An Effective Field Theory for Large Oscillons

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based on: Levkov, VM, Nugaev, Panin, arXiv:2208.04334

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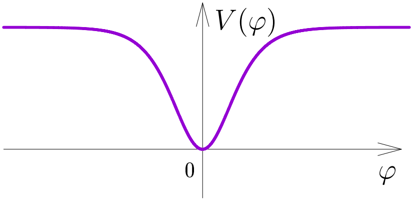
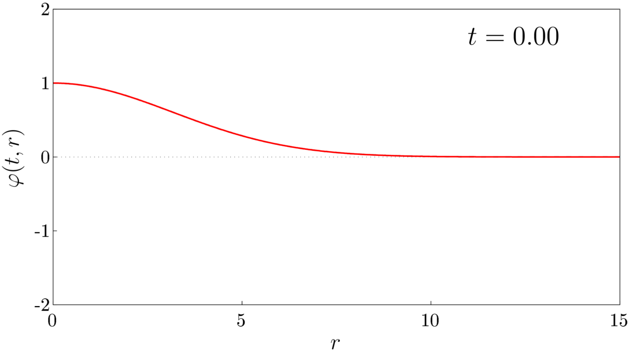
Oscillons: introduction

Example: scalar field theory

$$\partial_t^2 \varphi - \Delta\varphi = -V'(\varphi)$$

$$V(\varphi) = \frac{1}{2}\tanh^2\varphi$$





$d = 3$

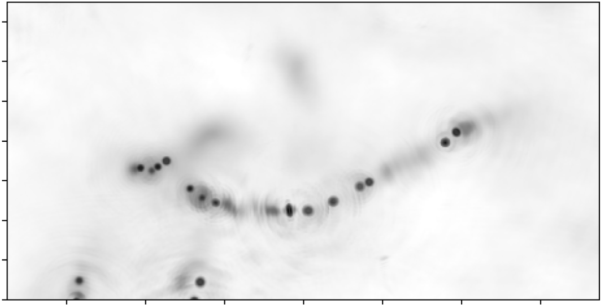
$$\varphi(0, r) = \varphi_0\, e^{-r^2/\sigma^2}$$

$$\varphi_0 = 1, \quad \sigma = 20$$

Lifetime:

$\gtrsim 10^5$ periods

- nucleate during generation of axion or ultra-light DM



Kolb, Tkachev '94

Vaquero, Redondo, Stadler '19

Buschmann, Foster, Safdi '20

- accompany cosmological phase transitions

Dymnikova, Kozel, Khlopov, Rubin '00
Gleiser, Graham, Stamatopoulos '10

- formed by inflaton field during preheating

Amin, Easther, Finkel, '10
Hong, Kawasaki, Yamazaki '18

Why are oscillons so long-lived? How to describe them?

Previous methods of describing oscillons

- Numerical methods

Kudryavtsev '75; Bogolyubsky, Makhankov '76
Piette, Zakrzewski '98
Gleiser et al. '10; Ollé, et al. '20

- Perturbative expansion: $|\varphi| \ll 1, \quad R \gg m^{-1}$

$$\Downarrow$$

Dashen, Hasslacher, Neveu '75
Kosevich, Kovalev '75
Fodor et al. '08; Fodor, et al. '09

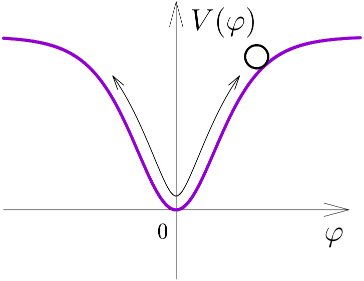
Small-amplitude oscillons

Goal.

Develop description of oscillons at large φ.

Effective Field Theory: action-angle variables

- Main idea: **consider large-size oscillons**
- Zero order approx.: $\partial_t^2\varphi - \cancel{\Delta\varphi} = -V'(\varphi) \implies$ Nonlinear oscillator



- Exactly solvable: action-angle variables

$$(\pi_\varphi \equiv \dot\varphi) \qquad \boxed{(\varphi, \dot\varphi) \to (I, \theta)}$$

- Hamiltonian: $h = \dfrac{\dot\varphi^2}{2} + V(\varphi) \equiv h(I)$
- Explicitly: $$I(\varphi, \dot\varphi) \propto \oint \sqrt{h - V}\, d\varphi$$ $$\theta(\varphi, \dot\varphi) \propto \frac{\partial}{\partial I} \int \sqrt{h - V}\, d\varphi'$$

