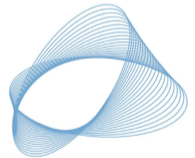


Conformal Renormalization of anti-de Sitter gravity

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Holographic Renormalization of AdS gravity action

Topological terms in AdS gravity

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{D-2}^2, \quad f^2(r) = 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Einstein-AdS gravity

$$I_{EH} = \frac{1}{16\pi G} \int_M d^D x \sqrt{-g} (R - 2\Lambda), \quad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

Black hole entropy S_{BH}

$$T I_{bulk}^E = \frac{(D-3)}{(D-2)} M - TS + \lim_{r \rightarrow \infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

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Holographic Renormalization [Henningson and Skenderis, 1998]

$$I_{\text{ren}} = I_{EH} - \frac{1}{8\pi G} \int_{\partial M} d^d x \sqrt{-h} K + \int_{\partial M} d^d x L_{ct}(h, \mathcal{R}, \nabla \mathcal{R})$$

Counterterms [de Boer, Gubser-Russo, 1999], [Papayan, Johnson, Myers, 1998]

$$8\pi G L_{ct} = \frac{d-1}{l} \sqrt{-h} + \frac{l\sqrt{-h}}{2(d-2)} \mathcal{R} + \frac{l^2 \sqrt{-h}}{2(d-2)^2(d-4)} \left(\mathcal{R}^{ij} \mathcal{R}_{ij} - \frac{d}{4(d-1)} \mathcal{R}^2 \right) \\ + \frac{l^3 \sqrt{-h}}{(d-2)^2(d-4)(d-6)} \left(\frac{8d-2}{4(d-1)} \mathcal{R} \mathcal{R}^{ij} \mathcal{R}_{ij} - \frac{d(d+2)}{16(d-1)^2} \mathcal{R}^3 \right. \\ \left. - 2\mathcal{R}^{ij} \mathcal{R}^{kl} \mathcal{R}_{ijkl} - \frac{d}{4(d-1)} \nabla_i \mathcal{R} \nabla^i \mathcal{R} + \nabla^k \mathcal{R}^{ij} \nabla_k \mathcal{R}_{ij} \right) + \dots$$

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$$8\pi G L_{ct} = \frac{d-1}{2} \sqrt{-h} + \frac{1}{2(d-2)} \sqrt{-h} \mathcal{R} + \frac{1}{2(d-2)^2(d-4)} \sqrt{-h} \left(\mathcal{R}^{\mu\nu} \mathcal{R}_{\mu\nu} - \frac{d}{4(d-1)} \mathcal{R}^2 \right) \\ + \frac{1}{(d-2)^2(d-4)(d-6)} \sqrt{-h} \left(\frac{d-2}{4(d-1)} \mathcal{R} \mathcal{R}^{\mu\nu} \mathcal{R}_{\mu\nu} - \frac{d(d+2)}{16(d-1)^2} \mathcal{R}^3 \right. \\ \left. - 2\mathcal{R}^{\mu\nu} \mathcal{R}^{\rho\sigma} \mathcal{R}_{\mu\rho} \mathcal{R}_{\nu\sigma} - \frac{d}{4(d-1)} \nabla_\mu \mathcal{R} \nabla^\mu \mathcal{R} + \nabla^\mu \mathcal{R}^\nu \nabla_\mu \mathcal{R}_\nu \right) + \dots$$

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$$\begin{aligned} 8\pi G L_{ct} = & \frac{d-1}{\ell} \sqrt{-h} + \frac{\ell \sqrt{-h}}{2(d-2)} \mathcal{R} + \frac{\ell^3 \sqrt{-h}}{2(d-2)^2(d-4)} \left(\mathcal{R}^{ij} \mathcal{R}_{ij} - \frac{d}{4(d-1)} \mathcal{R}^2 \right) \\ & + \frac{\ell^5 \sqrt{-h}}{(d-2)^3(d-4)(d-6)} \left(\frac{3d-2}{4(d-1)} \mathcal{R} \mathcal{R}^{ij} \mathcal{R}_{ij} - \frac{d(d+2)}{16(d-1)^2} \mathcal{R}^3 \right. \\ & \left. - 2\mathcal{R}^{ij} \mathcal{R}^{kl} \mathcal{R}_{ijkl} - \frac{d}{4(d-1)} \nabla_i \mathcal{R} \nabla^i \mathcal{R} + \nabla^k \mathcal{R}^{ij} \nabla_k \mathcal{R}_{ij} \right) + \dots \end{aligned}$$

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Counterterm Method reproduces BH Thermo

$$G = U - TS$$

Internal Energy

$$U = M + E_0$$

Counter Energy in $D = 2n + 1$ dimensions

$$E_0 = (-1)^n \frac{(2n-1)!!^2}{(2n)!} \frac{\text{Vol}(S^{2n-1})}{8\pi G} \ell^{2n-2}$$

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Counterterm Energy $E_0 = \int_{\partial M} \mathcal{L}_{CT}$

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4D AdS action [R. Aros et al, gr-qc/9909015]

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G} \int_M d^4x \sqrt{-g} \left[(R - 2\Lambda) + \frac{\ell^2}{4} (R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} R^{\mu\nu} + R^2) \right]$$

