Phase Transitions in the Dark Universe

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Outline

- Introduction to phase transitions + the main idea.
- Dark matter via inverse phase transition.
- Topological defects: domain walls.
- Axions.
- Summary.

Are there phase transitions in cosmology?

There is no electroweak phase transition, but rather a crossover, in the Standard Model $SU(3) \times SU(2) \times U(1)$.

NB. QCD $SU(3)$ phase transition leaves weak traces, if it took place.

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- Particle emission during phase transitions can be a source of Dark Matter (in this talk).

Phase transitions: symmetry of the system changes.

$$V_{eff}(\varphi, T) = \frac{\lambda}{4} \cdot \left(\varphi^2 - v^2\right)^2 + c_1 T^2 \varphi^2 + c_2 T \varphi^3 + \ldots.$$

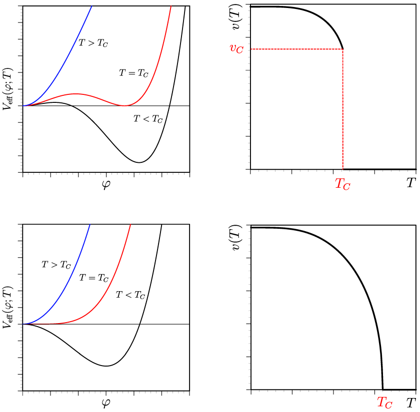

Figure is taken from Senaha'20

Common lore: the system evolves towards spontaneously broken phase described by a constant non-zero expectation value of a scalar as the Universe cools down.

Canonical example: electroweak phase transition in the early Universe is due to $\langle \text{Higgs} \rangle = \frac{v_{SM}}{\sqrt{2}} = \text{const.}$

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In this talk: scale of spontaneous symmetry breaking is time-dependent

$$\boxed{v(t) \propto T(t) \propto \frac{1}{a(t)}}$$

Weinberg'74, Vilenkin'81, Dodelson&Widrow'90

Other possible choices: $v(t) \propto H(t)$, $|\mathrm{B}(t)|$ (primordial magnetic fields)

Model with Z_2-symmetry

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{M^2 \cdot \chi^2}{2} - \frac{\lambda \cdot \chi^4}{4} + \frac{g^2 \chi^2 \phi^\dagger \phi}{2} \; .$$

χ is cold

ϕ is in thermal equilbrium with plasma

scalar portal

$0 < g^2 \ll 1$

$\frac{g^4}{\lambda_\phi} \lesssim \lambda \ll 1$

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$$\langle \phi^\dagger \phi \rangle_T = \frac{NT^2}{12} \Longrightarrow V_{eff} = \frac{M^2 \cdot \chi^2}{2} + \frac{\lambda \cdot \chi^4}{4} - \frac{Ng^2 T^2 \chi^2}{24}$$

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$T \propto \frac{1}{a(t)} \Longrightarrow Z_2$-symmetry breaking at early times

$$\langle \chi \rangle = \sqrt{\frac{Ng^2 T^2(t)}{12\lambda} - \frac{M^2}{\lambda}}$$

Symmetry restoration vs Symmetry non-restoration

$$\langle \chi \rangle = \sqrt{\frac{Ng^2T^2}{12\lambda} - \frac{M^2}{\lambda}}$$

- $M^2 > 0$ symmetry is restored at $T_{sym} \sim M/g$

$\langle \chi \rangle \to 0$ Inverse Phase Transition!

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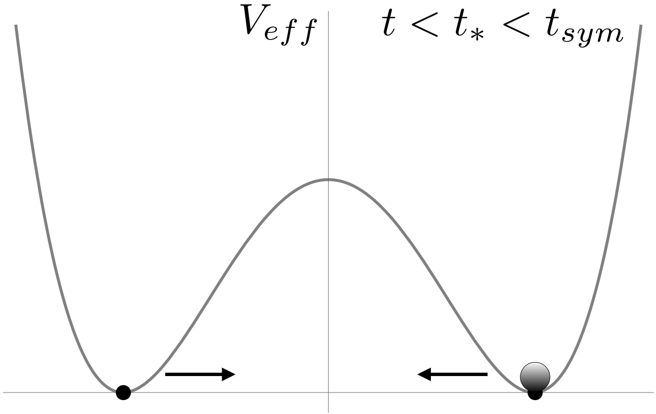
- $M^2 < 0 \Longrightarrow$ symmetry remains broken at late times.

