Exceptional world-volume currents and their algebras

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Motivation

- duality groups:
  - string/M-theory on T^d: $\mathrm{O}(d,d;\mathbb{Z})$ T-duality; $E_{d(d)}(\mathbb{Z})$ U-duality $\supset T, S$
    (equivalences of *different* theories in *different* backgrounds)
  - type II/11-dim. supergravities on T^d: (global) $\mathrm{O}(d,d)$, $E_{d(d)}$
  - gauge structure of supergravity fields: gauge trafos of metric&higher form gauge fields $\subset$ (local) $\mathrm{O}(d,d)$ resp. $E_{d(d)}$
- typical: world-volume dualities $\rightarrow$ space-time geometry
  s.t. setup is duality covariant
  - generalised geometry: extended (generalised) tangent bundles
  - double/exceptional field theory:
    extended target space + duality covariant constraint
  - why? including non-geometric/non-commutative backgrounds, alternative organising principle to supersymmetry,
- reversed logic: *generalised geometries* $\rightarrow$ *world-volume theories in string/M-theory?*

  $\rightarrow$ *so far*: mostly for strings in $\mathrm{O}(d,d)$ GG

  generalised dualities, classical integrability, ...

  $\rightarrow$ *now*: extend to arbitrary world-volume theories in $E_{d(d)}$

generalised geometry: basics, conventions

geometry	object	dim	relevant bundles		invariant	structure	metric
Riemannian	point	d	TM	0	–	$\mathrm{GL}(d)$	G_{MN}
generalised	string	d	$(T\oplus T^\star)M$	$C^\infty(M)$	η_{KL}	$\mathrm{O}(d,d)$	$\mathcal{H}_{MN}(G,B)$
|	|	4	$(T\oplus\Lambda^2T^\star)M$	|	$\epsilon_{\mathcal{KLMNP}}$	$\mathrm{SL}(5)$	|
except. gen.	misc.	5	$(T\oplus\Lambda^2T^\star\oplus\Lambda^5T^\star)M$	$(T^\star\oplus\Lambda^4T^\star)M$	$(\gamma_{\mathcal{M}})_{KL}$	$\mathrm{Spin}(5,5)$	$\mathcal{H}_{MN}(G,C)$
|	|	6	|	|	d_{KLM}	$E_{6(6)}$	|
			rep.: $\mathcal{R}_1$: $K,L,\dots$	rep.: $\mathcal{R}_2$: $\mathcal{K},\mathcal{L},\dots$	$\eta_{\mathcal{M},KL}/\eta^{\mathcal{M},KL}$		

- generalised Lie derivative: e.g. for $\phi = v+\xi \in (T\oplus\bigwedge^p T^\star)M$

  $$[\phi_1,\phi_2]_D = [v_1,v_2]+\mathcal{L}_{v_1}\xi_2 - \mathrm{d}(\iota_{v_2}\xi_1)$$

  $$[\phi_1,\phi_2]_D^K = \phi^L_{[1}\partial_L\phi^K_{2]} - Y^{KL}{}_{MN}\phi^M_1\partial_L\phi^N_2 \text{ with } Y^{KL}{}_{MN} = \eta^{\mathcal{P},KL}\eta_{\mathcal{P},MN}$$

  duality covariant, not symmetric, fulfils Leibniz instead Jacobi identity.

