Intersecting D-brane models and the anomalous magnetic moment of the muon

François Rondeau

Work in collaboration with:
P. Anastasopoulos, I. Antoniadis, E. Niederwieser

ITMP Seminar

November 9, 2022

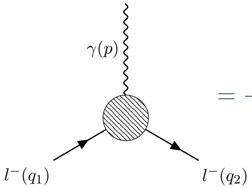
Plan

1. Anomalous magnetic moment of the muon
2. Physics of intersecting D-branes
3. Leptophilic extra $U(1)$ boson and the muon $g-2$
4. Light stringy states and the muon $g-2$

Anomalous magnetic moment of the muon

Landé g-factor

$$\vec{\mu} = g\frac{e}{2m}\vec{s},$$

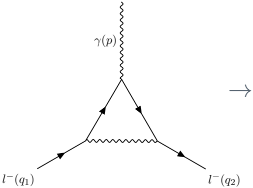


$$= -ie\,\bar{u}(q_2)\left[\gamma^\mu F_1\left(\frac{p^2}{m^2}\right) + \frac{i\gamma^{\mu\nu}p_\nu}{2m}F_2\left(\frac{p^2}{m^2}\right)\right]u(q_1)$$

Form factors:

- $F_1(0) \to$ Electric charge
- $F_2(0) = \frac{g-2}{2} \to$ Anomalous magnetic moment

- One-loop correction to F_2: $a \equiv \frac{g-2}{2} = F_2(0)$



$$\rightarrow F_2\left(\frac{p^2}{m^2}\right) = \frac{g_L^2 m^2}{4\pi^2}\int_0^1 dxdydz\,\frac{\delta(x+y+z-1)z(1-z)}{-xyp^2+(1-z)^2m^2+zM^2}$$

$$a = \frac{g_L^2 m^2}{4\pi^2}\int_0^1 dxdydz\,\delta(x+y+z-1)\frac{z(1-z)}{(1-z)^2m^2+zM^2}$$

Muon g-2 discrepancy

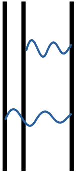
$$\Delta a_\mu \equiv a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (2.51 \pm 0.59)\times 10^{-9}$$

Intersecting D-brane physics

Single Dp-brane

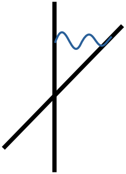
- $U(1)$ vector
- $9 - p$ scalars $\mathbb{R}$
- Non-chiral fermions

N coincident Dp-branes



- $U(N)$ vectors
- $N^2(9 - p)$ scalars $\mathbb{R}$
- Non-chiral fermions

Intersecting stacks of D6-branes



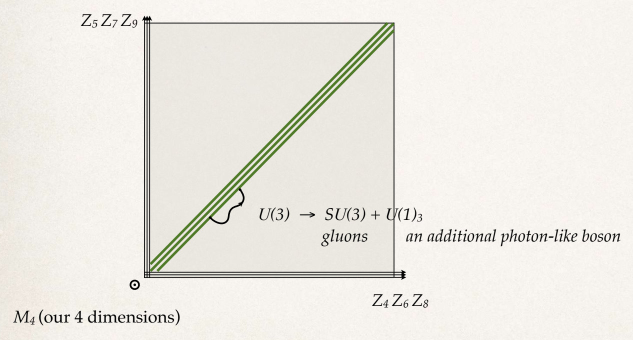
- Complex scalars
- Massless $4D$ chiral fermion

- Stack of N coincident D-branes $\rightarrow$ $U(N)$ gauge symmetry
- Intersecting D-branes $\rightarrow$ Chiral matter

Gauge sector of the Standard Model

- Open string tied to a stack of N parallel D-branes
- Gauge group:

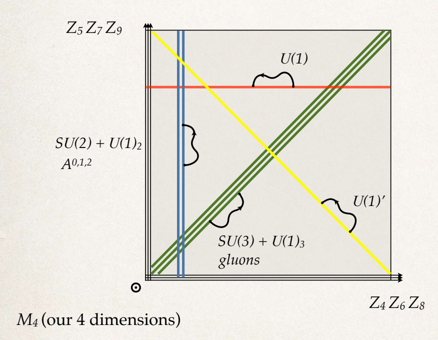
$$U(N) \simeq SU(N) \times U(1)$$



Gauge bosons live in the worldvolume of parallel D-branes

Gauge sector of the Standard Model

- Model with 4 stacks of $(3,2,1,1)$ D-branes
- Gauge group:
  $U(3)_c \times U(2)_w \times U(1) \times U(1)' \supset SU(3)_c \times SU(2)_w \times U(1)_Y$



- $SU(3)_c$ and $SU(2)_w$ gauge bosons
- Hypercharge: linear combination of the abelian factors

