Conformal Renormalization of anti-de Sitter gravity

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Rodrigo Olea

UNAB, Chile

HOLOGRAPHYCL

Outline

Holographic Renormalization of AdS gravity action

Topological terms in AdS gravity

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

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Black Hole Thermodynamics in AdS gravity

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{D-2}^2 \,, \qquad f^2(r) = 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Einstein-AdS gravity

$$I_{EH} = \frac{1}{16\pi G}\int_M d^D x \sqrt{-g}\,(R - 2\Lambda)\,, \qquad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

Gibbs free energy $G = TI^E$

$$TI^E_{bulk} = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

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Counterterms in AdS gravity

Holographic Renormalization [Henningson and Skenderis, 1998]

$$I_{\text{ren}} = I_{EH} - \frac{1}{8\pi G}\int_{\partial M} d^d x\,\sqrt{-h}\,K + \int_{\partial M} d^d x\, L_{ct}(h,\mathcal{R},\nabla\mathcal{R})$$

Counterterms [Balasubramanian-Kraus, 1999], [Emparan, Johnson, Myers, 1999]

$$\begin{aligned}8\pi G\, L_{ct} &= \frac{d-1}{\ell}\sqrt{-h} + \frac{\ell\sqrt{-h}}{2(d-2)}\mathcal{R} + \frac{\ell^3\sqrt{-h}}{2(d-2)^2(d-4)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d}{4(d-1)}\mathcal{R}^2\right)\\ &\quad + \frac{\ell^5\sqrt{-h}}{(d-2)^3(d-4)(d-6)}\left(\frac{3d-2}{4(d-1)}\mathcal{R}\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d(d+2)}{16(d-1)^2}\mathcal{R}^3\right.\\ &\quad \left. -2\mathcal{R}^{ij}\mathcal{R}^{kl}\mathcal{R}_{ijkl} - \frac{d}{4(d-1)}\nabla_i\mathcal{R}\nabla^i\mathcal{R} + \nabla^k\mathcal{R}^{ij}\nabla_k\mathcal{R}_{ij}\right) + \ldots\end{aligned}$$

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Black Hole Thermodynamics and Counterterms

Counterterm Method reproduces BH Thermo

$$G = U - TS$$

Internal Energy

$$U = M + E_0$$

Vacuum Energy in $D = 2n + 1$ dimensions

$$E_0 = (-1)^n \frac{(2n-1)!!^2}{(2n)!} \frac{\mathrm{Vol}(S^{2n-1})}{8\pi G} \ell^{2n-2}$$

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Topological terms in AdS gravity

4D AdS action [R. Aros et al, gr-qc/9909015]

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G} \int\limits_M d^4x \sqrt{-g} \left[(R - 2\Lambda) + \frac{\ell^2}{4} (R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2) \right]$$

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Euler-Gauss-Bonnet Theorem in 4D

$$\int_{M} d^4x\, GB = \int_{\partial M} d^3x\, B_3(K,\mathcal{R}) + 32\pi^2\chi(M)$$

Euler Theorem in $2n$ dimensions

$$\int_{M_{2n}} d^{2n}x\, (\text{Euler})_{2n} = \int_{\partial M_{2n}} d^{2n-1}x\, B_{2n-1} + (4\pi)^n n!\,\chi(M_{2n})$$

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Kounterterms in AdS gravity

Extrinsic counterterms

$$\tilde{I}_{ren} = I_{EH} + c_d \int\limits_{\partial M} d^d x \, B_d(h, K, \mathcal{R})$$

Kounterterms = counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}^{kl}_{ij}(h)$)

$D = 2n$ dimensions [R.O., hep-th/0504233]

$$B_{2n-1} = 2n\sqrt{-h} \int\limits_0^1 dt \; \delta^{[i_1 \cdots i_{2n-1}]}_{[j_1 \cdots j_{2n-1}]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3} \right) \times \cdots$$