- background geometry: generalised metric

$$\mathcal{H}^{-1} = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G-BG^{-1}B \end{pmatrix} \text{ or } G^{-\frac{1}{3}}\begin{pmatrix} G^{\kappa\lambda} & -G^{\kappa\mu}C_{\mu\lambda\lambda'} \\ -C_{\kappa\kappa'\mu}G^{\mu\lambda} & 2G_{\kappa[\lambda}G_{\lambda']\kappa'} + C_{\kappa\kappa'\mu}G^{\mu\mu'}C_{\mu'\lambda\lambda'} \end{pmatrix}$$

  encoding metric G, and 2/3-form gauge fields B, C in type II/11d supergravity

- extended target space: $X^M = (x^\mu, \tilde{x}_\mu)$ or $(x^\mu, \tilde{x}_{\mu_1\mu_2}, \tilde{x}_{\mu_1\dots\mu_5})$
  - *section condition:* $\eta^{\mathcal{M},KL}\partial_K f(X)\partial_L g(X) = 0$, for all f,g
  - *setting in this talk:* M-theory section, no tensor hierarchy

generalised geometry: basics, conventions

geometry	object	dim	relevant bundles		invariant	structure	metric
Riemannian	point	d	TM	0	$-$	$\mathrm{GL}(d)$	G_{MN}
generalised	string	d	$(T \oplus T^\star)M$	$C^\infty(M)$	η_{KL}	$\mathrm{O}(d,d)$	$\mathcal{H}_{MN}(G,B)$
except. gen.	misc.	4	$(T \oplus \Lambda^2 T^\star)M$	|	$\epsilon_{\mathcal{KLMNP}}$	$\mathrm{SL}(5)$	|
		5	$(T \oplus \Lambda^2 T^\star \oplus \Lambda^5 T^\star)M$	$(T^\star \oplus \Lambda^4 T^\star)M$	$(\gamma_{\mathcal{M}})_{KL}$	$\mathrm{Spin}(5,5)$	$\mathcal{H}_{MN}(G,C)$
	|	6	|	|	d_{KLM}	$E_{6(6)}$	|
			rep.: $\mathcal{R}_1$: $K, L, \ldots$	rep.: $\mathcal{R}_2$: $\mathcal{K}, \mathcal{L}, \ldots$	$\eta_{\mathcal{M},KL} / \eta^{\mathcal{M},KL}$		

- additional ingredient: ω-symbol: $P^{KL}{}_{MN} = \frac{1}{2}\eta^{\mathcal{P},KL}\left(\eta_{\mathcal{P},MN} + \omega_{\mathcal{P},MN}\right)$
  - defining canonical Lie bracket on extended tangent bundle $[\phi_1, \phi_2]^K_L = \phi^L_{[1}\partial_L \phi^K_{2]} - P^{KL}{}_{MN}\phi^M_1 \partial_L \phi^N_2$
  - not duality-invariant, $\omega_{\mathcal{M},KL} \neq -\omega_{\mathcal{M},LK}$ in general
  - choice of ω $\hat{=}$ choice of section (up to B/C-shift)

- η- and ω-symbols:

$$\mathrm{O}(d,d):\ \eta_{KL} = \begin{pmatrix} 0 & \delta^\lambda_\kappa \\ \delta^\kappa_\lambda & 0 \end{pmatrix}, \quad \omega_{KL} = \begin{pmatrix} 0 & -\delta^\lambda_\kappa \\ \delta^\kappa_\lambda & 0 \end{pmatrix}$$

$$E_{d(d)}:\ \eta_{\mu,KL} = \begin{pmatrix} 0 & \delta^{\lambda_1\lambda_2}_{\mu\kappa} & 0 \\ \delta^{\kappa_1\kappa_2}_{\mu\lambda} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \omega_{\mu,KL} = \begin{pmatrix} 0 & - & 0 \\ + & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\eta_{\mu_1\mu_2\mu_3\mu_4,KL} = \begin{pmatrix} 0 & 0 & \delta^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5}_{\mu_1\mu_2\mu_3\mu_4\kappa} \\ 0 & \delta^{\kappa_1\kappa_2\lambda_1\lambda_2}_{\mu_1\mu_2\mu_3\mu_4} & 0 \\ \delta^{\kappa_1\kappa_2\kappa_3\kappa_4\kappa_5}_{\mu_1\mu_2\mu_3\mu_4\lambda} & 0 & 0 \end{pmatrix}, \quad \omega_{\mu_1\mu_2\mu_3\mu_4,KL} = \begin{pmatrix} 0 & 0 & - \\ 0 & - & 0 \\ + & 0 & 0 \end{pmatrix}, \quad \ldots$$