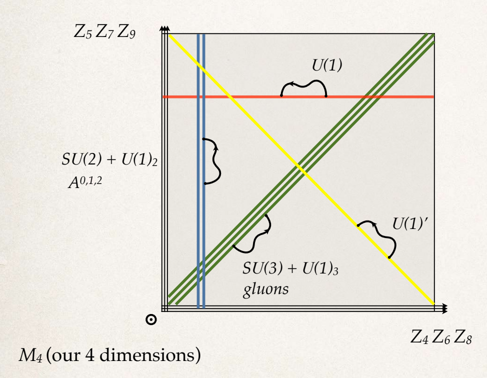
$$U(1)_Y = \sum_{i=1}^{4} c_i U(1)_i$$

- 3 extra abelian gauge fields: anomalous

$\rightarrow$ Become massive through the Green-Schwarz mechanism

In this talk: Leptophilic extra $U(1)$ boson

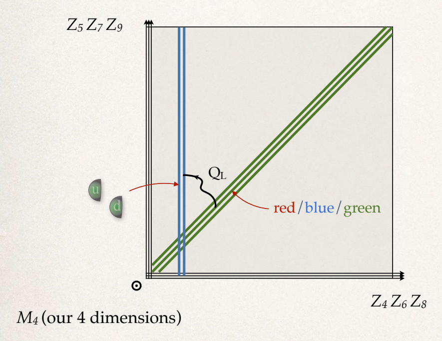
First part of this talk:
Contributions to the muon $g-2$ coming from the Kaluza-Klein excitations of a leptophilic extra $U(1)$ gauge boson



Matter sector of the Standard Model

- Open string stretched between a stack of M and a stack of N D-branes

$\rightarrow$ Transforms in the bifundamental $(\mathbf{N};\overline{\mathbf{M}})_{+1,-1}$ of $U(N)\times U(M)$

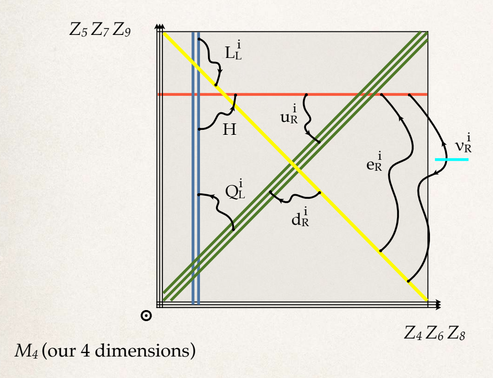


- Stacks of 3 and 2 branes
- $(\mathbf{3};\overline{\mathbf{2}})$ particle in the intersection

$\rightarrow$ Left-handed quark doublet Q_L of the SM

Matter sector of the Standard Model

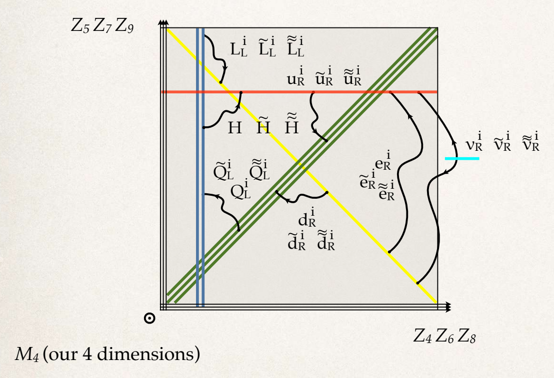
- Model with 4 stacks of $(3, 2, 1, 1)$ D-branes



SM matter particles live at intersections of D-branes

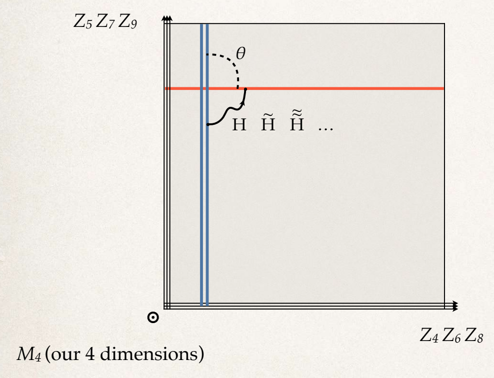
Massive copies at intersections

- SM particles: lighest (massless) modes at each intersections
- Followed by tower of massive copies



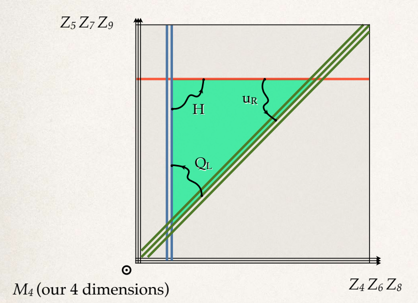
In this talk: Light scalar copies at intersections

Second part of this talk:
Contributions to the muon $g-2$ coming from (light) scalar excitations



