On Holographic Vacuum Misalignment

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Overview

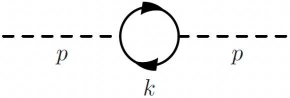
Composite Higgs Models

$$\frac{m_H^2}{\Lambda_{\mathrm{SM}}^2} \sim 10^{-28} \lll 1 \quad \to \text{Naturalness problem}$$

- Investigations into the fundamental nature and origin of the Higgs boson are among the topical subjects in theoretical physics.
- The Higgs being a composite object with a compositeness scale of TeV order, is one of the few options for "Naturally" generating its mass.
- The Higgs boson itself, as a pseudo-Nambu-Goldstone boson associated with symmetry breaking pattern, is a reasonable candidate.

Mass terms and naturalness

$$\mathcal{L} = -\frac{1}{2}\phi(\Box + m^2)\phi + \lambda\phi\bar{\psi}\psi + \bar{\psi}(i\partial\!\!\!/ - M)\psi$$



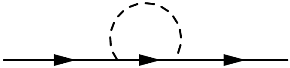
$$\Sigma_2\left(p^2\right) = \frac{3\lambda^2}{4\pi^2}\int_0^1 dx\left(\left[M^2 - p^2x(1-x)\right]\ln\frac{M^2 - p^2x(1-x)}{\Lambda^2} + \Lambda^2\right) + finite$$

$$m^2 = m_P^2 + \Sigma_2\left(p^2\right) \sim m_P^2 + \Lambda^2 \qquad m_p = 125\,GeV\,, \Lambda \sim 10^{19}\,GeV$$

$$\Downarrow$$

$m^2 = (1 + 10^{-34})\,\Lambda^2 \quad \rightarrow$ **Fine-tuning (Naturalness issue)**

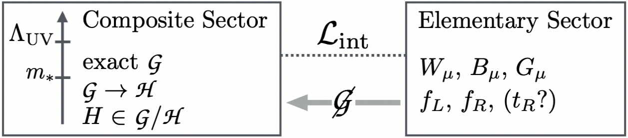
Fermion masses

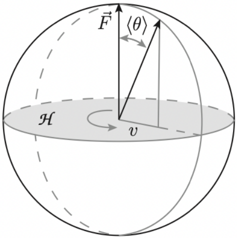


$$\Sigma_2\left(\not p\right) = i\frac{\lambda^2}{16\pi^2}\left[\frac{\not p + 2M}{\varepsilon} - \int_0^1 dx[(1-x)\not p + M]\ln\frac{M^2 x + (1-x)\left(m^2 - p^2 x\right)}{\tilde{\mu}^2}\right]$$

$M \to 0 \quad \Rightarrow$ **Custodial chiral symmetry** $\psi \to e^{i\alpha\gamma_5}\psi$

Vacuum misalignment





$\mathcal{G} \to \mathcal{H}$ spontaneous breaking $\to$ massless NGB's in the coset $\mathcal{G}/\mathcal{H}$

$$G_{\mathrm{EW}} = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subseteq \mathcal{H}$$

G is large enough to contain at least one Higgs doublet in the coset

[Panico, Wulzer '15]

Correspondance Dictionary

CHM	**Gravity**
D=4 ↓ Confining	D=5 ↓ D=6 compactified on a circle *[Witten '98]*
Global SO(5) ↓ SO(4)	Local SO(5) isometry ↓ Local SO(4)
Weakly gauged SO(4) (misaligned)	**Boundary terms**

Instead of $SU(2) \times U(1)$, our study is gauging an SO(4) group.

6D gravity action

$$
\begin{aligned}
\mathcal{S}_6 &= \mathcal{S}_6^{(bulk)} + \sum_{i=1,2} \mathcal{S}_{5,i}\,, \\
\mathcal{S}_6^{(bulk)} &= \frac{1}{2\pi}\int \mathrm{d}^6x\sqrt{-\hat{g}_6}\left\{\frac{\mathcal{R}_6}{4} - \frac{1}{2}\hat{g}^{\hat{M}\hat{N}}\left(D_{\hat{M}}\mathcal{X}\right)^T D_{\hat{N}}\mathcal{X} - \mathcal{V}_6(\mathcal{X}) \right.\\
&\left. - \frac{1}{2}\mathrm{Tr}\left[\hat{g}^{\hat{M}\hat{P}}\hat{g}^{\hat{N}\hat{Q}}\mathcal{F}_{\hat{M}\hat{N}}\mathcal{F}_{\hat{P}\hat{Q}}\right]\right\},
\end{aligned}
$$