the point particle

- Hamiltonian formulation in terms of Riemannian geometry
  - 'current': $\mathbf{e}_M = p_M \in TM = \mathcal{R}_1$
  - Hamiltonian: $H = \frac{1}{2}\mathbf{e}_M \mathbf{e}_N G^{MN}(x)$
- Poisson structure
  - 'current algebra': $\{\mathbf{e}_M, \mathbf{e}_N\} = 0$
    $\{\mathbf{e}_M, x^N\} = -\delta_M^N$ and $\{x^M, x^N\} = 0$
  - for sections $\phi = -\phi^M(x)\mathbf{e}_M$: $\{\phi_1, \phi_2\}^K = [\phi_1, \phi_2]_L^K$
- 'fluxes': $\mathbf{e}_A = e_A^M \mathbf{e}_M$
  s.t. $H = \frac{1}{2}\mathbf{e}_A \mathbf{e}_B \delta^{AB}$, $\{\mathbf{e}_A, \mathbf{e}_B\}_D = \mathbf{f}^C{}_{AB}(x)\mathbf{e}_C$

review: the O(d,d) string

[Duff 90; Tseytlin 90/91; Siegel 92/93; Alekseev/Strobl 04; Blair/Malek/Routh 13; d.o. 19; ...]

- generic string σ-model

$$S \sim \frac{1}{2}\int \mathrm{d}x^\mu \wedge \star \mathrm{d}x^\nu G_{\mu\nu}(x) + \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu B_{\mu\nu}(x)$$

- Hamiltonian formulation in terms of O(d,d) gen. geometry
  (σ : spatial world-sheet coordinate, $\partial = \partial_\sigma$)
  - current: $\mathbf{E}_M(\sigma) = (p_\mu(\sigma), \partial x^\mu(\sigma)) \in \mathcal{R}_1$
  - Hamiltonian: $H = \frac{1}{2}\int \mathrm{d}\sigma \mathbf{E}_M(\sigma)\mathbf{E}_N(\sigma)\mathcal{H}^{MN}(G(x), B(x))$
  - spatial Virasoro constraint: $\mathbf{E}_M(\sigma)\mathbf{E}_N(\sigma)\eta^{MN} = 0$
- current algebra
  - (canonical) Poisson bracket:
    $\{\mathbf{E}_M(\sigma), \mathbf{E}_N(\sigma')\}_L = \frac{1}{2}\eta_{MN}(\partial - \partial')\delta(\sigma - \sigma') + \frac{1}{2}\omega_{MN}(\partial + \partial')\delta(\sigma - \sigma')$
  - Dorfman Poisson bracket: $\hat{=} PB$ up to $\int \mathrm{d}\sigma\, \partial(\ldots)$ / $(\partial + \partial')\delta(\sigma - \sigma')$
    $\{\mathbf{E}_M(\sigma), \mathbf{E}_N(\sigma')\}_D = \eta_{MN}\partial\delta(\sigma - \sigma')$