Yukawa couplings

Anastasopoulos; Bianchi; Consoli; Fortsch. Phy. 65 (2017) [1609.09299]



$$
\begin{aligned}
|Y_{QuH}| &= g_{\mathrm{op}}(2\pi)^{-\frac{3}{4}}\left(\Gamma_{\pi-\theta_H,\pi-\theta_Q,-\theta_u}\right)^{\frac{3}{4}}\prod_{i=1}^{3} e^{-\frac{A^{(i)}_{QuH}}{2\pi\alpha'}} \\
|Y_{Qu\tilde{H}}| &= \frac{1}{\sqrt{\theta_H}}\sqrt{2\Gamma_{\pi-\theta_H,\pi-\theta_Q,-\theta_u}A_{QuH}M_s^2/\pi}\;|Y_{QuH}| \\
|Y_{Qu\tilde{\tilde{H}}}| &= \frac{\sqrt{2}}{\pi\theta_H}\Gamma_{\pi-\theta_H,\pi-\theta_Q,-\theta_u}\left(A_{QuH}M_s^2-\pi/2\right)\;|Y_{QuH}|
\end{aligned}
$$

Leptophilic extra $U(1)$ boson and the muon $g-2$

- L.A. Anchordoqui, I. Antoniadis, X. Huang, D. Lüst, FR and T.R. Taylor,
  Leptophilic U(1) Massive Vector Bosons from Large Extra Dimensions: Reexamination of Constraints from LEP Data,
  Phys. Lett. B **828** (2022) 137014 [2110.01247].
- I. Antoniadis and FR,
  Minimal embedding of the Standard Model into intersecting D-brane configurations with a bulk leptonic U(1),
  Eur. Phys. J. C **82** (2022) 701 [2112.07587].

Muon $(g-2)$ discrepancy from leptophilic Z' boson

Anchordoqui; Antoniadis; Huang; Lust; Taylor; PLB 820 (2021) 136585

- Gauge the lepton number $U(1)$ symmetry
- Assume the associated gauge boson to propagate along at least one extra dimension

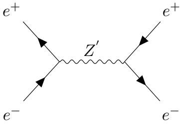
Zero mode

Anomalous: gets a mass $\mathcal{O}(M_s)$ through the Green-Schwarz anomaly cancellation mechanism
$\rightarrow$ negligible contribution to the $(g-2)_\mu$

Kaluza-Klein modes

- Couple only to leptons at tree level
- Evade the LHC bounds and can be sufficiently light

Bound from LEP data



$$\left|\frac{1}{4\pi}\sum_n \frac{g_L^2(n)}{s - M_n^2}\right| < B \sim (10\ \mathrm{TeV})^{-2}$$

- $M >> \sqrt{s}\big|_{\mathrm{LEP}} \sim 200\ \mathrm{GeV}$

  - LEP constraint: $\frac{1}{4\pi}\sum_n \frac{g_L^2}{M_n^2} < (10\ \mathrm{TeV})^{-2}$
  - $\delta a_\mu = \sum_n \frac{g_L^2}{12\pi^2}\frac{m_\mu^2}{M_n^2}$ $\Rightarrow \delta a_\mu < 10^{-11}$ ✗

One needs at least one KK state with mass
$M << \sqrt{s}\big|_{\mathrm{LEP}} \sim 200\ \mathrm{GeV}$

Muon $(g-2)$ discrepancy from leptophilic Z' boson

Anchordoqui; Antoniadis; Huang; Lust; FR; Taylor; PLB 828 (2022) 137014

- Gauge coupling:

$$g_L^2 = \frac{g_s}{V_\perp} = \frac{g_s}{(RM_s)^d} \underset{\text{1 extra dim}}{=} \frac{g_s}{RM_s} = g_s \frac{M}{M_s}$$

- $m_\mu \ll M \ll \sqrt{s}$:

$$\delta a_\mu = \sum_n \frac{g_L^2}{12\pi^2} \frac{m_\mu^2}{M_n^2} \underset{\text{1 extra dim}}{=} \frac{g_s m_\mu^2}{72 M M_s}$$

$$\Delta a_\mu = \delta a_\mu \Leftrightarrow M\, M_s \sim g_s \times 5 \times 10^4\ \mathrm{GeV}^2$$

with $M_s \gtrsim 10\ \mathrm{TeV}$, $g_s \lesssim 4\pi$.
$\Rightarrow$ Highest compactification scale $M \sim 60\ \mathrm{GeV}$

- Assumptions: $m_\mu \ll M \ll \sqrt{s}$ ✓
- LEP bound: $\left| \sum_n \frac{g_L^2}{4\pi(s - n^2 M^2)} \right| \sim 10^{-2}\ \mathrm{TeV}^{-2}$ ✓

"D-brane basis" versus "Hypercharge basis"

- Consider 3 stacks of D-branes $\rightarrow U(N_1) \times U(N_2) \times U(N_3)$
- "D-brane basis": $(B^1_\mu, B^2_\mu, B^3_\mu)$; "Hypercharge basis": (Y_μ, A_μ, A'_μ)
- $U(1)$ charges:

$$Q_Y = \sum c_i Q_i, \qquad Q_A = \sum a_i Q_i, \qquad Q_{A'} = \sum a'_i Q_i$$

- Covariant derivatives:

$$\begin{aligned} \mathcal{D}_\mu &= \partial_\mu - ig_1 Q_1 B^1_\mu - ig_2 Q_2 B^2_\mu - ig_3 Q_3 B^3_\mu \\ &= \partial_\mu - ig_Y Q_Y Y_\mu - ig_A Q_A A_\mu - ig_{A'} Q_{A'} A'_\mu . \end{aligned}$$