$\rho_1 < \rho < \rho_2$ and the space-time index is $\hat{M} = 0, 1, 2, 3, 5, 6$.

$$
\mathcal{V}_6(\phi) = -5 - \frac{\Delta(5-\Delta)}{2}\phi^2 - \frac{5\Delta^2}{16}\phi^4\,,
$$

$$
\mathcal{X} = \exp\left[2i\sum_{\hat{A}}\pi^{\hat{A}}t^{\hat{A}}\right]\mathcal{X}_0\,\phi\,, \qquad \text{where} \qquad \mathcal{X}_0 = (0,0,0,0,1)^T\,,
$$

with $\hat{A} = 1, \cdots, 4$, labelling the generators of the $SO(5)/SO(4)$ coset. For the gauge field and the scalar we have:

$$
\boxed{10 \to 6 \oplus 4 \to 3 \oplus 3 \oplus 3 \oplus 1, \quad 5 \to 4 \oplus 1 \to 3 \oplus 1 \oplus 1.}
$$

Asymptotically in the UV, the dual field theory flows towards a CFT in 5 dimensions, deformed by the insertion of an operator $\mathcal{O}$ with scaling dimension given by $\max(\Delta, 5-\Delta)$.
The two parameters appearing in the solution of the corresponding second-order classical equations correspond in field-theory terms to the coupling and condensate associated with $\mathcal{O}$.

Asymptotics and dimensional reduction to five dimensions

$$\mathrm{d}s_6^2 = e^{-2\chi}\mathrm{d}x_5^2 + e^{6\chi}\left(\mathrm{d}\eta + \chi_M \mathrm{d}x^M\right)^2 \,,$$

where the space-time index is $M = 0, 1, 2, 3, 5$.
A part of the reduced action then becomes

$$\mathcal{S}_5^{1\,(bulk)} = \int \mathrm{d}^5 x \sqrt{-g_5} \left\{ \frac{R}{4} - \frac{1}{2} g^{MN} \left[6\partial_M \chi \partial_N \chi \,+\, \partial_M \phi \partial_N \phi \right] - e^{-2\chi} \mathcal{V}_6(|\phi|) - \frac{1}{16} e^{8\chi} g^{MP} g^{NQ} F^{(\chi)}_{MN} F^{(\chi)}_{PQ} \right\} \,,$$

We consider background solutions in which $\chi_M = 0$, while the metric g_{MN}, ϕ, and χ depend on the radial coordinate only. The metric in five dimensions takes the domain-wall (DW) form

$$\mathrm{d}s_5^2 = \mathrm{d}r^2 + e^{2A(r)}\mathrm{d}x_{1,3}^2 = e^{2\chi(\rho)}\mathrm{d}\rho^2 + e^{2A(\rho)}\mathrm{d}x_{1,3}^2 \,,$$

with $\mathrm{d}\rho = e^{-\chi}\mathrm{d}r$.

UV expansions

$$\rho \to +\infty, \quad \phi = 0 \quad \text{critical point of } \mathcal{V}_6, \quad \chi \simeq \frac{1}{3}\rho, \quad A \simeq \frac{4}{3}\rho$$

We classify all the solutions in terms of a power expansion in the small parameter $z \equiv e^{-\rho}$. The expansion depends on five free parameters.

$$\begin{aligned}
\phi(z) &= \phi_J z^{\Delta_J} + \cdots + \phi_V z^{\Delta_V} + \cdots , \\
\chi(z) &= \chi_U - \frac{1}{3}\log(z) + \cdots + (\chi_5 + \cdots) z^5 + \cdots , \\
A(z) &= A_U - \frac{4}{3}\log(z) + \cdots .
\end{aligned}$$

The two parameters appearing in the solution of the corresponding second-order classical equations correspond in field-theory terms to the coupling and condensate associated with $\mathcal{O}$.

