A New Gravitational Action For The Trace Anomaly

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Quantum Anomalies

(e.g., Adler, Bell, Jackiw axial anomaly)

- Two or more global symmetries in a classical theory (e.g., vector & axial symm.)
- Not all of these symmetries can be preserved by the respective quantum theory.
- How to capture these anomalies in a "quantum effective theory"?

A toy example

(1+1) D massless electrodynamics (Schwinger)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\Psi} i \gamma^\mu (\partial_\mu + ie A_\mu) \Psi$$

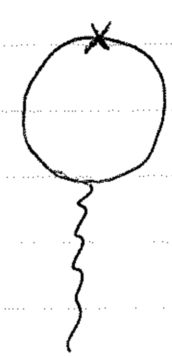
$(+\ -)$, $\mu, \nu = 0, 1$ $\qquad [e] = [\text{mass}]$

Classical symmetries:

$U_V(1) \qquad U_A(1)$

Full quantum theory

$$\partial_\mu J_\mu = 0$$



$$\partial_\mu J_\mu^5 = \frac{e}{2\pi} \varepsilon_{\alpha\beta} F^{\alpha\beta}$$

Bosonization: (Coleman '75 ...)

$$J_\mu = \frac{1}{\sqrt{\pi}} \varepsilon_{\mu\alpha} \partial^\alpha \Phi \qquad J_\mu^5 = \frac{1}{\sqrt{\pi}} \partial_\mu \Phi$$

Eq's of motion + anomaly:

$$\begin{cases} \partial^\mu F_{\mu\nu} = \dfrac{e}{\sqrt{\pi}} \varepsilon_{\nu\alpha} \partial^\alpha \Phi \\ \Box \phi = \dfrac{e}{2\sqrt{\pi}} \varepsilon_{\alpha\beta} F^{\alpha\beta} \end{cases}$$

Lagrangian that gives rise to these equations as its "classical" eom's

$$\mathcal{L}_B = -\frac{1}{4} F_{\mu\nu}^2 + \frac{e}{2\sqrt{\pi}} \Phi \varepsilon F + \frac{1}{2}(\partial_\mu \Phi)^2$$

Axial anomaly "captured" by classical equations of $\mathcal{L}_B$

4D Trace Anomaly

(Crewther '72
Chanowitz, Ellis, '72)

$$T^{\mu}_{\mu} = \frac{\beta(e)}{2e} F_{\alpha\beta}^{2}$$ no gravity

$\mu, \nu = 0, 1, 2, 3 \quad (- + + +)$

Including gravitational field

$$\langle T^{\mu}_{\mu} \rangle = aE + cW^2 + b\Box R$$

(Capper Duff 75
Duff 77)

$$E \equiv R_{\mu\nu\alpha\beta}^{2} - 4R_{\alpha\beta}^{2} + R^2$$

$$W^2 \equiv R_{\mu\nu\alpha\beta}^{2} - 2R_{\alpha\beta}^{2} + R^2/3$$

$$\boxed{M_p^2 R = aE + cW^2}$$

Need an effective action that has this as its equation of motion.

The Riegert Action:

(Riegert 84)
(Fradkin Tseytlin 84)

Find $\tilde{S}_A(g)$:

$$g^{\alpha\beta}\frac{2}{\sqrt{-g}}\frac{\delta\tilde{S}_A(g)}{\delta g^{\alpha\beta}} = a\,E(g) + c\,W^2$$

Riegert: $\tilde{S}_A(g)$ cannot be local.

$$g_{\alpha\beta} = e^{2\sigma}\,\bar{g}_{\alpha\beta}(x)$$

$$S' = S_{GR}(e^{2\sigma}\bar{g}) + S_A(\bar{g},\sigma)$$

$$S_A = a\int d^4x\,\sqrt{-\bar{g}}\left(\sigma\bar{E} - 4G_{\alpha\beta}\bar{\nabla}^\alpha\sigma\bar{\nabla}^\beta\sigma - 4(\bar{\nabla}^2\sigma)(\bar{\nabla}\sigma)^2 - 2(\bar{\nabla}\sigma)^4\right)$$

$$+\,c\int d^4x\,\sqrt{-\bar{g}}\;\bar{W}^2$$

(Komargodski, Schwimmer)
(GG, Tukhashvili $SO(2,4)/ISO(1,3)$)

$$M_p^2\,R = a\,E(g) + c\,W^2(g)$$

The issue:

$$S_{GR}(e^{2\sigma}\bar{g}) + S_A(\sigma, \bar{g})$$

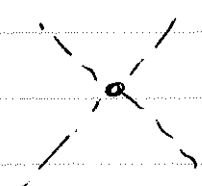
$$= S_{GR}(g) + S_A(\sigma, g)$$

no kinetic term for σ

$$G\nabla\sigma\nabla\sigma, \quad (\nabla^2\sigma)(\nabla\sigma)^2, \quad (\nabla\sigma)^4$$