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Kounterterms in $D=2n+1$ [R.O., hep-th/0610230]

$$
\begin{aligned}
B_{2n} \;=\;& 2n\sqrt{-h}\int_0^1 dt\int_0^t ds\,\delta^{[j_1\cdots j_{2n}]}_{[i_1\cdots i_{2n}]}\,K^{i_1}_{j_1}\delta^{i_2}_{j_2}\left(\frac{1}{2}\mathcal{R}^{i_3i_4}_{j_3j_4}-t^2K^{i_3}_{j_3}K^{i_4}_{j_4}+\frac{s^2}{\ell^2}\,\delta^{i_3}_{j_3}\delta^{i_4}_{j_4}\right)\times\cdots\\
&\cdots\times\left(\frac{1}{2}\mathcal{R}^{i_{2n-1}i_{2n}}_{j_{2n-1}j_{2n}}-t^2K^{i_{2n-1}}_{j_{2n-1}}K^{i_{2n}}_{j_{2n}}+\frac{s^2}{\ell^2}\,\delta^{i_{2n-1}}_{j_{2n-1}}\delta^{i_{2n}}_{j_{2n}}\right).
\end{aligned}
$$

Renormalized Action = Renormalized Volume

Black Hole Thermodynamics

$$T I_{bulk}^{E} = \frac{(D-3)}{(D-2)} M - TS + \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Euclidean Kounterterms

$$T c_d \int_{\partial M} B_d = \frac{M}{(D-2)} + E_0 - \lim_{r\to\infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Correct Black Hole Thermo with $U = M + E_0$

Renormalized Action = Renormalized Volume

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Boundary conditions in AdS gravity

Fefferman-Graham expansion for AAdS Einstein spaces

$$ds^2 = \frac{\ell^2}{z^2}\,dz^2 + \frac{1}{z^2}\,g_{ij}(x,z)\,dx^i dx^j\,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots$$

Dirichlet b.c. δh_{ij} — it does not make sense in AAdS spaces [Papadimitriou and Skenderis, 2004]

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Renormalization — variational problem in $g_{(0)ij}$

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Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell}\frac{g_{(0)ij}}{z^2} + \ldots$$

Counterterms of a different sort

$$\tilde{I}_{ren} = I_{EH} + c_d \int\limits_{\partial M} d^d x\, B(f(h), K)$$

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From extrinsic to intrinsic renormalization in 4D

AdS gravity action + KTs

$$\tilde{I}_{\text{ren}} = I_{EH} + \frac{\ell^2}{16\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} \right).$$

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Fefferman-Graham expansion

$$K^i_j = \frac{1}{\ell}\delta^i_j - \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2), \qquad S^i_j(h) = \frac{1}{d-2}\left(\mathcal{R}^i_j(h) - \frac{1}{2(d-1)}\delta^i_j \mathcal{R}(h)\right)$$

Kounterterms turn into counterterms [O.Miskovic and R.O., 0902.2082]

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Higher even dimensions

EH-AdS gravity +KTs

$$\mathrm{B}_{2n-1} = 2n\sqrt{-h}\int_0^1 dt\ \delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots$$
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$$\tilde{I}_{\text{ren}} = I_{Dir} + \int_{\partial M} d^{2n-1}x\, L_{ct}$$

Expanding and collecting

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \right.$$
$$\left. + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n+1)}{4(2n-2)}\mathcal{R}^2 - \frac{(2n-3)}{4}\mathcal{R}^{ijkl}\mathcal{R}_{ijkl}\right) + \cdots\right].$$

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Higher even dimensions

Boundary Weyl tensor $\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}$ implies

$$\mathcal{R}^{ijkl}\mathcal{R}_{ijkl} = \mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \frac{4}{(2n-3)}(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{1}{2(2n-2)}\mathcal{R}^2)$$

And, finally

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n-1)}{4(2n-2)}\mathcal{R}^2\right)\right.$$
$$\left. - \frac{\ell^3}{8(2n-3)(2n-5)}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \cdots\right].$$

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$$\left. - \frac{\ell^3}{8(2n-3)(2n-5)}\mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \cdots\right].$$

Higher even dimensions

Boundary Weyl tensor $\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}$ implies

$$\mathcal{R}^{ijkl}\mathcal{R}_{ijkl} = \mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \frac{4}{(2n-3)}(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{1}{2(2n-2)}\mathcal{R}^2)$$

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Holographic Renormalization = Kounterterms?

Mismatch with HR. [G.Anastasiou, O.Miskovic, R.O. and I.Papadimitriou, 2003.06425]

$$\tilde{I}_{\text{ren}} = I_{\text{HR}} - \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int\limits_{\partial M} \sqrt{-h}\, \mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \ldots$$

A similar result in $D = 2n+1$ dimensions.