$$\langle \chi \rangle \to \text{const} \neq 0$$

Storyline 1:
Inverse phase transition and beyond freeze-in Dark Matter

Based on SR, Babichev, Gorbunov, Vikman'21



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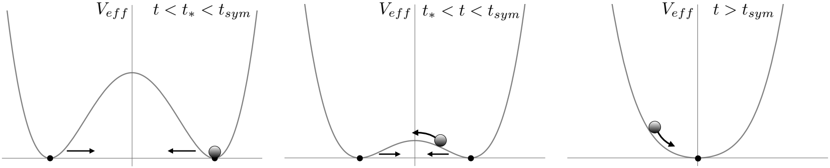

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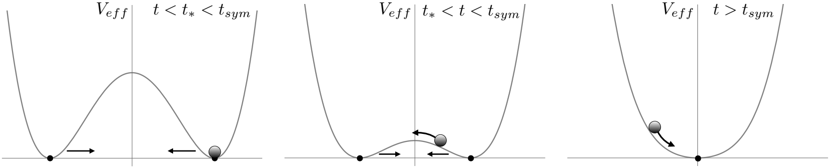
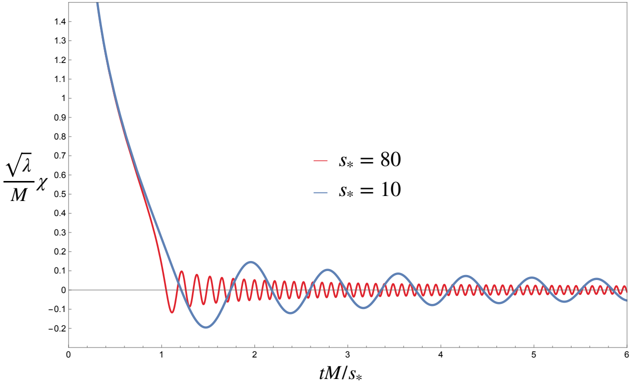


Z_2-symmetry + feeble couplings involved protect stability
$\Longrightarrow$ these oscillations naturally feed into Dark Matter



Dark Matter abundance is fulfilled provided that

$$M \simeq 25 \text{ eV} \cdot \frac{\beta^{3/5}}{\sqrt{N}} \cdot \left(\frac{g}{10^{-8}}\right)^{7/5} \qquad \beta \equiv \frac{\lambda}{g^4} \gtrsim 1/\lambda_\phi \gtrsim 1$$

Why being interested in so small masses and/or feeble couplings?

Freeze-out $\to$ freeze-in $\to$ inverse phase transition

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{M^2 \cdot \chi^2}{2} - \frac{\lambda \cdot \chi^4}{4} + \frac{g^2 \chi^2 \phi^\dagger \phi}{2} \, .$$

- Freeze-out: relatively large $10^{-8} \ll |g^2| \ll 1$

  WIMPS: masses 1 MeV $\lesssim M \lesssim$ 100 TeV

  Initially in thermal equilibrium by sufficiently large couplings $\Longrightarrow$ good prospects for testing.

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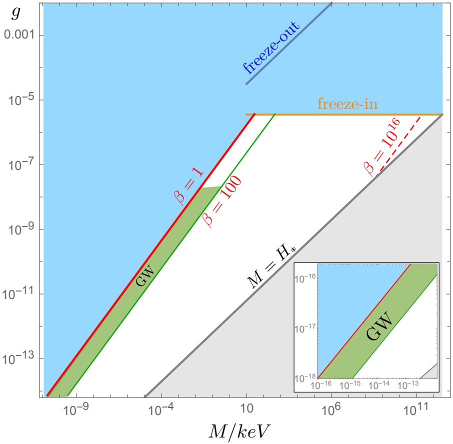
- For $0 < g^2 \lesssim 10^{-11}$ Dark Matter from inverse phase transition.

Is there a life beyond freeze-in?

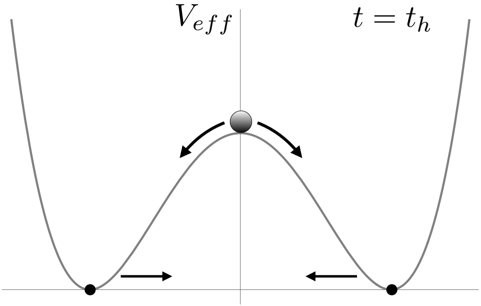


$$\beta \equiv \frac{\lambda}{g^4} \qquad\qquad 1 \lesssim \frac{1}{\lambda_\phi} \lesssim \beta \lesssim 10^{18}$$

Storyline 2: melting domain walls.