$D = 2n$ AdS action [R. Aros et al, gr-qc/9909015]

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4D - 2D AdS action [R. Aros et al, gr-qc/9909015]

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Euler-Gauss-Bonnet Theorem in 4D

$$\int_M d^4x GB = \int_{\partial M} d^3x B_3(K, \mathcal{R}) + 32\pi^2 \chi(M)$$

Euler Theorem in $2n$ dimensions

$$\int_{M_{2n}} d^{2n}x (\text{Euler})_{2n} = \int_{\partial M_{2n}} d^{2n-1}x B_{2n-1} + (4\pi)^n n! \chi(M_{2n})$$

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Extrinsic counterterms

$$\tilde{I}_{ren} = I_{EH} + c_d \int_{\partial M} d^d x B_d(h, K, \mathcal{R})$$

Kounterterms = counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}_{ij}^{kl}(h)$)

$$B_{2n-1} = 2n\sqrt{-h} \int_0^1 dt \delta_{[01 \dots j_{2n-1}]}^{[i_1 \dots i_{2n-1}]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3} - t^2 K_{i_2}^{j_2} K_{i_3}^{j_3} \right) \times \dots$$

$$\dots \times \left(\frac{1}{2} \mathcal{R}_{i_{2n-2} i_{2n-1}}^{j_{2n-2} j_{2n-1}} - t^2 K_{i_{2n-2}}^{j_{2n-2}} K_{i_{2n-1}}^{j_{2n-1}} \right)$$

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$D = 2n$ dimensions [R.O., hep-th/0504233]

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Kounterterms in $D = 2n + 1$ [R.O., hep-th/0610230]

$$B_{2n} = 2n\sqrt{-h} \int_0^1 dt \int_0^t ds \delta_{[i_1 \dots i_{2n}] }^{[j_1 \dots j_{2n}]} K_{j_1}^{i_1} \delta_{j_2}^{i_2} \left(\frac{1}{2} \mathcal{R}_{j_3 j_4}^{i_3 i_4} - t^2 K_{j_3}^{i_3} K_{j_4}^{i_4} + \frac{s^2}{\ell^2} \delta_{j_3}^{i_3} \delta_{j_4}^{i_4} \right) \times \dots$$
$$\dots \times \left(\frac{1}{2} \mathcal{R}_{j_{2n-1} j_{2n}}^{i_{2n-1} i_{2n}} - t^2 K_{j_{2n-1}}^{i_{2n-1}} K_{j_{2n}}^{i_{2n}} + \frac{s^2}{\ell^2} \delta_{j_{2n-1}}^{i_{2n-1}} \delta_{j_{2n}}^{i_{2n}} \right) .$$

Black Hole Thermodynamics

$$T I_{bulk}^E = \frac{(D-3)}{(D-2)} M - TS + \lim_{r \rightarrow \infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Euclidean Counterterms

$$T c_d \int_{\partial M} B_d = \frac{M}{(D-2)} + E_0 - \lim_{r \rightarrow \infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

Correct Black Hole Thermo with $U = M + E_0$

Black Hole Thermodynamics

$$TI_{bulk}^E = \frac{(D-3)}{(D-2)}M - TS + \lim_{r \rightarrow \infty} \frac{V(S^{D-2})}{8\pi G} \frac{r^{D-1}}{\ell^2}$$

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Fefferman-Graham expansion for AAdS Einstein spaces

$$ds^2 = \frac{\ell^2}{z^2} dz^2 + \frac{1}{z^2} g_{ij}(x, z) dx^i dx^j, \quad g_{ij}(x, \rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \dots$$

Dirichlet boundary conditions for metric tensor in AdS spaces

(Fefferman-Graham and Witten, 1998)

$$h_{ij} = \frac{g_{(0)ij}}{z^2} + \dots$$

Renormalization = variational problem in $g_{(0)ij}$

$$\delta I_{ren} = \frac{1}{2} \int_{\partial M} \sqrt{-g_{(0)}} T^{ij}[g_{(0)}] \delta g_{(0)ij}$$

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Dirichlet b.c. $\delta h_{ij} = 0$ does not make sense in AAdS spaces
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Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell} \frac{g_{(0)ij}}{z^2} + \dots$$

Counterterms of a dual field theory

$$\tilde{I}_{ren} = I_{EH} + c_d \int_{\partial M} d^d x B(f(h), K)$$

as long as the theory is holographic

$$\delta \tilde{I}_{ren} = \frac{1}{2} \int_{\partial M} \sqrt{-g_{(0)}} \tau^{ij} \delta g_{(0)ij}$$

Asymptotic form of the extrinsic curvature

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AdS gravity action + KTs

$$\tilde{I}_{\text{ren}} = I_{EH} + \frac{\ell^2}{16\pi G} \int_{\partial M} d^3x \sqrt{-h} \delta_{[j_1 j_2 j_3]}^{[i_1 i_2 i_3]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3}(h) - \frac{1}{3} K_{i_2}^{j_2} K_{i_3}^{j_3} \right).$$