$V(\varphi) = \frac{1}{2}\tanh^2\varphi$

$$\varphi = \operatorname{arcsinh}\left(\frac{\sqrt{I(2-I)}}{1-I}\cos\theta\right)$$

$$h(I) = I - I^2/2.$$

- Classical solution:

$$I = \text{const}, \ \theta = \Omega t + \text{const}, \ \boxed{\Omega = \frac{\partial h}{\partial I}}$$

- General case: $\varphi = \Phi(I, \theta), \quad \dot\varphi = \Pi(I, \theta)$

Effective Field Theory: leading-order effective action

- **BUT** oscillon depends on $\mathbf{x} \Longrightarrow I = I(\mathbf{x})$, $\theta = \theta(t, \mathbf{x})$, but slowly
- Let us derive effective action:

$$\mathcal{S} = \int dt\, d^d\boldsymbol{x} \left(\underbrace{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}_{I\partial_t\theta - h} - \underbrace{\frac{1}{2}(\partial_i\varphi)^2}_{\text{subleading}} \right)$$

- Averaging over period

$$(\partial_i\varphi)^2 \longrightarrow \langle(\partial_i\varphi)^2\rangle = \frac{1}{2\pi}\int_0^{2\pi} (\partial_i\Phi(I,\theta))^2\, d\theta$$

- Slow-varying $\partial_i I$, $\partial_i\theta$ are moved *out* of the average

$$\langle(\partial_i\varphi)^2\rangle \approx \frac{(\partial_i I)^2}{\mu_I(I)} + \frac{(\partial_i\theta)^2}{\mu_\theta(I)} + \cancel{\langle\partial_I\Phi\,\partial_\theta\Phi\rangle\partial_i I\partial_i\theta}$$

$$\mu_I \equiv \left\langle(\partial_I\Phi)^2\right\rangle^{-1}, \quad \mu_\theta \equiv \left\langle(\partial_\theta\Phi)^2\right\rangle^{-1}$$

Effective Field Theory: properties

Effective action in the leading order

$$\mathcal{S}_{\rm eff} = \int dt\, d^d\boldsymbol{x} \left(I\partial_t\theta - h(I) - \frac{(\partial_i I)^2}{2\mu_I(I)} - \frac{(\partial_i\theta)^2}{2\mu_\theta(I)} \right)$$

- Introducing complex field $\psi(t,\, \boldsymbol{x}) = \sqrt{I} \cdot e^{-i\theta}$ gives

Example: $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

$\mu_I = I(2-I)^2(1-I)^2, \quad \mu_\theta = I^{-1} - 1$

$$\mathcal{S}_{\rm eff} = \int dt\, d^d\boldsymbol{x} \left(i\psi^*\partial_t\psi - h - \frac{|\partial_i\psi|^2}{2\mu_1} - \frac{1}{2\mu_2}\left[\psi^{*2}(\partial_i\psi)^2 + \mathrm{h.c.}\right] \right)$$

nonlinear Schrödinger model

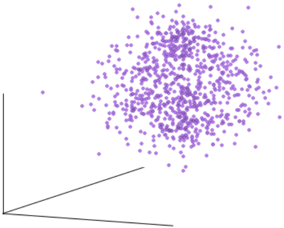
Form factors: $\mu_i = \mu_i(|\psi|^2), \quad h = h(|\psi|^2)$

- Global symmetry: $\theta \to \theta + \alpha \iff \psi \to \mathrm{e}^{-i\alpha}\psi$

$$\Downarrow$$

- Conserved charge: $N = \int d^d\mathbf{x}\, I = \int d^d\mathbf{x}\, |\psi|^2$

Attraction + charge conservation = solitons!



- Stationary ansatz:

$$\psi = \psi(r)\, e^{-i\omega t}, \quad \text{or} \quad \theta = \omega t$$

- Alternatively:

minimize the energy E at fixed charge N.