For $\Delta = 3$:

$$\phi(z) = \phi_J z^2 + \phi_V z^3 - \frac{25}{48}\phi_J^3 z^6 - \frac{57}{80}\phi_J^2\phi_V z^7 + \mathcal{O}\left(z^8\right) \ ,$$

$$\chi(z) = \chi_U - \frac{1}{3}\log(z) - \frac{1}{24}\phi_J^2 z^4 + \left(\chi_5 - \frac{2}{25}\phi_J\phi_V\right) z^5 - \frac{1}{24}\phi_V^2 z^6 + \mathcal{O}\left(z^8\right) \ ,$$

$$A(z) = A_U - \frac{4}{3}\log(z) - \frac{1}{6}\phi_J^2 z^4 + \left(\frac{1}{4}\chi_5 - \frac{8}{25}\phi_J\phi_V\right) z^5 - \frac{1}{6}\phi_V^2 z^6 + \mathcal{O}\left(z^8\right) \ .$$

Background solutions

Confining solutions The confining solutions are such that the circle parametrised by η shrinks to zero size at some point ρ_o of the radial direction ρ and there is no conical singularity. For small $(\rho - \rho_o)$, we find that such solutions have the following form

$$\phi(\rho) = \phi_I - \frac{1}{16}\Delta\phi_I \left(20 + \Delta\left(5\phi_I^2 - 4\right)\right)(\rho - \rho_o)^2 + \mathcal{O}\left((\rho - \rho_o)^2\right) \,,$$
$$\chi(\rho) = \chi_I + \frac{1}{3}\log(\rho - \rho_o) + O\left((\rho - \rho_o)^4\right) \,,$$
$$A(\rho) = A_I + \frac{1}{3}\log(\rho - \rho_o) + \mathcal{O}\left((\rho - \rho_o)^2\right) \,,$$

Singular domain-wall solutions They obey the DW ansatz $A = 4\chi = \frac{4}{3}\mathcal{A}$. In six dimensions, they take the form of Poincaré domain walls.

$$\phi(\rho) = \phi_I - \sqrt{\frac{2}{5}}\log(\rho - \rho_o) + \mathcal{O}((\rho - \rho_o)^2) \,,$$
$$\mathcal{A}(\rho) = \frac{1}{5}\log(\rho - \rho_o) + \mathcal{O}((\rho - \rho_o)^2) \,.$$

The system of equations is symmetric under the change $\phi \to -\phi$, hence a second branch of solutions can be obtained by just changing the sign of ϕ. These solutions are singular at the end of space.

Boundary-localised interactions

We add to the bulk action, $\mathcal{S}_5^{(bulk)}$, several boundary terms—denoted as in order to obtain the desired five-dimensional action,

$$\mathcal{S}_5 = \mathcal{S}_5^{(bulk)} + \sum_{i=1,2} \Big(\mathcal{S}_{\mathrm{GHY},i} + \mathcal{S}_{\lambda,i} \Big) + \mathcal{S}_{P_5,2} + \mathcal{S}_{\mathcal{V}_4,2} + \mathcal{S}_{\mathcal{A},2} + \mathcal{S}_{\chi,2} + \mathcal{S}_{\mathcal{X},2} \,,$$

$$\mathcal{S}_{P_5,2} = \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left\{ -\frac{1}{2} K_5\, \tilde{g}^{\mu\nu} \left(D_\mu P_5\right) D_\nu P_5 - \lambda_5 \left(P_5^T P_5 - v_5^2\right)^2 \right\}\Bigg|_{\rho=\rho_2} ,$$

$$\mathcal{S}_{\mathcal{V}_4,2} = -\int \mathrm{d}^4x \sqrt{-\tilde{g}}\, \mathcal{V}_4(\mathcal{X}, \chi, P_5)\Big|_{\rho=\rho_2} ,$$

$$\mathcal{S}_{\mathcal{A},2}\Big|_{P_5=\overline{P_5}} = \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left\{ -\frac{1}{4} \hat{D}_2\, \tilde{g}^{\mu\rho} \tilde{g}^{\nu\sigma} \mathcal{F}^{\hat{A}}_{\mu\nu} \mathcal{F}^{\hat{A}}_{\rho\sigma} - \frac{1}{4} \bar{D}_2\, \tilde{g}^{\mu\rho} \tilde{g}^{\nu\sigma} \mathcal{F}^{\bar{A}}_{\mu\nu} \mathcal{F}^{\bar{A}}_{\rho\sigma} \right\}\Bigg|_{\rho=\rho_2} .$$

$$\mathcal{S}_{\mathcal{X},2} = \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left\{ -\frac{1}{2} K_{\mathcal{X},2}\, \tilde{g}^{\mu\nu} (D_\mu \mathcal{X})^T D_\nu \mathcal{X} \right\}\Bigg|_{\rho=\rho_2} .$$