review: the O(d,d) string

- for sections $\phi = -\int \mathrm{d}\sigma \phi^M(x(\sigma))\mathbf{E}_M(\sigma)$:
  $\{\phi_1,\phi_2\}_L^K = [\phi_1,\phi_2]_L^K, \quad \{\phi_1,\phi_2\}_D^K = [\phi_1,\phi_2]_D^K$
- non-geometric/extended target space interpretation
  $\mathbf{E}_M = \eta_{MN}\partial X^N = (\partial\tilde{x}_\mu, \partial x^\mu) \equiv (p_\mu, \partial x^\mu)$
  dual coord.: non-local contribution of momentum
- generalised fluxes: $\mathbf{E}_A = E_A^M \mathbf{E}_M$ s.t. $H = \frac{1}{2}\int \mathrm{d}\sigma \mathbf{E}_A\mathbf{E}_B\delta^{AB}$
  $\{\mathbf{E}_A(\sigma), \mathbf{E}_B(\sigma')\}_D = \eta_{AB}\partial\delta(\sigma-\sigma') - \mathbf{F}^C{}_{AB}(x(\sigma))\mathbf{E}_C(\sigma)\delta(\sigma-\sigma')$
  - e.o.m. in simple form $\mathrm{d}\mathbf{E}^C + \frac{1}{2}\mathbf{F}^C{}_{AB}\mathbf{E}^B \wedge \mathbf{E}^C = 0$
  - generalised T-dualities as canonical transformations
    [Lozano 95, Klimcik/Severa 95, Sfetsos 97, ... , Borsato/Driezen 21]
  - non-commutative/non-associative interpretation
- alternative derivation (when neglecting $\int \mathrm{d}\sigma\ \partial(\ldots)$-terms)
  $\{\ ,\ \}_D =$ Dirac bracket on extended canonical phase space:
  $\{X^M(\sigma), P_N(\sigma')\} = \delta_N^M\delta(\sigma-\sigma'), \quad \{X^M(\sigma), X^N(\sigma')\} = \{P_M(\sigma), P_N(\sigma')\} = 0$
  w.r.t. constraint $\Phi_M = P_M - \mathbf{E}_M = P_M - \eta_{MN}\partial X^N = 0$

generalisation to the exceptional case

Aim: $E_{d(d)}$-invariant Hamiltonian formulations, challenges:

- different kinds of world-volume theories
  M2/M5/... in M-theory, string/D-branes/... in type II sections
- Hamiltonian $\leftarrow$ **generalised metric**
  current algebra (diff. constraints) $\leftarrow$ **??**
  closest: η-symbol $\eta_{\mathcal{M},KL}$ with extra $\mathcal{R}_2$-index, cf. η_{KL} for $\mathrm{O}(d,d)$
  $\rightarrow$ extra object $\in \mathcal{R}_2$ needed
- currents/extended coordinates in $\mathcal{R}_1$,
  $\dim\mathcal{R}_1 > 2d = \sharp(\text{coord. fields}) + \sharp(\text{momenta})$, for $d \geq 4$.
  $\rightarrow$ role of extended coordinates not clear
- existing literature on $E_{d(d)}$-invariant world-volume theories
  - Hamiltonian formalism: membrane in $E_{3(3)}$ and $E_{4(4)}$ [Duff/Lu 90, Hatsuda/Kamimura 12, Duff/Lu/Percacci/Pope/Samtleben/Sezgin 15, Sakatani/Uehara 20], M5-brane in $E_{5(5)}$, D-branes [Hatsuda/Kamimura 12/13]
  - actions [Blair/Musaev 17, Sakatani/Uehara 18/20, Arvanitakis/Blair 18]

the membrane in SL(5)

[Hatsuda/Kamimura 12, d.o. 21]

- membrane σ-model

$$S \sim \int \frac{1}{2}\mathrm{d}x^\mu \wedge \star \mathrm{d}x^\nu G_{\mu\nu}(x) + \frac{1}{3!}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \wedge \mathrm{d}x^\rho C_{\mu\nu\rho}(x) + \frac{1}{2}\star 1$$