An infinitely strongly coupled theory

$$\lim_{z\to 0}\left(z^2(\nabla\sigma)^2 - (\nabla\sigma)^4 \rightarrow (\nabla\hat{\sigma})^2 - \frac{(\nabla\hat{\sigma})^4}{z^4} \right)$$



$$\frac{s^2}{z^4} \xrightarrow[z\to 0]{} \infty$$

Origin of the problem:

the conformal mode is not a propagating dof in GR:

$$\sqrt{-g}\, R$$

$$h_{\alpha\beta} = g_{\alpha\beta} - \eta_{\alpha\beta}$$

$$h_{00} = 2n \qquad h_{0j} = v_j + \partial_j u$$

$$h_{ij} = t_{ij} + \partial_i \omega_j + \partial_j \omega_i + 2\partial_i \partial_j \rho - 2\delta_{ij}\tau$$

$$-3\dot{\tau}^2 + (\partial_j \tau)^2 - 2n\, \partial_j^2 \tau$$

$$\frac{\delta}{\delta n} \quad \Rightarrow \quad \Delta\tau = 0$$

τ does not propagate

The $R - \bar{R}$ theory:

$$\boxed{S_{R-\bar{R}} = M^2 \int d^4x \sqrt{-g}\, R - \bar{M}^2 \int d^4x \sqrt{-\bar{g}}\, \bar{R}}$$

$$\boxed{g_{\alpha\beta} = e^{2\sigma} \bar{g}_{\alpha\beta}} \qquad \epsilon^2 \equiv \bar{M}^2 / M^2 \ll 1$$

$$\underbrace{-3\dot{\tau}^2 + (\partial_j \tau)^2 - 2h\Delta\tau}\; - \epsilon^2 \Big(-3(\dot{\tau} + \dot{\sigma})^2$$
$$+ (\partial_j(\tau+\sigma))^2 - 2(h+\sigma)\Delta(\tau+\sigma) \Big)$$

Constraint: $\quad \Delta\left(\tau - \frac{\epsilon^2}{1-\epsilon^2}\sigma\right) = 0$

$$\text{Lagrangian} = -6M^2 \frac{1-\epsilon^2}{\epsilon^2} (\partial\tau)^2 = -\frac{6\bar{M}^2}{1-\epsilon^2} (\partial\sigma)^2$$

proper kinetic term
$(-+++)$

Symmetries

$$S_{R-\bar{R}} = \int d^4x \sqrt{-g} \left(M^2 R - \phi^2 R - 6 (\nabla \phi)^2 \right)$$

$$\phi = \bar{M} e^{-\sigma} \qquad \bar{M} << M = M_p/\sqrt{2}$$

$$\phi \to \phi + \frac{\omega M (1 - \phi^2/M^2)}{1 + \omega \phi / M}$$

$\omega = \text{const.}$

$$g_{\mu\nu} \to g_{\mu\nu} \frac{(1 + \omega \phi/M)^2}{1 - \omega^2}$$

invariant $\hat{g} = g - \epsilon^2 \bar{g} = g(1 - \phi^2/M^2)$

$$\hat{g} \simeq g + O(\epsilon^2)$$

Matter couples to $\hat{g} \simeq g$

* Duality: $M \leftrightarrow \bar{M}$ $\quad g \leftrightarrow -\bar{g}$ $\quad \sigma \to -\sigma$

Toward the quantum effective action

$$H \equiv M(g-\eta) \qquad \Sigma = \bar{M} - \Phi$$

① $M \to \infty$ $\bar{m}$ is fixed

$$-\frac{1}{4} H G(H) - 6(\partial\Sigma)^2$$

Hence, loop generated terms $\to 0$; $M \to \infty$

② $M \to \infty$ $\bar{M} \to \infty$ $\epsilon = \bar{M}/M \ll 1$ fixed

$$-\frac{1-\epsilon^2}{4} HG(H) + 2\epsilon \Sigma R_L - 6(\partial\Sigma)^2$$

Loop generated terms should vanish in ②

Hence, all local terms $\frac{R^{\#}}{M^{\#}}$

but nonlocal terms can exist

Trace anomaly

$R - \bar{R}$ combined with Riegert

$$S_{eff} = M^2 \int d^4x \sqrt{-\bar{g}}\, e^{2\sigma} \left(\bar{R} + 6(\bar{\nabla}\sigma)^2\right) - \bar{M}^2 \int d^4x \sqrt{-\bar{g}}\, \bar{R}$$

$$+ a \int d^4x \sqrt{-\bar{g}} \left(\sigma \bar{E} - 4 \bar{G} \bar{\nabla}\sigma \bar{\nabla}\sigma - 4 \bar{G}_3 - 2 \bar{G}_4\right)$$

$$+ c \int d^4x \sqrt{-\bar{g}}\, \sigma \bar{W}^2$$

$$\Rightarrow \quad 2M^2 R = a E(g) + c W^2(g) + O(\epsilon^2)$$

(P. Fernandes '21)

Based on a classical requirement that eom's be conf. inv.

The anomaly terms are down by scales $<< M_P$!

Strong coupling @ $\bar{M}$

„Decoupling" limit $M \to \infty$ $\bar{M}$ fixed

$$\pi \equiv \bar{M}\sigma$$

$$S_{eff}\Big|_{\bar{g}=e^{-2\pi/\bar{M}}\eta} \simeq \int d^4x\left(-6\,e^{-2\pi/\bar{M}}(\partial\pi)^2\right.$$

$$\left.+\,4a\int d^4x\left(-\frac{\partial^2\pi(\partial\pi)^2}{\bar{M}^3}+\frac{(\partial\pi)^4}{2\bar{M}^4}\right)+\dots\right)$$

cubic Galileon ↑ ↑ quartic Gal

Conformal Galileon

Nicolis, Rattazzi, Trincherini

(BH solution, S. Tsujikawa 23)

Fifth force $\sim \alpha\left(\bar{M}/M_p\right)^2$

Conclusions:

$$S_{GR}(M, e^{2\sigma}\bar{g}) - S_{GR}(\bar{M}, \bar{g})$$

$$+ S_A(\sigma, \bar{g})$$

An additional propagating degree of freedom σ. Strongly coupled @ $\bar{M} \ll M_p$.

Physical consequences of σ.

$$S_{GR}(M, g) - S_{GR}(\bar{M}, e^{-2\sigma} g)$$

$$+ \bar{S}_A(\sigma, g)$$

Matter couples to $\simeq g$