Black holes and vacuum decay

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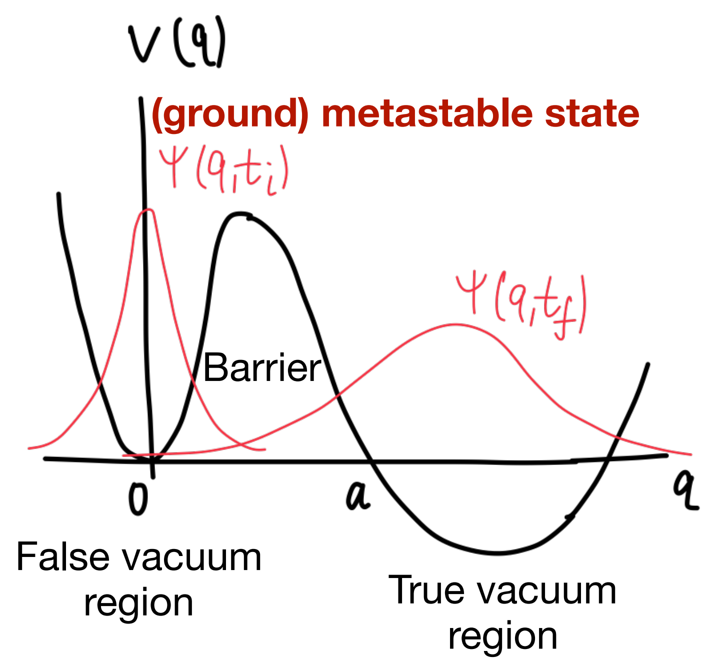
Decay of metastable state

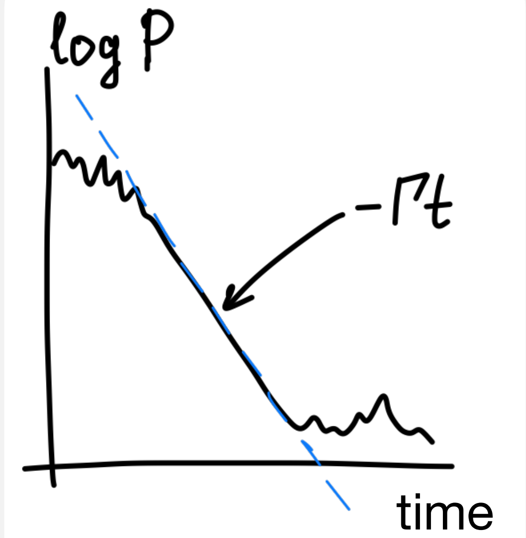
- Consider the quantum-mechanical system with the Hamiltonian $H = \frac{p^2}{2m} + V(q)$ and the "tunneling" potential.



- Probability of survival of the metastable state is $P \sim e^{-\Gamma t}$

WKB: $\Gamma \sim e^{-B}$ — decay rate

$$B = 2\int_0^a \sqrt{2m V(q)}\, dq$$ — suppression exponent



Survival probability as a function of time

See, e.g., Andreassen, Farhi, Frost, Schwartz 18

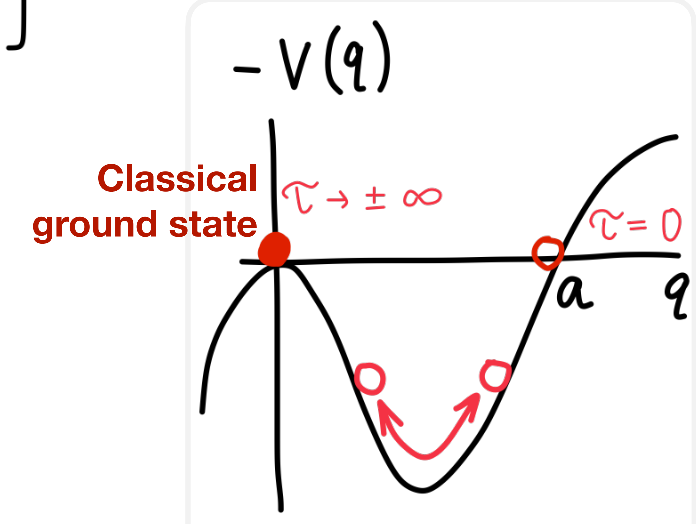
- One can write in terms of the **bounce** trajectory: $B = S_E[q_B]$

Euclidean action: $$S_E = \int d\tau \left(\frac{m}{2}\left(\frac{dq}{d\tau}\right)^2 + V(q) \right)$$

Classical equation of motion: $$m\frac{d^2 q}{d\tau^2} = \frac{\partial V}{\partial q} \equiv -\frac{\partial(-V)}{\partial q}$$

(False) vacuum boundary conditions: $q_B(\pm\infty) = 0$,

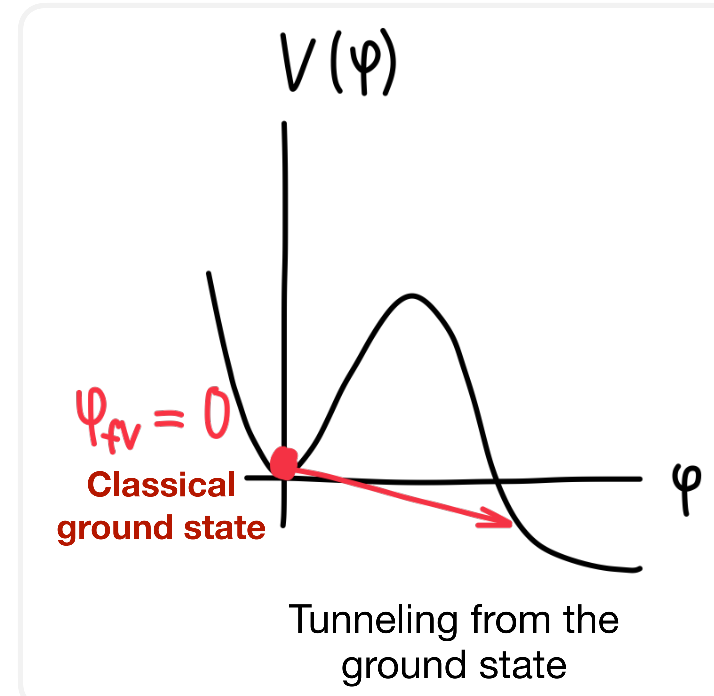
Turning point: $\dot{q}_B(0) = 0$



The classical particle moves along the bounce trajectory

Decay of metastable state in field theory

- Consider the scalar field theory with the Lagrangian $\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$ (in flat space) and the tunneling (configuration-space) potential



In the WKB approximation, the decay rate is $\Gamma \sim e^{-B}$

Coleman 77; Callan, Coleman 77

$$B = S_E[\varphi_B]$$

$$S_E = \frac{1}{g^2}\int d\vec{x}\, d\tau \left(\frac{1}{2}\left(\frac{\partial \varphi}{\partial \tau}\right)^2 + \frac{1}{2}\left(\frac{\partial \varphi}{\partial \vec{x}}\right)^2 + V(\varphi) \right)$$