- Hamiltonian formulation in terms of SL(5) gen. geometry
  ($\sigma = (\sigma_1, \sigma_2)$ spatial world-vol. coordinates, d spatial world-vol. differential)
  - current (spatial 2-form): $\mathbf{Z}_M(\sigma) = \big(p_\mu(\sigma), \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu(\sigma)\big) \in \mathcal{R}_1$
  - Hamiltonian: $H = \frac{1}{2}\int \mathbf{Z}_M(\sigma) \wedge \star\mathbf{Z}_N(\sigma)\mathcal{H}^{MN}(G(x), C(x))$
  - spatial diffeomorphism constraints: $\mathbf{Z}_M(\sigma) \wedge \star\mathbf{Z}_N(\sigma)\eta^{\mathcal{K},MN} = 0$
- current algebra
  - (canonical) Poisson bracket:

$$\{\mathbf{Z}_M(\sigma), \mathbf{Z}_N(\sigma')\}_L = \mathrm{d}x^\mu \wedge \left(\tfrac{1}{2}\eta_{\mu MN}(\mathrm{d} - \mathrm{d}')\delta(\sigma - \sigma') + \tfrac{1}{2}\omega_{\mu,MN}(\mathrm{d} + \mathrm{d}')\delta(\sigma - \sigma')\right)$$

  - Dorfman Poisson bracket:
    $\{\mathbf{Z}_M(\sigma), \mathbf{Z}_N(\sigma')\}_D = \eta_{\mu,MN}\mathrm{d}x^\mu(\sigma) \wedge \mathrm{d}\delta(\sigma - \sigma')$
    $\hat{=} PB$ up to $\int \mathrm{d}x^\mu \wedge \mathrm{d}(\ldots)$ / $\mathrm{d}x^\mu \wedge (\mathrm{d} + \mathrm{d}')\delta(\sigma - \sigma')$

the membrane in SL(5)

- for sections $\phi = -\int \phi^M(x(\sigma))\mathbf{Z}_M(\sigma)$:
  $\{\phi_1, \phi_2\}_L^K = [\phi_1, \phi_2]_L^K, \quad \{\phi_1, \phi_2\}_D^K = [\phi_1, \phi_2]_D^K$
- non-geometric/extended target space interpretation
  $\mathbf{Z}_M = \frac{1}{2}\eta_{\lambda,MK}\mathrm{d}x^\lambda \wedge \mathrm{d}X^K = \left(\frac{1}{2}\mathrm{d}x^\lambda \wedge \mathrm{d}\tilde{x}_{\lambda\kappa}, \mathrm{d}x^{\mu_1} \wedge \mathrm{d}x^{\mu_2}\right)$
  dual coord.: non-local contribution of momentum
- generalised fluxes: $\mathbf{Z}_A = E_A^M \mathbf{Z}_M$, $[E_A, E_B]_D = \mathbf{F}^C{}_{AB} E_C$
  $\{\mathbf{Z}_A(\sigma), \mathbf{Z}_B(\sigma')\}_D = \eta_{\mathcal{C},AB} j^{\mathcal{C}} \wedge \mathrm{d}\delta(\sigma - \sigma') - \mathbf{F}^C{}_{AB}(x)\mathbf{E}_C(\sigma)\delta(\sigma - \sigma')$
- alternative derivation (when neglecting total derivative terms)
  $\{\ ,\ \}_D =$ Dirac bracket on extended canonical phase space:
  $\{X^M(\sigma), P_N(\sigma')\} = \delta_N^M \delta(\sigma - \sigma'), \quad \{X^M(\sigma), X^N(\sigma')\} = \{P_M(\sigma), P_N(\sigma')\} = 0$
  w.r.t. constraint $\Phi_M = P_M - \mathbf{Z}_M = P_M - \eta_{\kappa,MN}\mathrm{d}x^\kappa \wedge \mathrm{d}X^N = 0$
- double reduction to IIa string: $x^4(\sigma) = \sigma^2$, $x^{\underline{\mu}} \equiv x^{\underline{\mu}}(\sigma^1)$
  $\mathbf{Z}_M \to \mathbf{z}_M(\sigma_1) \wedge \mathrm{d}\sigma^2, \quad \mathbf{z}_M(\sigma) = (p_{\underline{\mu}}, \mathrm{d}x^{\underline{\mu}})$
  $\{\mathbf{z}_M(\sigma), \mathbf{z}_N(\sigma')\}_D = \eta_{MN}\mathrm{d}\delta(\sigma - \sigma')$
- SL(5)-invariance? (e.g. $\mathrm{d}x^\mu \wedge \mathrm{d}\delta(\sigma - \sigma')$)?