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Efforton-Graham expansion

$$K_j^i = \frac{1}{\ell} \delta_j^i - \ell S_j^i(h) + \mathcal{O}(\mathcal{R}^2), \quad S_j^i(h) = \frac{1}{d-2} (\mathcal{R}_j^i(h) - \frac{1}{2(d-1)} \delta_j^i \mathcal{R}(h))$$

Conformal anomaly and counterterms [O. Miskovic and R.O., 0902.2082]

$$L_{ct} = \frac{1}{8\pi G} \sqrt{-h} \left(\frac{2}{\ell} + \frac{\ell}{2} \mathcal{R}(h) \right).$$

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Conformal Killing vectors [O. Miskovic and R.O., 0902.2082]

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Conformal renormalization of anti-de Sitter gravity [O. Miškovic and R.O., 0902.2082]

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Conformal anomaly in 4D [Q. Mishkov and R.O., 0902.2082]

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EH-AdS gravity +KTs

$$B_{2n-1} = 2n\sqrt{-h} \int_0^1 dt \delta_{[j_1 \dots j_{2n-1}]^{[i_1 \dots i_{2n-1}]} K_{i_1}^{j_1} \left(\frac{1}{2} \mathcal{R}_{i_2 i_3}^{j_2 j_3} - t^2 K_{i_2}^{j_2} K_{i_3}^{j_3} \right) \times \dots$$

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Expanding and collecting

$$L_{\text{ct}} = \frac{\sqrt{-h}}{8\pi G} \left[\frac{(2n-2)}{t} + \frac{t}{2(2n-3)} \mathcal{R} + \frac{t^2}{2(2n-3)^2(2n-5)} \left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n+1)}{4(2n-2)} \mathcal{R}^2 - \frac{(2n-3)}{4} \mathcal{R}^{ijkl}\mathcal{R}_{ijkl} \right) + \dots \right].$$

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Boundary Weyl tensor $\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}$ implies

$$\mathcal{R}^{ijkl}\mathcal{R}_{ijkl} = \mathcal{W}^{ijkl}\mathcal{W}_{ijkl} + \frac{4}{(2n-3)}(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{1}{2(2n-2)}\mathcal{R}^2)$$

Mod. Analy.

$$L_4 = \frac{\sqrt{k}}{8\pi G} \left[\frac{(2n-2)}{l} + \frac{l}{2(2n-3)}\mathcal{R} + \frac{l^2}{2(2n-3)^2(2n-5)} \left(\mathcal{R}^i{}^j\mathcal{R}_j{}^i - \frac{(2n-1)}{2(2n-3)}\mathcal{R}^2 \right) - \frac{l^2}{8(2n-3)(2n-5)} \mathcal{W}^{ijkl}(h)\mathcal{W}_{ijkl}(h) + \dots \right]$$

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Mismatch with HR. [G.Anastasiou, O.Miskovic, R.O. and I.Papadimitriou, 2003.06425]

$$\tilde{I}_{\text{ren}} = I_{\text{HR}} - \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int_{\partial M} \sqrt{-h} \mathcal{W}^{ijkl} \mathcal{W}_{ijkl} + \dots$$

A similar result in $D = 2n + 1$ dimensions.

Last term is zero for most AAdS spaces (Schwarzschild, Kerr, black strings).

Gravitational instantons: non-trivial boundary geometries

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Gravitational instantons: non-trivial boundary geometries

Mismatch with HR. [G.Anastasiou, O.Miskovic, R.O. and I.Papadimitriou, 2003.06425]

$$\tilde{I}_{\text{ren}} = I_{\text{HR}} - \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int_{\partial M} \sqrt{-h} \mathcal{W}^{ijkl} \mathcal{W}_{ijkl} + \dots$$

A similar result in $D = 2n + 1$ dimensions.

Last term is zero for most AAdS spaces (Schwarzschild, Kerr, black strings).

Gravitational instantons: non-trivial boundary geometries



Is there any other principle to renormalize AdS gravity action, which includes topological terms as a particular case.

Renormalized AdS action = MacDowell-Mansouri action (1977)

$$I_{\text{ren}} = \frac{\ell^2}{256\pi G} \int_M d^4x \sqrt{-g} \delta_{[\mu_1 \dots \mu_4]}^{[\nu_1 \dots \nu_4]} \left[R_{\nu_1 \nu_2}^{\mu_1 \mu_2} + \frac{\delta_{\nu_1 \nu_2}^{\mu_1 \mu_2}}{\ell^2} \right] \left[R_{\nu_3 \nu_4}^{\mu_3 \mu_4} + \frac{\delta_{\nu_3 \nu_4}^{\mu_3 \mu_4}}{\ell^2} \right],$$

Weyl tensor

$$W_{\mu\nu}^{\alpha\beta} = R_{\mu\nu}^{\alpha\beta} - 4S_{[\mu}^{\alpha} \delta_{\nu]}^{\beta]}, \quad \text{Schouten } S_{\mu}^{\alpha} = \frac{1}{D-2} \left(R_{\mu}^{\alpha} - \frac{1}{2(D-1)} \delta_{\mu}^{\alpha} R \right)$$

