Trace Anomalies, RG Flow and Scattering Amplitude

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Based on the papers

(1) arXiv: 2204.01786: Bootstrapping the a-anomaly in 4d QFTs, with **Denis Karateev** , **Jan Marucha** & **Joao Penedones.**

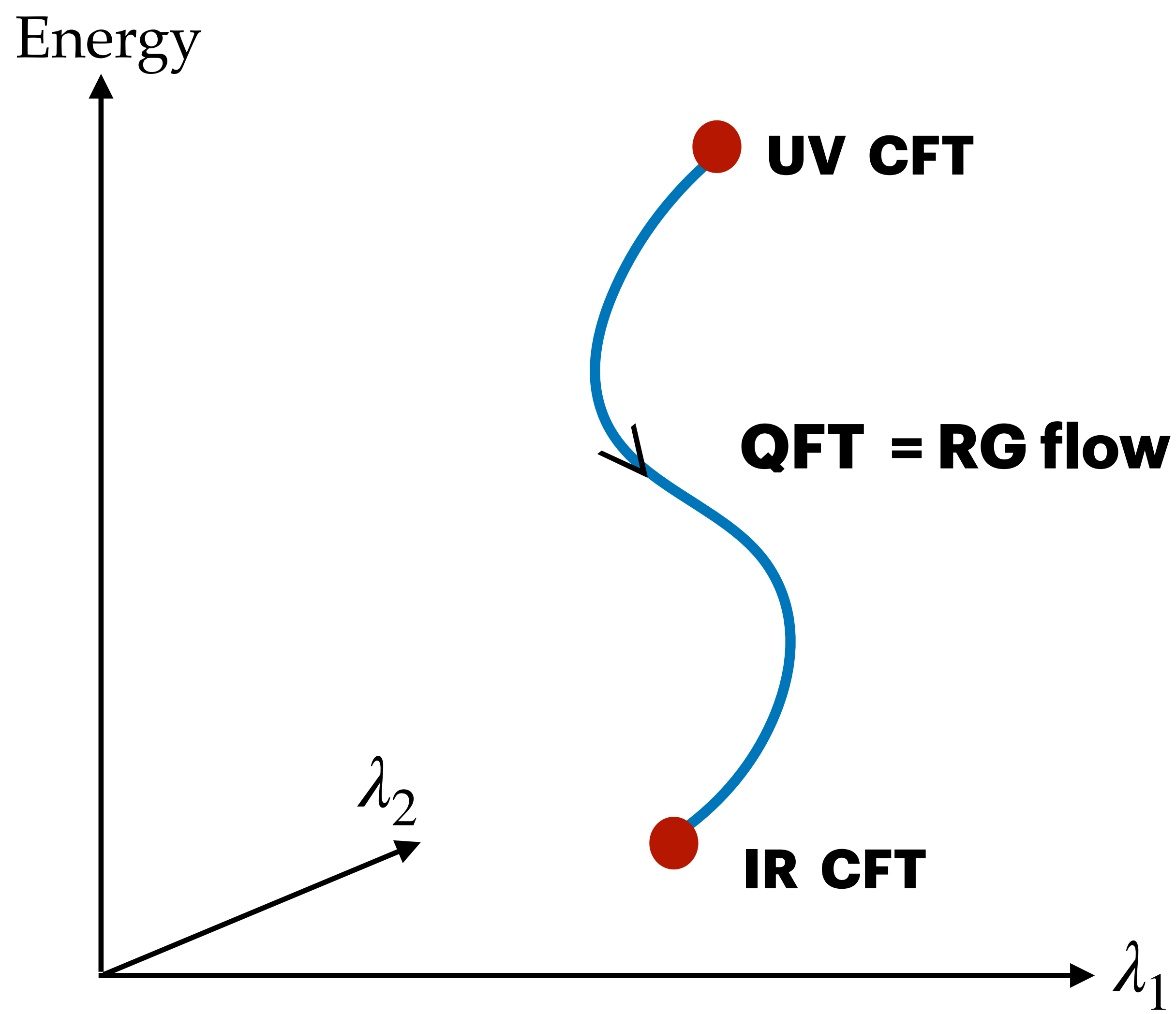
(2) arXiv: 2312.09308: Trace Anomalies and the Graviton-Dilaton Amplitude, with **Denis Karateev** , **Zohar Komargodski** & **Joao Penedones.**

(3) arXiv: 2402.xxxxx: Trace Anomalies in Perturbative Flows, with **Denis Karateev.**

Motivation

Why use scattering amplitude to study trace anomalies
along renormalization group (RG) flow?

Quantum Field Theories (QFTs) can be non-perturbatively defined as a renormalization group flow between UV fixed point and IR fixed point which are assumed to enjoy conformal symmetry.

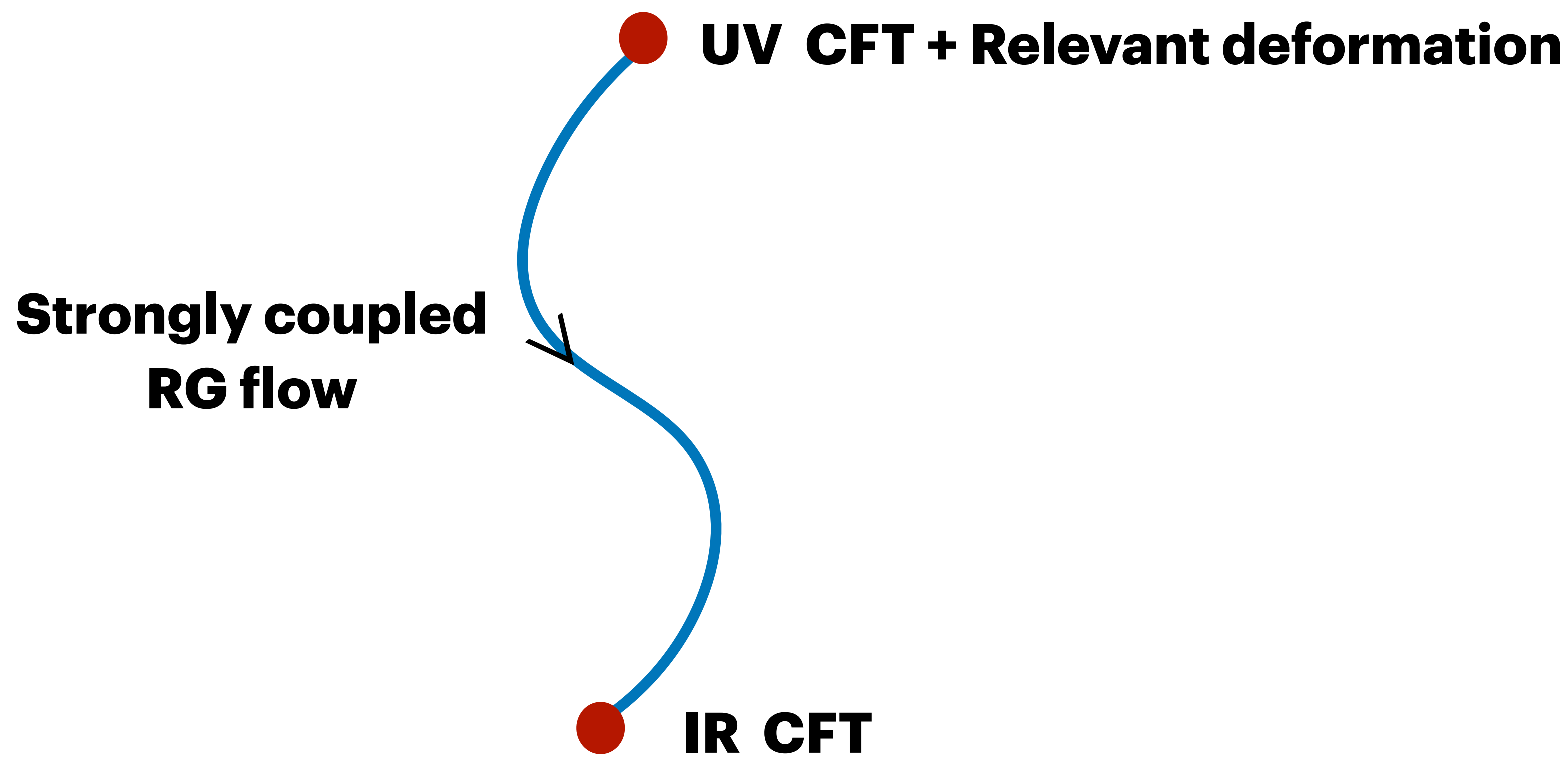


To specify a particular QFT it is sufficient to provide the UV CFT, and

(1) the relevant deformation triggering the RG flow in the explicit conformal symmetry breaking case,

OR

(2) the VEV of the scalar primary operator in the spontaneous symmetry breaking case.



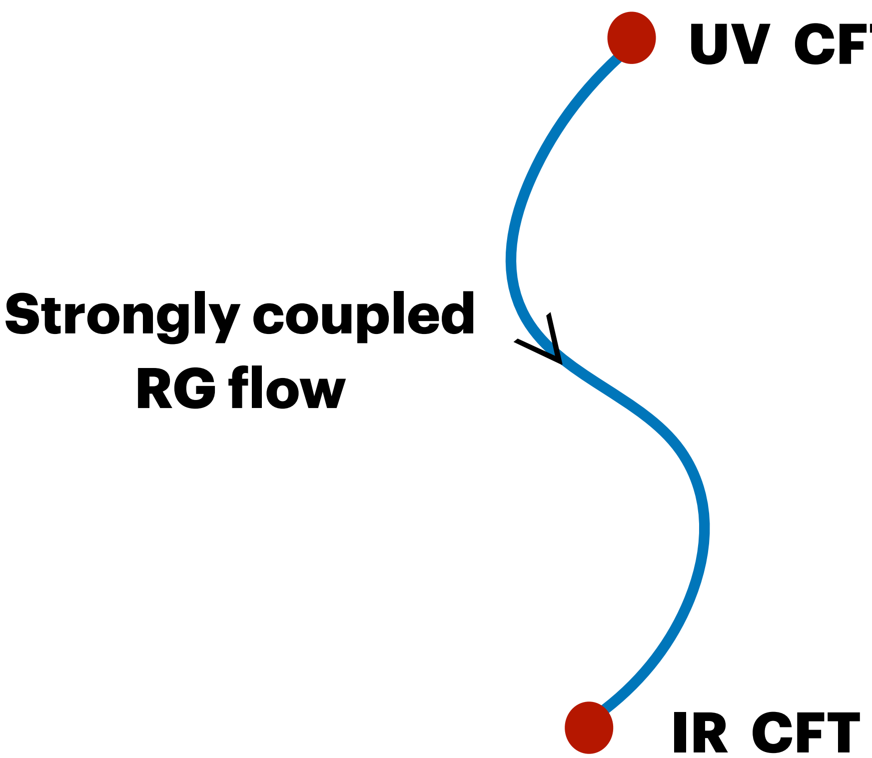
We can non-perturbatively study:

the UV fixed point using conformal bootstrap program,
(to put bounds on the allowed values of scaling dimensions, OPE coefficients,...)

and

the scattering amplitudes of light d.o.f. near IR fixed point using
S-matrix bootstrap program.
(to bound ratios of masses of stable particles, coupling constants, EFT parameters,...)

The parameters are different



QFT spectrum or scattering amplitudes at any energy scale can be computed using various numerical methods like lattice field theory, Hamiltonian truncation, tensor networks, etc [requires introduction of UV cut-off and costly extrapolation to continuum limit]

Very Hard

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the scattering amplitudes of light d.o.f. near IR fixed point using S-matrix bootstrap program.
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The parameters are different

Questions we like to ask: Can we identify a set of observables which are determined by the UV CFT and IR CFT data, and do not depend on the details of the RG flow (``protected")?