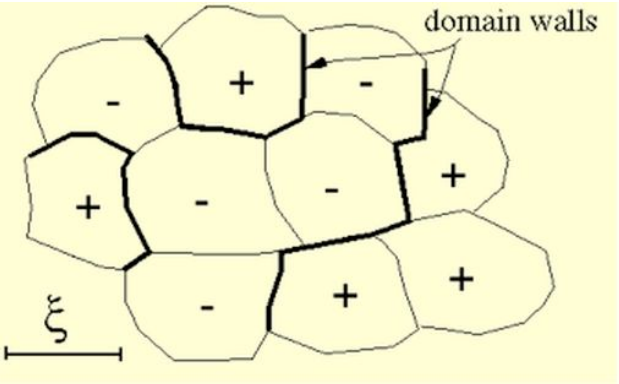
Babichev, Gorbunov, SR, Vikman'21

Domain walls are common in models with spontaneous breaking of Z_2-symmetry Zel'dovich et al'74



NB Setting to zero through $\xi R\chi^2/2$ for $\xi \gtrsim 1$

Domain walls separate regions, where $\chi = \pm\langle\chi\rangle$



The picture is taken from http://www.ctc.cam.ac.uk/

Domain wall problem

$$\text{Kink :} \quad \chi(z) = \langle\chi\rangle \cdot \tanh\left(\sqrt{\frac{\lambda}{2}} \cdot \langle\chi\rangle \cdot z\right)$$

$$\text{Domain wall tension:} \quad \sigma_{wall} = \frac{M_{wall}}{S} = \frac{2\sqrt{2\lambda}\langle\chi\rangle^3}{3}$$

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In the scaling regime: one or a few domain walls in the horizon volume.

Ryden, Press, Spergel'89

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$$\rho_{wall} \sim M_{wall} H^3 \sim \sigma_{wall} H$$

Constant tension domain walls: $\rho_{wall} \simeq \sigma_{wall} H \propto T^2$

$\frac{\rho_{wall}}{\rho_{rad}} \propto \frac{1}{T^2(t)} \propto a^2(t) \Longrightarrow$ domain walls overclose the Universe!

No domain wall problem in our case

$$\langle \chi \rangle \propto T \Longrightarrow \sigma_{wall} \sim \sqrt{\lambda} \langle \chi \rangle^3 \propto T^3$$

$$\rho_{wall} \simeq \sigma_{wall} H \propto T^5 \qquad \frac{\rho_{wall}}{\rho_{rad}} \propto T(t) \propto \frac{1}{a(t)}$$

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Energy density of domain walls redshifts faster than radiation

Domain walls completely vanish at inverse phase transition

Vilenkin'81

Do melting domain walls leave any trace?

Domain walls emit gravitational waves

Einstein quadrupole formula+dimensional considerations

Power of gravitational radiation: $P \sim \frac{\dddot{Q}_{ij}\dddot{Q}_{ij}}{40\pi M_{Pl}^2}$

Quadrupole moment: $Q_{ij} \sim \frac{M_{wall}}{H^2} \sim \frac{\sigma_{wall}}{H^4}$

$$\rho_{gw} \sim (P \cdot t) \cdot H^3 \sim \frac{\sigma_{wall}^2}{M_{Pl}^2} \Longrightarrow \rho_{gw} \propto T^6(t) \propto \frac{1}{a^6(t)}$$

Most energetic gravitational waves are emitted right after domain wall formation

Weaker coupled means more visible

Numerical simulations: Hiramatsu, Kawasaki, Saikawa'13

$$F_{gw,peak} \simeq H(t) \qquad \rho_{gw} \simeq \frac{\sigma_{wall}^2}{M_{Pl}^2}$$

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$$F_{gw,peak} \simeq H(t) \qquad \rho_{gw} \simeq \frac{\sigma_{wall}^2}{M_{Pl}^2}$$