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Weyl tensor for Einstein spaces $S_{\mu}^{\alpha} = -\frac{1}{2\ell^2} \delta_{\mu}^{\alpha}$

$$W_{(E)\mu\nu}^{\alpha\beta} = R_{\mu\nu}^{\alpha\beta} + \frac{1}{\ell^2} \delta_{[\mu\nu]}^{[\alpha\beta]}$$

Renormalized action for Einstein spaces

$$I_{\text{ren}} = \frac{\ell^2}{64\pi G} \int_M d^4x \sqrt{-g} W_{(E)\mu\nu\alpha\beta} W_{(E)}^{\mu\nu\alpha\beta}$$

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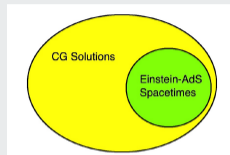
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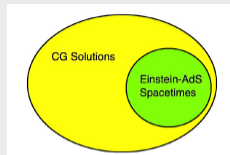
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Embedding Einstein theory in Conformal Gravity



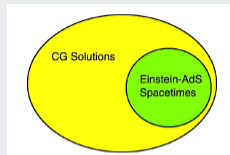
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- What for?: Renormalization should be inherited by the Einstein sector.
- How?: a *holographic* mechanism to turn CG into Einstein

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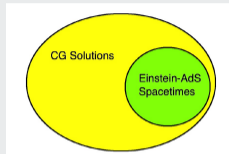
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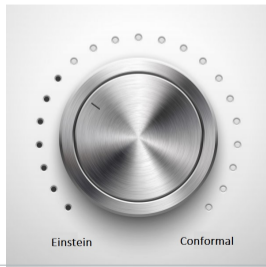


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Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG} = \alpha_{CG} \int_M d^4x \sqrt{-g} W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

Foliation-Graham expansion for AAdS spaces in CG

$$ds^2 = \frac{\ell^2}{z^2} dz^2 + \frac{1}{z^2} g_{ij}(x, z) dx^i dx^j, \quad g_{ij}(x, z) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \dots \\ + z g_{(1)ij}(x) + \dots$$

Linear Conformal Gravity: D'Alembert

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\nu\sigma} = 0, \quad C_{\mu\lambda}^{\nu} = \nabla_\nu S_{\lambda}^{\mu} - \nabla_\lambda S_{\nu}^{\mu}$$

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Patchwork-Galilean expansion in AdS space in CG

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Weyl tensor and Neumann boundary conditions

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EOM for Conformal Gravity: Bach tensor

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0, \quad C^\mu_{\nu\lambda} = \nabla_\nu S^\mu_\lambda - \nabla_\lambda S^\mu_\nu$$

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Einstein-AdS spaces

$$S_{\nu}^{\mu} = -\frac{1}{2\ell^2}\delta_{\nu}^{\mu}, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$D_{\nu}^{\mu} = R_{\nu}^{\mu} - \frac{1}{D}R\delta_{\nu}^{\mu} = 0$$

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CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{CG}[E] = I_{HR}$$

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EH Action+Euler term

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G} \int_M d^6x \sqrt{-g} \left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72} (\text{Euler})_6 \right),$$

In terms of fully-antisymmetric objects

$$\begin{aligned} \tilde{I}_{\text{ren}} = & \frac{1}{16\pi G \times 192} \int_M d^6x \sqrt{-g} \delta_{[\mu_1 \dots \mu_3]}^{[\nu_1 \dots \nu_3]} \left[R_{\nu_1 \nu_2}^{\mu_1 \mu_2} \delta_{[\mu_3 \mu_4]}^{[\nu_3 \nu_4]} \delta_{[\mu_5 \mu_6]}^{[\nu_5 \nu_6]} \right. \\ & \left. + \frac{2}{3/2} \delta_{[\mu_1 \mu_2]}^{[\nu_1 \nu_2]} \delta_{[\mu_3 \mu_4]}^{[\nu_3 \nu_4]} \delta_{[\mu_5 \mu_6]}^{[\nu_5 \nu_6]} - \frac{\ell^4}{3} R_{\nu_1 \nu_2}^{\mu_1 \mu_2} R_{\nu_3 \nu_4}^{\mu_3 \mu_4} R_{\nu_5 \nu_6}^{\mu_5 \mu_6} \right], \end{aligned}$$

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In terms of fully-antisymmetric objects

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G \times 192} \int_M d^6x \sqrt{-g} \left[R_{[12][34][56]} \delta^{[1234][56]} + \frac{2}{3} \delta^{[1234][56]} \delta^{[1234][56]} \delta^{[1234][56]} - \frac{\ell^4}{3} R_{[12][34][56]} R_{[12][34][56]} R_{[12][34][56]} \right]$$

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Polynomial of $W_{(E)}$

$$\begin{aligned} \tilde{I}_{\text{ren}} = & \frac{\ell^4}{16\pi G \times 4!} \int_M d^6 x \sqrt{-g} \left[\frac{1}{2\ell^2} \delta_{[\mu_1 \dots \mu_4]}^{[\nu_1 \dots \nu_4]} W_{(E)\nu_1\nu_2}^{\mu_1\mu_2} W_{(E)\nu_3\nu_4}^{\mu_3\mu_4} \right. \\ & \left. - \frac{1}{4!} \delta_{[\mu_1 \dots \mu_6]}^{[\nu_1 \dots \nu_6]} W_{(E)\nu_1\nu_2}^{\mu_1\mu_2} W_{(E)\nu_3\nu_4}^{\mu_3\mu_4} W_{(E)\nu_5\nu_6}^{\mu_5\mu_6} \right], \end{aligned}$$