- Oscillon profile equation

$$\boxed{\Omega = \partial h / \partial I}$$

$$-\frac{2\psi^2}{\mu_I}\,\Delta\psi - (\partial_i\psi)^2 \frac{d}{d\psi}\left(\psi^2/\mu_I\right) + \Omega\psi = \omega\psi$$

- At the oscillon: $\delta E = \omega\delta N$ for all variations $\Longrightarrow \omega = dE/dN$.
- Physical interpretation: N — „number of particles"

  ω — „energy of a particle"

Example: $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

$\boxed{d = 1} \quad : \quad (t, x)$

- ODE is integrable

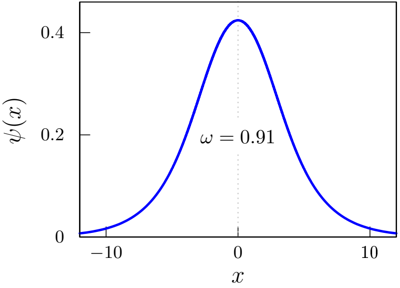
$$x = \frac{2}{\sqrt{2\omega - 1}} \arctan \frac{\zeta(\psi)}{\sqrt{2\omega - 1}} - \frac{1}{\sqrt{2\omega}} \arctan \frac{\zeta(\psi)}{\sqrt{2\omega}} + \frac{1}{\sqrt{2 - 2\omega}} \operatorname{arctanh} \frac{\zeta(\psi)}{\sqrt{2 - 2\omega}},$$

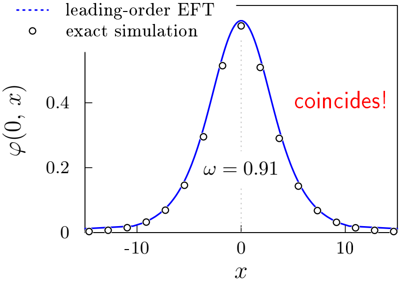
where $\zeta(\psi) = \sqrt{2 - 2\omega - \psi^2}$

- Oscillon profile:

$$\varphi(t, x) = \Phi(\psi^2, \omega t)$$

- $\underbrace{\omega \to 1}_{\text{NR limit}}$: $R \propto (1 - \omega)^{-1/2} \to \infty$



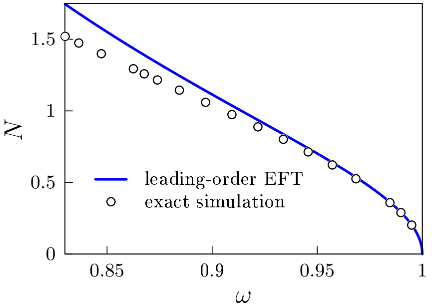


Example: $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

- N and E — also explicitly computed:

$$N = \frac{4}{\sqrt{2\omega - 1}}\ \arctan\frac{\sqrt{1-\omega}}{\sqrt{\omega - 1/2}} - \frac{4}{\sqrt{2\omega}}\ \arctan\sqrt{1/\omega - 1}\,,$$

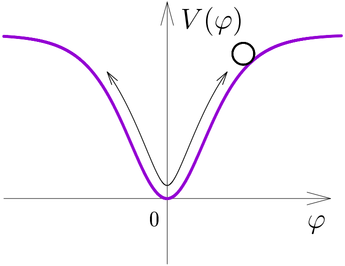
$$E = \frac{4(1-\omega)}{\sqrt{2\omega - 1}}\ \arctan\frac{\sqrt{1-\omega}}{\sqrt{\omega - 1/2}} + 2\sqrt{2\omega}\ \arctan\sqrt{1/\omega - 1}\,,$$



- For $d \neq 1$ — profile $\psi(r)$, N and E are found numerically.

Oscillons: existence, longevity, stability

- In the EFT — existence conditions can be derived *Coleman '85*



$I_0 \equiv I(0), \quad \Omega(I) = \partial h / \partial I$

Existence conditions.

$\Omega(I_0) < m$

$h(I_0)/I_0 < m$

&

$\underbrace{\mu_I \big|_{I \le I_0} \neq 0}_{\text{non-singular EFT}}$

- Longevity. Profile eqs. $\Longrightarrow \quad \omega - \Omega \sim (mR)^{-2} \ll 1$

$$\Downarrow$$

$\boxed{|d\Omega/dI| \ll \Omega/I}$
– conditions for EFT
– longevity of oscillon

- Linear stability

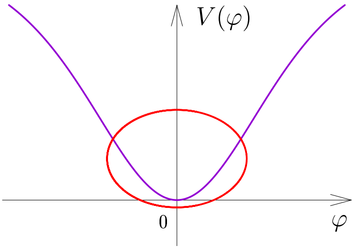
$\boxed{dN(\omega)/d\omega < 0}$ (Vakhitov-Kolokolov crit.)