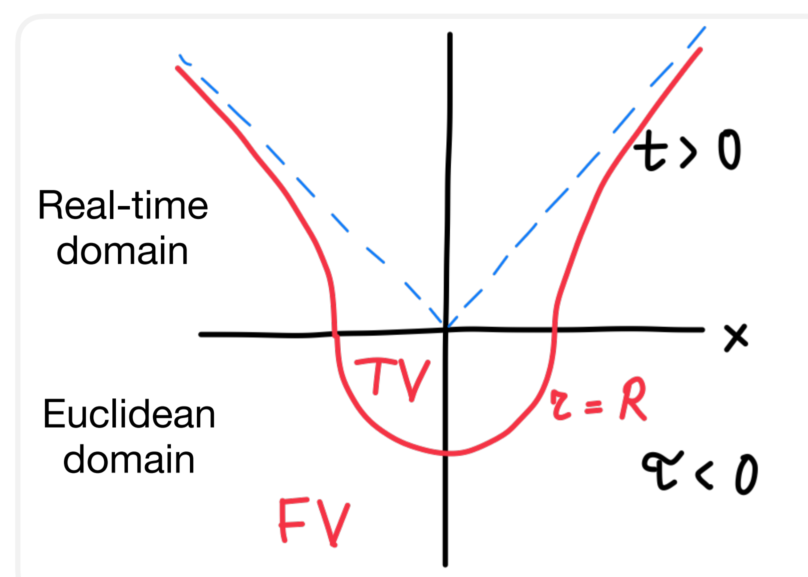
$g \ll 1$ — coupling constant

$$\partial_\mu \partial^\mu \varphi - V'(\varphi) = 0$$

- The vacuum bounce is spherically symmetric in d+1 dimensions,

Coleman, Glacer, Martin 78;
Blum, Honda, Sato, Takimoto, Tobioka 16

$$r = \sqrt{\tau^2 + \vec{x}^2}$$



- Vacuum boundary conditions: $\varphi_B(r \to \infty) \to 0$

Turning point: $\dot{\varphi}_B(\tau = 0, \vec{x}) = 0$

Decay of metastable state at finite temperature

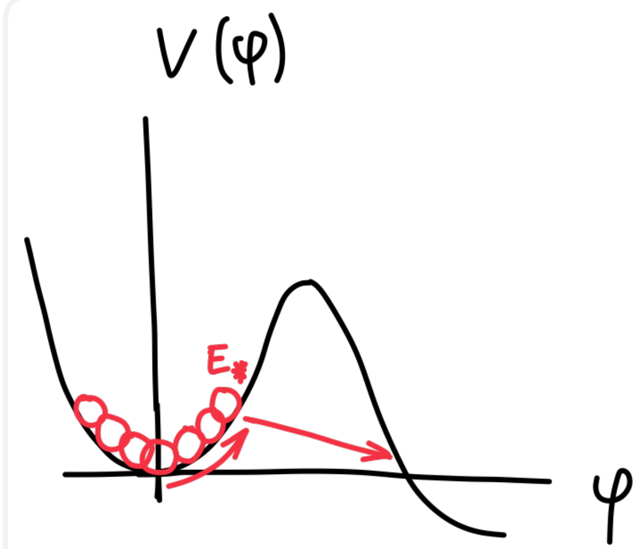
- Consider the scalar field theory with the Lagrangian $\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$
  and tunneling from the **thermally-populated** initial state

As usual (at not too high temperatures), $\Gamma \sim e^{-B}$

$$B = S_E[\varphi_B]$$

$$\partial_\mu \partial^\mu \varphi - V'(\varphi) = 0$$

Boundary conditions for the thermal bounce?



Tunneling from an excited state

- Thermal partition function implies **periodic** boundary conditions

Thermal averaging: $\Gamma \sim \int dE \, e^{-\frac{E}{T}} e^{-S_E[\varphi_{B,E}]} \sim e^{-\frac{E_*}{T} - S_E[\varphi_{B,E^*}]} = e^{-B}$

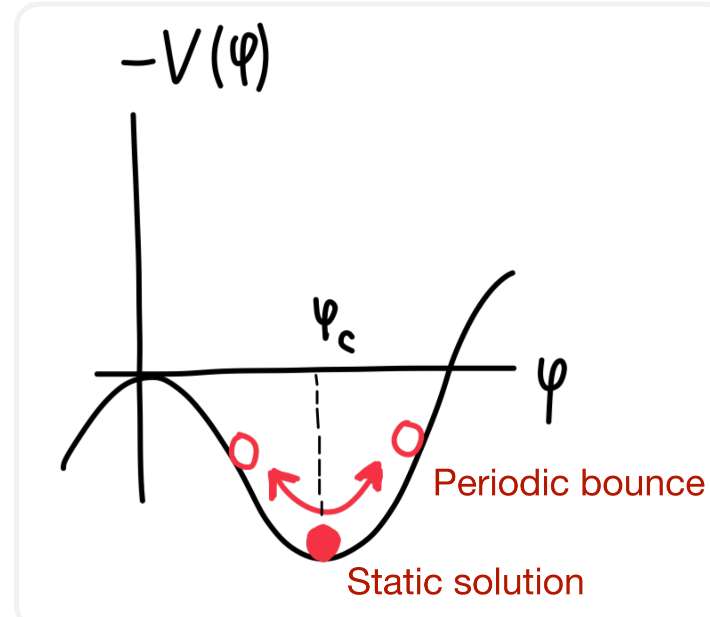
$$\longrightarrow \quad \varphi_B(\tau + 1/T, \vec{x}) = \varphi_B(\tau, \vec{x})$$

Linde 82;
Brown, Weinberg 07

Decay of metastable state via thermal activation

- At large T, one expects the decay to occur via classical thermal jumps of the field over the barrier.

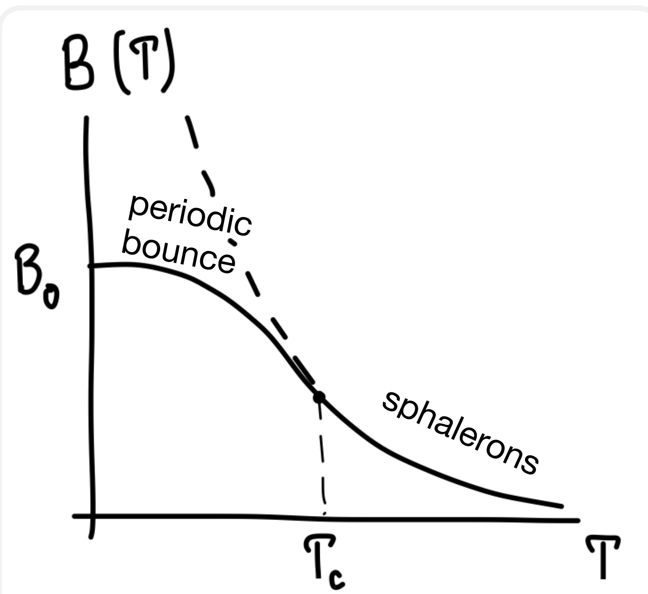
  In the WKB, this is described by the static solution — **sphaleron**.