There are three Conformal Invariants in 6D

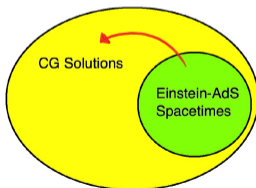
$$\begin{aligned}I_1 &= W_{\alpha\beta\mu\nu} W^{\alpha\sigma\lambda\nu} W_{\sigma}{}^{\beta\mu}{}_{\lambda}, \\I_2 &= W_{\mu\nu\alpha\beta} W^{\alpha\beta\sigma\lambda} W_{\sigma\lambda}{}^{\mu\nu}, \\I_3 &= W_{\mu\rho\sigma\lambda} \left(\delta_{\nu}^{\mu} \square + 4R_{\nu}^{\mu} - \frac{6}{5}R\delta_{\nu}^{\mu} \right) W^{\nu\rho\sigma\lambda} + \nabla_{\mu} J^{\mu},\end{aligned}$$

with

$$\begin{aligned}J_{\mu} &= 4R_{\mu}{}^{\lambda\rho\sigma} \nabla^{\nu} R_{\nu\lambda\rho\sigma} + 3R^{\nu\lambda\rho\sigma} \nabla_{\mu} R_{\nu\lambda\rho\sigma} - 5R^{\nu\lambda} \nabla_{\mu} R_{\nu\lambda} \\&\quad + \frac{1}{2}R \nabla_{\mu} R - R_{\mu}^{\nu} \nabla_{\nu} R + 2R^{\nu\lambda} \nabla_{\nu} R_{\lambda\mu}.\end{aligned}$$

Einstein	→	Conformal	CI's
$\delta_{[\mu_1 \dots \mu_6]}^{[\nu_1 \dots \nu_6]} \mathbf{W}_{(E)\nu_1\nu_2}^{\mu_1\mu_2} \mathbf{W}_{(E)\nu_3\nu_4}^{\mu_3\mu_4} \mathbf{W}_{(E)\nu_5\nu_6}^{\mu_5\mu_6}$	→	$\delta_{[\mu_1 \dots \mu_6]}^{[\nu_1 \dots \nu_6]} \mathbf{W}_{\nu_1\nu_2}^{\mu_1\mu_2} \mathbf{W}_{\nu_3\nu_4}^{\mu_3\mu_4} \mathbf{W}_{\nu_5\nu_6}^{\mu_5\mu_6}$	32(2I ₁ + I ₂)
$-\frac{1}{\ell^2} \delta_{[\mu_1 \dots \mu_4]}^{[\nu_1 \dots \nu_4]} \mathbf{W}_{(E)\nu_1\nu_2}^{\mu_1\mu_2} \mathbf{W}_{(E)\nu_3\nu_4}^{\mu_3\mu_4}$	→	$\delta_{[\mu_1 \dots \mu_5]}^{[\nu_1 \dots \nu_5]} \mathbf{W}_{\nu_1\nu_2}^{\mu_1\mu_2} \mathbf{W}_{\nu_3\nu_4}^{\mu_3\mu_4} \mathbf{S}_{\nu_5}^{\mu_5} + 16\mathbf{C}^{\mu\nu\lambda} \mathbf{C}_{\mu\nu\lambda} + \nabla^\mu \mathbf{J}_\mu$	4I ₁ - I ₂ - I ₃

$$\mathbf{J}_\mu = 16\mathbf{W}_\mu^{\kappa\lambda\nu} \mathbf{C}_{\kappa\lambda\nu} - 2\mathbf{W}_{\nu\sigma}^{\kappa\lambda} \nabla_\mu \mathbf{W}_{\kappa\lambda}^{\nu\sigma}$$



6D CG with an Einstein sector [Lu, Pang and Pope, 2013]

$$I_{CG} = \alpha_{CG} \int_M d^6 x \sqrt{-g} \left(\frac{1}{4!} \delta_{[\mu_1 \dots \mu_6]}^{[\nu_1 \dots \nu_6]} W_{\nu_1 \nu_2}^{\mu_1 \mu_2} W_{\nu_3 \nu_4}^{\mu_3 \mu_4} W_{\nu_5 \nu_6}^{\mu_5 \mu_6} + \frac{1}{2} \delta_{[\mu_1 \dots \mu_5]}^{[\nu_1 \dots \nu_5]} W_{\nu_1 \nu_2}^{\mu_1 \mu_2} W_{\nu_3 \nu_4}^{\mu_3 \mu_4} S_{\nu_5}^{\mu_5} \right. \\ \left. + 8C^{\mu\nu\lambda} C_{\mu\nu\lambda} \right) + \alpha_{CG} \int_{\partial M} d^5 x \sqrt{-h} n^\mu \left(8W_\mu^{\kappa\lambda\nu} C_{\kappa\lambda\nu} - W_{\nu\sigma}^{\kappa\lambda} \nabla_\mu W_{\kappa\lambda}^{\nu\sigma} \right).$$