Last term is zero for most AAdS spaces (Schwarzschild, Kerr, black strings).

Gravitational instantons: non-trivial boundary geometries

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Gravitational instantons: non-trivial boundary geometries

Motivation

Is there any other principle to renormalize AdS gravity action, which includes topological terms as a particular case.

Renormalized AdS Action

Renormalized AdS action = MacDowell-Mansouri action (1977)

$$I_{\text{ren}} = \frac{\ell^2}{256\pi G}\int\limits_M d^4x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]}\left[R^{\mu_1\mu_2}_{\nu_1\nu_2}+\frac{\delta^{\mu_1\mu_2}_{\nu_1\nu_2}}{\ell^2}\right]\left[R^{\mu_3\mu_4}_{\nu_3\nu_4}+\frac{\delta^{\mu_3\mu_4}_{\nu_3\nu_4}}{\ell^2}\right],$$

Weyl tensor

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - 4S^{[\alpha}_{[\mu}\delta^{\beta]}_{\nu]}, \qquad \text{Schouten } S^{\alpha}_{\mu} = \frac{1}{D-2}\left(R^{\alpha}_{\mu} - \frac{1}{2(D-1)}\delta^{\alpha}_{\mu}R\right)$$

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Renormalized AdS Action

Weyl tensor for Einstein spaces $S^{\alpha}_{\mu} = -\frac{1}{2\ell^2}\delta^{\alpha}_{\mu}$

$$W^{\alpha\beta}_{(E)\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\,\delta^{[\alpha\beta]}_{[\mu\nu]}$$

Renormalized action for Einstein spaces

$$I_{\text{ren}} = \frac{\ell^2}{64\pi G}\int\limits_M d^4x\,\sqrt{-g}\,W_{(E)\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}_{(E)}$$

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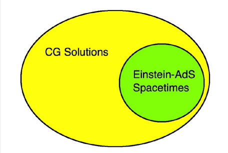
Conformal Renormalization

Embedding Einstein theory in Conformal Gravity



- Why?: Conformal Gravity is finite for AAdS conditions. [Grumiller et al., 2013]
- What for?: Renormalization should be inherited by the Einstein sector.
- How?: a *holographic* mechanism to turn CG into Einstein

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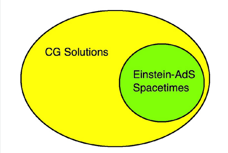

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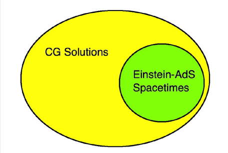

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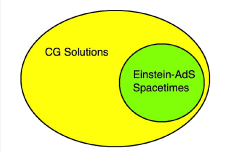

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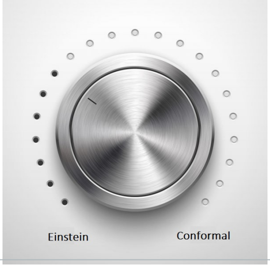

Einstein Gravity from Conformal Gravity in 4D

Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG}=\alpha_{CG}\int\limits_{M}d^4x\,\sqrt{-g}\,W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}$$

Fefferman-Graham expansion for AAdS spaces in CG

$$ds^2=\frac{\ell^2}{z^2}\,dz^2+\frac{1}{z^2}\,g_{ij}(x,z)\,dx^i dx^j\,,\quad g_{ij}(x,\rho)=g_{(0)ij}(x)+z^2 g_{(2)ij}(x)+\cdots$$
$$+z g_{(1)ij}(x)+\cdots$$

EOM for Conformal Gravity - Bach tensor

$$B_{\mu\nu}=\nabla^\lambda C_{\mu\nu\lambda}+S^{\lambda\sigma}W_{\lambda\mu\sigma\nu}=0\,,\qquad C^{\mu}_{\nu\lambda}=\nabla_\nu S^{\mu}_{\lambda}-\nabla_\lambda S^{\mu}_{\nu}$$

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Einstein spaces: holographic prescription

Einstein-AdS spaces

$$S^{\mu}_{\nu} = -\frac{1}{2\ell^2}\delta^{\mu}_{\nu}, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$D^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{D}R\delta^{\mu}_{\nu} = 0$$

$$D_{\mu\nu} = 0 \Longleftrightarrow \partial_z g_{ij} = g_{(1)ij} = 0 \text{ and } tr(\partial_z^3 g_{ij}) \sim tr(g_{(3)ij}) = 0$$