Klinkhamer, Manton 84

$$\Gamma \sim e^{-\frac{E_{sph}}{T}} \qquad B = \frac{1}{T} E_{sph} \qquad (T \gtrsim T_c)$$

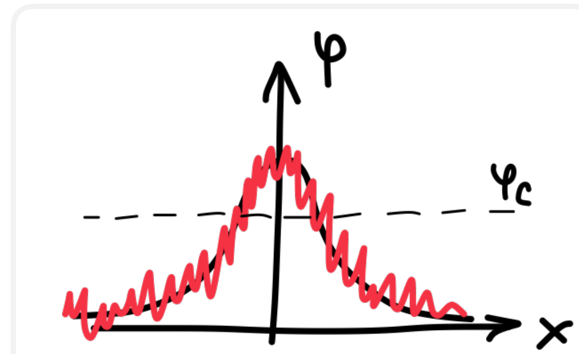
- Periodic bounces dominate at low T — tunneling
  Sphaleron dominates at large T — thermal jumps



Exponential suppression of vacuum decay as a function of temperature

- Phase transition driven by classical fluctuations can be studied in real-time lattice simulations

Grigoriev, Rubakov, Shaposhnikov 89
Khlebnikov, Kofman, Linde, Tkachev 98



Field configuration triggering the phase transition

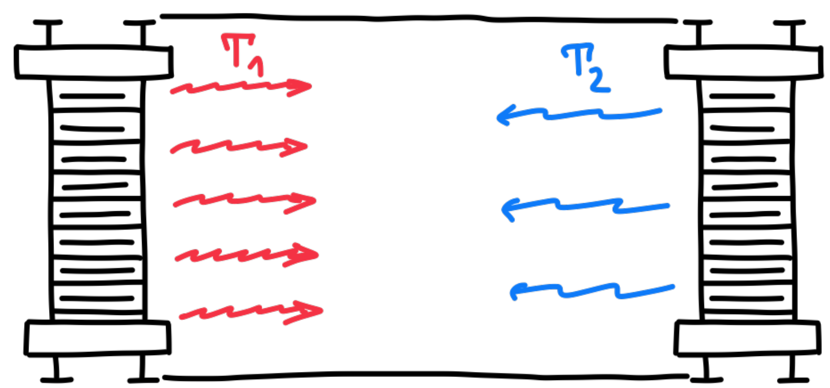
- Depending on the tunneling potential, the transition point can be smooth (“2nd order”) or sharp (“1st order”).

Out-of-equilibrium metastable state

- Euclidean time prescription is derived from the equilibrium initial condition.

  What about out-of-equilibrium initial states?

Anisotropic flux of radiation



Multicomponent radiation

- Thermal activation can be studied in real-time simulations $(T \gtrsim T_c)$.

  What is the corresponding WKB solution?

- What WKB solution is responsible for tunneling?

- It is important to understand **boundary conditions.**

Transition amplitude

- $$\mathcal{A}_{i,f} = \langle f | i \rangle$$

  where $| i \rangle$ — initial state associated with the vacuum of free theory

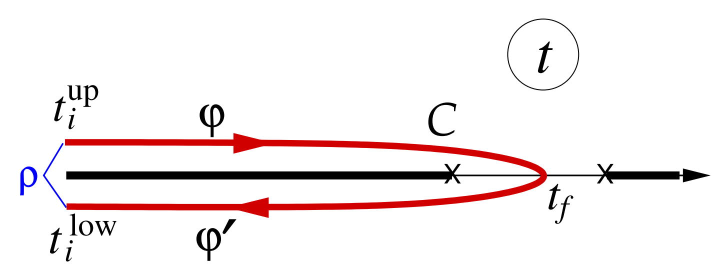
  $| f \rangle$ — final state in the “basin of attraction” of true vacuum.

$$\mathcal{A}_{if} = \int_{\substack{\varphi(t_i,x) = \varphi_i(x) \\ \varphi(t_f,x) = \varphi_f(x)}} D\varphi_i(x)\, D\varphi_f(x)\, D\varphi(t,x)\; e^{iS[\varphi]} \langle f | \varphi_f t_f \rangle \langle \varphi_i t_i | i \rangle$$

where $\hat{\varphi}(t,x)\, | \varphi, t \rangle = \varphi(x)\, | \varphi, t \rangle$ are eigenstates of the field operator

- Decay probability $\mathcal{P}_{decay} = \sum_{\text{“}f \in TV\text{”}} \mathcal{A}_{if} \mathcal{A}^*_{if}$

$$\langle \mathcal{P}_{decay} \rangle_{\rho_i} = \int_{\substack{\varphi(t_i^{up},x) = \varphi_i(x) \\ \varphi(t_i^{low},x) = \varphi_i'(x)}} D\varphi_i(x)\, D\varphi_i'(x)\, D\varphi_c(t,x)\; e^{iS[\varphi_c]} \langle \varphi_i t_i^{up} | \rho_i | \varphi_i' t_i^{low} \rangle$$



Contour in the complex time plane for the calculation of the decay probability. Thick black lines denote singularities of the bounce.

Bounce

- One can evaluate the path integral in the saddle-point approximation.

  $g \ll 1$ is the semiclassical parameter.

  This leads to $\Gamma \sim e^{-B}$, $B = -iS[\varphi_B]$ (+ boundary terms)

  where φ_B is a bounce.

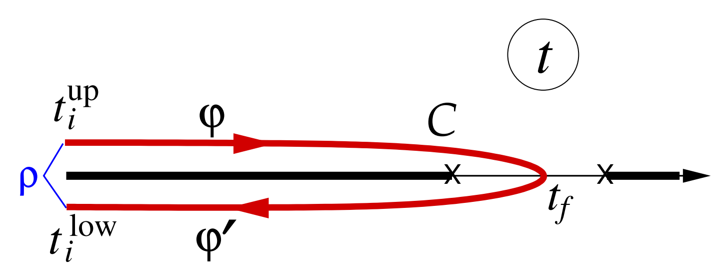
- It lives on the contour *C,* its values on the upper and lower parts of the contour are complex conjugate.

  As a consequence of its reality and uniqueness.

  It is real at $t = t_f$ (turning point). At $t > t_f$ it describes the evolution of the field after tunneling.

- It linearises in the limit $t \to t_i^{up/low}$, where it satisfies the vacuum boundary conditions.

  These are the same as for the **time-ordered Green's function** in the corresponding vacuum.



Contour in the complex time plane for the calculation of the decay probability. Thick black lines denote singularities of the bounce.

Vacuum boundary conditions

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- The time-ordered vacuum Green's function: $G_\times(\vec{x},t,\vec{x}',t') = \frac{1}{g^2}\, {}_\times\langle T(\hat{\varphi}(t,\vec{x})\,\hat{\varphi}(t',\vec{x}'))\rangle_\times$

$$(\partial_\mu\partial^\mu - m^2)\, G_\times(\vec{x},t,\vec{x}',t') = i\,\delta(\vec{x}-\vec{x}')\,\delta(t-t')$$

- The general solution of the equation $\partial_\mu\partial^\mu\varphi - m^2\varphi - V'_{int}(\varphi) = 0$

is written as

it provides boundary conditions for $\varphi(t,x)$

$$\varphi(t,x) = -i\int dt'\,dx'\, G(t,x,t',x')\, V'_{int}(\varphi(t',x'))$$

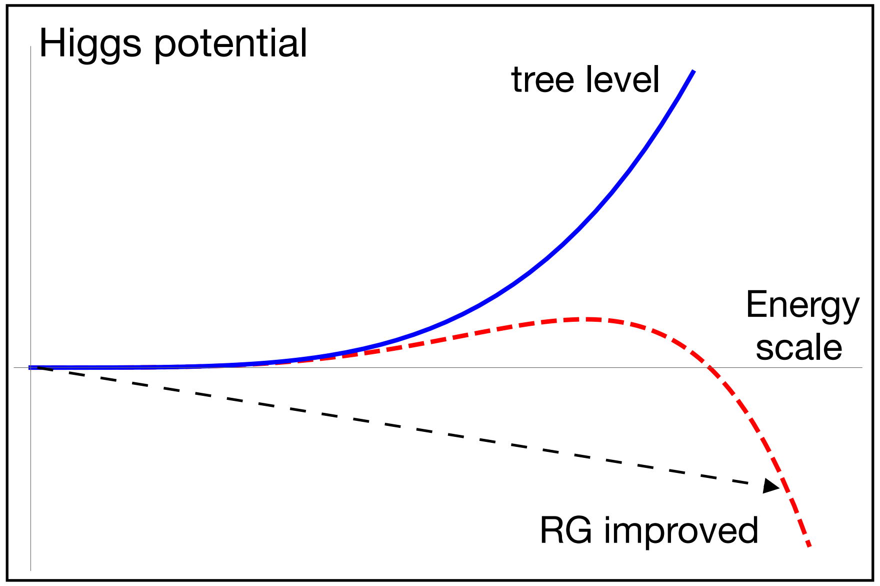
- To obtain the particular solution — bounce — one should specify the time-integration contour and Green's function:

$$\varphi_B(t,x) = -i\int_C dt'\,dx'\, G_\times(t,x,t',x')\, V'_{int}(\varphi_B(t',x'))$$

⟶ The Euclidean prescription follows for initial states in equilibrium.