Boundary term	Parameters	Free parameters
$\mathcal{S}_{\mathrm{GHY},i}$	—	—
$\mathcal{S}_{\lambda,i}$	$m^2_{\mathcal{X},i}$	—
$\mathcal{S}_{\chi,2}$	$D_{\chi,2}$	—
$\mathcal{S}_{P_5,2}$	$K_5(k_5), \lambda_5, v_5$	—
$\mathcal{S}_{\mathcal{V}_4,2}$	$\partial^2_v \mathcal{V}_4(v, m^2_4)$	v, m^2_4
$\mathcal{S}_{\mathcal{A},2}$	$\bar{D}_2(\bar{\varepsilon}), \hat{D}_2(\hat{\varepsilon})$	$\bar{\varepsilon}$
$\mathcal{S}_{\mathcal{X},2}$	$K_{\mathcal{X},2}(k_{\mathcal{X}})$	$k_{\mathcal{X}}$

Linearised equations for scalar fluctuations

We write here the action of the two-derivative sigma-model in D dimensions, consisting of n scalars Φ^a, with $a = 1, \cdots, n$, coupled to gravity:

$$\mathcal{S} = \int \mathrm{d}^D x \sqrt{-g} \left[\frac{R}{4} - \frac{1}{2} g^{MN} G_{ab} \partial_M \Phi^a \partial_N \Phi^b - \mathcal{V}(\Phi^a) \right]. \tag{1}$$

Where $M = 0, \cdots, 3, 5, \cdots, D$ the D-dimensional space-time indexes.
If we adopt the ansatz for the background solutions, that the metric is of the DW form, and that the scalar fields only depend on the radial coordinate,

$$\mathrm{d}s_D^2 = \mathrm{d}r^2 + e^{2A(r)} \mathrm{d}x_{1,D-2}^2 \,, \tag{2}$$

$$\Phi^a = \Phi^a(r) \,, \tag{3}$$

The fluctuations around the classical background of the active scalars and gravity are treated with the gauge-invariant formalism [*Bianchi et al. '20, Berg et al. '20, Elander '20, Elander, Piai '20 ...]*:

$$\Phi^a(x^\mu, r) = \Phi^a(r) + \varphi^a(x^\mu, r)\,,$$

By decomposing the metric according to the ADM formalism, one writes

$$\mathrm{d}s_D^2 \quad = \quad \left((1+\nu)^2 + \nu_\sigma \nu^\sigma\right)\mathrm{d}r^2 + 2\nu_\mu \mathrm{d}x^\mu \mathrm{d}r + e^{2A(r)}\left(\eta_{\mu\nu} + h_{\mu\nu}\right)\mathrm{d}x^\mu \mathrm{d}x^\nu\,,$$

and

$$h^\mu{}_\nu \quad = \quad \mathfrak{e}^\mu{}_\nu + iq^\mu \epsilon_\nu + iq_\nu \epsilon^\mu + \frac{q^\mu q_\nu}{q^2}H + \frac{1}{D-2}\delta^\mu{}_\nu h,$$

where $\mathfrak{e}^\mu{}_\nu$ *is transverse and traceless,* ϵ^μ *is transverse.*
We form the following gauge-invariant (under diffeomorphisms) combinations:

$$\begin{aligned}
\mathfrak{a}^a \quad &= \quad \varphi^a - \frac{\partial_r \Phi^a}{2(D-2)\partial_r A}h\,,\\
\mathfrak{b} \quad &= \quad \nu - \partial_r\left(\frac{h}{2(D-2)\partial_r A}\right)\,,\\
\mathfrak{c} \quad &= \quad e^{-2A}\partial_\mu \nu^\mu - \frac{e^{-2A}q^2 h}{2(D-2)\partial_r A} - \frac{1}{2}\partial_r H\,,\\
\mathfrak{d}^\mu \quad &= \quad e^{-2A}P^\mu{}_\nu \nu^\nu - \partial_r \epsilon^\mu\,,
\end{aligned}$$