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AdS gravity in 6D

EH Action+Euler term

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G} \int\limits_M d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right),$$

In terms of fully antisymmetric objects

$$\begin{aligned}\tilde{I}_{\text{ren}} &= \frac{1}{16\pi G \times 192} \int\limits_M d^6x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\Big[R^{\mu_1\mu_2}_{\nu_1\nu_2}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} \\ &\quad + \frac{2}{3\ell^2}\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} - \frac{\ell^4}{3}R^{\mu_1\mu_2}_{\nu_1\nu_2}R^{\mu_3\mu_4}_{\nu_3\nu_4}R^{\mu_5\mu_6}_{\nu_5\nu_6}\Big],\end{aligned}$$

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In terms of fully-antisymmetric objects

$$\begin{aligned}\tilde{I}_{\text{ren}} &= \frac{1}{16\pi G\times 192}\int\limits_{M} d^6x\sqrt{-g}\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\left[R^{\mu_1\mu_2}_{\nu_1\nu_2}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]}\right.\\ &\left.+\frac{2}{3\ell^2}\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} - \frac{\ell^4}{3}R^{\mu_1\mu_2}_{\nu_1\nu_2}R^{\mu_3\mu_4}_{\nu_3\nu_4}R^{\mu_5\mu_6}_{\nu_5\nu_6}\right],\end{aligned}$$

AdS gravity in 6D

Polynomial of $W_{(E)}$

$$\tilde{I}_{\text{ren}} = \frac{\ell^4}{16\pi G\times 4!}\int\limits_M d^6x\sqrt{-g}\Big[\frac{1}{2\ell^2}\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]}W^{\mu_1\mu_2}_{(E)\nu_1\nu_2}W^{\mu_3\mu_4}_{(E)\nu_3\nu_4} - \frac{1}{4!}\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}W^{\mu_1\mu_2}_{(E)\nu_1\nu_2}W^{\mu_3\mu_4}_{(E)\nu_3\nu_4}W^{\mu_5\mu_6}_{(E)\nu_5\nu_6}\Big],$$

Conformal Gravity in 6D

There are three Conformal Invariants in 6D

$$I_1 = W_{\alpha\beta\mu\nu} W^{\alpha\sigma\lambda\nu} W_{\sigma}{}^{\beta\mu}{}_{\lambda} \,,$$
$$I_2 = W_{\mu\nu\alpha\beta} W^{\alpha\beta\sigma\lambda} W_{\sigma\lambda}{}^{\mu\nu} \,,$$
$$I_3 = W_{\mu\rho\sigma\lambda} \left(\delta^{\mu}_{\nu} \Box + 4R^{\mu}_{\nu} - \frac{6}{5} R \delta^{\mu}_{\nu} \right) W^{\nu\rho\sigma\lambda} + \nabla_{\mu} J^{\mu} \,,$$

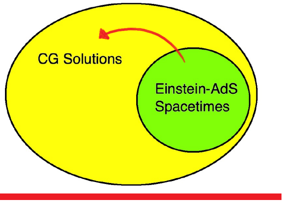
with

$$\begin{aligned} J_{\mu} \quad &= \quad 4R_{\mu}{}^{\lambda\rho\sigma} \nabla^{\nu} R_{\nu\lambda\rho\sigma} + 3R^{\nu\lambda\rho\sigma} \nabla_{\mu} R_{\nu\lambda\rho\sigma} - 5R^{\nu\lambda} \nabla_{\mu} R_{\nu\lambda} \\ & \quad + \frac{1}{2} R \nabla_{\mu} R - R^{\nu}_{\mu} \nabla_{\nu} R + 2R^{\nu\lambda} \nabla_{\nu} R_{\lambda\mu} . \end{aligned}$$