⟶ We can handle out-of-equilibrium and time-dependent initial states.

Decay of Higgs vacuum



Vacuum decay in field theory is relevant for phenomenology.

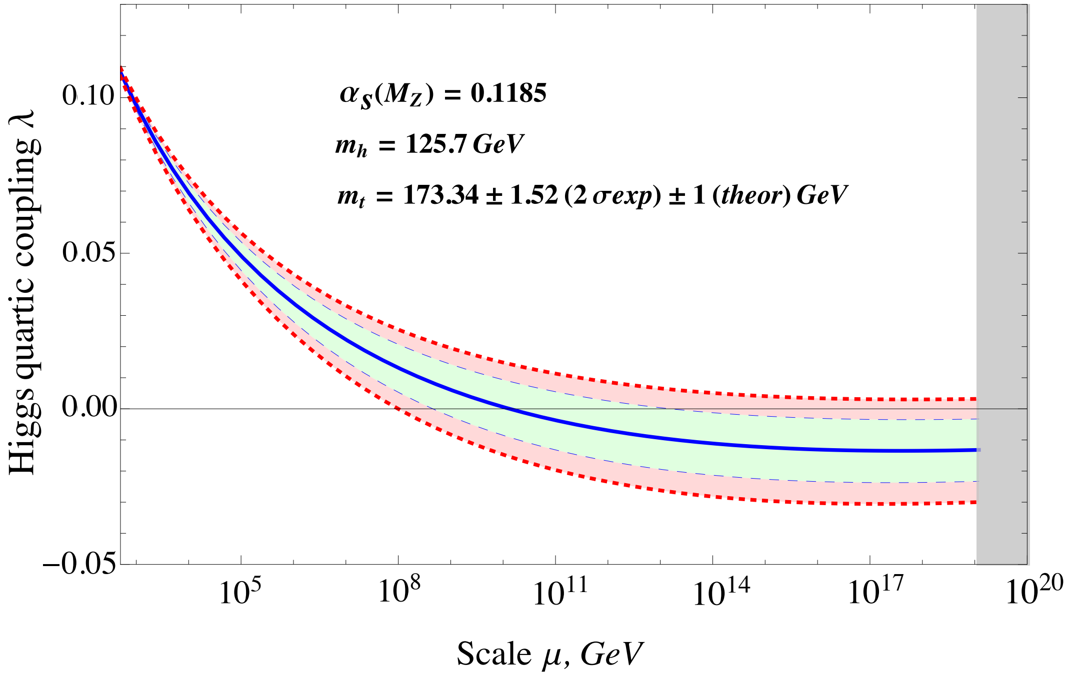
- Standard Model Higgs vacuum may not be absolutely stable.

  In the present-day Universe, the decay probability is small enough.
  But this can change in extreme environments.

- Stability of the Higgs vacuum has been studied in various setups:

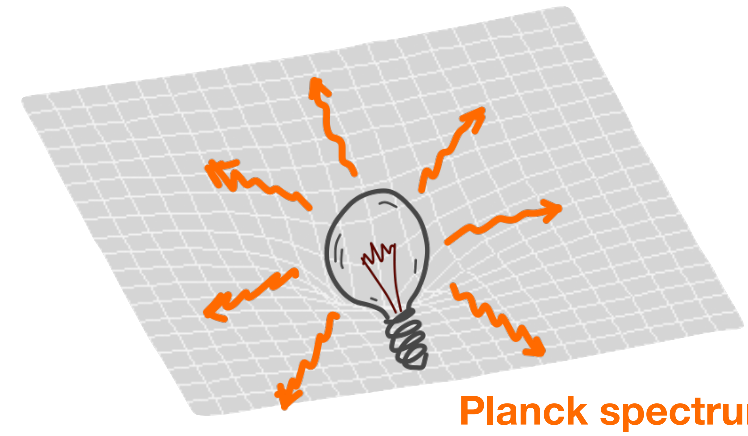
  in the Standard Model and its modifications,
  with or without gravitational corrections,
  in thermal bath and during inflation,
  near local inhomogeneities such as **black holes**

Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia 13;
Herranen, Markkanen, Nurmi, Rajantie 15;
Salvio, Strumia, Tetradis, Urbano 16;
Rajantie, Stopyra 17;
Andreassen, Frost, Schwartz 18 …



Standard Model running of the Higgs quartic coupling

Black holes and vacuum decay



Planck spectrum, corrected by greybody factors

BH features:

- It's a **simple** gravitational impurity — **curved geometry**

  S. Chandrasekhar:

  > "The black holes are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time… They are the simplest objects as well."

- It's a **simple** source of (almost) thermal radiation — **quantum vacuum**

These two facts are equally important for vacuum decay.

The problem is not new: Hiscock 87; Berezin, Kuzmin, Tkachev 88, 91; Arnold 90

…but the interest has been revived recently: Gregory, Moss, Withers 14; Burda, Gregory, Moss 15, 16; Tetradis 16
Gorbunov, Levkov, Panin 17; Mukaida, Yamada 17; Kohri, Matsui 17 …
Strumia 23

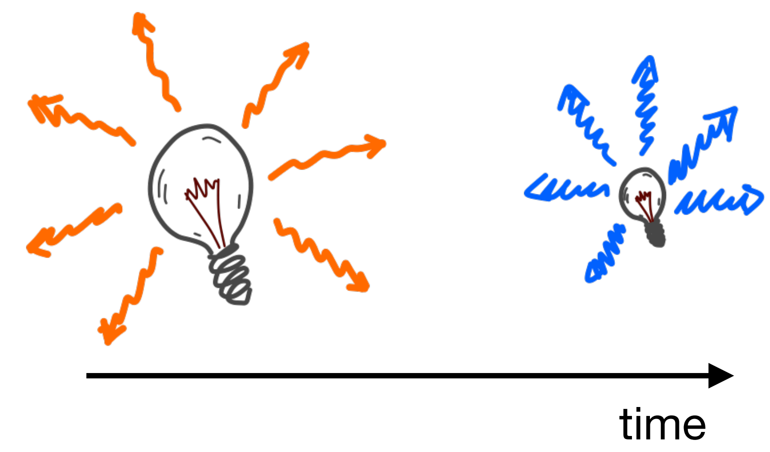
Black holes and vacuum decay

Claim: BHs can actually make vacuum decay **unsuppressed.**

Burda, Gregory, Moss: from “**BHs as bubble nucleation cites**” to “**Fate of the Higgs vacuum**”

They use Euclidean approach and look for the minimal-action configuration.

Their calculation can be improved, but what do we expect?