- Rotation matrix:

$$\begin{pmatrix} Y_\mu \\ A_\mu \\ A'_\mu \end{pmatrix} = \begin{pmatrix} c_3 \frac{g_Y}{g_3} & c_2 \frac{g_Y}{g_2} & c_1 \frac{g_Y}{g_1} \\ . & . & . \\ . & . & . \end{pmatrix} \begin{pmatrix} B^1_\mu \\ B^2_\mu \\ B^3_\mu \end{pmatrix} .$$

"D-brane basis" versus "Hypercharge basis"

- Orthogonality condition for $\mathcal{R}$: $\sum_{j=1}^{3} \mathcal{R}_{ij}^2 = 1, \forall i = 1, 2, 3$

$$\frac{1}{g_Y^2} = \frac{c_1^2}{g_1^2} + \frac{c_2^2}{g_2^2} + \frac{c_3^2}{g_3^2}$$

- $U(1)$ factor extended in the bulk:

$$g_1^2 = \frac{g_{1(D)}^2}{V_\perp}$$

- Large volume limit:

$$g_Y^2 \sim \frac{g_{1(D)}^2}{c_1^2 V_\perp} \ll 1$$

Any $U(1)$ factor extended in the bulk should not contribute to the hypercharge

D-brane construction

Requirements

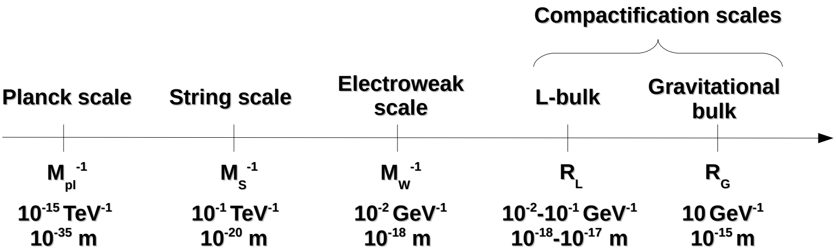
- $U(1)_L$ extended in the bulk
- $U(1)_L$ not participating to the hypercharge

Minimal configuration

- 5 stacks of D-branes
- Gauge group:

$$\begin{aligned} G &= U(3)_c \times U(2)_w \times U(1) \times U(1)' \times U(1)_L \\ &= SU(3)_c \times SU(2)_w \times U(1)_c \times U(1)_w \times U(1) \times \underbrace{U(1)' \times U(1)_L}_{\text{Extended in the bulk}} \end{aligned}$$

Scale hierarchy



D-brane construction

- Hypercharge: $q_Y = c_3 q_3 + c_2 q_2 + c_1 q_1$
- Lepton sector:

	q_3	q_2	q_1	q_1'	q_L	q_Y
L	0	1	0	0	ϵ_L	$-\frac{1}{2}$
e^C	0	0	1	0	$-\epsilon_L$	1

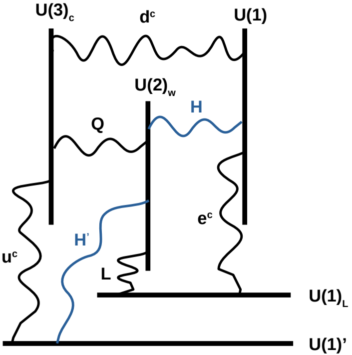
$\rightarrow$ Fixes $c_1 = 1$ and $c_2 = -\frac{1}{2}$.

- Quark sector:

	q_3	q_2	q_1	q_1'	q_L	q_Y
Q	1	v	0	0	0	$\frac{1}{6}$
u^C	-1	0	x	z	0	$-\frac{2}{3}$
d^C	-1	0	y	w	0	$\frac{1}{3}$

$\rightarrow$ Fixes $c_3 = \frac{1}{6} + \frac{v}{2} \begin{cases} v = +1 : c_3 = 2/3, x = 0, z = \pm 1, y = 1, w = 0 \\ v = -1 : c_3 = -1/3, x = -1, z = 0, y = 0, w = \pm 1 \end{cases}$