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Schwarzschild BH is a solution of LPP CG. [Lu, Pang and Pope, 1301.7083]

EoM in terms of W , C and S tensors \rightarrow any Einstein-AdS spacetime is a solution
[Anastasiou, Araya and RO, 2010.15146]

Energy-momentum tensor

$$E_{\mu}^{\nu} = E_{\mu\lambda}^{\alpha\beta} R_{\alpha\beta}^{\nu\lambda} - \frac{1}{2} \delta_{\mu}^{\nu} \mathcal{L} + 2 \nabla^{\lambda} \nabla_{\sigma} E_{\mu\lambda}^{\nu\sigma}$$
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EoM in terms of W , \mathcal{L} and \mathcal{E} tensors \rightarrow any Einstein-like spacetime equations
[Anastasiou, Araya and RO, 2010.15146]

$$E_{\mu}^{\nu} = E_{\mu\lambda}^{nd} R_{\sigma\delta}^{\nu\lambda} - \frac{1}{2} \delta_{\mu}^{\nu} \mathcal{L} + 2 \nabla^{\lambda} \nabla_{\sigma} E_{\mu\lambda}^{\nu\sigma}$$
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For an arbitrary gravity theory

$$\begin{aligned}\mathcal{E}_\mu^\nu &= E_{\mu\lambda}^{\alpha\beta} R_{\alpha\beta}^{\nu\lambda} - \frac{1}{2} \delta_\mu^\nu \mathcal{L} + 2 \nabla^\lambda \nabla_\sigma E_{\mu\lambda}^{\nu\sigma} \\ E_{\mu\lambda}^{\nu\sigma} &= \frac{\partial \mathcal{L}}{\partial R_{\nu\sigma}^{\mu\lambda}} - \nabla_\alpha \left(\frac{\partial \mathcal{L}}{\partial \nabla_\alpha R_{\nu\sigma}^{\mu\lambda}} \right)\end{aligned}$$

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For LPP CG theory

$$E_{\lambda\sigma}^{\mu\nu} = \frac{1}{8} \delta_{\lambda\sigma\nu_1\cdots\nu_4}^{\mu\nu\mu_1\cdots\mu_4} (W_{\mu_1\mu_2}^{\nu_1\nu_2} + 8S_{\mu_1}^{\nu_1} \delta_{\mu_2}^{\nu_2}) W_{\mu_3\mu_4}^{\nu_3\nu_4} - 8\delta_{\alpha\nu_1\cdots\nu_4}^{\beta\mu_1\cdots\mu_4} (\Delta_{\beta}^{\alpha})_{\lambda\sigma}^{\mu\nu} S_{\mu_1}^{\nu_1} W_{\mu_2\mu_3}^{\nu_2\nu_3} + 8\nabla^{[\mu} C_{\lambda\sigma}^{\nu]}$$

$$S_{\beta}^{\alpha} = (\Delta_{\beta}^{\alpha})_{\lambda\sigma}^{\mu\nu} R_{\mu\nu}^{\lambda\sigma}$$

Obstruction tensor

$$E_{\lambda\sigma}^{\mu\nu}[E] = \frac{1}{8} \delta_{\lambda\sigma\nu_1\cdots\nu_4}^{\mu\nu\mu_1\cdots\mu_4} (R_{\mu_1\mu_2}^{\nu_1\nu_2} R_{\mu_3\mu_4}^{\nu_3\nu_4} - \delta_{\mu_1\mu_2}^{\nu_1\nu_2} \delta_{\mu_3\mu_4}^{\nu_3\nu_4})$$

Obstruction (Hawking) tensor

[Anastasiou, Araya, Corral and RO, 2308.09140]

$$H_{\mu}^{\nu} = E_{\mu\lambda}^{\alpha\beta} R_{\alpha\beta}^{\nu\lambda} - \frac{1}{2} \delta_{\mu}^{\nu} \mathcal{L}_{LPP} + 2\nabla^{\lambda} \nabla_{\sigma} E_{\mu\lambda}^{\nu\sigma}$$

For LPP CG theory

$$\mathbf{E}_{\lambda\sigma}^{\mu\nu} = \frac{1}{8} \delta_{\lambda\sigma\nu_1\cdots\nu_4}^{\mu\nu\mu_1\cdots\mu_4} (W_{\mu_1\mu_2}^{\nu_1\nu_2} + 8S_{\mu_1}^{\nu_1} \delta_{\mu_2}^{\nu_2}) W_{\mu_3\mu_4}^{\nu_3\nu_4} - 8\delta_{\alpha\nu_1\cdots\nu_4}^{\beta\mu_1\cdots\mu_4} (\Delta_{\beta}^{\alpha})_{\lambda\sigma}^{\mu\nu} S_{\mu_1}^{\nu_1} W_{\mu_2\mu_3}^{\nu_2\nu_3} + 8\nabla^{[\mu} C_{\lambda\sigma}^{\nu]}$$

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Construction of energy tensor