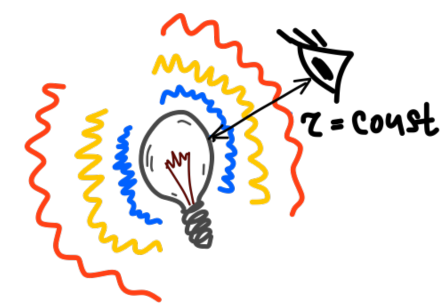
- Intuition in favour:

Smaller BHs have larger temperature: $T_{BH} = \dfrac{M_{Pl}^2}{8\pi M_{BH}}$

→ BHs evaporate, thus probing a wide range of energy scales.

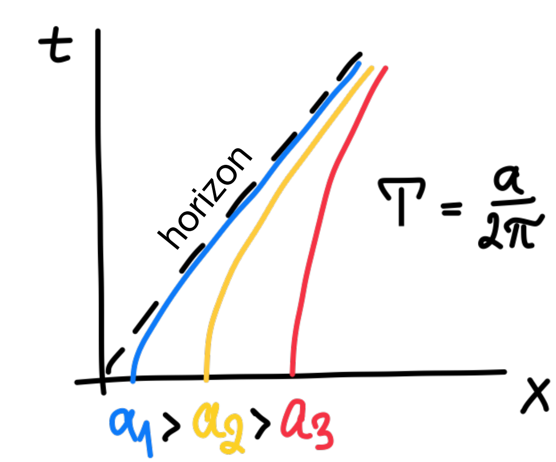
→ (Small) primordial BHs could be produced abundantly in the Early Universe.

Carr, Kohri, Sendouda, Yokoyama, 2002.12778



The BH temperature is an observer-dependent quantity: $\tilde{T}_{BH}(r) = \dfrac{T_{BH}}{\sqrt{1 - \frac{r_s}{r}}}$

→ Close to the horizon, field fluctuations become extremely energetic.



Trajectories of uniformly accelerating observers in flat spacetime.

- Counter-arguments:

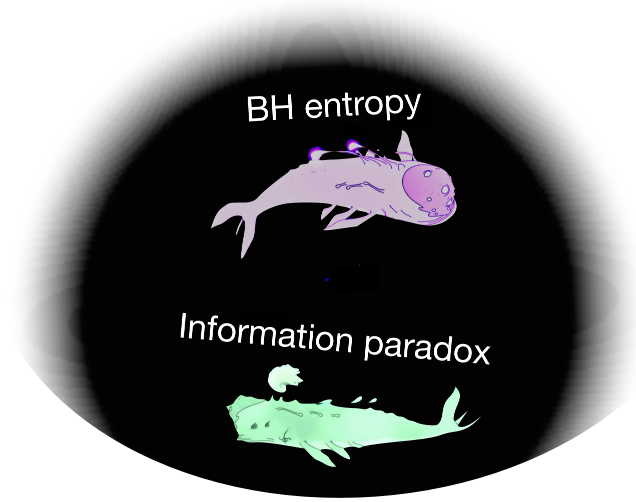
→ $\Delta E\, \Delta x \sim \hbar$, but this is not enough for vacuum decay: one must form a **coherent field configuration**: tunneling bounce or sphaleron.

→ Accelerating observers in flat space also see thermal radiation of arbitrary high T (**Unruh effect**), but the decay rate is observer-independent.

Black holes and vacuum decay

- BH catalysis of vacuum decay is an interesting theoretical problem with applications to phenomenology.

  Semiclassical gravity, Nonperturbative quantum gravity (once back-reaction is taken into account)…



- For the semiclassical analysis, we need to know vacuum states associated with BHs.

  These are, in general, out of thermal equilibrium; also they live in a curved background.

  We can use the general method to associate the vacuum states with the boundary conditions for the bounce solution.

Black hole vacuum

Boulware

Boulware 75

Eternal BH

BH mimicker

Unruh

Unruh in
"Notes on BH evaporation" 76

BH from the collapse of matter

Hartle-Hawking

Hartle, Hawking 76

BH in thermal equilibrium

One can expect that $\Gamma_B < \Gamma_U < \Gamma_{HH}$

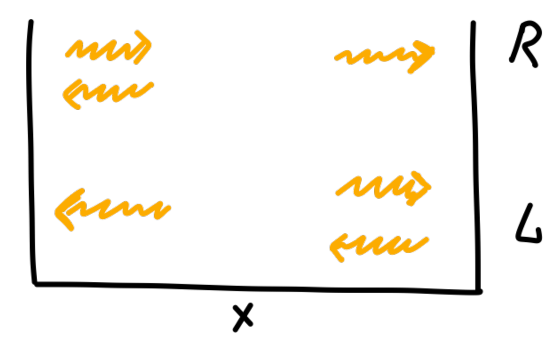
Arnold 90:

> "…I do not know how to handle the question of false vacuum decay in a nonequilibrium situation such as this one. I merely note that, since radiation helps the system cross the barrier, the results should lie somewhere between the two extremes of zero radiation and thermal equilibrium."

Black hole vacuum

- Scalar field in 1+1 dimensions:

$$\hat{\varphi}(t,x) = g \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} \sum_{I=L,R} \left(\hat{a}_{I,\omega}\, \varphi^{+}_{I,\omega}(t,x) + \hat{a}^{\dagger}_{I,\omega}\, \varphi^{-}_{I,\omega}(t,x) \right)$$



Right-moving modes

Left-moving modes

near-horizon region

asymptotically flat space

- Define the following initial state:

$$\langle \hat{a}^{\dagger}_{L,\omega}\, \hat{a}_{L,\omega'} \rangle_X = \frac{\delta(\omega-\omega')}{e^{\frac{2\pi\omega}{\lambda_L}} - 1} \qquad \langle \hat{a}^{\dagger}_{R,\omega}\, \hat{a}_{R,\omega'} \rangle_X = \frac{\delta(\omega-\omega')}{e^{\frac{2\pi\omega}{\lambda_R}} - 1} \qquad \lambda_{L,R} = 2\pi T_{L,R}$$

The three BH vacuum states can then be specified as follows:

Boulware: $\lambda_L = \lambda_R = 0$

Hartle-Hawking: $\lambda_L = \lambda_R = \lambda$

Unruh: $\lambda_L = 0, \ \lambda_R = \lambda$

Black hole Green's functions

- In the static BH background $\varphi^+_\omega(t,x) = f(x)\, e^{-i\omega t}, \quad \varphi^-_\omega(t,x) = f^*(x)\, e^{i\omega t}, \quad \omega > 0\,.$

  And the general Green's function takes the form

$$G_{\times}(x,t,x',t') = \int_0^\infty \frac{d\omega}{4\pi\omega}\Big(Q_L f_L(x) f_L^*(x') + Q_R f_R(x) f_R^*(x') + (|\beta|^2-1)\big(f_R(x) f_R^*(x') - f_L(x) f_L^*(x')\big)\Delta_{RL}$$

$$+ \sqrt{\frac{k}{\omega}}\big(\beta^*\gamma f_R(x) f_L^*(x') + \beta\gamma^* f_L(x) f_R^*(x')\big)\Delta_{RL}\Big)$$

$$Q_{L,R} = \frac{e^{-i\omega|t|}}{1 - e^{-\frac{2\pi\omega}{\lambda_{L,R}}}} + \frac{e^{i\omega|t|}}{e^{\frac{2\pi\omega}{\lambda_{L,R}}} - 1}$$

$$\Delta_{RL} = \left[\frac{1}{e^{\frac{2\pi\omega}{\lambda_R}} - 1} - \frac{1}{e^{\frac{2\pi\omega}{\lambda_L}} - 1}\right] e^{i\omega t}$$