Minimal D-brane configuration



$$
\begin{array}{ll}
Q & (\mathbf{3},\mathbf{2};1,1,0,0,0)_{1/6} \\
u^C & (\bar{\mathbf{3}},\mathbf{1};-1,0,0,\epsilon_u,0)_{-2/3} \\
d^C & (\bar{\mathbf{3}},\mathbf{1};-1,0,1,0,0)_{1/3} \\
L & (\mathbf{1},\mathbf{2};0,1,0,0,\epsilon_L)_{-1/2} \\
e^C & (\mathbf{1},\mathbf{1};0,0,1,0,-\epsilon_L)_{1} \\
H & (\mathbf{1},\mathbf{2};0,1,1,0,0)_{-1/2} \\
H' & (\mathbf{1},\mathbf{2};0,-1,0,-\epsilon_u,0)_{-1/2}
\end{array}
$$

$$q_Y = \frac{2}{3}q_3 - \frac{1}{2}q_2 + q_1$$

Anomaly analysis

- 2 anomaly-free $U(1)$'s in $4D$:
  - Hypercharge: $q_Y = \frac{2}{3}q_3 - \frac{1}{2}q_2 + q_1$
  - $q = q_3 - q_2 + q_1 + q_1^{'} + q_L \quad \rightarrow \quad$ invisible from the SM particles

- $6D$ anomalies:

An anomaly-free $U(1)$ in $4D$ is not necessarily massless

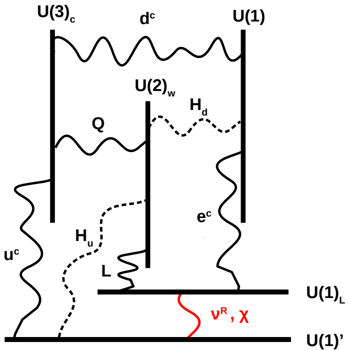
$\rightarrow$ Impose additional model-dependent constraints to ensure that q_Y remains massless in $4D$

- Example of type IIA orientifold compactification with:
  - D6-branes
  - Orientifold O6-planes

$\rightarrow$ Massless condition for q_Y ✓

Other phenomenological applications

Two extra branes beyond the SM ones $\rightarrow$ Additional fermions with vanishing hypercharge

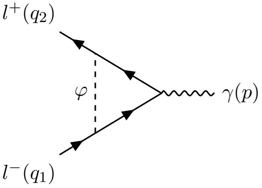


- Right-handed neutrino ν^R
- Dark Matter candidate χ

Light stringy states and the muon $g-2$

- P. Anastasopoulos, E. Niederwieser and FR,
  Light stringy states and the $g-2$ of the muon, [2209.11152].

Scalar contribution to the muon $g-2$



$$\rightarrow \delta a_\ell = \frac{\lambda_\ell^2 m_\ell^2}{8\pi^2}\int_0^1 dz\, \frac{(1+z)(1-z)}{m_\ell^2(1-z)^2 + m_\varphi^2 z}$$

- Standard Model Higgs contribution:

  $m_\mu = 105\ \mathrm{MeV},\ m_h = 125\ \mathrm{GeV},\ \lambda_\mu \sim 10^{-4} \rightarrow \delta a_\mu^{(h)} \sim 10^{-14}$ ✗

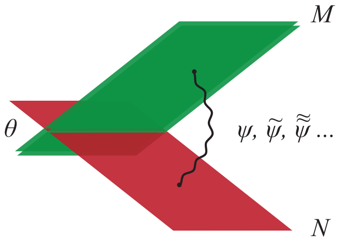
- Scalar field coupling only to the muon:

$$\left.\begin{array}{l} m_\varphi \gtrsim 2m_\mu \\ \lambda_\mu \lesssim 0.1 \end{array}\right\} \rightarrow \text{Significant contribution to the } (g-2)_\mu \quad ✓$$

How can we get such light scalar state in intersecting D-brane models?

Intersecting D-branes framework

- Open string stretched between two stacks of intersecting D-branes:



Whole tower of states with masses:

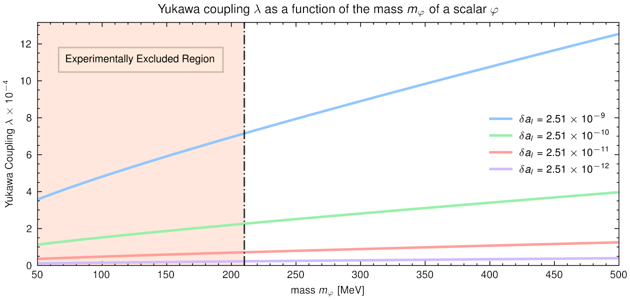
$$m_n^2 = n\theta/\pi M_s^2$$

- Light stringy states: $\theta << 1 \Rightarrow m_n << M_s$
- Upper bound for the intersection angle:

$$m_{\tilde{h}} = 250 \text{ MeV}, \; M_s \gtrsim 10 \text{ TeV} \quad \rightarrow \quad \theta/\pi \lesssim 6 \times 10^{-10}$$

Scalar contribution to the muon $g - 2$

- Yukawa coupling in terms of the scalar mass:



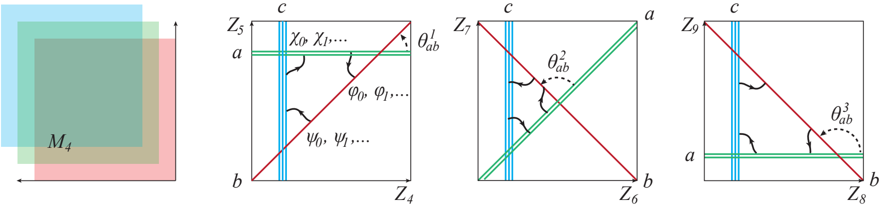
- Require $\tilde{\lambda}_\mu \lesssim \lambda_\mu = 4\sqrt{2} \times 10^{-4}$

Assume $\tilde{h}$ to give a significant contribution to the $(g-2)_\mu$, but smaller than the total discrepancy $\Delta a_\mu = 2.5 \times 10^{-9}$

Typically, consider $\delta a_\mu = 2.5 \times 10^{-10}$

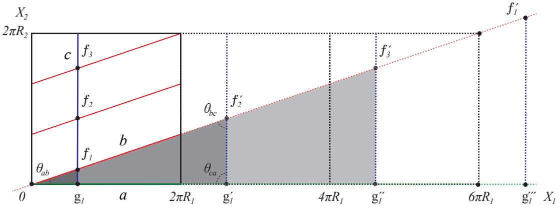
Intersecting D-branes framework

- Internal space factorizable: $\mathbb{T}^6 = \mathbb{T}_1^2 \times \mathbb{T}_2^2 \times \mathbb{T}_3^2$



- Assumptions:
  - Lighest excitations of all SM particles: Higgs excitations
  - $\theta_{ab}^1 << \theta_{ab}^2, \theta_{ab}^3 \rightarrow$ Focus on the tower of massive states in the *ab* intersection in the first 2-torus $\mathbb{T}_1^2$

Intersecting D-branes framework



- Yukawa coupling of the first excited mode:

$$|\widetilde{\lambda}_\mu| = |\lambda_\mu| \sqrt{2\Gamma_{1-a^1_{ab},1-a^1_{bc},-a^1_{ca}} A^{(1)}_{\phi\chi^I\psi^J} M_s^2/(\pi a^1_{ab})}$$

- $\Gamma_{a,b,c} \equiv \dfrac{\Gamma(a)\Gamma(b)\Gamma(c)}{\Gamma(1-a)\Gamma(1-b)\Gamma(1-c)}$
- $A^{(i)}_{\phi\chi^I\psi^J} = \dfrac{1}{2}\left|\dfrac{\sin\pi a^i_{bc}\sin\pi a^i_{ca}}{\sin\pi a^i_{ab}}\right| \left|f_{\chi^I\psi^J,i}\right|^2$
- $f_{\chi^I\psi^J,i} = g^i_I - f^i_J$

Summary

- Fixed parameters:

$$m_{\tilde{h}} = 250\ \mathrm{MeV},\ M_s = 10\ \mathrm{TeV} \quad \rightarrow \quad a^1_{ab} = 6 \times 10^{-10}$$

- Contribution to the $(g-2)_\mu$ discrepancy:

$$\delta a_\mu = 2.5 \times 10^{-10} \rightarrow \tilde{\lambda}_\mu = 2.5 \times 10^{-4}$$

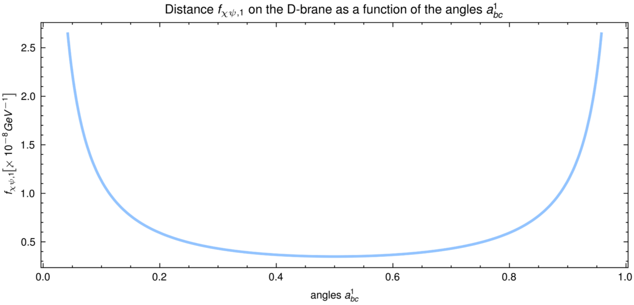
- Supersymmetric configuration:

$$a^1_{ab} + a^1_{bc} + a^1_{ca} = 0 \rightarrow \text{Two independent angles}$$

- Two free parameters: Length $f_{\chi\psi,1}$ and angle a^1_{bc}

$$f_{\chi\psi,1}(a^1_{bc}) = \frac{\tilde{\lambda}_\mu}{\lambda_\mu}\frac{1}{M_s}\sqrt{\frac{\pi a^1_{ab}}{\Gamma_{1-a^1_{ab},1-a^1_{bc},a^1_{ab}+a^1_{bc}}}\left|\frac{\sin \pi a^1_{ab}}{\sin \pi a^1_{bc} \sin \pi (a^1_{ab}+a^1_{bc})}\right|}$$

Parameter space



$$\forall a^1_{bc} \in [0,1],\, f_{\chi\psi,1} \sim 10^{-8}\ \mathrm{GeV}^{-1} \sim 10^{-21}\ \mathrm{mm}$$