$$\boxed{\varphi(V) = \int_0^V \frac{dh}{\Omega(h)\sqrt{2V-2h}}}$$

2 ways of achieving $|d\Omega/dI| \ll \Omega/I$:

- Small-amplitude approximation:

$$V = \frac{1}{2}m^2\varphi^2 + \frac{g_3}{4}\varphi^4 + \frac{g_5}{6}\varphi^6 + \ldots$$



- Special cases of flat $\Omega(I)$, e.g.

$$\Omega = \Omega_0 + \delta\Omega(I), \quad \delta\Omega(I) \ll \Omega_0.$$

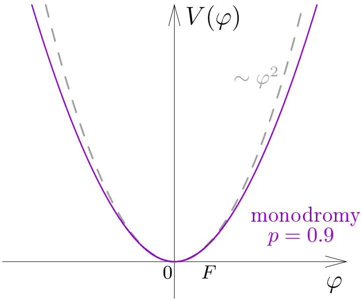
$$\Downarrow$$

$$V = \frac{1}{2}\tilde{\Omega}^2(\varphi)\,\varphi^2, \quad \tilde{\Omega}^2 \approx \Omega_0^2 + \delta\tilde{\Omega}(\varphi)$$

- E.g. monodromy potential

$$V_p(\varphi) = \frac{1}{2p}m^2F^2\left[(1+\frac{\varphi^2}{F^2})^p - 1\right],$$

$p \to 1$ *Ollé et al. '20*



Corrections

- Goal: Develop asymptotic expansion in R^{-2}:

$$\mathcal{S}_{\text{eff}} = \underbrace{\mathcal{S}^{(1)}_{\text{eff}}}_{R^0+R^{-2}} + \overbrace{\underbrace{\mathcal{S}^{(2)}_{\text{eff}}}_{R^{-4}} + \underbrace{\mathcal{S}^{(3)}_{\text{eff}}}_{R^{-6}} + \ldots}^{\text{corrections}}$$

- Field corrections:

$$I = \underbrace{\bar{I}}_{\text{slow}} + \underbrace{\delta I}_{\textit{fast}}, \qquad \theta = \underbrace{\bar{\theta}}_{\text{slow}} + \underbrace{\delta\theta}_{\text{fast}}$$

$$\langle \delta I \rangle = \langle \delta\theta \rangle = 0, \quad \delta I \ll I, \quad \delta\theta \ll \theta$$

- Solve eqs. for δI, $\delta\theta$

$$\delta I \approx \frac{\mathcal{I}[\partial_{\bar{\theta}}\Phi\Delta\Phi]}{\partial_t\bar{\theta}}, \qquad \delta\theta \approx \frac{1}{\partial_t\bar{\theta}}\left\{\partial_{\bar{I}}\Omega\cdot\mathcal{I}[\delta I] - \mathcal{I}\left[\partial_{\bar{I}}\Phi\Delta\Phi - \langle\partial_{\bar{I}}\Phi\Delta\Phi\rangle\right]\right\}$$

$$\mathcal{I}[f] = \int^{\bar{\theta}} f(\bar{\theta}')\,d\bar{\theta}' - A \quad \text{— primitive}, \qquad \Phi = \Phi(\bar{I},\bar{\theta}), \qquad \Omega = \Omega(\bar{I})$$

Second-order EFT: oscillons

- Plug δI, $\delta\theta$ into action $\Longrightarrow \mathcal{S}_{\text{eff}} = \mathcal{S}_{\text{eff}}^{(1)} + \mathcal{S}_{\text{eff}}^{(2)}$,

$\boxed{\bar\theta = \omega t}$

$$\mathcal{S}_{\text{eff}}^{(2)} = \int dt\, d^d\boldsymbol{x}\, \left[d_1\, (\partial_i\psi)^4 + d_2\, \psi\Delta\psi(\partial_i\psi)^2 + d_3\, (\Delta\psi)^2\right]$$

Note. Four spatial derivatives

$d_i(\psi^2)$ — form factors

- Oscillon profile $\bar I = \psi^2(\mathbf{x})$ can be found perturbatively:

$$\bar I = \bar I^{(1)} + \bar I^{(2)}, \quad S_{\text{eff}}^{(1)}[\bar I^{(1)}] - \text{min}$$