β — reflection amplitude

γ — transmission amplitude

Unruh Green's function

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(Massive scalar) **Unruh Green's function** in two dimensions:

$$G_U(t,x,t',x') = \int_0^\infty \frac{d\omega}{4\pi\omega}\Bigg\{ f_{R,\omega}(x) f^*_{R,\omega}(x')\left[\frac{e^{-i\omega|t-t'|}}{1-e^{-\frac{2\pi\omega}{\lambda}}} + \frac{e^{i\omega|t-t'|}}{e^{\frac{2\pi\omega}{\lambda}}-1}\right]$$ — right modes in thermal bath

$$+ f_{L,\omega}(x) f^*_{L,\omega}(x')\, e^{-i\omega|t-t'|}$$ — left modes in vacuum

$$+ \left(|\beta_\omega|^2 - 1\right)\left[f_{R,\omega}(x) f^*_{R,\omega}(x') - f_{L,\omega}(x) f^*_{L,\omega}(x')\right]\frac{e^{i\omega(t-t')}}{e^{\frac{2\pi\omega}{\lambda}}-1}$$

$$+ \sqrt{\frac{k}{\omega}}\left[\gamma_\omega \beta^*_\omega f_{R,\omega}(x) f^*_{L,\omega}(x') + \gamma^*_\omega \beta_\omega f_{L,\omega}(x) f^*_{R,\omega}(x')\right]\frac{e^{i\omega(t-t')}}{e^{\frac{2\pi\omega}{\lambda}}-1}\Bigg\}$$ — greybody factor

Toy model: CGHS black hole

Callan, Giddings, Harvey, Strominger 92

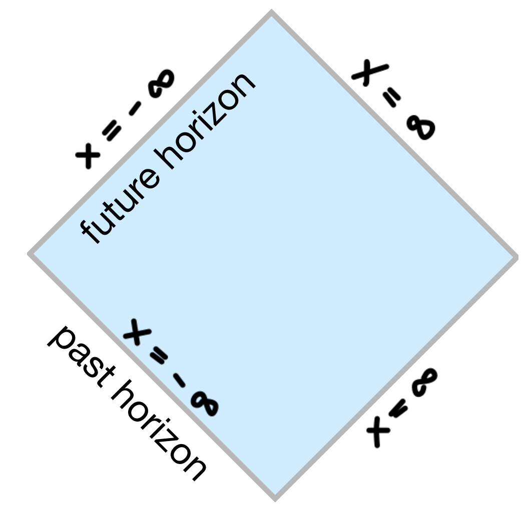
- BH metric in 1+1 dimensions: $ds^2 = \Omega(x)\left(-dt^2 + dx^2\right)$

  x is a "tortoise" coordinate

$$\Omega(x) = \frac{1}{1+e^{-2\lambda x}} \qquad \lambda = 2\pi T_{BH}$$

$\Omega(x) \approx 1 \qquad x \to +\infty$ — asymptotically flat space

$\Omega(x) \approx e^{2\lambda x} \qquad x \to -\infty$ — near-horizon region



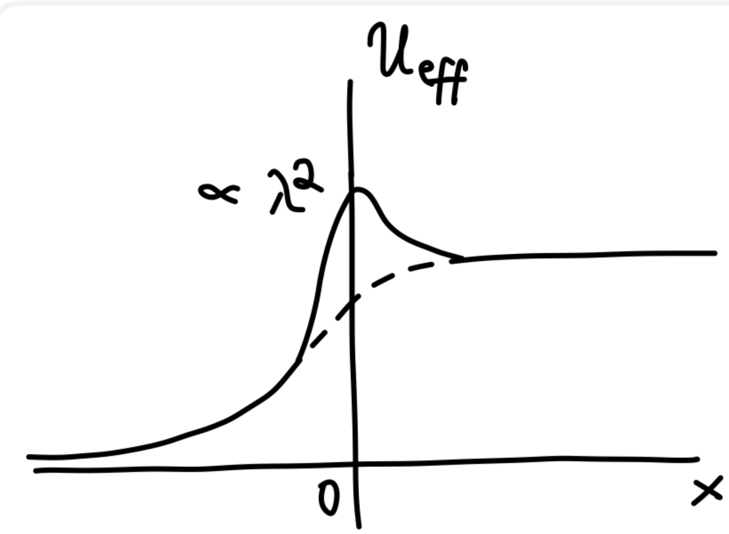
The part of the BH spacetime relevant for vacuum decay

- Tunneling system:

$$S = \frac{1}{g^2}\int d^2x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right)$$

$$= \frac{1}{g^2}\int d^2x\left(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\Omega\varphi^2 - \Omega V_{int}(\varphi)\right)$$

We neglect back-reaction of the tunneling field on the BH.



Effective potential for massive scalar modes

- To emulate the 4d centrifugal barrier, we also add the "dilaton barrier":

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$$\Delta S = -Q\int d^2x\sqrt{-g}\, e^{2\phi}\varphi^2$$

dimensionfull coupling

dilation field compounding the 2d BH

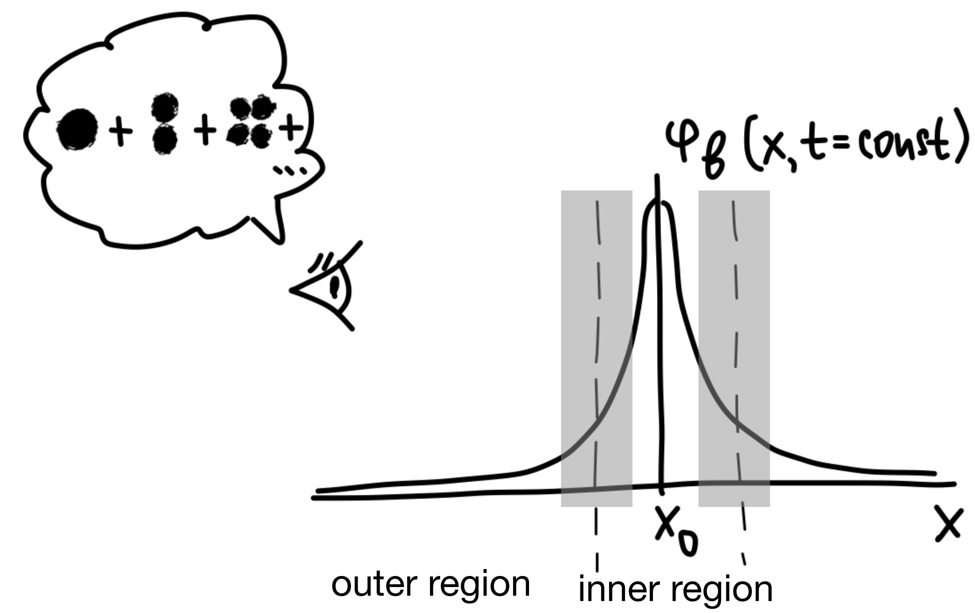
Toy model: Liouville potential

- Equation to solve: $\varphi_B(t,x) = -i \int_C dt' dx' G_{X}(t,x,t',x')\, \Omega(x')\, V'_{int}(\varphi_B(t',x'))$

  This is hard.

  Way around: find a model in which the bounce has a narrow nonlinear core (inner region) and a broad linear tail (outer region).

  Then $V'_{int}(\varphi_B(t',x')) \propto \delta^{(2)}(t',x')$. far enough from the core, and one can solve the equation separately in the two regions.