Then the CFT datas can be used as parameters of both the conformal and S-matrix bootstrap to derive non-perturbative bounds.

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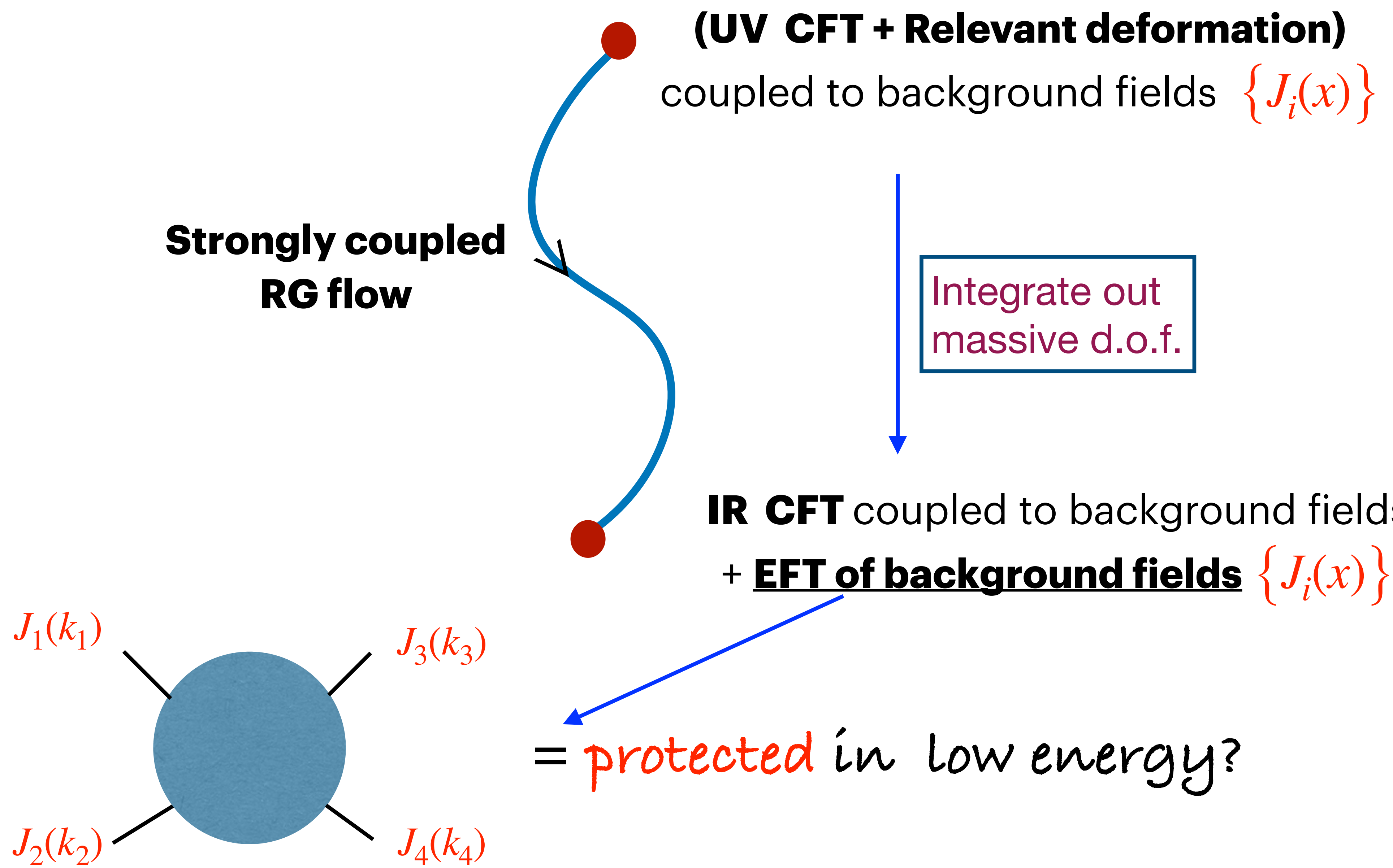

The answer we provide: Introduce two background fields → dilaton and graviton.

Dilaton compensate explicit conformal symmetry breaking near UV (or the Goldstone boson of SSB)

Graviton is the quanta of background metric (after providing dynamics) introduced such that the QFT in the curved background in classically Weyl invariant.

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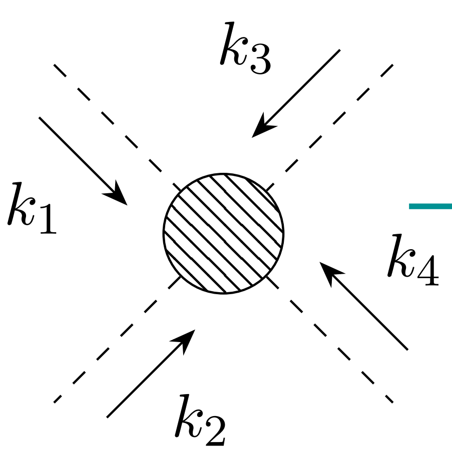
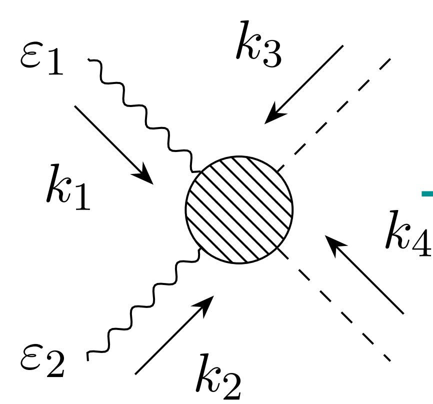


$\rightarrow \Delta a \times$ **(momenta)**^{4} ,

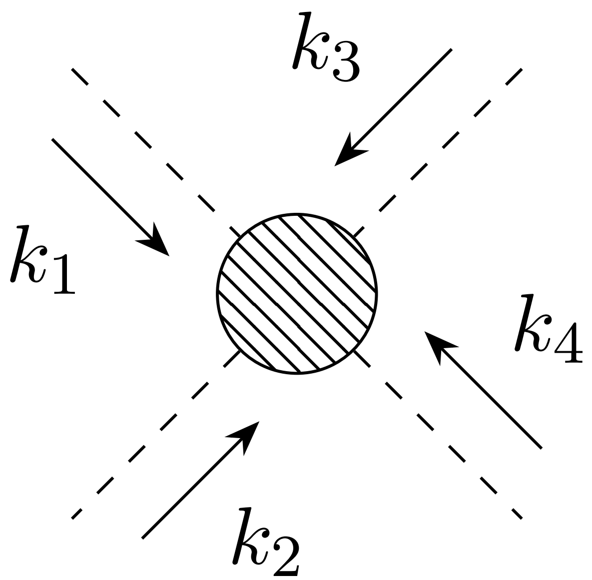


$\rightarrow (\Delta c - \Delta a) \times$ **(momenta)**^{4}

$$\Delta a \equiv a_{UV} - a_{IR} \qquad and \qquad \Delta c \equiv c_{UV} - c_{IR}$$

The 'a' and 'c' are trace/Weyl anomaly coefficients of CFT determined in terms of OPE coefficients of stress tensor correlation function.

Applications in 4d



Using unitarity proved $a-$theorem i.e. $a_{UV} \geq a_{IR}$ for any RG flow

Komargodski & Schwimmer

Derived lower bound on a_{UV} in the space of CFTs which can flow to a gapped QFT

Karateev, Marucha, Penedones, B.S.

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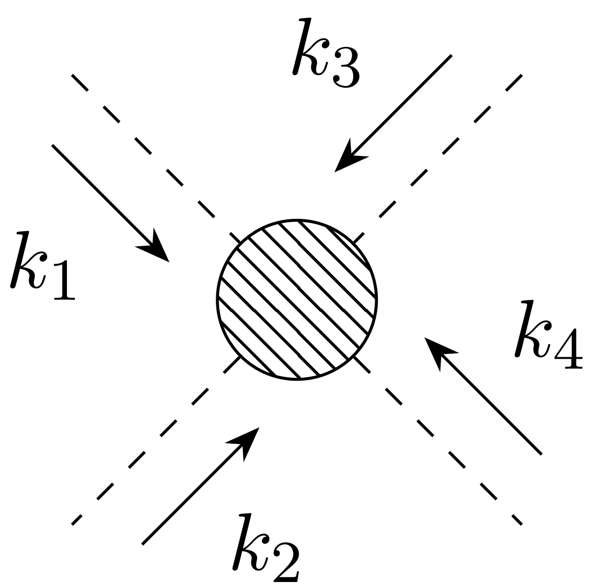

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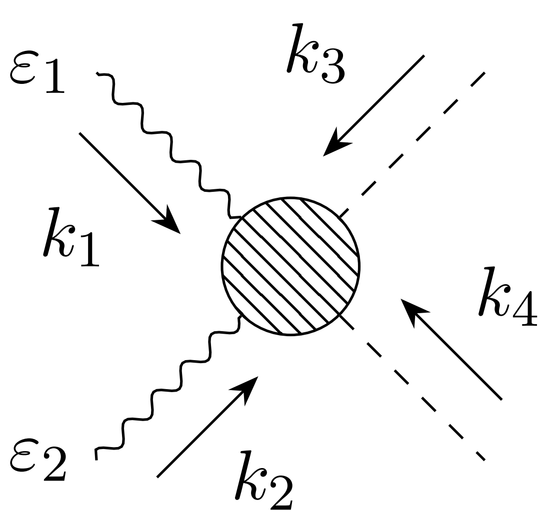

Shown how $(\Delta c - \Delta a)$ is related to the massive spinning states of the QFT

Karateev, Komargodski, Penedones, B.S.