Conformal Covariantization

Einstein	$\to$	**Conformal**	**CI's**
$\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\mathbf{W}^{\mu_1\mu_2}_{(E)\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{(E)\nu_3\nu_4}\mathbf{W}^{\mu_5\mu_6}_{(E)\nu_5\nu_6}$	$\to$	$\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\mathbf{W}^{\mu_1\mu_2}_{\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{\nu_3\nu_4}\mathbf{W}^{\mu_5\mu_6}_{\nu_5\nu_6}$	$32(2\mathbf{I}_1+\mathbf{I}_2)$
$-\frac{1}{\ell^2}\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]}\mathbf{W}^{\mu_1\mu_2}_{(E)\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{(E)\nu_3\nu_4}$	$\to$	$\delta^{[\nu_1\cdots\nu_5]}_{[\mu_1\cdots\mu_5]}\mathbf{W}^{\mu_1\mu_2}_{\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{\nu_3\nu_4}\mathbf{S}^{\mu_5}_{\nu_5}+\mathbf{16C}^{\mu\nu\lambda}\mathbf{C}_{\mu\nu\lambda}+\nabla^\mu\mathbf{J}_\mu$	$4\mathbf{I}_1-\mathbf{I}_2-\mathbf{I}_3$

$$\mathbf{J}_\mu=\mathbf{16W}^{\kappa\lambda\nu}_{\mu}\mathbf{C}_{\kappa\lambda\nu}-\mathbf{2W}^{\kappa\lambda}_{\nu\sigma}\nabla_\mu\mathbf{W}^{\nu\sigma}_{\kappa\lambda}$$



Lu-Pang-Pope CG in 6D

6D CG with an Einstein sector [Lu, Pang and Pope, 2013]

$$I_{CG} = \alpha_{CG} \int\limits_{M} d^6x\sqrt{-g}\left(\frac{1}{4!}\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]} W^{\mu_1\mu_2}_{\nu_1\nu_2} W^{\mu_3\mu_4}_{\nu_3\nu_4} W^{\mu_5\mu_6}_{\nu_5\nu_6} + \frac{1}{2}\delta^{[\nu_1\cdots\nu_5]}_{[\mu_1\cdots\mu_5]} W^{\mu_1\mu_2}_{\nu_1\nu_2} W^{\mu_3\mu_4}_{\nu_3\nu_4} S^{\mu_5}_{\nu_5} + 8C^{\mu\nu\lambda}C_{\mu\nu\lambda}\right) + \alpha_{CG}\int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(8W^{\kappa\lambda\nu}_{\mu}C_{\kappa\lambda\nu} - W^{\kappa\lambda}_{\nu\sigma}\nabla_{\mu}W^{\nu\sigma}_{\kappa\lambda}\right).$$

- LPP action appears as type-B anomaly and one-loop divergences in 7D

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$$\left.+8C^{\mu\nu\lambda}C_{\mu\nu\lambda}\right)+\alpha_{CG}\int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(8W^{\kappa\lambda\nu}_{\mu}C_{\kappa\lambda\nu}-W^{\kappa\lambda}_{\nu\sigma}\nabla_{\mu}W^{\nu\sigma}_{\kappa\lambda}\right).$$

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$$\left. + 8C^{\mu\nu\lambda}C_{\mu\nu\lambda}\right) + \alpha_{CG} \int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(8W^{\kappa\lambda\nu}_{\mu} C_{\kappa\lambda\nu} - W^{\kappa\lambda}_{\nu\sigma}\nabla_{\mu} W^{\nu\sigma}_{\kappa\lambda}\right).$$

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Lu-Pang-Pope CG in 6D (EoM)

Schwarzschild BH is a solution of LPP CG. [Lu, Pang and Pope, 1301.7083]

EOM in terms of W, C and S tensors → any Einstein AdS spacetime is a solution

[Anastasiou, Araya and RO, 2010.15146]

For an arbitrary gravity theory

$$\begin{aligned} \mathcal{E}_{\mu}^{\nu} &= E_{\mu\lambda}^{\alpha\beta} R_{\alpha\beta}^{\nu\lambda} - \frac{1}{2}\delta_{\mu}^{\nu}\mathcal{L} + 2\nabla^{\lambda}\nabla_{\sigma} E_{\mu\lambda}^{\nu\sigma} \\ E_{\mu\lambda}^{\nu\sigma} &= \frac{\partial \mathcal{L}}{\partial R_{\nu\sigma}^{\mu\lambda}} - \nabla_{\alpha}\left(\frac{\partial \mathcal{L}}{\partial \nabla_{\alpha} R_{\nu\sigma}^{\mu\lambda}}\right) \end{aligned}$$

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Lu-Pang-Pope CG in 6D (EoM)