For the scalar fluctuations $\mathfrak{a}^a = \mathfrak{a}^a(q,\rho)$, where q^μ is the four-momentum:

$$\left[\partial_\rho^2 + (4\partial_\rho A - \partial_\rho\chi)\partial_\rho - e^{2\chi - 2A} q^2\right]\mathfrak{a}^a - e^{2\chi}\mathcal{X}^a{}_c\mathfrak{a}^c \quad = \quad 0\,.$$

With $G_{ab} = \mathrm{diag}(1,\, 6)$, $\mathcal{X}^a{}_c$ reads as follows:

$$\begin{aligned}\mathcal{X}^a{}_c \quad \equiv \quad & \frac{\partial}{\partial\Phi^c}\left(G^{ab}\frac{\partial(e^{-2\chi}\mathcal{V}_6)}{\partial\Phi^b}\right) + \frac{4}{3\partial_\rho A}\left[\partial_\rho\Phi^a\frac{\partial(e^{-2\chi}\mathcal{V}_6)}{\partial\Phi^c} + G^{ab}\frac{\partial(e^{-2\chi}\mathcal{V}_6)}{\partial\Phi^b}\partial_\rho\Phi^d G_{dc}\right] \\ & + \frac{16(e^{-2\chi}\mathcal{V}_6)}{9(\partial_\rho A)^2}\partial_\rho\Phi^a\partial_\rho\Phi^b G_{bc}\,.\end{aligned}$$

A discrete spectrum can be found by imposing the following boundary conditions:

$$e^{-2\chi}\partial_\rho\Phi^c\partial_\rho\Phi^d G_{db}\partial_\rho\mathfrak{a}^b\Big|_{\rho_i} = \left[\,\frac{3\partial_\rho A}{2}e^{-2A}q^2\delta^c{}_b + \partial_\rho\Phi^c\left(\frac{4\mathcal{V}_6}{3\partial_\rho A}\partial_\rho\Phi^d G_{db} + \frac{\partial\mathcal{V}_6}{\partial\Phi^b}\right)\right]\mathfrak{a}^b\Big|_{\rho_i}\,,$$

Physical composite states in the dual theory have mass $M^2 = -q^2$.

The equations of motion for the (transverse and traceless) tensor fluctuations $\mathfrak{e}^{\mu}{}_{\nu}$:

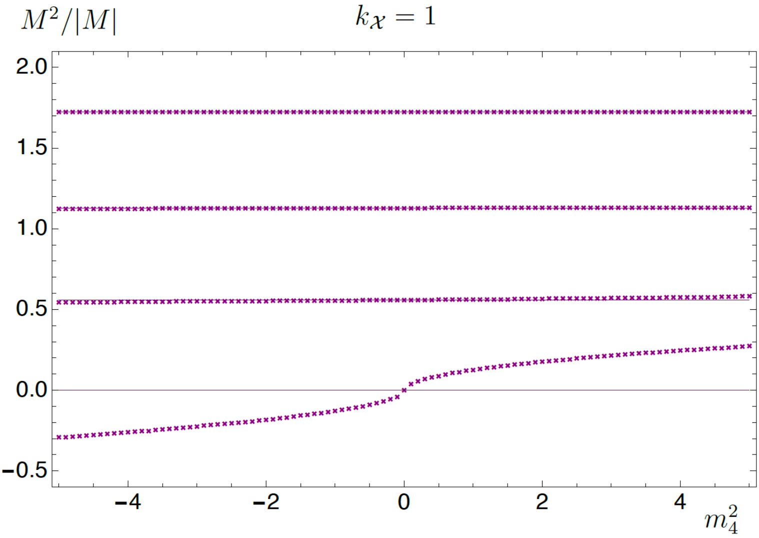
$$0 \;=\; \left[\partial_\rho^2 + (4\partial_\rho A - \partial_\rho \chi)\partial_\rho - e^{2\chi - 2A} q^2\right] \mathfrak{e}^{\mu}{}_{\nu}\,,$$

For the vector χ_M one looks at the gauge-invariant transverse polarisations,

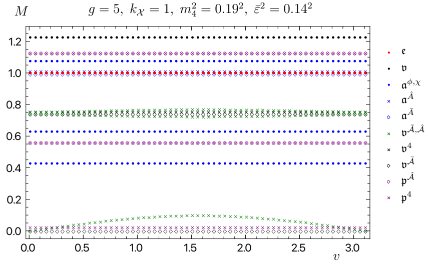
$$0 \;=\; P^{\mu\nu}\left[\partial_\rho^2 + (2\partial_\rho A + 7\partial_\rho \chi)\partial_\rho - e^{2\chi - 2A} q^2\right] \chi_\nu\,,$$

where $P^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$. Neumann boundary conditions is imposed on these two kinds of fluctuations.