Outline of the seminar

- Trace anomalies in CFT
- Proposal of background field method to probe trace anomalies along RG flow
- Testing the proposal in QFTs: free massive scalar and Dirac fermion, weakly relevant flow
- Dilaton-Dilaton and Graviton-Dilaton scattering amplitudes
- S-matrix Bootstrap applications
- Outlook for future

Trace anomalies in CFT

Based on:

Capper, Duff and Isham (1974-76)

Osborn and Petkos (1993)

CFT in curved spacetime and trace anomaly

CFT conformally coupled to background geometry → Invariant under Weyl transformation

Weyl transformations: $g_{\mu\nu}(x) \to e^{2\sigma(x)} g_{\mu\nu}(x)$ $\qquad \mathcal{O}_{\mu_1\ldots}(x) \to e^{-\Delta_{\mathcal{O}}\sigma(x)} \mathcal{O}_{\mu_1\ldots}(x)$

Parameter of Weyl transformation

Local primary operator

curved space CFT action

For a Weyl invariant theory, classically: $T^\mu{}_\mu(x) = 0\,,$ where $T^\mu_\mu(x) \equiv \frac{1}{\sqrt{-g}} \frac{\delta_W \boxed{A^g}}{\delta\sigma(x)}$

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Connected functional

$$e^{i\boxed{W[g_{\mu\nu}]}} = Z[g_{\mu\nu}] \equiv \int [d\phi]_g \, e^{iA^g[\phi, g_{\mu\nu}]}$$

Trace anomaly

Euler density $\qquad$ Weyl tensor square

$$\delta_W W[g_{\mu\nu}] = \int d^4x \sqrt{-g}\, \sigma(x) \, \langle 0 | T^\mu_{\ \mu}(x) | 0 \rangle_g = \int d^4x \sqrt{-g}\, \sigma(x) \big(-a \times E_4 + c \times \mathscr{W}^2 \big)$$

'a' and 'c' as OPE coefficients

In flat background: $\partial_\mu T^{\mu\nu}(x) = 0$ $\quad T^\mu_{\ \mu}(x) = 0$

Two and three point correlators:

Central charge

$$\langle 0 | T^{\mu\nu}(x_1) T^{\rho\sigma}(x_2) | 0 \rangle = \frac{C_T}{x_{12}^8} \times \mathbf{T}_0^{\mu\nu;\rho\sigma}$$

OPE coefficients

$$\langle 0 | T^{\mu\nu}(x_1) T^{\rho\sigma}(x_2) T^{\alpha\beta}(x_3) | 0 \rangle = \frac{1}{x_{12}^4 x_{23}^4 x_{31}^4} \left(\mathbb{A} \mathbf{T}_1^{\mu\nu;\rho\sigma;\alpha\beta} + \mathbb{B} \mathbf{T}_2^{\mu\nu;\rho\sigma;\alpha\beta} + \mathbb{C} \mathbf{T}_3^{\mu\nu;\rho\sigma;\alpha\beta} \right)$$

– $\mathbf{T}_I$ for $I = 0,1,2,3$ are known tensor structures.

a and c anomalies in terms of central charge and OPE coefficients :

$$a \equiv \frac{\pi^4}{64 \times 90} \left(9\mathbb{A} - 2\mathbb{B} - 10\mathbb{C} \right) \qquad c \equiv \frac{\pi^4}{64 \times 30} \left(14\mathbb{A} - 2\mathbb{B} - 5\mathbb{C} \right) = \frac{\pi^2}{64 \times 10} C_T$$

Proposal of background field method to probe trace anomalies along RG flow

Based on:

Fradkin and Tseytlin (1984)

Schwimmer and Theisen (2010)

Komargodski and Schwimmer (2011)

Luty, Polchinski and Rattazzi (2012)

Karateev, Komargodski, Penedones and B.S.

Dilaton as a conformal compensator

Weyl invariant

$$A_{\text{QFT}}[\phi, g_{\mu\nu}] = A_{\text{UV CFT}}[\phi, g_{\mu\nu}] + A_{\text{deformation}}(M_i)$$

$$A_{\text{deformation}}(M_i) = \sum_i \int d^4x \sqrt{-g} \left(\lambda_i M_i^{4-\Delta_i} \mathcal{O}_i(x) \right)$$

$\boxed{\Delta_i < 4}$

Weyl symmetry is explicitly broken due to $A_{\text{deformation}}$ which triggers the RG flow

$$T^\mu{}_\mu(x) = \sum_i \lambda_i (4 - \Delta_i) M_i^{4-\Delta_i} \mathcal{O}_i(x)$$

Dilaton as a conformal compensator

Restore the Weyl symmetry by introducing a compensator field $\Omega(x)$ and scaling all the mass parameters $M_i \to M_i(x) \equiv \Omega(x) M_i$

$$A_{\text{QFT}}^{\text{compensated}}[\phi, g_{\mu\nu}, \Omega] = A_{\text{UV CFT}}[\phi, g_{\mu\nu}] + A_{\text{deformation}}^{\text{compensated}}(M_i)$$

$$A_{\text{deformation}}^{\text{compensated}}(M_i) = \sum_i \int d^4x \sqrt{-g}\ \lambda_i \left(M_i \Omega(x)\right)^{4-\Delta_i} \mathcal{O}_i(x)$$

Under Weyl transformation $\quad \Omega(x) \to e^{-\sigma(x)} \Omega(x)\ .$

source for $T_\mu^\mu(x)$

$\Omega(x) = e^{-\tau(x)}$, where $\tau(x)$ is the **dilaton field** under Weyl transformation : $\tau(x) \to \tau(x) + \sigma(x)$

Trace anomaly in compensated QFT

Connected functional

$$e^{\boxed{iW[g_{\mu\nu},\Omega]}} = \int [d\phi]_g \, e^{A^{\text{compensated}}_{\text{QFT}}[\phi, g_{\mu\nu}, \Omega]}$$

$$\delta_W W[g_{\mu\nu}, \Omega] = \int d^4x \sqrt{-g}\,\sigma(x)\Big(-a_{UV} \times E_4 + c_{UV} \times \mathscr{W}^2 + \mathscr{A}_{\text{coupling space}}\Big)$$

$$\mathscr{A}_{\text{coupling space}} = \sum_{\substack{i \\ \Delta_i \in \mathbb{Z}_+}} \mathbf{c}_i^{UV} M_i^{8-2\Delta_i}\, \Omega(x)^{4-\Delta_i}\big(\Box^{\Delta_i - 2} + \dots\big)\, \Omega(x)^{4-\Delta_i}$$

Extra scale anomaly for integer dimension operators, won't be discussed in this talk

Trace anomaly matching and graviton-dilaton EFT

$$A_{IR}[\Phi, g_{\mu\nu}, \tau] = A_{IR\ CFT}[\Phi, g_{\mu\nu}] + A_{EFT}[\tau, g_{\mu\nu}] + \sum_{1\leq\Delta\leq 2} \lambda_\Delta \int d^4x\sqrt{-\hat{g}}\ M^{2-\Delta} R(\hat{g})\ \widehat{\mathbf{O}}_\Delta(x)$$

$+$ irrelevant terms

$$\boxed{\hat{g}_{\mu\nu}(x) \equiv e^{-2\tau(x)} g_{\mu\nu}(x) \qquad \widehat{\mathbf{O}}_\Delta(x) \equiv e^{\Delta\tau(x)}\mathbf{O}(x)}$$

$$\delta_W W[g_{\mu\nu}, \Omega] = \int d^4x\sqrt{-g}\,\sigma(x)\left(-a_{UV} \times E_4 + c_{UV} \times \mathscr{W}^2\right)$$

$$\Big\downarrow$$

$$\delta_W A_{EFT}[g_{\mu\nu}, \tau] = \int d^4x\sqrt{-g}\,\sigma(x)\left(-\Delta a \times E_4 + \Delta c \times \mathscr{W}^2\right)$$

$$\boxed{\Delta a \equiv a_{UV} - a_{IR} \qquad \textit{and} \qquad \Delta c \equiv c_{UV} - c_{IR}}$$

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$$\boxed{\widehat{g}_{\mu\nu}(x) \equiv e^{-2\tau(x)} g_{\mu\nu}(x) \qquad \widehat{\mathbf{O}}_\Delta(x) \equiv e^{\Delta\tau(x)}\mathbf{O}(x)}$$

Solution

$$A_{EFT}[\tau, g_{\mu\nu}] = -\Delta a \times A_a[\tau, g_{\mu\nu}] + \Delta c \times A_c[\tau, g_{\mu\nu}] + A_{invariant}[\widehat{g}_{\mu\nu}]$$

$$A_a[\tau, g_{\mu\nu}] = \int d^4x\sqrt{-g}\left(\tau E_4 + 4\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\partial_\mu\tau\partial_\nu\tau + 2(\partial\tau)^4 - 4(\partial\tau)^2\Box\,\tau\right)$$

$$A_c[\tau, g_{\mu\nu}] = \int d^4x\sqrt{-g}\,\tau\,\mathscr{W}^2$$

$$A_{invariant}[\widehat{g}_{\mu\nu}] = \int d^4x\ \sqrt{-\widehat{g}}\left(M^4\lambda + M^2 r_0\,\widehat{R} + r_1\,\widehat{R}^2 + r_2\,\widehat{W}^2 + r_3\,\widehat{E}_4\right)$$

Vertices to probe anomaly coefficients

$$e^{-\tau(x)} \equiv 1 - \frac{\varphi(x)}{\sqrt{2}f}$$

dilaton field

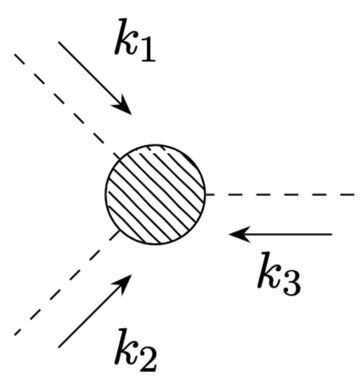
graviton field (traceless and transverse)

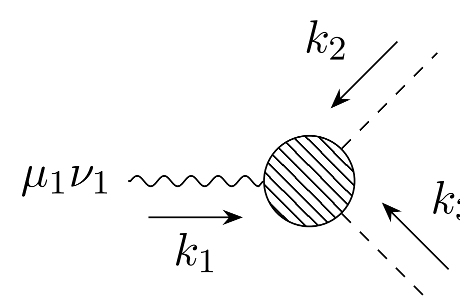
$$g_{\mu\nu}(x) \equiv \eta_{\mu\nu} + 2\kappa\, h_{\mu\nu}(x)$$

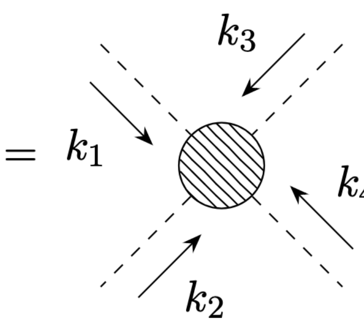
$\boxed{\textbf{graviton-dilation vertex}}$

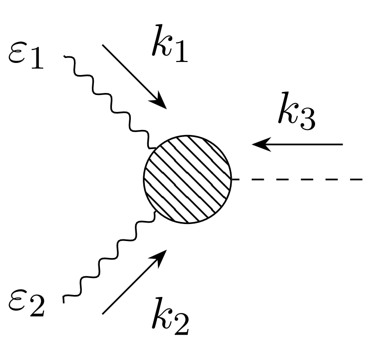
$$(2\pi)^4\delta^{(4)}(p_1 + \ldots + p_m + q_1 + \ldots + q_n) \times V^{\mu_1\nu_1,\ldots,\mu_m\nu_m}_{(h\ldots h\varphi\ldots\varphi)}(p_1,\ldots,p_m,q_1,\ldots,q_n)$$

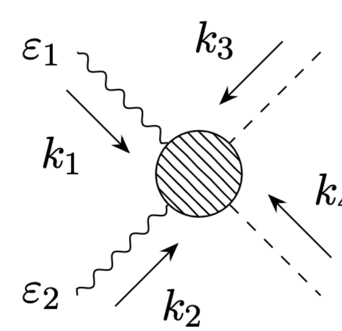
$$\equiv \left. \frac{i\,\delta^{m+n} A_{EFT}[\tau, g_{\mu\nu}]}{\delta h_{\mu_1\nu_1}(p_1)\ldots\delta h_{\mu_m\nu_m}(p_m)\delta\varphi(q_1)\ldots\delta\varphi(q_n)} \right|_{h,\varphi=0}$$

$V_{(\varphi\varphi\varphi)}(k_1,k_2,k_3) =$ 

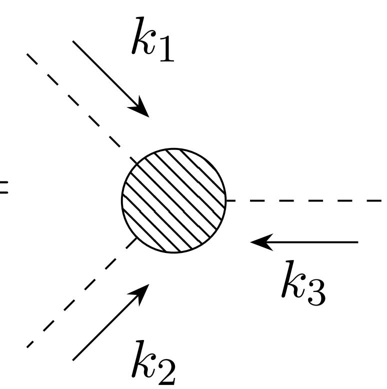
$V^{\mu_1\nu_1}_{(h\varphi\varphi)}(k_1,k_2,k_3) =$ 