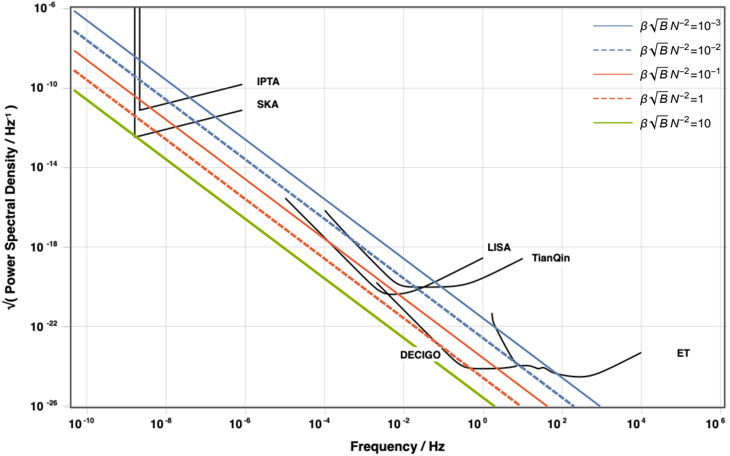
$$\boxed{f_{gw,peak} \simeq 60 \text{ Hz} \cdot \sqrt{\frac{N}{B}} \cdot \left(\frac{g}{10^{-8}}\right)} \qquad \boxed{\Omega_{gw,peak} \cdot h_0^2 \approx \frac{4 \cdot 10^{-14} \cdot N^4}{B \cdot \beta^2}}$$

$B = \ln^2 \frac{2\langle\chi\rangle}{\delta\chi} \simeq 1 - 100$ takes into account finite time of roll

Vanilla region: $\boxed{\beta \equiv \frac{\lambda}{g^4} \simeq 1}$ $\qquad \boxed{N \gg 1}$

NB $\quad \frac{Ng^4}{16\pi^2} \lesssim \lambda \lesssim 1$

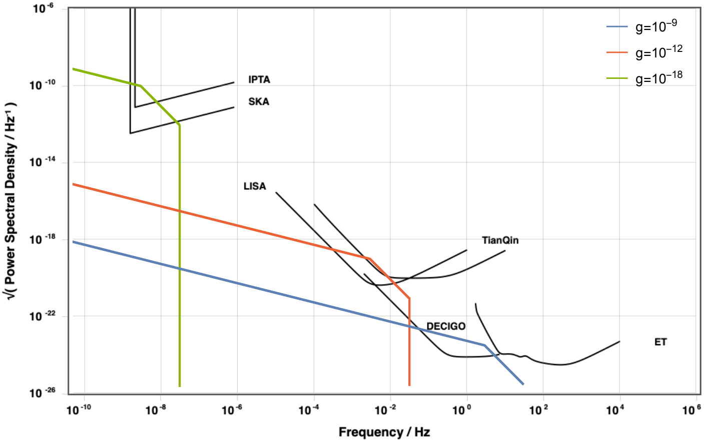
Gravitational waves vs sensitivity curves



Strain $\sqrt{S_h}$ $\qquad \Omega_{gw} H_0^2 = \frac{2\pi^2 f^3}{3} S_h$

gwplotter.com Moore, Cole, and Berry'14

Preliminary spectrum of GWs



$$\sqrt{S_h} \sim \frac{1}{\sqrt{f}} \ (f < f_{gw,peak}) \qquad \sqrt{S_h} \sim \frac{1}{f^2} \ \ (f_{gw,peak} < f < f_{cutoff})$$

Different spectrum compared to the case of constant tension domain walls.

Storyline 3: Symmetry remains broken at low temperatures. Axions. Based on SR and R. Samanta'22.

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- Axions are introduced to resolve strong CP-problem of QCD.

  Peccei&Quinn'77

- They are pseudo-Nambu-Goldstone bosons of spontaneously broken anomalous $U(1)$ Peccei-Quinn symmetry.

$$S = |S| \cdot e^{\frac{ia}{f_{PQ}}}$$

- Axions are often considered as promising dark matter candidates.

$$V(a) \simeq \Lambda_{QCD}^4 \cdot \left[1 - \cos\left(\frac{a}{f_{PQ}}\right)\right] \Longrightarrow m_a \simeq \frac{\Lambda_{QCD}^2}{2f_{PQ}}$$

Preskill et al'83, Abbott&Sikivie'83, Dine&Fischler'83

$$\mathcal{L} = |\partial_\mu S|^2 - \lambda_S \cdot |S|^4 + g^2 |S|^2 \phi^\dagger \phi \ .$$