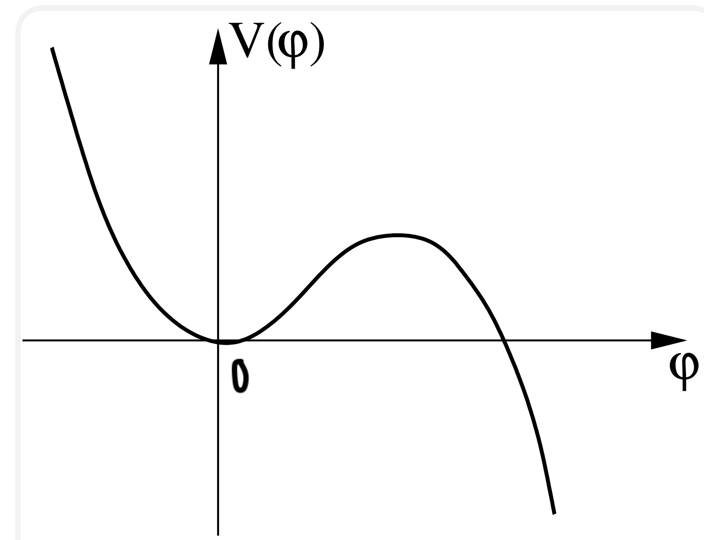


- For the nonlinear part take the Liouville potential:

$$V_{int}(\varphi) = -2\kappa\,(e^{\varphi} - 1)\,, \quad \kappa > 0 \qquad \ln \frac{m}{\sqrt{\kappa}} \gg 1$$

to ensure the existence of the overlap region

Then we can find the bounce and the decay rate **analytically**, for all three initial vacuum states, in the near-horizon region and far from the BH.



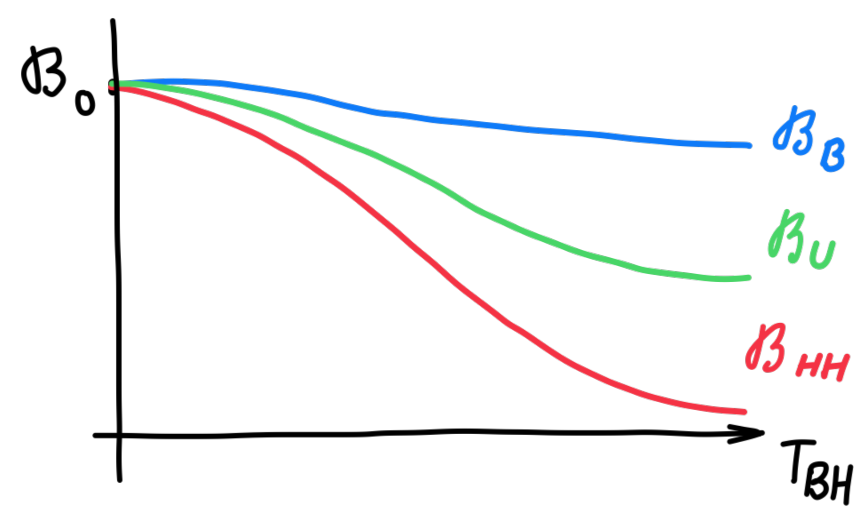
The scalar field potential

Toy model: results

- For the **Boulware** and **Hartle-Hawking** vacua our method is equivalent to the known prescriptions of looking for vacuum and finite-temperature bounces.
- The catalysing effect is both due to geometry and due to excitations of the field modes by the BH. Both effects are closely intertwined.

$$\mathcal{B}_{HH} < \mathcal{B}_{U} < \mathcal{B}_{B} \qquad\qquad \mathcal{B}_{near} < \mathcal{B}_{far}$$

“vacuum excitations” “curved geometry”



Suppression of the **Boulware**, **Hartle-Hawking** and **Unruh** vacuum decay as a function of BH temperature.

Stochastic regime for the Unruh vacuum

- We could only find the Unruh bounce up to a certain temperature T_{CU}.

  This is not surprising: thermal bounces also cease to exist at $T = T_c$, yielding to the sphaleron.

  So we expect that at $T > T_{CU}$ the decay proceeds via stochastic jumps.

- We couldn't find the "Unruh sphaleron" analytically, but we were able to use a simple stochastic estimate:

$$\Gamma \sim \exp\left(-\frac{\varphi_{max}^2}{2\,\delta\varphi^2}\right)$$

where the variance of the field fluctuations is

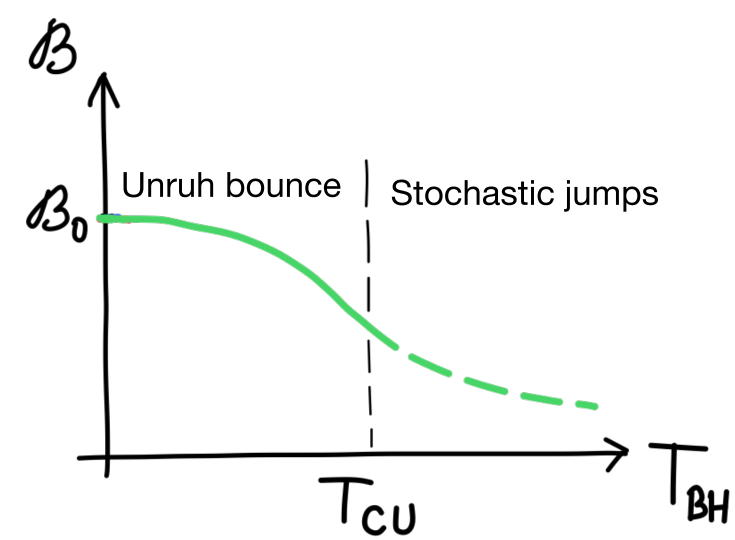
$$\delta\varphi^2_X = g^2 \lim_{t,x\to 0}\left(G_{2X}(t,x,0,0) - G_F(t,x,0,0)\right)$$

This works in our particular model because

→ almost any relevant field fluctuation is Gaussian

→ modes with relevant frequencies $\omega \sim m$ are highly populated

In general, classical simulation is needed.



Reflections and outlook

- Contrary to the Hartle-Hawking vacuum, the decay of the Unruh vacuum remains exponentially suppressed at all BH temperatures. **This result probably holds in four dimensions.**
- Gravitational backreaction must be included at some point.
- So far we focused on the semiclassical method. Real-time simulations might also be useful.
- The method is quite general and can be applied to time-dependent systems as well.