$V_{(\varphi\varphi\varphi\varphi)}(k_1,k_2,k_3,k_4) =$ 

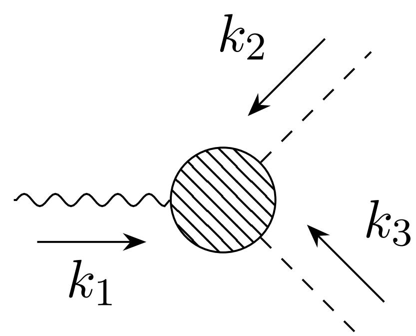
$V_{(hh\varphi)}(k_1,k_2,k_3;\varepsilon_1,\varepsilon_2) =$ 

$V_{(hh\varphi\varphi)}(k_1,k_2,k_3,k_4;\varepsilon_1,\varepsilon_2) =$ 

3-dilaton and 1-graviton-2-dilaton vertices

$V_{(\varphi\varphi\varphi)}(k_1,k_2,k_3) =$ 

$$= \frac{i\sqrt{2}}{f^3}\Bigg(\Delta a \left(\left(k_1^2\right)^2 + \left(k_2^2\right)^2 + \left(k_3^2\right)^2\right) + 2(18r_1 - \Delta a)\left(k_1^2 k_2^2 + k_2^2 k_3^2 + k_3^2 k_1^2\right) + \ldots\Bigg)$$

$V^{\mu_1\nu_1}_{(h\varphi\varphi)}(k_1,k_2,k_3) =$ $\mu_1\nu_1$ 

$$= \frac{i\kappa}{f^2}\eta^{\mu\nu}\Big[-12r_1 k_1^2 k_1.k_2 + (2\Delta a - 12r_1)k_1^2 k_2^2 + 36r_1(k_2^2)^2 - 6r_1(k_1^2)^2\Big]$$

$$-\frac{i\kappa}{f^2}k_2^\mu k_2^\nu\Big[(4\Delta a + 72r_1)k_1^2 + 144r_1(k_2^2 + k_1.k_2)\Big]$$

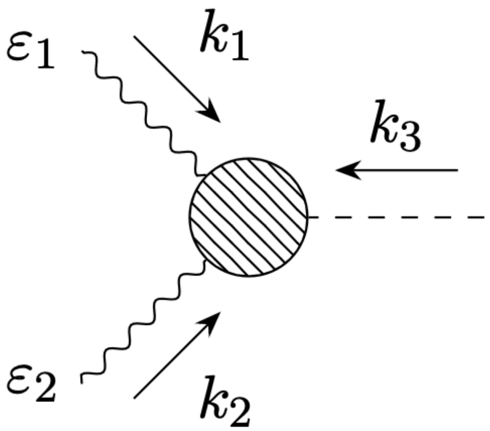
$(\partial_{k_{2u}} - \partial_{k_{3u}})^2\vec{\partial}_{k_2}\cdot\vec{\partial}_{k_3}\ V^{uu}_{(h\varphi\varphi)}$ $\longrightarrow$ related to Hartman-Mathys ANEC sum-rule

graviton-graviton-dilaton vertex

$$(\varepsilon_1 . \varepsilon_2) \equiv \varepsilon_{1\mu\nu}\varepsilon_2^{\mu\nu}$$

$$(k_i . \varepsilon_j . k_k) \equiv k_{i\mu}\varepsilon_j^{\mu\nu}k_{k\nu}$$

$$(k_i . \varepsilon_1 . \varepsilon_2 . k_j) \equiv k_{i\mu}\varepsilon_1^{\mu\rho}\varepsilon_{2\rho\nu}k_j^{\nu}$$



$$= f_1(k_1,k_2)\times(\varepsilon_1 . \varepsilon_2) + f_2(k_1,k_2)\times(k_1 . \varepsilon_2 . k_1)(k_2 . \varepsilon_1 . k_2) + f_3(k_1,k_2)\times(k_1 . \varepsilon_2 . \varepsilon_1 . k_2)$$

at four power in momenta:

$$f_1(k_1,k_2) = \frac{4i\kappa^2}{\sqrt{2}f}\Big(2(-\Delta a + \Delta c + 18r_1)(k_1 . k_2)^2 + (2\Delta a - \Delta c + 24r_1)k_1^2k_2^2 + 12r_1(k_1^4 + k_2^4) + 42r_1(k_1 . k_2)(k_1^2 + k_2^2)\Big)$$

$$f_2(k_1,k_2) = \frac{8i\kappa^2}{\sqrt{2}f}\,(-\Delta a + \Delta c)$$

$$f_3(k_1,k_2) = \frac{8i\kappa^2}{\sqrt{2}f}\,\Big(2(\Delta a - \Delta c - 6r_1)(k_1 . k_2) - 6r_1(k_1^2 + k_2^2)\Big)$$

Testing the proposal in QFTs

Based on:

Cappeli and Latorre (1989)

Klebanov, Pufu and Safdi (2011)

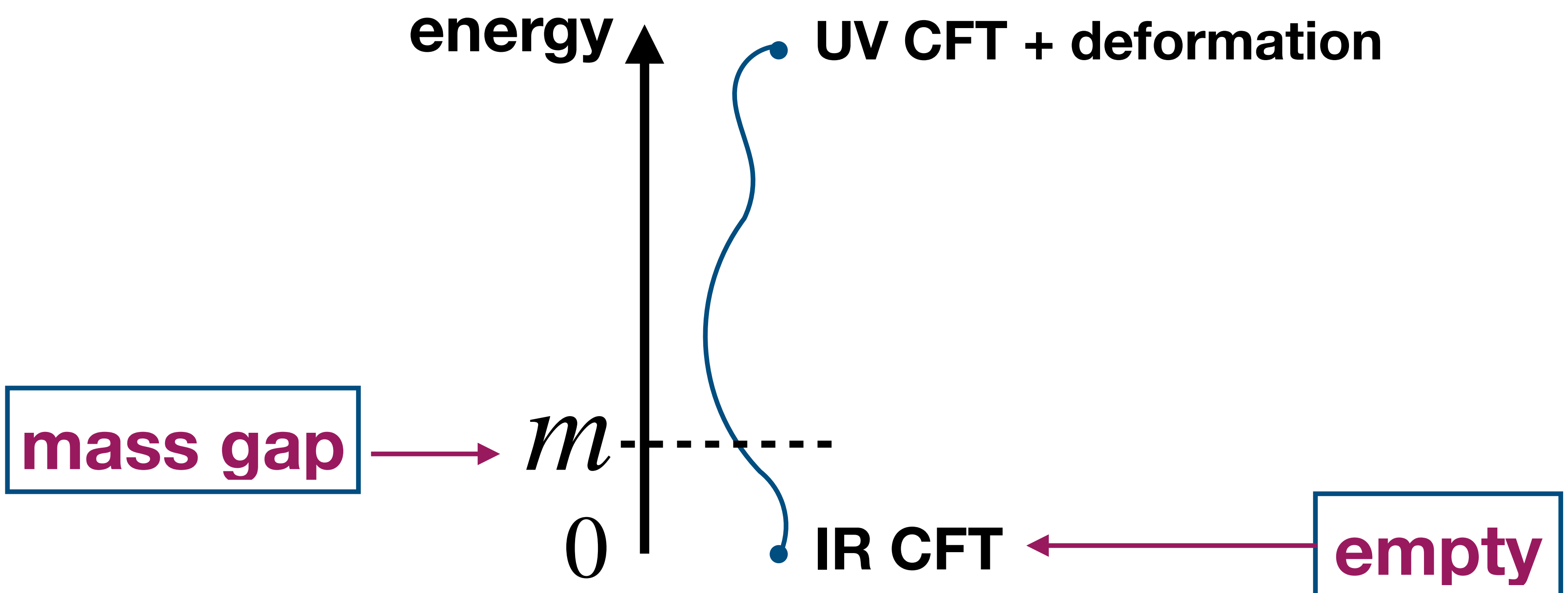
Komargodski (2011)

Karateev, Komargodski, Penedones and B.S.

Karateev and B.S. (to appear)

Example 1: Free massive scalar

$$a_{UV} = \frac{1}{5760\pi^2}, \quad c_{UV} = \frac{3}{5760\pi^2}$$



$$a_{IR} = 0, \quad c_{IR} = 0$$

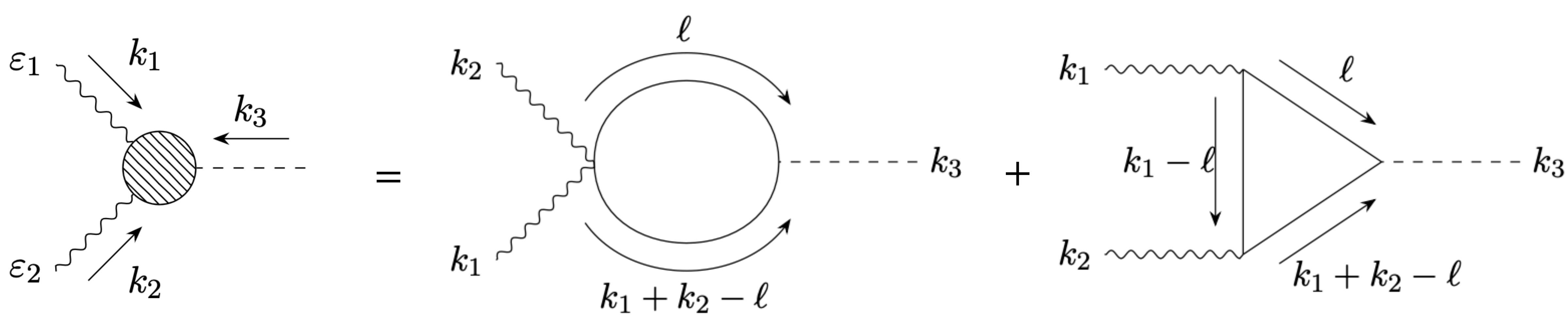
$$A_{QFT} = \int d^4x \left(\underbrace{-\frac{1}{2}\big(\partial\Phi(x)\big)^2}_{\text{UV CFT}} \underbrace{-\frac{1}{2}m^2\Phi(x)^2}_{\text{deformation}} \right)$$