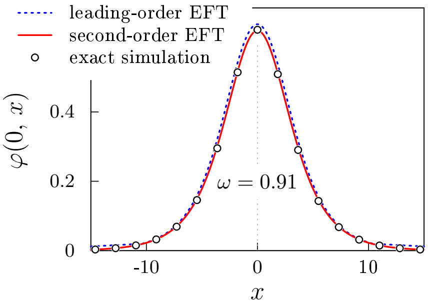
$$\Downarrow$$

$$\frac{\delta^2 S_{\text{eff}}^{(1)}}{\delta \bar I^2} \cdot \bar I^{(2)} = -\frac{\delta S_{\text{eff}}^{(2)}}{\delta \bar I}$$

- Oscillon field:

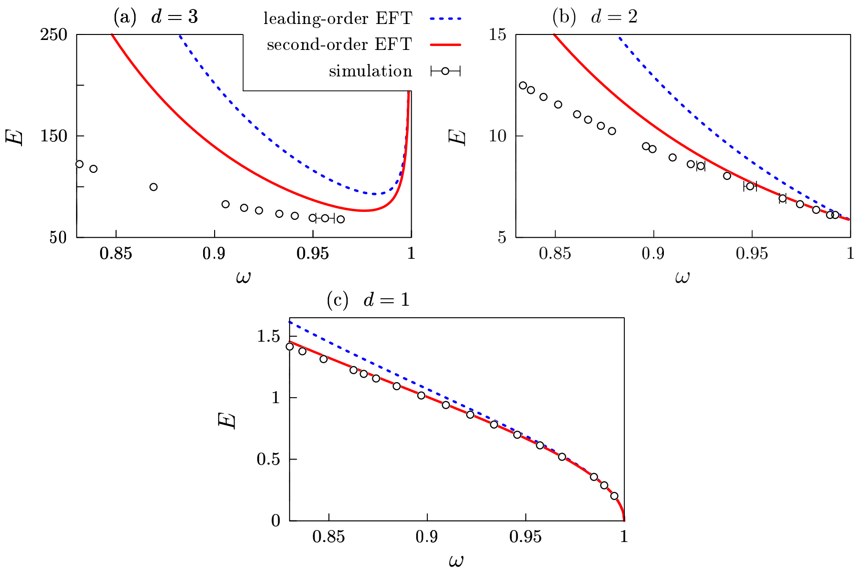
$$\varphi(t, \boldsymbol{x}) = \Phi\left(\bar I + \delta I,\, \omega t + \delta\theta\right),$$

$$\delta I = \delta I(\bar I, \omega t), \quad \delta\theta = \delta\theta(\bar I, \omega t)$$



$$V(\varphi) = \frac{1}{2}\tanh^2\varphi, \quad d = 1$$

Oscillons: $E(\omega)$ comparison for $V(\varphi) = \frac{1}{2}\tanh^2\varphi$



EFT, even for lesser ω, works better at smaller d. Why?

Limit $d \to 0$

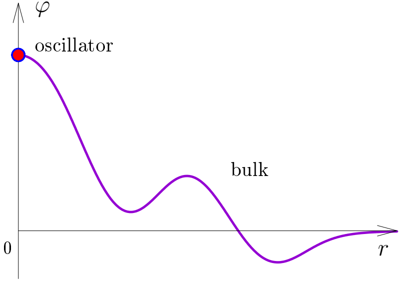
- Analytically continue field equation to $d < 1$:

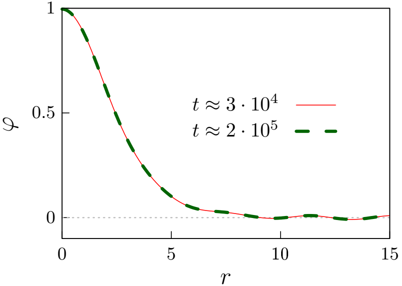
$$\partial_t^2 \varphi - \partial_r^2 \varphi - \frac{d-1}{r}\,\partial_r \varphi = -V'(\varphi)$$