$\rightarrow$ Lower bound for one size of $\mathbb{T}^2_1$

Contribution from the second excited state

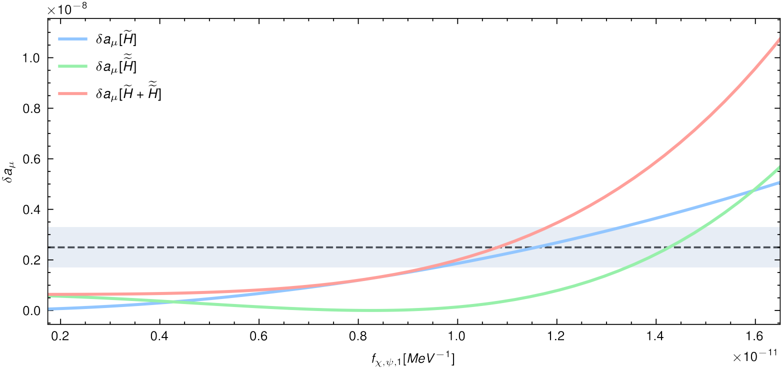
- Mass and Yukawa coupling for the second excited state $\tilde{\tilde{h}}$:

$$
\begin{aligned}
m^2_{\tilde{\tilde{h}}} &= 2a^1_{ab} M_s^2, \\
|\tilde{\tilde{\lambda}}_\mu| &= \frac{\sqrt{2}}{\pi a^1_{ab}} |\lambda_\mu| \, \Gamma_{1-a^1_{ab}, 1-a^1_{bc}, -a^1_{ca}} \left(A^{(1)}_{\varphi\bar{\psi}\psi} M_s^2 - \pi/2 \right).
\end{aligned}
$$

- Regions where $\tilde{\tilde{\lambda}}_\mu > \tilde{\lambda}_\mu \to \tilde{\tilde{h}}$ contributes more than $\tilde{h}$

Contribution from the second excited state

- Fix $a^1_{ab} = 6 \times 10^{-10}$, $a^1_{bc} = 0.5$, $M_s = 10\ \mathrm{TeV}$



Region of the parameter space

$$4.25 \times 10^{-12}\ \mathrm{MeV}^{-1} \lesssim f_{\chi\psi,1} \lesssim 1.1 \times 10^{-11}\ \mathrm{MeV}^{-1}$$

D-brane construction

Call H_d: Higgs with ultra-light excitations

Requirements

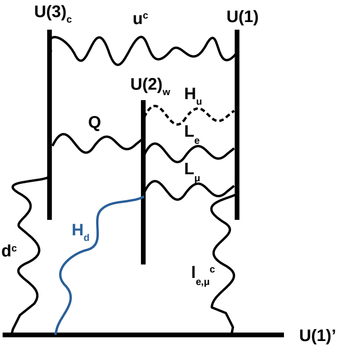
- No tree-level coupling between the electron and H_d and the quarks and H_d
- No other fields in the intersection where H_d lives

Minimal configuration

- 4 stacks of D-branes
- Gauge group:

$$\begin{aligned} G &= U(3)_c \times U(2)_w \times U(1) \times U(1)' \\ &= SU(3)_c \times SU(2)_w \times U(1)_c \times U(1)_w \times U(1) \times U(1)' \end{aligned}$$

Minimal D-brane configuration



$$
\begin{array}{ll}
Q & (\mathbf{3},\mathbf{2};1,{\color{red}1},0,0)_{1/6} \\
u^C & (\bar{\mathbf{3}},\mathbf{1};-1,0,-1,0)_{-2/3} \\
d^C & (\bar{\mathbf{3}},\mathbf{1};-1,0,0,1)_{1/3} \\
L_e & (\mathbf{1},\mathbf{2};0,{\color{red}1},-1,0)_{-1/2} \\
L_\mu & (\mathbf{1},\mathbf{2};0,{\color{red}-1},-1,0)_{-1/2} \\
\ell^C_{e,\mu} & (\mathbf{1},\mathbf{1};0,0,1,1)_{1} \\
H_d & (\mathbf{1},\mathbf{2};0,1,0,-1)_{-1/2} \\
H_u & (\mathbf{1},\mathbf{2};0,1,1,0)_{1/2}
\end{array}
$$

$$q_Y = \frac{1}{6}q_3 + \frac{1}{2}q_1 + \frac{1}{2}q_1'$$

Conclusion and outlooks

Discrepancy in the muon $g-2$ with:

- **Extra** $U(1)$ **gauge boson** and **large extra dimensions**
- **Light scalar stringy states**

Results:

- Existence of some regions in the parameter space, compatible with the experimental bounds, able to accommodate this discrepancy
- Minimal D-brane configurations: 5 and 4 stacks of branes respectively

Outlook:

- String theory embedding of our D-brane constructions
- Muon $g-2$ as a hint for low string scale and large extra dimensions scenarios?