the general setting

p-brane world-volume theory with $E_{d(d)}$-symmetry

($\sigma = (\sigma_1, \ldots, \sigma_p)$ spatial world-vol. coordinates, d spatial world-vol. differential)

- $(p-1)$-form charge $\mathcal{Q}^{\mathcal{M}} \in \mathcal{R}_2$, with $\mathrm{d}\mathcal{Q} = 0$
- *p*-form current: $\mathbf{Z}_M = \frac{1}{2}\eta_{\mathcal{L},MK}\mathcal{Q}^{\mathcal{L}} \wedge \mathrm{d}X^M \in \mathcal{R}_1$
- postulates:
  - current algebra: $\{\mathbf{Z}_M(\sigma), \mathbf{Z}_N(\sigma')\}_D = \eta_{\mathcal{K},MN}\mathcal{Q}^{\mathcal{K}}(\sigma) \wedge \mathrm{d}\delta(\sigma - \sigma')$
  - Hamiltonian: $H = \frac{1}{2}\int \mathbf{Z}_M(\sigma) \wedge \star\mathbf{Z}_N(\sigma)\mathcal{H}^{MN}(X)$
  - spatial diffeomorphism constraints: $\mathbf{Z}_M(\sigma) \wedge \star\mathbf{Z}_N(\sigma)\eta^{\mathcal{K},MN} = 0$
- require $\{\phi_1, \phi_2\}_D^K = [\phi_1, \phi_2]_D^K$ for $\phi = -\int \phi^M(\sigma)\mathbf{Z}_M(\sigma)$
  $\Rightarrow$ **charge condition**:

$$\mathcal{Q}^{\mathcal{M}} \wedge \mathrm{d}X^N\partial_N = \eta^{\mathcal{M},NP}\mathbf{Z}_P\partial_N = \frac{1}{2}\eta^{\mathcal{M},NP}\eta_{\mathcal{K},LP}\mathcal{Q}^{\mathcal{K}} \wedge \mathrm{d}X^L\partial_N$$

- role of charge:
  - specifies object (and with that also the kind of section)
  - projects out certain parts of generalised geometry description:
    e.g. in current $\rightarrow$ generalised metric (via Hamiltonian), generalised Lie derivative (via current algebra)

solutions to SL(5) charge condition

[Arvanitakis/Blair 18]

- for SL(5): $M = [\mathcal{M}\mathcal{M}']$ and the charges take particularly easy form
  - $p = 0$: $\mathcal{Q}^{\mathcal{M}} = 0$ – no currents, no current algebra
  - $p = 1$: $\mathcal{Q}^{\mathcal{M}} = \mathbf{q}^{\mathcal{M}} \in \mathcal{R}_2$
  - $p = 2$: $\mathcal{Q}^{\mathcal{M}} = \mathbf{q}_{\mathcal{M}'} \mathrm{d}X^{\mathcal{M}\mathcal{M}'}$, $\mathbf{q}_{\mathcal{M}'} \in \overline{\mathcal{R}}_2 = \mathcal{R}_3$,
    $\mathbf{q}_5 \neq 0 \to \mathcal{Q} = \mathrm{d}x^{\mu}$
  - $p = 3$: $\mathcal{Q}^{\mathcal{M}} = \eta^{\mathcal{M} M_3 M_4} \eta_{\mathcal{N}_1, M_1 M_3} \eta_{\mathcal{N}_2, M_2 M_4} \mathbf{q}^{\mathcal{N}_1 \mathcal{N}_2} \mathrm{d}X^{M_1} \wedge \mathrm{d}X^{M_2}$
    with $\mathbf{q}^{\mathcal{N}_1 \mathcal{N}_2} \in \bigwedge^2 \mathcal{R}_2 = \mathcal{R}_4$
- the charge conditions:
  $\mathbf{q}^{\mathcal{M}} \partial_{\mathcal{M}\mathcal{M}'} = 0 \in \mathcal{R}_3$, $\mathbf{q}_{[\mathcal{M}_1} \partial_{\mathcal{M}_2 \mathcal{M}_3]} = 0 \in \mathcal{R}_4$, $\mathbf{q}^{\mathcal{M}_1 \mathcal{M}_2} \partial_{\mathcal{M}_1 \mathcal{M}_2} = 0 \in \mathcal{R}_5$
  p=1 – 2-dim solution space in IIb section: F1/D1,