- Substitute $r = 0$: $\quad \partial_t^2 \varphi(0) \;-\; \underbrace{d\,\partial_r^2 \varphi(0)}_{\text{vanishes at } d=0} \;=\; -V'(\varphi(0))$

$$\Downarrow$$

Oscillator at $d = 0,\ r = 0$



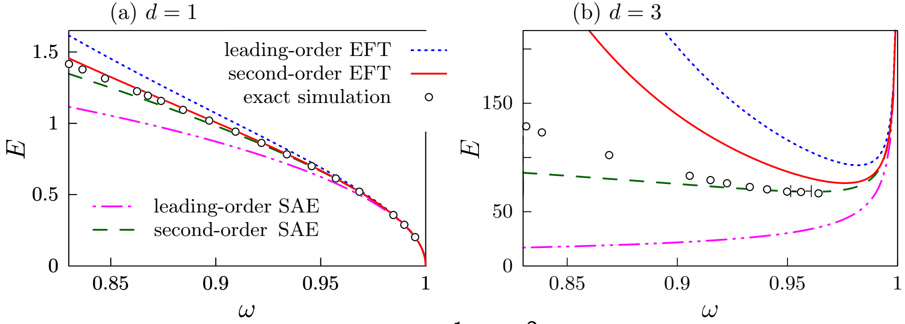


- There are exactly periodic solutions at $d = 0$.

Small-amplitude expansion vs. EFT

Reminder. different expansion parameters:

$R^{-1} \to 0$ at finite φ (EFT), $\qquad R^{-1} \sim \varphi \to 0$ (small-amplitude)



$$V(\varphi) = \frac{1}{2}\tanh^2\varphi$$

EFT.

- Sole parameter of the expansion: $(mR)^{-2}$
- Global $U(1)$–symmetry $\Longrightarrow$ oscillons
- Conditions for existence of long-lived oscillons:

$$V(\varphi) \begin{cases} \text{attractive} \\ \text{nearly quadratic potential} \end{cases}$$

- $d = 0$: exact oscillons and exact EFT
- Generic models: $\varphi = \Phi(I, \theta) = c_1 I^{1/2} + c_2 I^{3/2} + \ldots$
  systematic small-amplitude expansion

Perspective.

- EFT for monodromy potentials φ^{2n}, $n \to 1$
- Decay of oscillons — nonperturbative in EFT?

Thank you for your attention!

- Auxiliary field $\chi(I) = \int_0^I dI' \, [\mu_I(I')]^{-1/2} = 2 \int_0^\psi \psi' d\psi' / \sqrt{\mu_I}$

normalizes the gradient term: $F = \int d^d \boldsymbol{x} \left[\frac{1}{2} (\partial_i \chi)^2 - U_\omega(\chi) \right]$,

where $U_\omega \equiv \omega I - h$.

- Spherically symmetric oscillon:

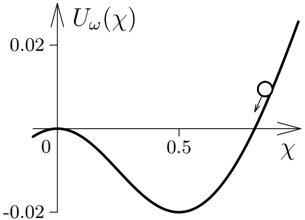
$$\partial_r^2 \chi + \frac{d-1}{r} \partial_r \chi = -dU_\omega(\chi)/d\chi$$

Existence conditions:

- max at $\chi = 0 \implies \omega < m$
- $\underset{\chi_0 \equiv \chi(0),\ I_0 \equiv I(0)}{\underline{\underline{\partial_{\chi_0} U_\omega(\chi_0) > 0}}}$ & $\begin{cases} U_\omega(\chi_0) > 0, & d > 1 \\ U_\omega(\chi_0) = 0, & d = 1 \end{cases}$

$$\Downarrow$$

$\omega > \Omega(I_0)$, $\quad \underset{\text{,,=" for } d=1}{\omega \geq h(I_0)/I_0}$, $\quad \mu_I|_{I<I_0} \neq 0$



$V = \frac{1}{2} \tanh^2 \varphi$, $\quad \omega = 0.8$

Oscillons: possibility of parametric resonance

- Possible parametric resonance: for wave vectors $k \sim O(m)$:

$$(\partial_t^2 + k^2)\delta\varphi = -V''(\Phi(I_0, \omega t))\,\delta\varphi \qquad (I(r) \approx I_0)$$