[Anastasiou, Araya, Corral and RO, 2308.09140]

$$H_{\mu}^{\nu} = E_{\mu\lambda}^{\alpha\beta} R_{\alpha\beta}^{\nu\lambda} - \frac{1}{2} \delta_{\mu}^{\nu} C_{LPP} + 2\nabla^{\lambda} \nabla_{\sigma} E_{\mu\lambda}^{\nu\sigma}$$

For LPP CG theory

$$\mathbf{E}_{\lambda\sigma}^{\mu\nu} = \frac{1}{8} \delta_{\lambda\sigma\nu_1\cdots\nu_4}^{\mu\nu\mu_1\cdots\mu_4} (W_{\mu_1\mu_2}^{\nu_1\nu_2} + 8S_{\mu_1}^{\nu_1} \delta_{\mu_2}^{\nu_2}) W_{\mu_3\mu_4}^{\nu_3\nu_4} - 8\delta_{\alpha\nu_1\cdots\nu_4}^{\beta\mu_1\cdots\mu_4} (\Delta_{\beta}^{\alpha})_{\lambda\sigma}^{\mu\nu} S_{\mu_1}^{\nu_1} W_{\mu_2\mu_3}^{\nu_2\nu_3} + 8\nabla^{[\mu} C_{\lambda\sigma}^{\nu]}$$

$$S_{\beta}^{\alpha} = (\Delta_{\beta}^{\alpha})^{\mu\nu} R_{\mu\nu}^{\lambda\sigma}$$

For Einstein spaces

$$\mathbf{E}_{\lambda\sigma}^{\mu\nu}[E] = \frac{1}{8} \delta_{\lambda\sigma\nu_1\cdots\nu_4}^{\mu\nu\mu_1\cdots\mu_4} (R_{\mu_1\mu_2}^{\nu_1\nu_2} R_{\mu_3\mu_4}^{\nu_3\nu_4} - \delta_{\mu_1\mu_2}^{\nu_1\nu_2} \delta_{\mu_3\mu_4}^{\nu_3\nu_4})$$

Obstruction (Weyl) tensor:

[Anastasiou, Araya, Corral and RO, 2308.09140]

$$H_{\mu}^{\nu} = E_{\mu\lambda}^{\sigma\delta} R_{\sigma\delta}^{\nu\lambda} - \frac{1}{2} \delta_{\mu}^{\nu} \mathcal{L}_{LPP} + 2\nabla^{\lambda} \nabla_{\sigma} E_{\mu\lambda}^{\nu\sigma}$$

For LPP CG theory

$$\mathbf{E}_{\lambda\sigma}^{\mu\nu} = \frac{1}{8} \delta_{\lambda\sigma\nu_1\cdots\nu_4}^{\mu\nu\mu_1\cdots\mu_4} (W_{\mu_1\mu_2}^{\nu_1\nu_2} + 8S_{\mu_1}^{\nu_1} \delta_{\mu_2}^{\nu_2}) W_{\mu_3\mu_4}^{\nu_3\nu_4} - 8\delta_{\alpha\nu_1\cdots\nu_4}^{\beta\mu_1\cdots\mu_4} (\Delta_{\beta}^{\alpha})_{\lambda\sigma}^{\mu\nu} S_{\mu_1}^{\nu_1} W_{\mu_2\mu_3}^{\nu_2\nu_3} + 8\nabla^{[\mu} C_{\lambda\sigma}^{\nu]}$$

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$$\mathbf{E}_{\lambda\sigma}^{\mu\nu}[E] = \frac{1}{8} \delta_{\lambda\sigma\nu_1\cdots\nu_4}^{\mu\nu\mu_1\cdots\mu_4} (R_{\mu_1\mu_2}^{\nu_1\nu_2} R_{\mu_3\mu_4}^{\nu_3\nu_4} - \delta_{\mu_1\mu_2}^{\nu_1\nu_2} \delta_{\mu_3\mu_4}^{\nu_3\nu_4})$$

Conformal anomaly tensor

[Anastasiou, Araya, Corral and RO, 2308.09140]

$$H_{\mu}^{\nu} = E_{\mu\lambda}^{\sigma\theta} R_{\sigma\theta}^{\nu\lambda} - \frac{1}{2} \delta_{\mu}^{\nu} C_{LPP} + 2\nabla^{\lambda} \nabla_{\sigma} E_{\mu\lambda}^{\sigma\nu}$$

For LPP CG theory

$$\mathbf{E}_{\lambda\sigma}^{\mu\nu} = \frac{1}{8} \delta_{\lambda\sigma\nu_1\cdots\nu_4}^{\mu\nu\mu_1\cdots\mu_4} (W_{\mu_1\mu_2}^{\nu_1\nu_2} + 8S_{\mu_1}^{\nu_1} \delta_{\mu_2}^{\nu_2}) W_{\mu_3\mu_4}^{\nu_3\nu_4} - 8\delta_{\alpha\nu_1\cdots\nu_4}^{\beta\mu_1\cdots\mu_4} (\Delta_{\beta}^{\alpha})_{\lambda\sigma}^{\mu\nu} S_{\mu_1}^{\nu_1} W_{\mu_2\mu_3}^{\nu_2\nu_3} + 8\nabla^{[\mu} C_{\lambda\sigma}^{\nu]}$$

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Obstruction (Haendel) tensor.