  p=2 – 1-dim solution space in M-theory section: M2, ...

  in general: specify section → find dimensionality of charge solution
- two special properties, that are not clear to hold for $d \geq 5$
  - charge $\in \mathcal{R}_{(p+1)}$, charge condition $\in \mathcal{R}_{(p+2)}$
  - charge condition simplifies before specifying section

$$\mathcal{Q}^{\mu} = -\frac{1}{6}\, \mathrm{d}x^{\mu} \wedge \mathrm{d}x^{\mu_1} \wedge \mathrm{d}x^{\mu_2} \wedge \mathrm{d}\tilde{x}_{\mu_1\mu_2}$$

$$\mathcal{Q}^{\mu_1\mu_2\mu_3\mu_4} = \mathrm{d}x^{\mu_1} \wedge \mathrm{d}x^{\mu_2} \wedge \mathrm{d}x^{\mu_3} \wedge \mathrm{d}x^{\mu_4}$$

- the corresponding current is: (simplifying $Z_\mu \equiv p_\mu$)

$$\mathbf{Z}_M = \left(p_\mu \ , \ \frac{1}{6}\mathrm{d}x^{\mu_1} \wedge \ldots \wedge \mathrm{d}x^{\mu_4} \wedge \mathrm{d}\tilde{x}_{\mu_3\mu_4} \ , \ \mathrm{d}x^{\mu_1} \wedge \ldots \wedge \mathrm{d}x^{\mu_5} \right)$$

  - examplary for $p > 2$-brane solutions
  - unique 5-brane solution in M-theory section

- canonical choice: $\mathbf{Z} = \mathbf{Z}^{(2)} + \mathbf{Z}^{(5)}$ for $\mathbf{Z}_M^{(p)} = \left(p_\mu \ , \ \mathrm{d}x^{\mu_1} \wedge \ldots \wedge \mathrm{d}x^{\mu_p} \right)$
  leads to $\{\mathbf{Z}_M(\sigma), \mathbf{Z}_N(\sigma')\}_L =$
  $\mathcal{Q}^{\mathcal{L}} \wedge \left(\left(\eta_{\mathcal{L},MN} + \omega_{\mathcal{L},(MN)} \right) \frac{1}{2} \left(\mathrm{d} - \mathrm{d}' \right) + \omega_{\mathcal{L},[MN]} \frac{1}{2} \left(\mathrm{d} + \mathrm{d}' \right) \right) \delta(\sigma - \sigma')$
  - $\{\omega_1, \omega_2\}^K = 0 \neq [\omega_1, \omega_2]_D^K = (\omega_1 \wedge \mathrm{d}\omega_2)^K$ for 2-forms ω
    not even up to total world-volume derivatives
  - $\mathbf{Z}_M^{(p)} = \left(p_\mu \ , \ \mathrm{d}x^{\mu_1} \wedge \ldots \wedge \mathrm{d}x^{\mu_p} \right)$ + can. PB leads to current algebra
    $\hat{=}$ gen. Lie derivative of para-Hermitian version of SL$(p+3)$ GG

$\Rightarrow$ not exceptional enough!