Example 1: Free massive scalar

$$A_{QFT}^{compensated}[\Phi, g_{\mu\nu}, \tau] = \int d^dx\, \sqrt{-g}\left(\boxed{-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{d-2}{8(d-1)}R\Phi^2} - \frac{1}{2}\boxed{m^2e^{-2\tau}}\Phi^2\right)$$

conformally coupled to background metric

compensation of explicit breaking



Low energy expansion:

$$f_1(k_1, k_2) = \frac{i\kappa^2}{1440\sqrt{2}\pi^2 f}\left(2(k_1^2)^2 + 2(k_2^2)^2 + 10(k_1 \,.\, k_2)^2 + 7k_1 \,.\, k_2(k_1^2 + k_2^2) + 3k_1^2k_2^2\right)$$

$$f_2(k_1, k_2) = +\frac{i\kappa^2}{360\sqrt{2}\pi^2 f}$$

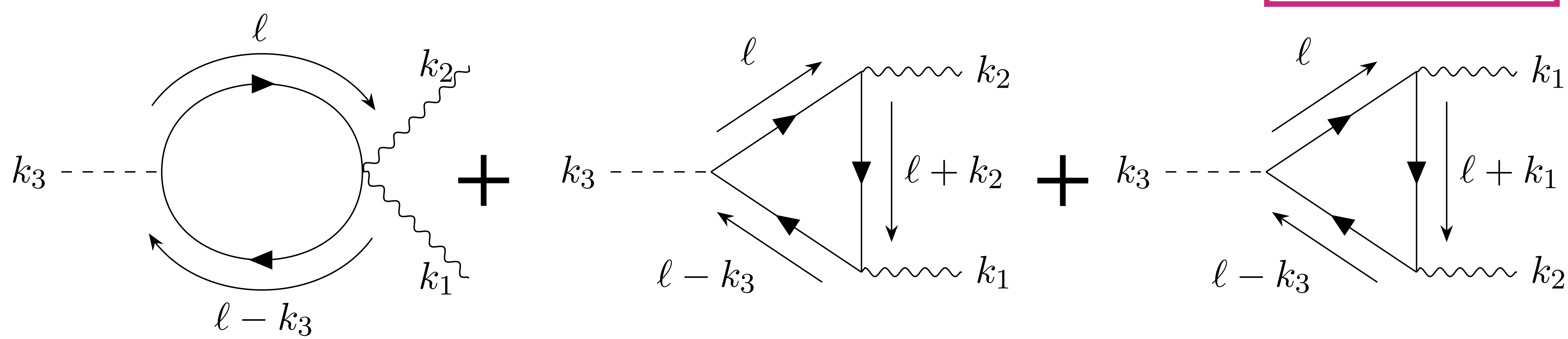
$$f_3(k_1, k_2) = -\frac{i\kappa^2}{720\sqrt{2}\pi^2 f}\left(k_1^2 + k_2^2 + 6(k_1 \,.\, k_2)\right)$$

$$\boxed{\Delta a = \frac{1}{5760\pi^2}, \qquad \Delta c = 3\Delta a, \qquad r_1 = \frac{\Delta a}{6}}$$

Example II: Free massive Dirac fermion

$$A_{QFT}^{compensated}[\Psi, g_{\mu\nu}, \tau] = \int d^d x \sqrt{-g}\ \overline{\Psi}(x)\left(i\gamma^a E_a^\mu D_\mu - \boxed{m e^{-\tau(x)}}\right)\Psi(x)$$

deformation + compensation

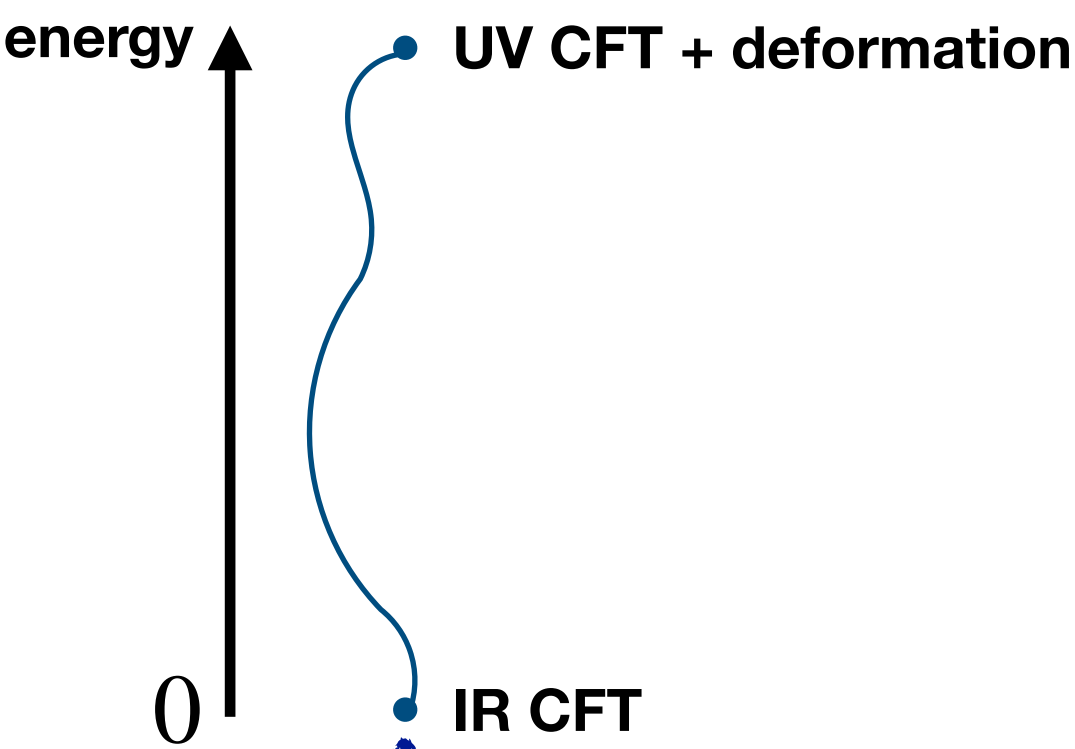


Low energy expansion:

$$\begin{aligned} & V_{(hh\varphi)}(k_1, k_2, k_3; \varepsilon_1, \varepsilon_2) \\ &= \frac{i\kappa^2}{720\sqrt{2}\pi^2 f}\Big[(\varepsilon_1 \cdot \varepsilon_2)\left\{14 k_1^2 k_2^2 + 6\left((k_1^2)^2 + (k_2^2)^2\right) + 25(k_1.k_2)^2 + 21 k_1.k_2(k_1^2 + k_2^2)\right\} \\ &\qquad -(k_2.\varepsilon_1.\varepsilon_2.k_1)\ \left\{6(k_1^2 + k_2^2) + 26 k_1.k_2\right\} + 7(k_2.\varepsilon_1.k_2)(k_1.\varepsilon_2.k_1)\Big] \end{aligned}$$

$$\boxed{\Delta a = \frac{11}{5760\pi^2}, \qquad \Delta c = \frac{18}{5760\pi^2}, \qquad r_1 = \frac{1}{5760\pi^2}.}$$

Example III: weakly relevant flow



$$A_{QFT} = A_{UV\,CFT} + \lambda_0 m_0^{4-\Delta} \int d^4x \; \mathcal{O}(x)$$

short flow if $\Delta = 4 - \delta \qquad 0 < \delta \ll 1$

perturbative parameter λ_0

"renormalization" at scale μ

$\lambda(\mu) \to$ renormalized coupling

$Z(\mu)^{-\frac{1}{2}} \; \mathcal{O}(x) \equiv \mathbf{O}(x) \to$ renormalized operator

$$\beta(\lambda) = -\,\delta\lambda + \frac{1}{2} C_{\mathcal{O}\mathcal{O}\mathcal{O}} \Omega_3 \lambda^2 + O(\lambda^3)$$

OPE coefficient

$Vol(\mathbb{S}^3) = 2\pi^2$

$$\gamma(\lambda) = C_{\mathcal{O}\mathcal{O}\mathcal{O}} \Omega_3 \lambda + O(\lambda^2)$$

$$\beta(\lambda_\star) = 0$$

$$\lambda_\star = \frac{2\delta}{C_{\mathcal{O}\mathcal{O}\mathcal{O}} \Omega_3}$$

Example III: weakly relevant flow

(Introducing background fields in 4d Euclidean QFT)

$$A_{\text{QFT}}^{\text{compensated}} = A_{\text{UV CFT}}[\phi, g_{\mu\nu}] + \lambda_0 \int d^4x \sqrt{-g}\, \left(m_0 \Omega(x)\right)^{\delta}\ \mathcal{O}_g(x)$$

$$A_{\text{UV CFT}} + \kappa \int d^4x h_{\mu\nu}(x) T^{\mu\nu} x + O(\kappa^2)$$

$$\Omega(x) = 1 - \frac{\varphi(x)}{\sqrt{2} f}$$

In the renormalized theory it demands: $\mu^{\delta}\lambda(\mu) \longrightarrow \mu^{\delta}\lambda\left(\Omega(x)^{-1}\mu\right)$

$$\lambda\left(\Omega(x)^{-1}\mu\right) = \lambda(\mu) + \frac{1}{\sqrt{2}f}\varphi(x)\beta(\lambda) + O(f^{-2})$$

Example III: weakly relevant flow

$$A_{\text{QFT}}^{\text{compensated}} = A_{\text{UV CFT}}[\phi, g_{\mu\nu}] + \lambda_0 \int d^4x \sqrt{g}\, \left(m_0 \Omega(x)\right)^{\delta}\, \mathcal{O}_g(x)$$

$$A_{\text{UV CFT}} + \kappa \int d^4x h_{\mu\nu}(x) T^{\mu\nu}(x) + O(\kappa^2)$$

$$\Omega(x) = 1 - \frac{\varphi(x)}{\sqrt{2} f}$$

$$\boxed{A_{\text{EFT}}[\varphi, h] = -\log \int [d\phi]_g\, e^{-A_{\text{QFT}}^{\text{compensated}}}}$$

$$A_{\text{EFT}}[\varphi, h] = -\frac{1}{4\sqrt{2} f^3}\, \delta^2 \left(m_0^{\delta} \lambda_0\right)^2 \int d^d x_1 \int d^d x_2\ \varphi(x_2)^2 \varphi(x_1) \times \langle \mathcal{O}(x_{12}) \mathcal{O}(0) \rangle_{\text{QFT}} \rightarrow V_{(\varphi\varphi\varphi)}$$

$$A_{\text{EFT}}[\varphi, h] = \frac{\kappa^2}{2\sqrt{2} f} (\delta m_0^{\delta} \lambda_0) \int d^d x_1 \int d^d x_2 \int d^d x_3\ h_{\mu\nu}(x_1) h_{\rho\sigma}(x_2) \varphi(x_3) \times \langle T^{\mu\nu}(x_1) T^{\rho\sigma}(x_2) \mathcal{O}(x_3) \rangle_{\text{QFT}} \rightarrow V_{(hh\varphi)}$$

Example III: weakly relevant flow

(computation of Δa)

Four derivative part of the relevant EFT action:

$$A_{\mathrm{EFT}}[\varphi, h] = -\frac{1}{4\sqrt{2} f^3} \int d^d x \; \varphi(x)^2 \partial_\mu \partial_\nu \partial_\rho \partial_\sigma \varphi(x) \, L_1^{\mu\nu\rho\sigma}(\delta)$$

$$L_1^{\mu\nu\rho\sigma}(\delta) \equiv \frac{1}{24} \delta^2 \left(m_0^\delta \lambda_0 \right)^2 \int d^d y \; y^\mu y^\nu y^\rho y^\sigma \, \langle \mathcal{O}(y) \mathcal{O}(0) \rangle_{\mathrm{QFT}}$$

$V_{(\varphi\varphi\varphi)}$

$$\boxed{\Delta a = \frac{\delta^3}{2304 \pi^2 C_{\mathcal{O}\mathcal{O}\mathcal{O}}^2} \left(1 + O(\lambda^2)\right), \qquad r_1 = \frac{\Delta a}{18}}$$

Agrees with the Δa result of Klebanov, Pufu and Safdi (2011) from free energy computation on $\mathbb{S}^4$.

