theory on an $\mathrm{AdS}_5 \times S^5$ space is dual to the large-*N* limit superconformal Yang Mills theory on the four-dimensional boundary of the geometry.


4-Dimensional flat spacetime
(hologram)
5-Dimensional anti–de Sitter spacetime
Superstrings
Black hole
Conformal fields
Hot radiation

Implications:

- Problems in strongly coupled quantum field theory can be solved in terms of weakly coupled gravity.
- The dynamics of the gauge theory is translated into interactions of metric fluctuations, as well as fields on the world-volume of D-branes (low energy effective actions)
- Still complicated, but much easier to solve than strongly coupled gauge theory!



- In supergravity approximation, the string picture disappears and the degrees of freedom are low energy effective fields, e.g. the metric and gauge fields
- Background geometry arises from stack of D3-branes

Problem: QCD is neither conformal nor supersymmetric, so the AdS/CFT correspondence is not immediately useful for formulating a theory of hadrons.

Solution: An alternate instance of the correspondence due to Witten, based on an $\mathrm{AdS}_7 \times S^4$ solution of 11-dimensional supergravity (M-theory) compactified on a circle in a supersymmetry-breaking way

Gravity side of the duality: geometry induced by a stack of N_c D4-branes. Fundamental degrees of freedom are fluctuations of the background metric obeying linearized Einstein equations.

Gauge side of the duality: five dimensional gauge-theory on $\mathbb{R}^{3,1} \times S^1$ that is effectively four-dimensional below a certain scale related to the size of the circle. Fundamental degrees of freedom are glueballs.

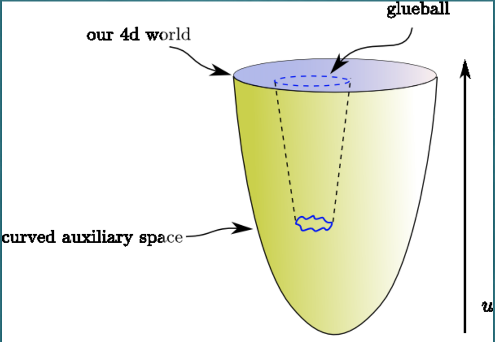
The gravity background is a solution to type IIA supergravity:

$$S_{\mathrm{IIA}} = S_{\mathrm{NS}} + S_{\mathrm{RR}} + S_{\mathrm{CS}},$$

$$S_{\mathrm{NS}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{g}e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H_3|^2\right),$$

$$S_{\mathrm{RR}} = -\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{g}\left(|F_2|^2 + |\tilde{F}_4|^2\right),$$

$$S_{\mathrm{CS}} = -\frac{1}{4\kappa_{10}^2}\int B_2 \wedge F_4 \wedge F_4$$



$$ds^2 = \frac{r^2}{L^2}\left(f(r)dx_4^2 + \eta_{\mu\nu}dx^\mu dx^\nu + dx_{11}^2\right) + \frac{L^2}{r^2}\frac{dr^2}{f(r)} + \frac{L^2}{4}d\Omega_4^2, \quad (1)$$

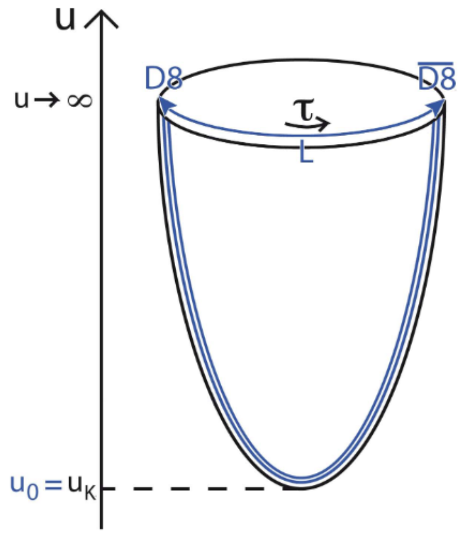
Strategy: linearize the metric as $g_{MN} \to g_{MN} + h_{MN}$ and solve eigenvalue problem for the modes

Results: there is a rich spectrum of glueballs of various quantum numbers, in particular there is a 0^{++} glueball with a mass of approximately 1.5 GeV, in close agreement with the lattice result, and close to the mass of the candidate states $f_0(1500)$ and $f_0(1710)$.