Thank you for your attention

Other regions in the parameter space

Anchordoqui; Antoniadis; Huang; Lust; FR; Taylor; PLB 828 (2022) 137014

- $M \sim m_\mu$: $\delta a_\mu^{(2)} = \frac{\alpha_L}{\pi}\frac{-9+2\sqrt{3}\pi}{18}$

Total contribution:

$$\delta a_\mu = \frac{g_s}{4\pi^2}\frac{m_\mu}{M_s}\left(\frac{-9+2\sqrt{3}\pi}{18} + \frac{1}{3}\sum_{n>1}\frac{1}{n^2}\right)$$

$$\Delta a_\mu = \delta a_\mu \Leftrightarrow M_s \sim g_s \times 3 \times 10^2 \ \mathrm{TeV}$$

$\Rightarrow g_L = \sqrt{g_s \frac{M}{M_s}} \sim 5 \times 10^{-4}$

- LEP bound: $\left|\sum_n \frac{g_L^2}{4\pi(s-n^2M^2)}\right| \sim 10^{-4}\ \mathrm{TeV}^{-2}$ ✓

Other regions in the parameter space

Anchordoqui; Antoniadis; Huang; Lust; FR; Taylor; PLB 828 (2022) 137014

- $M << m_\mu$: $\delta a_\mu^{(3)} = \frac{g_s}{8\pi^2}\frac{m_\mu}{M_s}$

Total contribution:

$$\delta a_\mu = \frac{g_s}{8\pi^2}\frac{m_\mu}{M_s}\left(1 + 2\cdot\frac{-9+2\sqrt{3}\pi}{18} + \frac{2}{3}\frac{m_\mu}{M_1}\sum_{n=\frac{m_\mu}{M_1}+2}\frac{1}{n^2}\right)$$

Example: $\frac{m_\mu}{M} = 10 \Rightarrow \delta a_\mu \sim \frac{g_s}{8\pi^2}\frac{m_\mu}{M_s}$

$$\Delta a_\mu = \delta a_\mu \Leftrightarrow M_s \sim g_s \times 5 \times 10^2 \ \mathrm{TeV}$$

$\Rightarrow g_L = \sqrt{g_s \frac{M}{M_s}} \sim 10^{-4}$

- LEP bound: $\left|\sum_n \frac{g_L^2}{4\pi(s-n^2M^2)}\right| \sim 6\times 10^{-4}\ \mathrm{TeV}^{-2}$ ✓

Type IIA orientifold compactification

- Basic elements:
  - D6-branes
  - Orientifold O6-planes
- Notation:
  - i: stack of N_i D6-branes
  - $\tilde{i}$: its orientifold image
- $D6_i$-branes: span $4D$ Minkowski space, wrap 3-cycles Π_i in the internal space X_6
- $[\Pi_i]$: 3-homology class of the 3-cycle Π_i
- Homological intersection number of the stacks i and j:

$$I_{ij} = [\Pi_i] \cdot [\Pi_j]$$

Type IIA orientifold compactification

- Open string sectors:
  - $ij \to I_{ij}$ $4D$ chiral fermions in the $(\mathbf{N_i}, \overline{\mathbf{N}}_\mathbf{j})$ of $U(N_i) \times U(N_j)$
  - $i\tilde{j} \to I_{i\tilde{j}}$ $4D$ chiral fermions in the $(\mathbf{N_i}, \mathbf{N_j})$ of $U(N_i) \times U(N_j)$

$\{N_i, I_{ij}, I_{i\tilde{j}}\}$ determine the massless chiral spectrum of the $4D$ theory

- Anomaly-free $U(1)$:

$$q_Y = \sum_i c_i q_i$$

Massless condition

$$\sum_{i \neq j} c_i N_i \left(I_{ji} - I_{j\tilde{i}} \right) = 0$$

Type IIA orientifold compactification

- Minimal supersymmetric extension of the model with $q_Y = -\frac{1}{3}q_3 - \frac{1}{2}q_2 + q_1$:

	q_3	q_2	q_1	q_1'	q_L	q_Y
Q	1	-1	0	0	0	$\frac{1}{6}$
u^C	-1	0	-1	0	0	$-\frac{2}{3}$
d^C	-1	0	0	1	0	$\frac{1}{3}$
L	0	1	0	0	1	$-\frac{1}{2}$
e^C	0	0	1	0	-1	1
$\tilde{H}_u$	0	1	1	0	0	$\frac{1}{2}$
$\tilde{H}_d$	0	1	0	-1	0	$-\frac{1}{2}$

- Intersection numbers:

$$I_{32} = 3, \quad I_{3\tilde{1}} = -3, \quad I_{31'} = -3, \quad I_{2\tilde{1}_L} = 3, \quad I_{11_L} = 3, \quad I_{2\tilde{1}} = 1, \quad I_{21'} = 1$$

- Massless conditions for $j = \{3, 1_L\}$ ✓
- Massless conditions for $j = \{2, 1, 1'\}$: require $I_{2\tilde{1}} = 3$ and $I_{21'} = 3$

$\rightarrow$ Introduce extra Higgs doublets in the $(\mathbf{2}, \mathbf{1})$ and $(\mathbf{2}, \mathbf{1}')$