At high temperatures: $$f_{PQ} \equiv \sqrt{2}\langle S \rangle = \sqrt{\frac{N}{24\beta}} \cdot \frac{T}{g}$$

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Peccei-Quinn scale is gliding in the early Universe from sub-Planck values to relatively small values, as the Universe cools down

Another characteristic feature: $M^0_S = g v_\phi \lesssim 1$ GeV

Typically, Peccei-Quinn field is much heavier, $M_S \simeq f_{PQ}$

This may have a dramatic effect on axion dark matter production mechanisms

Conventional axion dark matter production mechanisms:

- Misalignment mechanism.
- Decay of global topological defects Davis'86, Kawasaki et al'14

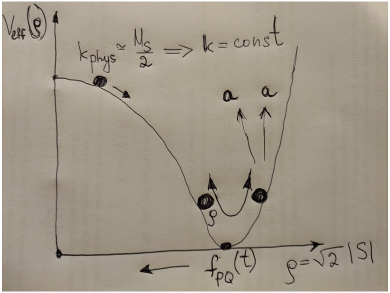
Both are efficient for $f_{PQ}^{0} \simeq 10^{11} - 10^{12}$ GeV.

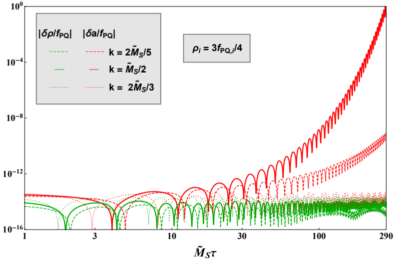
On the other hand, $f_{PQ}^{0} \sim 10^{8} - 10^{9}$ GeV can be interesting for supernovae and horizontal branches stars.

Temperature-dependence $f_{PQ}(t) \propto T(t)$
enables a new production mechanism for axions, efficient independently of low-energy scale f_{PQ}^{0}.

Resonant axion production in the early Universe, when the radial field relaxes to the minimum of its potential, cf. Co, Hall, and Harigaya'17

Similar to (narrow) parametric resonance after inflation.
Compared to inflation, resonance band is stable: redshift of produced axions is compensated by time-decrease of the Peccei-Quinn scale $f_{PQ}(t)$.





Parameter space strongly depends on the ratio $r \equiv \frac{\mathcal{E}_\rho}{\mathcal{E}_a+\mathcal{E}_\rho}$ of produced radial fluctuations $\rho \equiv \sqrt{2}|S|$ along with axions. For $r \gtrsim 0.01$, the scenario is on the way to being ruled out.

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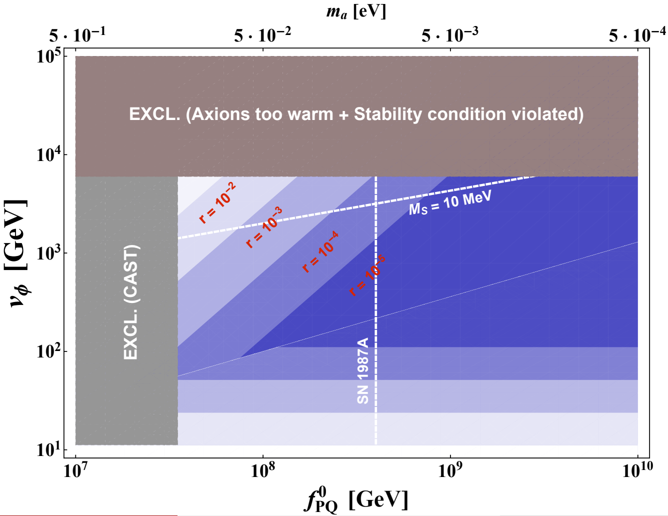

Summary

Spontaneous symmetry breaking in the early Universe by a time-varying expectation value of a scalar field allows:

- to generate the right abundance of Dark Matter in a feebly coupled regime.
- to get rid of domain wall problem, and at the same time enjoy potentially observable gravitational waves from domain walls. With these gravitational waves, one can probe the underlying field-theoretical model in a very weakly coupled regime inaccessible by other experiments.
- to generate axion dark matter in astrophysically interesting range of Peccei-Quinn scale values.

Thanks for your attention!!!