Solution of Callan-Symanzik equation + conformal perturbation theory

$$\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_{\mathrm{QFT}} = \frac{1 - \frac{4\lambda(\mu)}{\lambda_\star}}{r^{2(d-\delta)}} \times \left(\frac{\lambda_\star}{\lambda_\star + \left((\mu r)^\delta - 1 \right) \lambda(\mu)} \right)^4$$

Example III: weakly relevant flow

(computation of Δc)

conformal perturbation theory

$$\langle \mathbf{T}^{\mu\nu}(x_1)\mathbf{T}^{\rho\sigma}(x_2)\rangle_{\text{QFT}} = \langle T^{\mu\nu}(x_1)T^{\rho\sigma}(x_2)\rangle_{\text{UV CFT}} - m_0^\delta \lambda_0 \int d^d x_3 \langle T^{\mu\nu}(x_1)T^{\rho\sigma}(x_2)\mathcal{O}(x_3)\rangle_{\text{UV CFT}} + O(\lambda_0^2)$$

Computation at IR fixed point

$$\boxed{\frac{640}{\pi^2}\times c_{UV}}$$

$$\langle T^{\mu\nu}(x_1)T^{\rho\sigma}(x_2)\rangle = \boxed{C_T} \times \frac{\mathcal{I}^{\mu\nu,\rho\sigma}(x_{12})}{\left(x_{12}^2\right)^d},$$

$$\langle T^{\mu\nu}(x_1)T^{\rho\sigma}(x_2)\mathcal{O}(x_3)\rangle = C_{TT\mathcal{O}} \times \frac{\mathcal{I}^{\mu\nu,\alpha\beta}(x_{13})\mathcal{I}^{\rho\sigma,\gamma\delta}(x_{23})t_{\alpha\beta,\gamma\delta}(X_{12})}{\left(x_{12}^2\right)^{\frac{2d-\Delta}{2}}\left(x_{23}^2\right)^{\frac{\Delta}{2}}\left(x_{31}^2\right)^{\frac{\Delta}{2}}}$$

$$\boxed{\Delta c = \frac{\pi^2}{2304}\frac{C_{TT\mathcal{O}}}{C_{\mathcal{O}\mathcal{O}\mathcal{O}}}\delta}$$

Compute $V_{(hh\varphi)}$

Four derivative part of $A_{\text{EFT}}(\varphi, h)$ involving $\langle TT\mathcal{O}\rangle$

Dilaton-Dilaton and Graviton-Dilaton scattering amplitudes

Based on:

Komargodski and Schwimmer (2011)

Karateev, Komargodski, Penedones and B.S.

Providing dynamics to graviton and dilaton

$$A^{\varphi}_{kinetic} = -\frac{f^2}{6}\int d^4x\sqrt{-\widehat{g}}\ R(\widehat{g})$$

$$= \int d^4x\sqrt{-g}\ \left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{f^2}{6}R + \frac{\sqrt{2}f}{6}R\varphi - \frac{1}{12}R\varphi^2\right]$$

$$A^{h}_{kinetic} = \left(\frac{1}{2\kappa^2} + \frac{f^2}{6}\right)\int d^4x\sqrt{-g}\ R$$

Weyl symmetry is broken in the Planck scale κ^{-1} with the **decoupling limit**: $\kappa \to 0, \qquad f \to \infty, \qquad \kappa \ll \frac{1}{f}$

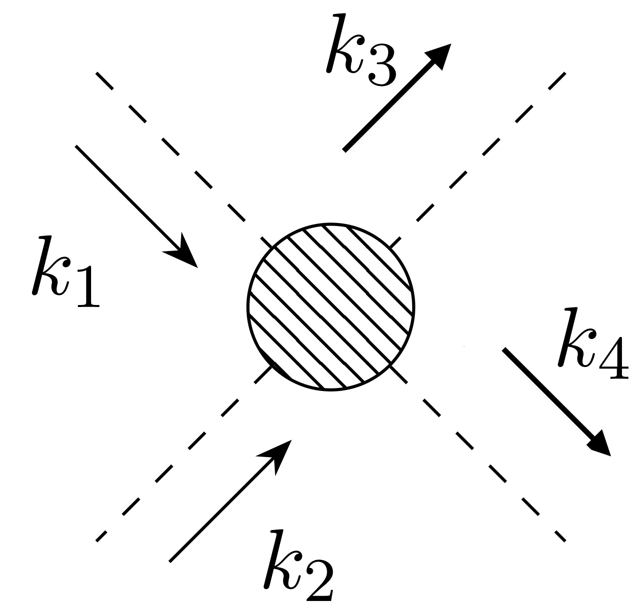
Compute scattering amplitudes for the given action:

$$A = A^{\varphi}_{kinetic} + A^{h}_{kinetic} + A_{EFT}[\tau, g_{\mu\nu}] + \sum_{1\leq\Delta\leq 2}\lambda_\Delta\int d^4x\sqrt{-\widehat{g}}\ M^{2-\Delta}R(\widehat{g})\ \widehat{\mathbf{O}}_\Delta(x)$$

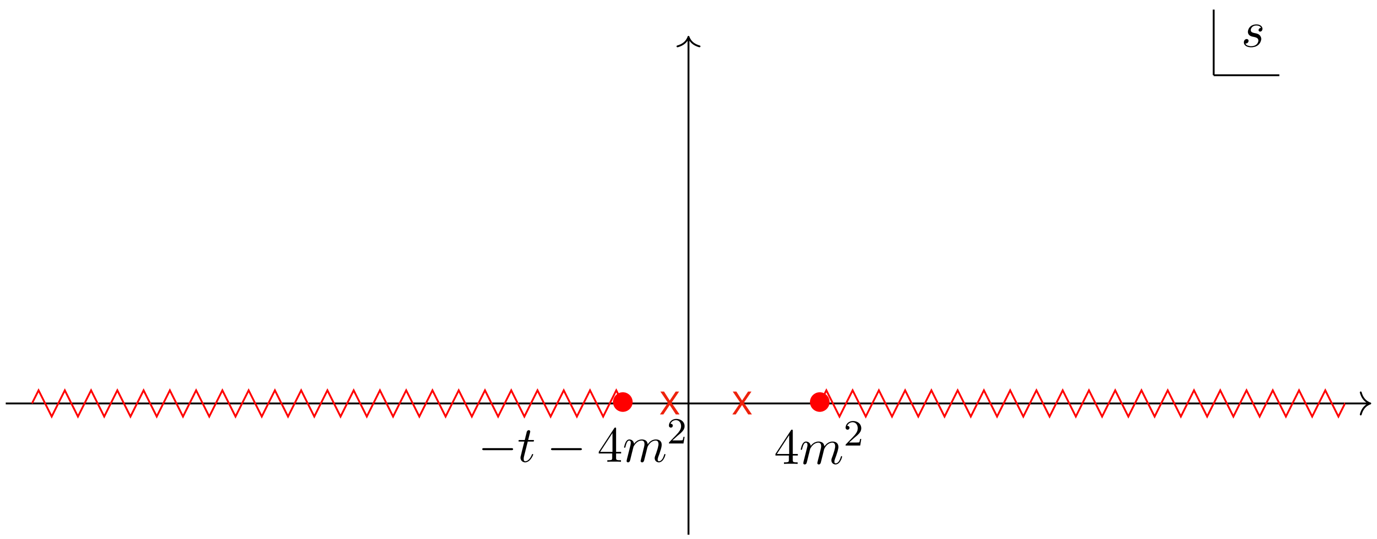
Four dilaton amplitude

At leading order in decoupling limit:

$$\mathcal{T}_{\varphi\varphi\to\varphi\varphi}(s,t,u) = \frac{\Delta a}{f^4}(s^2+t^2+u^2) + \cdots$$



$$s = -(k_1+k_2)^2$$
$$t = -(k_1-k_3)^2$$
$$u = -(k_1-k_4)^2$$



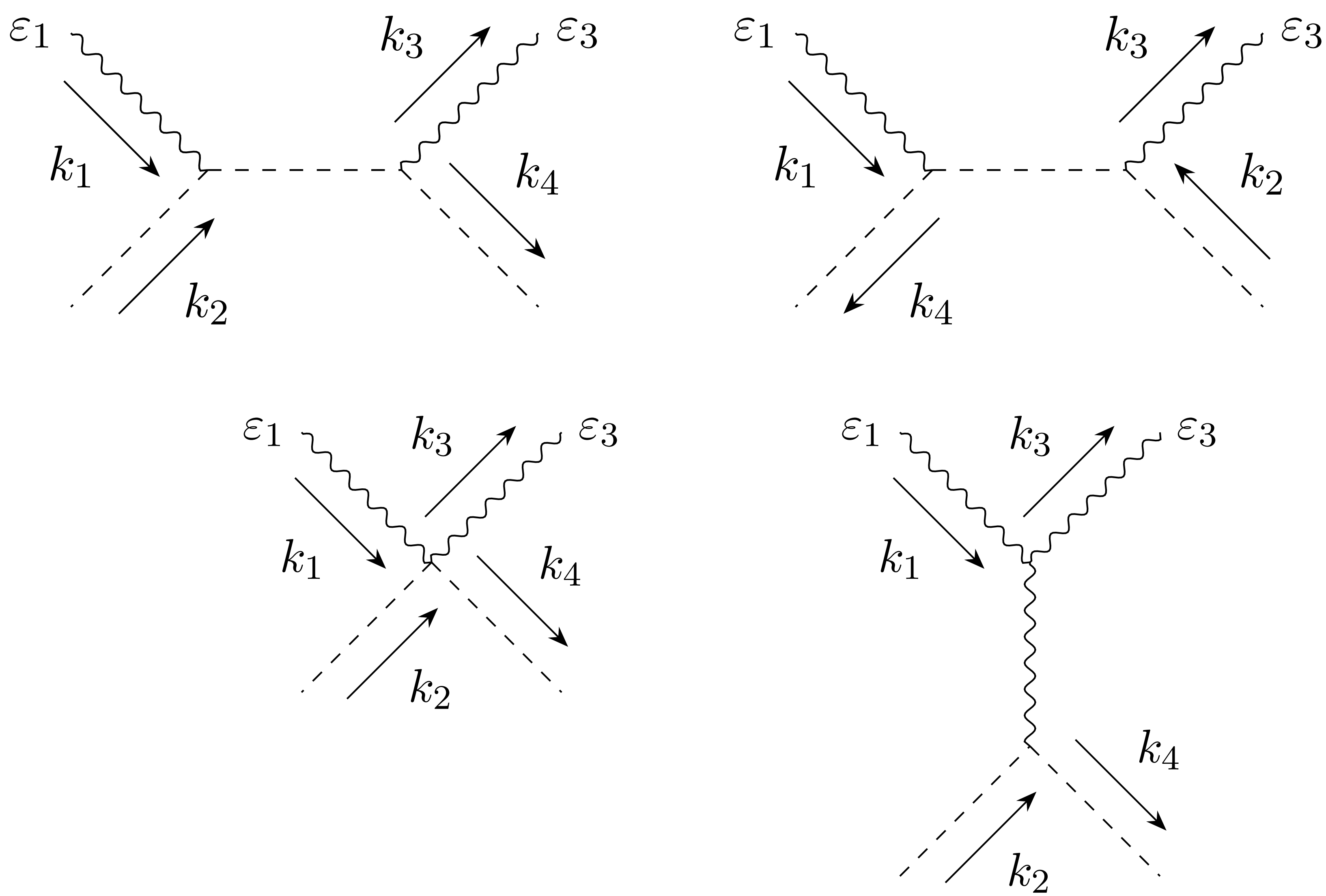
Dispersion relation with assumption $\lim_{|s|\to\infty}\frac{\mathcal{T}_{\varphi\varphi\to\varphi\varphi}(s,0,-s)}{s^2}=0:$

$$\boxed{\Delta a = f^4\int_{s>0}\frac{ds}{\pi}\frac{\textbf{Im}\,\mathcal{T}_{\varphi\varphi\to\varphi\varphi}(s,0,-s)}{s^3}\geq 0}$$