But what about interactions?

Quarks were introduced to Witten's gluonic background by Sakai and Sugimoto:

- Add stacks of N_f D8 and anti-D8-branes $\rightarrow$ pairs of quarks
- Geometric realization of chiral symmetry breaking $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$ by joining D8 and anti-D8 branes at the tip of the cigar-shape
- Free parameters (mass scale M_{KK} and 't Hoof coupling λ) are fixed to the experimental mass of the rho meson, as well as the pion decay constant/string tension from lattice


u
D8
$\overline{\text{D8}}$
$u \to \infty$
τ
L
$u_0 = u_K$

Low energy behaviour governed by DBI and CS actions on D8-branes:

$$S_{\mathrm{DBI}} = -T_8 \int d^9x \; e^{-\phi} \, \mathrm{Tr}\sqrt{\det\left(g_{MN} + \mathcal{F}_{MN}\right)}$$

$$S_{\mathrm{CS}} = iT_p \int \mathrm{Tr}\exp(B_2 + 2\pi\alpha' F_2) \wedge \sum_q C_q.$$

Identify the correct degrees of freedom:
vector mesons $\leftrightarrow A_\mu$, pseudoscalar mesons $\leftrightarrow A_r$

Integrate out extra-dimensions to obtain a four-dimensional effective action of hadrons at low energy (examples on next slide).

The important difference to conventional effective actions for QCD is that in this case, the actions are derived from an explicit duality relation, not from symmetry considerations

First important result: the chiral Lagrangian is accurately reproduced!

$$\mathcal{L}_{\text{chiral}} = \frac{f_\pi^2}{4} \mathrm{Tr} \left(U^{-1} \partial_\mu U \right)^2 .$$

The mass spectrum of vector mesons (ρ, a_1...) is in good agreement with experimental results.

Interactions between pions and rho-mesons:

$$\mathcal{L}_{\rho\pi\pi} = -g_{\rho\pi\pi}\epsilon_{abc}(\partial_\mu \pi^a)\rho^{b\mu}\pi^c$$

$$g_{\rho\pi\pi} \approx 33.98\lambda^{-1/2}N_c^{-1/2}$$

With $\lambda \approx 16.63 \ldots 12.55$ and $N_c = 3$ we obtain

$$\Gamma_\rho/m_\rho \approx 0.1535 \ldots 0.2034,$$

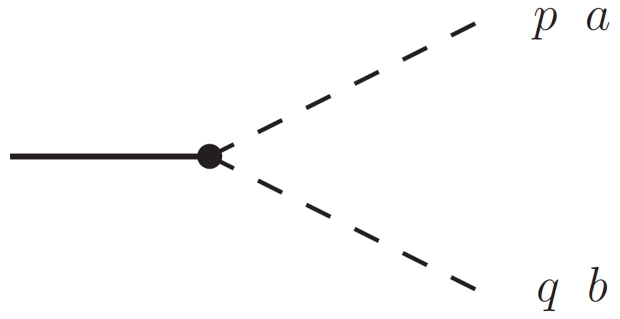
in agreement with the experimental value of $\Gamma_\rho/m_\rho = 0.191(1)$.