For LPP CG theory

$$\mathbf{E}^{\mu\nu}_{\lambda\sigma} = \frac{1}{8}\delta^{\mu\nu\mu_1\cdots\mu_4}_{\lambda\sigma\nu_1\cdots\nu_4}\left(W^{\nu_1\nu_2}_{\mu_1\mu_2} + 8S^{\nu_1}_{\mu_1}\delta^{\nu_2}_{\mu_2}\right)W^{\nu_3\nu_4}_{\mu_3\mu_4} - 8\delta^{\beta\mu_1\cdots\mu_4}_{\alpha\nu_1\cdots\nu_4}\left(\Delta^{\alpha}_{\beta}\right)^{\mu\nu}_{\lambda\sigma}S^{\nu_1}_{\mu_1}W^{\nu_2\nu_3}_{\mu_2\mu_3} + 8\nabla^{[\mu}C^{\nu]}_{\lambda\sigma}$$

$$S^{\alpha}_{\beta} = \left(\Delta^{\alpha}_{\beta}\right)^{\mu\nu}_{\lambda\sigma}R^{\lambda\sigma}_{\mu\nu}$$

For Einstein spaces

$$\mathbf{E}^{\mu\nu}_{\lambda\sigma}[E] = \frac{1}{8}\delta^{\mu\nu\mu_1\cdots\mu_4}_{\lambda\sigma\nu_1\cdots\nu_4}\left(R^{\nu_1\nu_2}_{\mu_1\mu_2}R^{\nu_3\nu_4}_{\mu_3\mu_4} - \delta^{\nu_1\nu_2}_{\mu_1\mu_2}\delta^{\nu_3\nu_4}_{\mu_3\mu_4}\right)$$

Obstruction (Haende!) tensor

[Anastasiou, Araya, Corral and RO, 2308.09140]

$$H^{\nu}_{\mu} = \mathbf{E}^{\alpha\beta}_{\mu\lambda}R^{\nu\lambda}_{\alpha\beta} - \frac{1}{2}\delta^{\nu}_{\mu}\mathcal{L}_{\text{LPP}} + 2\nabla^{\lambda}\nabla_{\sigma}\mathbf{E}^{\nu\sigma}_{\mu\lambda}$$

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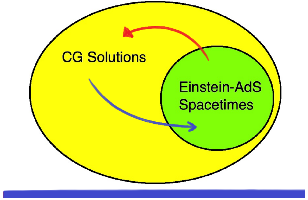
And back...(Einstein gravity from CG in 6D)

LPP CG action decomposed into Einstein and non-Einstein parts:

$$
\begin{aligned}
I_{CG} &= -4!\alpha_{CG}\int\limits_{M} d^6x\sqrt{-g}\left[P_6\left(W_{(E)}\right)+Q\left(W_{(E)},D\right)\right] \\
&\quad -\alpha_{CG}\int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W^{\kappa\lambda}_{(E)\nu\sigma}\nabla_{\mu}W^{\nu\sigma}_{(E)\kappa\lambda}\right).
\end{aligned}
$$



Back to Einstein gravity (with an extra term)

Einstein condition, and $\alpha_{\mathrm{E}} = -\frac{\ell^4}{384\pi G}$:

$$I_{CG}[E] = \frac{1}{16\pi G}\int_M d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right) + \frac{\ell^4}{384\pi G}\int_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W_{(E)\nu\sigma}^{\kappa\lambda}\nabla_{\mu}W_{(E)\kappa\lambda}^{\nu\sigma}\right),$$

Performing asymptotic expansions:

$$\Delta I = \frac{\ell^8}{192\pi G}\int_{\partial M} d^5x\sqrt{-h}W^{ijkl}(h)W_{ijkl}(h) + \ldots$$

CG action for Einstein spaces = Renormalized Einstein AdS action

$$I_{CG}[E] = I_{\mathrm{HR}}$$

Back to Einstein gravity (with an extra term)

HOLOGRAPHYCL

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CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{CG}\left[E\right] = I_{\mathsf{HR}}$$

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In 4D: Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in codimension-2

[Anastasiou, Araya and RO, 2209.02006]

In 4D: Renormalization in the Bulk $\Longrightarrow$ Renormalization of codimension-2 functionals

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

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(with N.Boulanger)

Co-dimension 2 functionals from 6D CG: Willmore energy, Reduced Hawking Mass

(with P.Bueno)

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HOLOGRAPHYCL

Anillos de Investigación en Ciencia y/o Tecnología