$a-$theorem

Graviton-dilaton amplitude

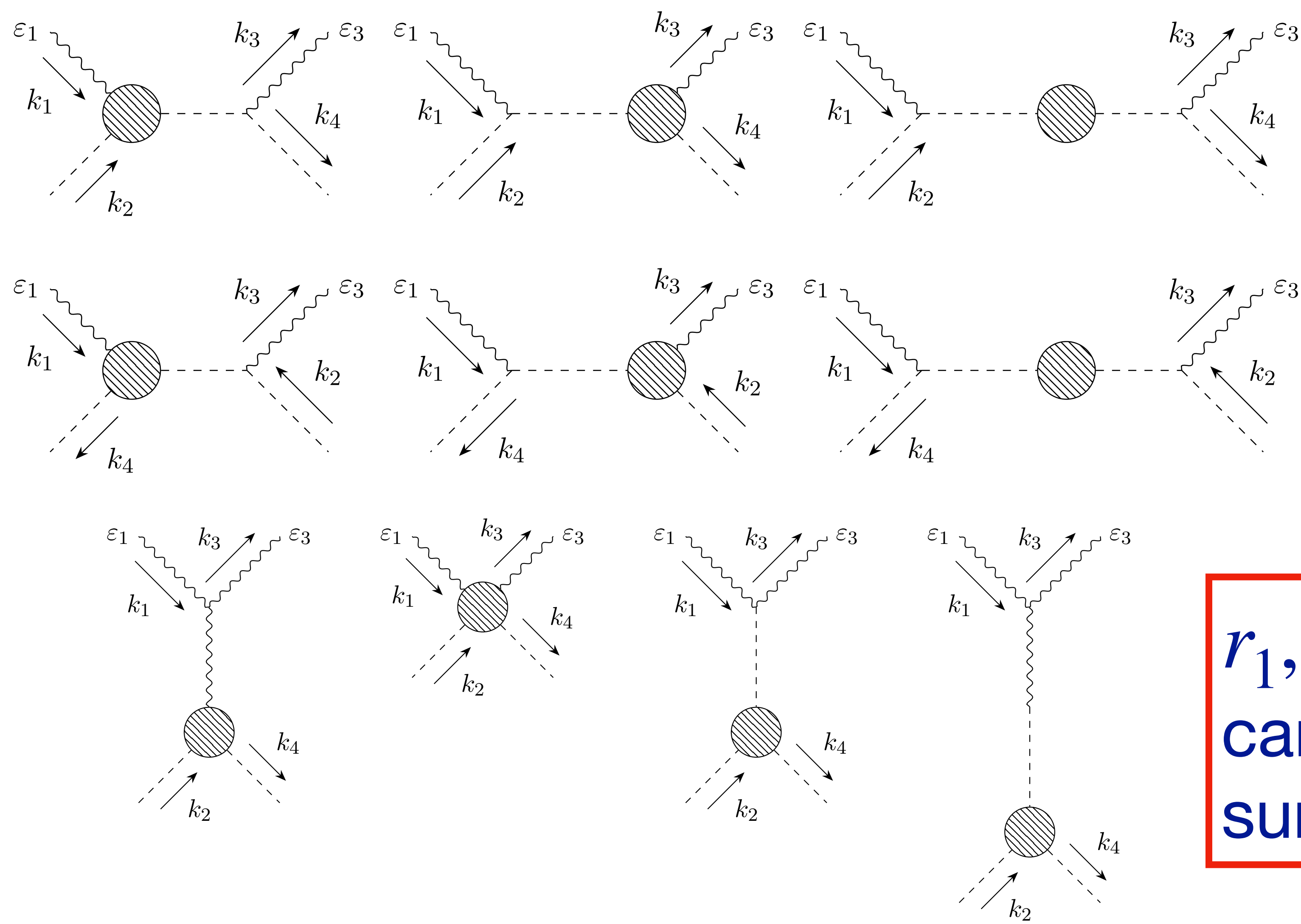
At leading order in decoupling limit (order κ^2)



It does not probe RG flow, determined by the graviton-dilaton dynamics we provided.

Graviton-dilaton amplitude

At subleading order in decoupling limit (order $\kappa^2 f^{-2}$)



r_1, λ_Δ dependence cancels out in the sum over diagrams

$$\mathcal{T}^{\text{sub-leading in } 1/f}_{h\varphi \longrightarrow h\varphi}(k_1, k_2, k_3, k_4; \varepsilon_1, \varepsilon_3) = \sum_{I=1}^{10} \mathcal{T}_I$$

$$= \frac{\kappa^2}{f^2} (\Delta c - \Delta a) \times \Big[t^2(\varepsilon_1.\varepsilon_3) - 4t(k_1.\varepsilon_3.\varepsilon_1.k_3) + 4(k_1.\varepsilon_3.k_1)(k_3.\varepsilon_1.k_3) \Big]$$

Graviton-dilaton amplitude

In COM frame:

$$\mathcal{T}^{+2}_{+2}(s,t,u) = \mathcal{T}^{-2}_{-2}(s,t,u) = \kappa^2 \left(1 - \frac{6 r_0 M^2}{f^2} \right) \frac{su}{t}$$

$$\mathcal{T}^{-2}_{+2}(s,t,u) = \mathcal{T}^{+2}_{-2}(s,t,u) = \frac{\kappa^2}{f^2} (\Delta c - \Delta a)\, t^2$$

Dispersion relation with assumption $\lim_{|s| \to \infty} \partial_t^2 \mathcal{T}^{-2}_{+2}(s,0,-s) = 0$:

$$\boxed{\Delta c - \Delta a = \frac{f^2}{\kappa^2} \int_{s>0} \frac{ds}{\pi} \frac{\mathrm{Im}\, \partial_t^2 \mathcal{T}^{-2}_{+2}(s,0,-s)}{s}}$$

helicity flipping amplitude

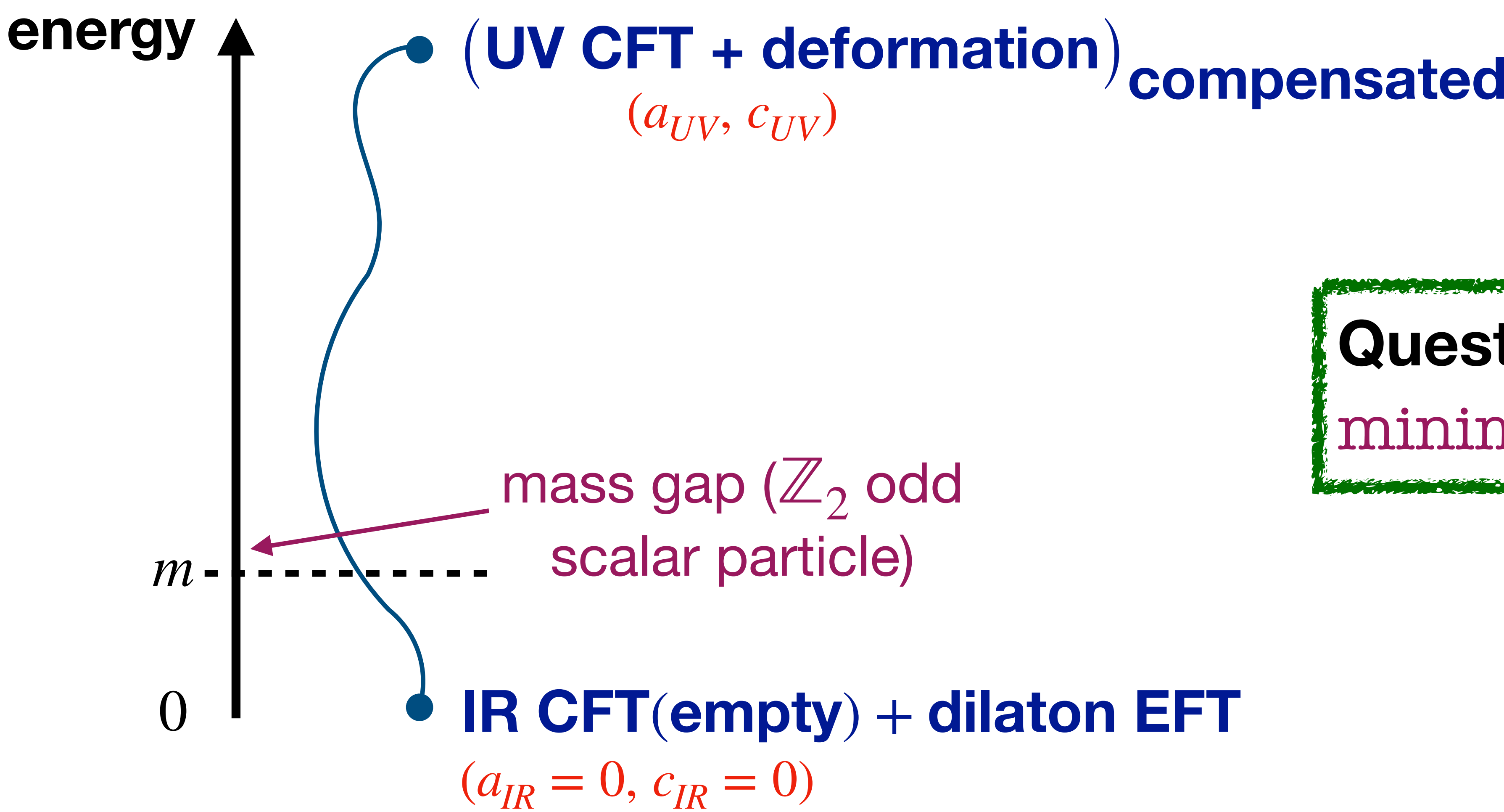
- $(\Delta c - \Delta a)$ probes spinning massive states with partial wave spin ≥ 2

S-matrix Bootstrap applications

Based on:

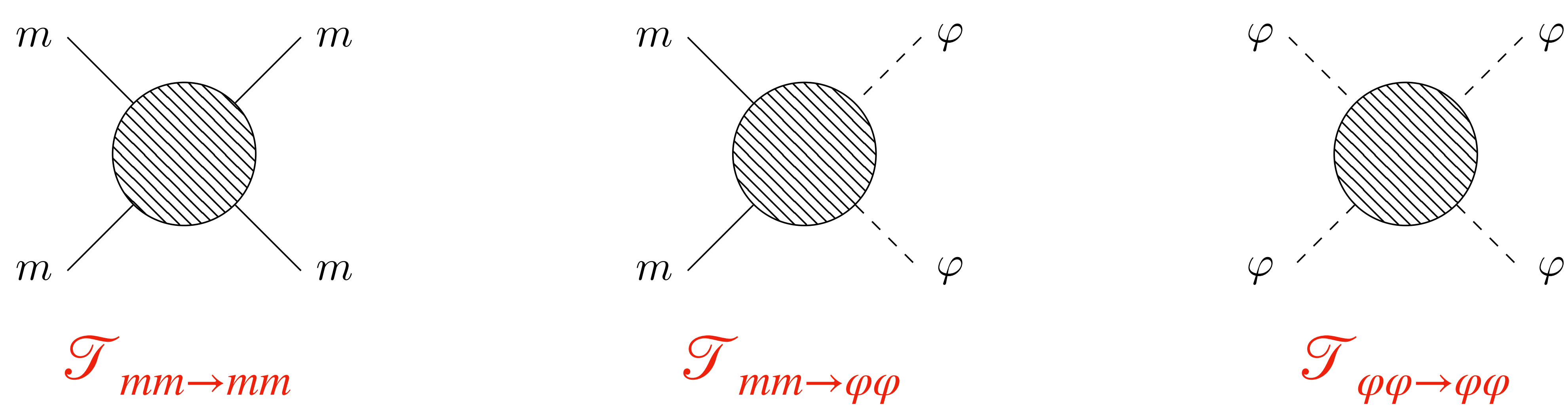
Karateev, Marucha, Penedones and B.S.

Bootstrap Setup



Question: what is the minimum value of a_{UV} ?