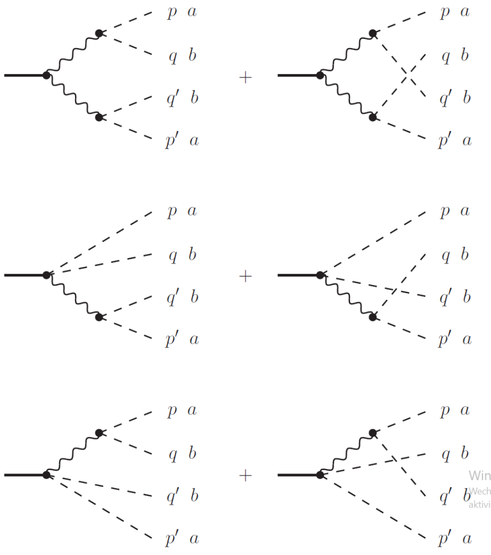
Interactions between glueballs and pions:

$$\mathcal{L}_{G\pi\pi} = \frac{1}{2}A * \partial_\mu \pi^a \partial_\nu \pi^a \left(\eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{M_G^2} \right) G + \frac{1}{2} B (\pi^a)^2 G$$

Decay into kaons is much stronger than that into pions, in agreement with data for the glueball candidate $f_0(1710)$:

$f_0(1710)$	exp.	prediction
$\frac{4}{3} \cdot \Gamma(\pi\pi)/\Gamma(K\bar{K})$	$0.55^{+0.15}_{-0.23}$	0.463
$4 \cdot \Gamma(\eta\eta)/\Gamma(K\bar{K})$	1.92 ± 0.60	1.12


p a
q b


p a
q b
q' b
p' a
+
p a
q b
q' b
p' a
p a
q b
q' b
p' a
+
p a
q b
q' b
p' a
p a
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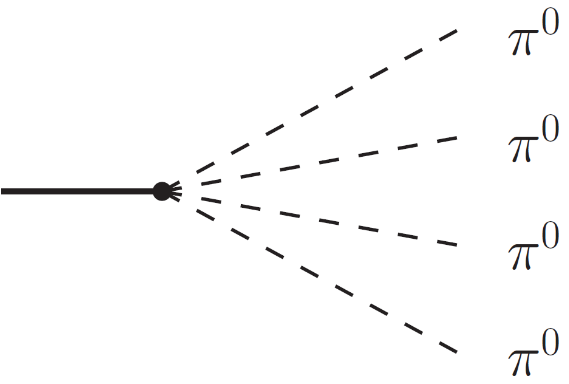
q' b
p' a
+
p a
q b
q' b
p' a



π^0
π^0
π^0
π^0

Additional features include

- A significant decay of the scalar glueball into 4π, to be confirmed by experiment
- Gluon condensate close to SVZ sum rule value
- A broad tensor glueball (2^{++}) above 2 GeV
- A broad pseudovector glueball (1^{+-})
- Specific decay patterns for the pseudoscalar glueball (0^{-+})

Word of warning:

The theory is similar to QCD at low energy but not identical: potential issues due to approximations (large N, λ), as well as the five-dimensional nature of the theory (low compactificiation scale)

Take-home message:

Gauge/gravity provides an interesting avenue for studying the low energy effective dynamics of gauge theories.

Despite all approximations and deviations from real-world QCD, surprisingly good agreement with experimental data and exciting predictions are achieved

In the meanwhile ...

GLUEBALL

A hypothetical particle composed solely of gluons, the **GLUEBALL** is held together because gluons carry charge color and experience the strong interaction. This glueball has eight eyes to represent the color octet, the eight color states proposed by Murray Gell-Mann.

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●○○○○○○○○○○○○
LIGHT HEAVY

GLUON PHOTON NEUTRINO TACHYON ELECTRON UP QUARK DOWN QUARK TAU NEUTRINO MUON UP Q
NEUTRON DOWN QUARK TAU GLUON **GLUEBALL** NEUTRINO TACHYON ELECTRON UP QUARK DOWN QU
NEUTRINO MUON UP QUARK PROTON NEUTRON DOWN QUARK TAU GLUON PHOTON NEUTRINO TACHY

The PARTICLE ZOO