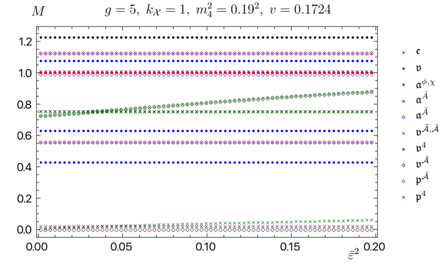
Results



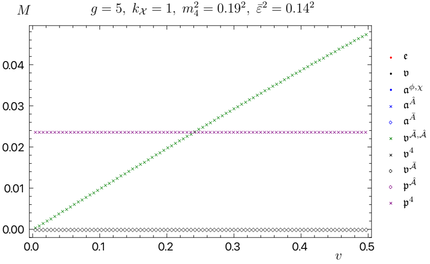
Results



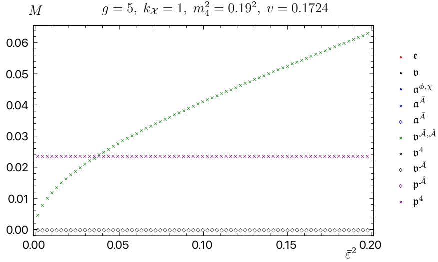
M
g = 5, k_χ = 1, m_4^2 = 0.19^2, ε̄^2 = 0.14^2
v



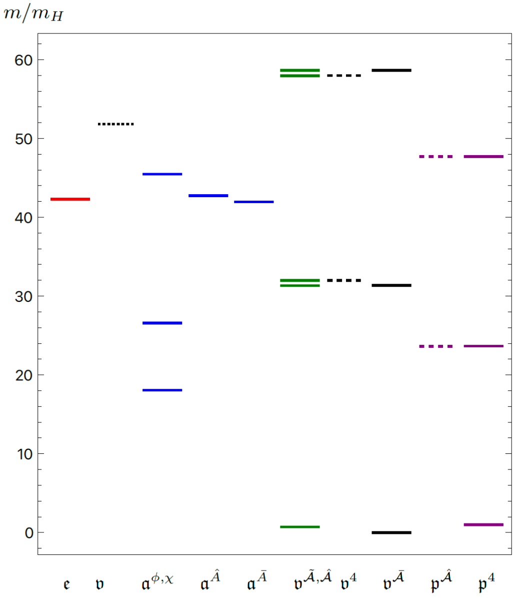
M
g = 5, k_χ = 1, m_4^2 = 0.19^2, v = 0.1724
ε̄^2



M
g = 5, k_χ = 1, m_4^2 = 0.19^2, ε̄^2 = 0.14^2
v



M
g = 5, k_χ = 1, m_4^2 = 0.19^2, v = 0.1724
ε̄^2



m/m_H
60
50
40
30
20
10
0
$\mathfrak{e}$
$\mathfrak{v}$
$\mathfrak{a}^{\phi,\chi}$
$\mathfrak{a}^{\hat{A}}$
$\mathfrak{a}^{\bar{A}}$
$\mathfrak{v}^{\tilde{\mathcal{A}},\hat{\mathcal{A}}}$
$\mathfrak{v}^4$
$\mathfrak{v}^{\bar{\mathcal{A}}}$
$\mathfrak{p}^{\hat{\mathcal{A}}}$
$\mathfrak{p}^4$

Summary and Outlook

- We have studied a bottom-up model, with a completely smooth gravity background, that implements a simple realisation of the holographic description of confinement in the dual gauge theory.
- We presented the mass spectrum of bosonic states in the field theory.
- All the new particles are parametrically heavy with respect to the bosons that play the role of the Z, W, and Higgs boson
- We can further extend this six-dimensional model to more realistic composite Higgs models with gauged $SU(2) \times U(1)$ and fermions.
- Extension to a top-down model is expected.

Thank you