Set of amplitudes considered in the non-perturbative S-matrix bootstrap program:



Bootstrap Setup

1. Map complex $s, t, u-$planes to three unit disks with complex coordinate ρ_s, ρ_t, ρ_u such that two particle branch cuts lie on the perimeters of unit discs.

2. Assuming Mandelstam **analyticity** and **crossing** property write down ansatz for all three amplitudes as a polynomial in ρ_s, ρ_t, ρ_u with unknown parameters.

3. Use double **soft dilaton theorem** constraint on the ansatz of $\mathcal{T}_{mm\to\varphi\varphi}$ to fix some of the unknown parameters, and fix one parameter of $\mathcal{T}_{\varphi\varphi\to\varphi\varphi}$ in terms of Δa to match the four dilaton EFT amplitude.

4. Use SDPB to impose **unitarity** on the partial wave amplitudes with the minimization demand on Δa.

Bootstrap bound on a_{UV}

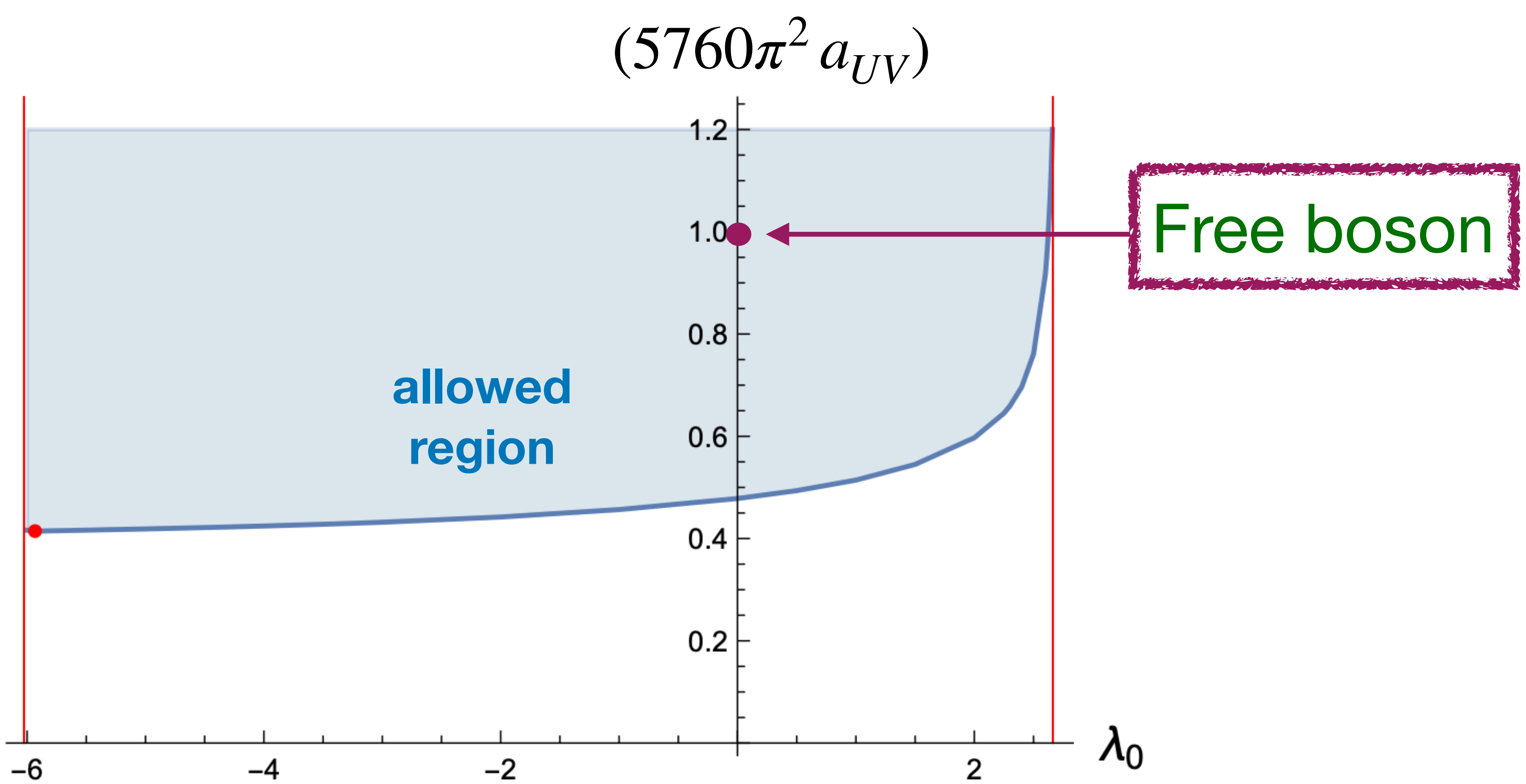
Non-perturbative observables:

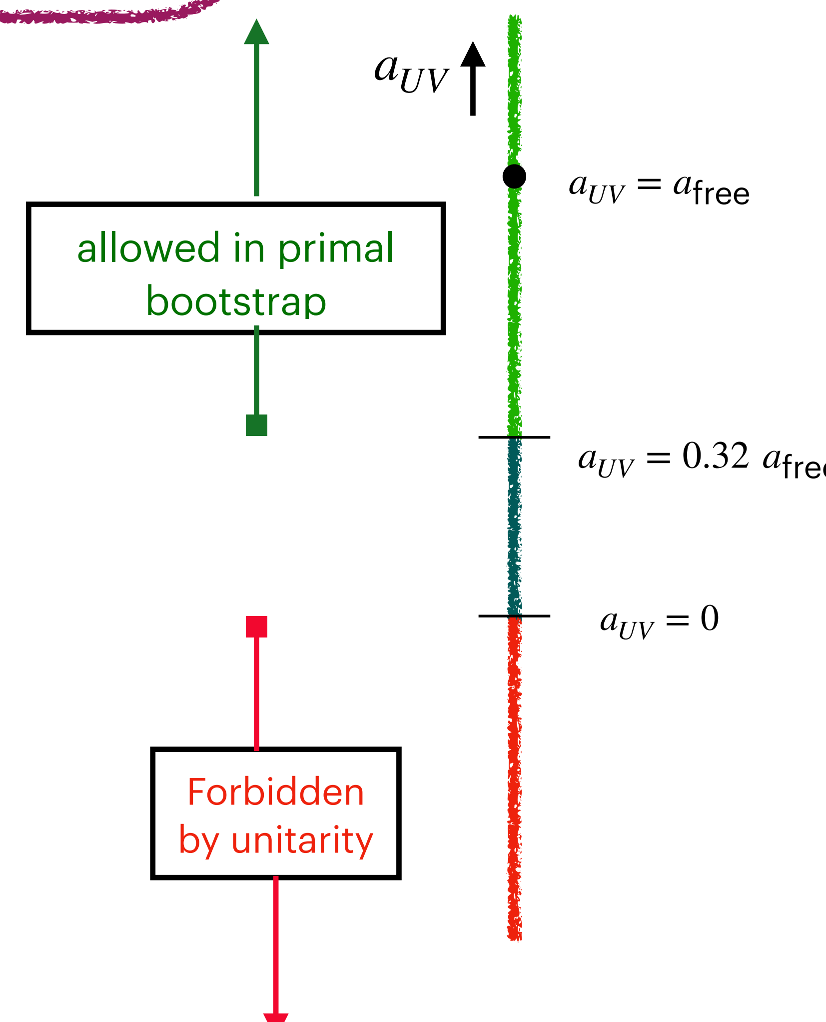
$$\lambda_0 \equiv \frac{1}{32\pi}\mathcal{T}_{mm\to mm}(4m^2/3, 4m^2/3, 4m^2/3)$$

$$\lambda_2 \equiv \frac{1}{32\pi}m^4\partial_s^2\mathcal{T}_{mm\to mm}(4m^2/3, 4m^2/3, 4m^2/3)$$

S-matrix bootstrap bounds:

$$-6.0253 \leq \lambda_0 \leq +2.6613$$
$$0 \leq \lambda_2 \leq +2.2568$$





Outlook for the future

Connection with literatures

In 4d CFT, $(c-a)$ combination appears in various context

(1) Counting specific operators in supersymmetric theories (Pietro & Komargodski; Beem & Rastelli; Ardehali, Martone & Rossello),

(2) Angular dependent part of the expectation value of the energy in the state produced by the $U_R(1)-$current (Hofman & Maldacena),

(3) Counting spinning primary operators in the large central charge, strong coupling limit of CFTs (Camanho, Edelstein, Maldacena & Zhiboedov),

(4) Logarithmic term in the entanglement entropy of Schwarzschild black hole (Solodukhin).

$$\boxed{\langle T^\mu_\mu \rangle_g = (c-a)R^2_{\mu\nu\rho\sigma} + 2(2c-a)R^2_{\mu\nu} + \left(\frac{c}{3}-a\right)R^2}$$

Can we thought of our $(\Delta c - \Delta a)$ sum rule as an RG flow generalization in such scenarios?

S-matrix bootstrap application

If we combine our result $a_{UV} \geq 0.32\ a_{\text{free}}$ with the conformal collider bound $\frac{31}{18} \geq \frac{a}{c} \geq \frac{1}{3}$ (Hofman & Maldacena), we find the following bound on the c-anomaly value for the set of UV CFTs which only flow to a gapped QFT

$$c_{UV} \geq 0.17\ a_{\text{free}}$$

Can this bound be improved by introducing graviton as another external probe in the S-matrix bootstrap setup we studied?

(Mis)matching of type-B anomaly in $\mathcal{N} = 2$ SCFT ?

Niarchos, Papageorgakis, Pini & Pomoni

Coupling space scale-anomaly involving integer dimension Coulomb branch operators between the unbroken phase and Higgs phase does NOT match in presence of non-trivial coulomb branch chiral ring in the IR.

Using general arguments based on background field method we think it should match. Need to re-investigate using our proposal.

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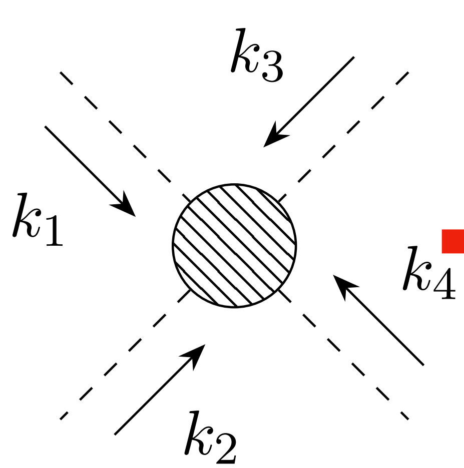
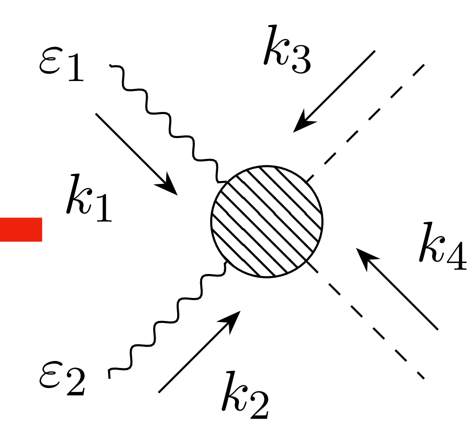
RG flow in six dimensions using scattering amplitudes ?

Elvang, Freedman, Hung, Kiermaier, Myers &Theisen



Both the amplitudes probes Δa at six power in momenta, but vanish in the forward limit $\Rightarrow$ probe only massive spinning states $\Rightarrow$ No $a-$theorem!



Thank You for your attention!