[Anastasiou, Araya, Corral and RO, 2308.09140]

$$H_{\mu}^{\nu} = \mathbf{E}_{\mu\lambda}^{\alpha\beta} R_{\alpha\beta}^{\nu\lambda} - \frac{1}{2} \delta_{\mu}^{\nu} \mathcal{L}_{\text{LPP}} + 2\nabla^{\lambda} \nabla_{\sigma} \mathbf{E}_{\mu\lambda}^{\nu\sigma}$$

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$$\mathbf{E}_{\lambda\sigma}^{\mu\nu} = \frac{1}{8} \delta_{\lambda\sigma\nu_1\cdots\nu_4}^{\mu\nu\mu_1\cdots\mu_4} (W_{\mu_1\mu_2}^{\nu_1\nu_2} + 8S_{\mu_1}^{\nu_1} \delta_{\mu_2}^{\nu_2}) W_{\mu_3\mu_4}^{\nu_3\nu_4} - 8\delta_{\alpha\nu_1\cdots\nu_4}^{\beta\mu_1\cdots\mu_4} (\Delta_{\beta}^{\alpha})_{\lambda\sigma}^{\mu\nu} S_{\mu_1}^{\nu_1} W_{\mu_2\mu_3}^{\nu_2\nu_3} + 8\nabla^{[\mu} C_{\lambda\sigma}^{\nu]}$$

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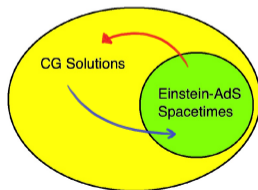
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LPP CG action decomposed into Einstein and non-Einstein parts:

$$I_{CG} = -4! \alpha_{CG} \int_M d^6 x \sqrt{-g} [P_6 (W_{(E)}) + Q (W_{(E)}, D)] \\ - \alpha_{CG} \int_{\partial M} d^5 x \sqrt{-h} n^\mu \left(W_{(E)\nu\sigma}^{\kappa\lambda} \nabla_\mu W_{(E)\kappa\lambda}^{\nu\sigma} \right).$$



Back to Einstein gravity (with an extra term)



Einstein condition, and $\alpha_E = -\frac{\ell^4}{384\pi G}$:

$$I_{CG}[E] = \frac{1}{16\pi G} \int_M d^6x \sqrt{-g} \left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72} (\text{Euler})_6 \right) + \frac{\ell^4}{384\pi G} \int_{\partial M} d^5x \sqrt{-h} n^\mu \left(W_{(E)\nu\sigma}^{\kappa\lambda} \nabla_\mu W_{(E)\kappa\lambda}^{\nu\sigma} \right),$$

Expanding asymptotic expansions

$$\Delta I = \frac{\ell^4}{192\pi G} \int_{\partial M} d^5x \sqrt{-h} W^{JM}(h) W_{JM}(h) + \dots$$

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{CG}[E] = I_{\text{ren}}$$

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Renormalized Einstein-AdS action

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Performing asymptotic expansions

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$$I_{CG}[E] = I_{HR}$$

In 4D and 6D: Conformal Invariance \implies Renormalization

In 4D: Conformal Invariance in the Bulk \implies Conformal Invariance in codimension-2
[Anastasiou, Araya and RO, 2209.02006]

In 6D: Renormalization in the Bulk \implies Renormalization of codimension-2 functions

Renormalized Volume \implies Renormalized Area (in convexly singular manifolds)

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Conformal Gravity in higher even dimensions $D \geq 8$?

(with N.Boulanger)

4-dimension? $\text{Tr}(R^2)$ from $SO(3,1)$ Wilson loop, R and Weyl invariance

(with P.Bueno)

UV extension of Conformal Invariants in $SO(3,1)$ (D'Auria, Ferrara, Horava and Trigiante, 2010)

(with L.Andrianopoli, R.D'Auria, M.Trigiante)

Conformal Gravity in higher even dimensions $D \geq 8$?

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Conformal Gravity in higher odd dimensions $D \geq 5$?

(with P.Buono)

UV completion of Conformal Gravity in $D=4$ dimensions

(with L.Andrianopoli, R.D'Auria, M.Trigiante)

Conformal Gravity in higher even dimensions $D \geq 8$?

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Co-dimension 2 functionals from 6D CG: Willmore energy, Reduced Hawking Mass

(with P.Bueno)

UV extension of Conformal Invariants in 6D gravity, M2-branes, M2 and M5

(with L.Andrianopoli, R.D'Auria, M.Trigiante)

Conformal Gravity in higher even dimensions $D \geq 8$?

(with N.Boulanger)

Co-dimension 2 functionals from 6D CG: Willmore energy, Reduced Hawking Mass

(with P.Bueno)

Conformal Gravity in odd dimensions $D \geq 5$

(with L.Andrianopoli, R.D'Auria, M.Trigiante)

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Acknowledgements

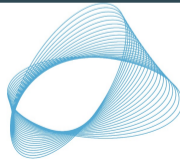


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