More on bounce at finite temperature

The bounce-sphaleron transition point was studied in QM and field theory

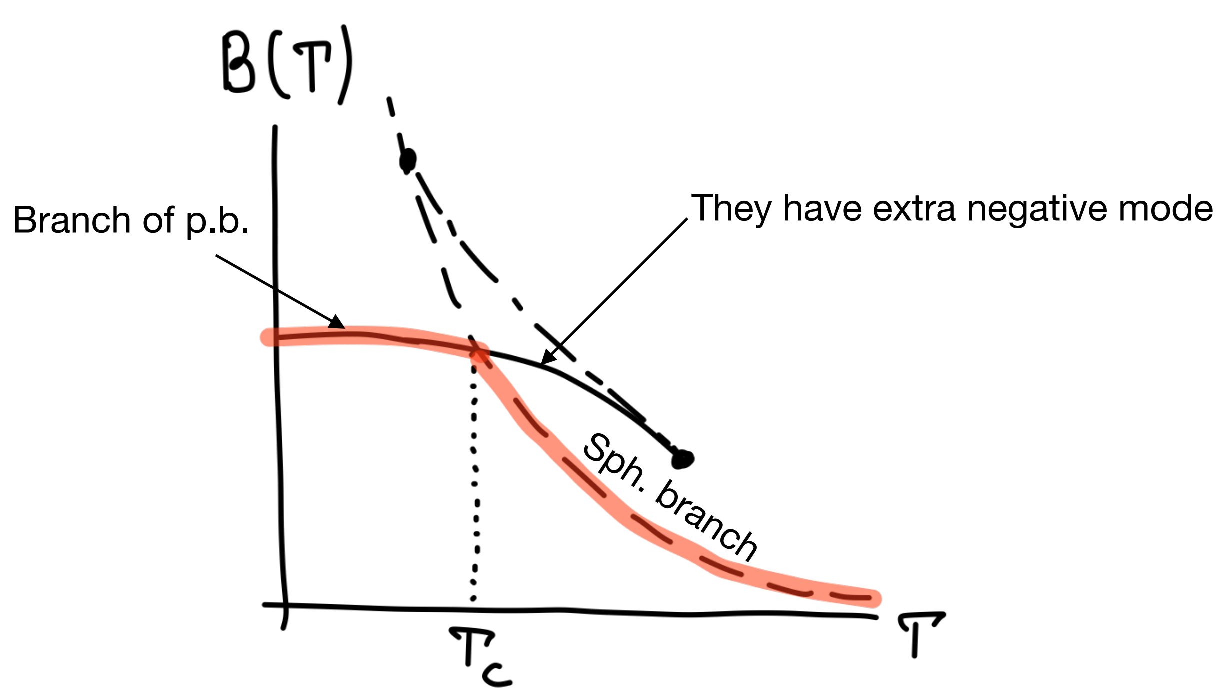
Chudnovsky 92

Garriga 94; Ferrera 95

In the thin-wall approximation, periodic bounces do not merge with the sphaleron — the transition is 1st order.

(The thin-wall sphaleron may not even exist)

This is not an artefact of the approximation.



Exponential suppression of vacuum decay in the thin-wall potential in 3+1 dimension.

Decay of the Unruh vacuum

In the model with the dilaton barrier, the effective potential for massive scalar modes is given by

$$U_{\text{eff}}(x) = \frac{m^2}{1+\mathrm{e}^{-2\lambda x}} + \frac{2q\lambda^2\mathrm{e}^{-2\lambda x}}{(1+\mathrm{e}^{-2\lambda x})^2}$$

Define $\quad q = \dfrac{a}{\ln(m/\sqrt{\kappa})}$

Then, the tunneling action behaves as follows:

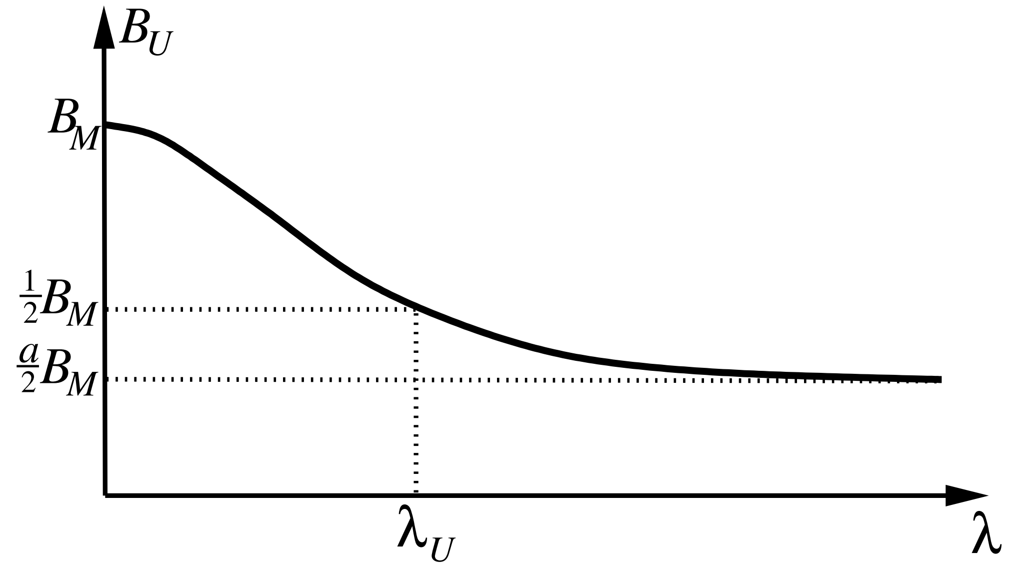
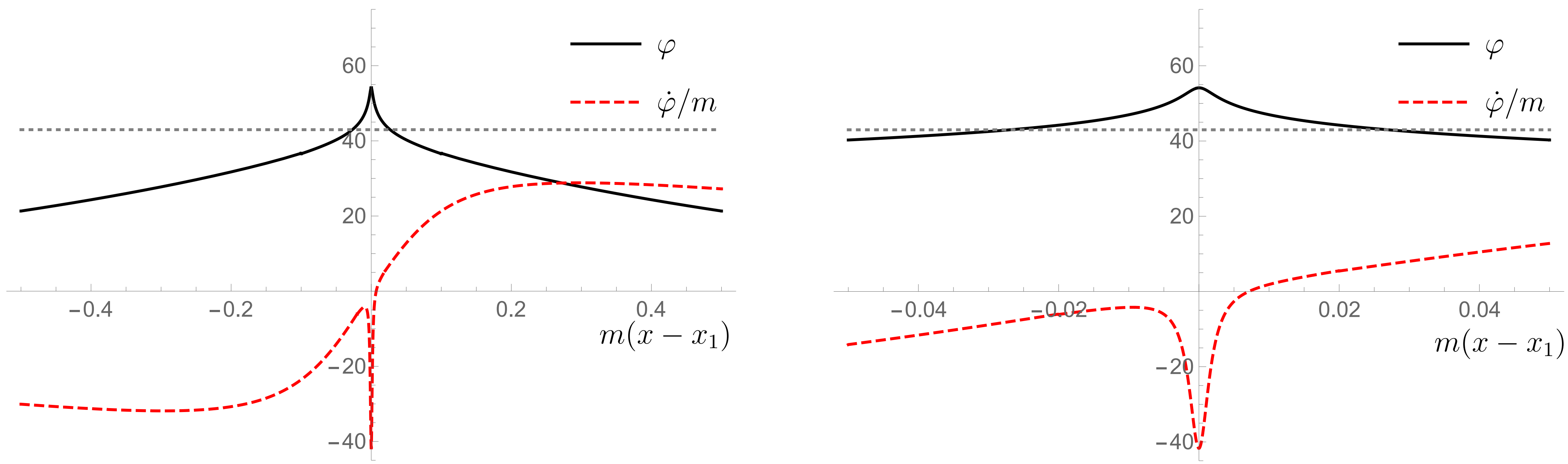


Figure 10: Exponential suppression of the Unruh vacuum decay as a function of BH temperature $T_{BH} = \lambda/(2\pi)$. Decay proceeds via tunneling in the near-horizon region at $\lambda < \lambda_U$, whereas at $\lambda > \lambda_U$ it is mediated by stochastic jumps in the vicinity and far away from BH.

Unruh bounce far from horizon



Bounce solution describing tunneling from the Unruh vacuum far away from the BH.
Left: Profiles of the bounce (black solid) and its time derivative (red dashed) at $t=0$ for $\lambda = 0.87\Lambda_{U1}$.
Right: Zoom-in on the central region of the left plot. We take $\ln \frac{m}{\sqrt{\kappa}} = 20$.
The grey dotted line marks the field value φ_{max} at the maximum of the potential barrier.

$$\Lambda_{U1} = \frac{3\pi m}{4}\left(\ln \frac{m}{\sqrt{\kappa}} + \gamma_E - \frac{1}{4}\right)$$

Here we consider the model without the dilaton barrier.