the M5-brane in $E_{6(6)}$

$$\mathbf{Z}_M = \left(p_\mu \ , \ \frac{1}{6} \mathrm{d}x^{\mu_1} \wedge \ldots \wedge \mathrm{d}x^{\mu_4} \wedge \mathrm{d}\tilde{x}_{\mu_3\mu_4} \ , \ \mathrm{d}x^{\mu_1} \wedge \ldots \wedge \mathrm{d}x^{\mu_5} \right)$$

- manifest non-geometry of the current:
  - extended coordinate fields related to momenta:

$$X^M = \Big(x^\mu, \underbrace{\tilde{x}_{\mu_1\mu_2}}_{\text{M2}}, \underbrace{\tilde{x}_{\mu_1\ldots\mu_5}}_{\text{M5}} \Big), \quad X^M = \Big(x^\mu, \underbrace{\tilde{x}^m_\mu}_{\text{F1/D1}}, \underbrace{\tilde{x}_{\mu_1\mu_2\mu_3}}_{\text{D3}}, \underbrace{\tilde{x}^m_{\mu_1\ldots\mu_5}}_{\text{NS5/D5}} \Big)$$

  - $\rightarrow$ interpretation 1: $\tilde{x}_{\mu_1\mu_2}$ in M5-current describes (somehow) coupling to M2-brane ending on M5-brane.
  - fundamental $\tilde{x}$-brackets seem inachievable
- comparison to the Hatsuda-Kamimura M5-current:
  - current of M5-brane action [Pasti/Sorokin/Tonin '97]
  - $\mathbf{Z}_M$ with identification $F = \frac{1}{6} \mathrm{d}x^{\mu_1} \wedge \mathrm{d}x^{\mu_2} \wedge \mathrm{d}\tilde{x}_{\mu_1\mu_2}$
  - $\{F(\sigma), F(\sigma')\} = \mathrm{d}\delta(\sigma - \sigma')$ – Dirac bracket of self-dual 2-form gauge field A, $F = \mathrm{d}A$, on M5 world-volume. On the canonical phase space $(p_\mu(\sigma), x^\mu(\sigma); E(\sigma), A(\sigma))$:

$$\{\mathbf{Z}_M(\sigma), \mathbf{Z}_N(\sigma')\}_{[F]} = \mathcal{Q}^{\mathcal{L}}_{[F]} \wedge \left(\eta_{\mathcal{L},MN} \tfrac{1}{2} \left(\mathrm{d} - \mathrm{d}'\right) + \omega_{\mathcal{L},[MN]} \tfrac{1}{2} \left(\mathrm{d} + \mathrm{d}'\right) \right) \delta(\sigma - \sigma')$$

  - $\rightarrow$ interpretation 2: $\tilde{x}_{\mu_1\mu_2}$ in M5-current describes coupling of M5-brane to gauge field F, at cost of $E_{d(d)}$-invariance

Outlook

- several conceptual questions open
  - $\{\mathbf{Z}_M(\sigma), \mathbf{Z}_N(\sigma')\} = \eta_{\mathcal{L},MN}\mathcal{Q}^{\mathcal{L}} \wedge \mathrm{d}\delta(\sigma - \sigma')$ postulated, but not derived yet from some underlying structure in general
    maybe QP-structures? [Arvanitakis 21]
  - manifest duality covariance, but non-pert. dualities still mysterious due to different dimensionality
  - $E_{7(7)}$, $E_{8(8)}$, ... (additional objects in current algebra)
    tensor hierarchy
- study of classical dynamics M5/M2 as coupled classical system ($\leftarrow$ generalised geodesics? [Strickland-Constable 21])
- non-geometric interpretation of M-theory backgrounds based on current algebra in generalised flux frame?
  (going from, e.g. $\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$ to x^μ problematic)
- 'non-abelian U-duality'
  $\leftrightarrow$ canonical transformation of world-volume theories?

Thank you for your attention!