- Resonance bands width: $\delta k \sim |V''_{osc}|/k$
- Growing modes $\delta\varphi(t, \mathbf{k})$ should be localized inside oscillon

$$\delta k^{-1} \lesssim R \implies |V''_{osc}|R \gtrsim m$$

- EFT regime: near-quadratic potential

$$|V''_{osc}|/m^2 \propto |d\ln\Omega/d\ln I| \sim (mR)^{-2}$$

$$\Downarrow$$

$$|V''_{osc}|R/m \sim (mR)^{-1} \ll 1 \implies \text{no parametric resonance}$$

Corrections: technicalities

- Solve eqs. for δI, $\delta\theta$

$$\begin{cases} \partial_t \delta I = j_\theta(I,\,\theta), \\ \partial_t \delta\theta = \Omega(I) - \langle\Omega\rangle - j_I(I,\,\theta)\,, \end{cases} \qquad \text{where} \qquad \begin{array}{l} \Omega = \partial_I h \\ j_\theta = \partial_\theta \Phi\,\Delta\Phi, \\ j_I = \partial_I \Phi\,\Delta\Phi - \langle \partial_I \Phi\,\Delta\Phi\rangle \end{array}$$

- For second-order EFT we find δI, $\delta\theta$ in the leading order

$$\Downarrow$$

$$\begin{array}{rl} \partial_t \approx (\partial_t\bar\theta)\,\partial_{\bar\theta}, & \quad j_I(I,\,\theta) \approx j_I(\bar I,\,\bar\theta), \\ \Omega - \langle\Omega\rangle \approx \delta I\cdot\partial_{\bar I}\Omega, & \quad j_\theta(I,\,\theta) \approx j_\theta(\bar I,\,\bar\theta) \end{array}$$

- Fast-oscillating parts in the leading order:

$$\delta I \approx \frac{\mathcal{I}[j_\theta(\bar I,\,\bar\theta)]}{\partial_t\bar\theta}\,, \qquad \delta\theta \approx \frac{1}{\partial_t\bar\theta}\left\{\partial_{\bar I}\Omega(\bar I)\,\mathcal{I}[\delta I(\bar I,\,\bar\theta)] - \mathcal{I}[j_I(\bar I,\,\bar\theta)]\right\}\,,$$

$$\mathcal{I}[f] = \int_0^{\bar\theta} f(\bar\theta')\,d\bar\theta' - \left\langle \int_0^{\bar\theta} f(\bar\theta')\,d\bar\theta' \right\rangle \qquad \text{satisfying} \quad \langle\mathcal{I}[f]\rangle = \langle f\rangle = 0\,.$$

- Second-order correction to the effective action:

$$\mathcal{S}^{(2)}_{\rm eff}[\bar I,\,\bar\theta\,] = \int dt\,d^d\boldsymbol{x}\left[\frac{1}{\partial_t\bar\theta}\,\langle j_I\mathcal{I}[j_\theta]\rangle - \frac{\partial_{\bar I}\Omega}{2(\partial_t\bar\theta)^2}\langle(\mathcal{I}[j_\theta])^2\rangle\right]\,.$$

Integral quantities (e.g. energy)

- Volume element: $d^d \boldsymbol{x} = S_{d-1}(\mu r)^d \, dr/r, \quad S_{d-1} = 2\pi^{d/2}/\Gamma(d/2),$
- $d \to 0$: $\quad S_{d-1} \propto d \to 0, \quad dr/r$ diverges at $r = 0$.

$$\Downarrow$$

$$E = E_0 + E_1 d + O(d^2) \qquad (\text{“core”} + \text{“bulk”})$$

$$E_0 = (\partial_t \varphi_0)^2/2 + V(\varphi_0), \quad E_1 = \int_\epsilon^\infty \rho \, dr/r + E_0 \ln(c\mu\epsilon), \quad \boxed{\epsilon \to 0}$$

$$\rho(r) = (\partial_t \varphi)^2/2 + (\partial_i \varphi)^2/2 + V(\varphi), \quad c = \mathrm{e}^{\gamma_E/2}\sqrt{\pi} \approx 2.37$$

- Bulk energy conservation:

$$\partial_t E_1 = J_0 - J$$

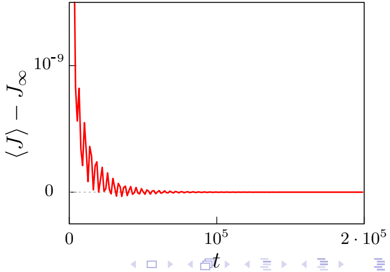
- Flux radiated to infinity:

$$J = -\lim_{r\to\infty} r^{-1} \partial_t \varphi \partial_r \varphi$$

- Flux from the origin:

$$J_0 = -\partial_t \varphi \partial_r^2 \varphi \Big|_{r=0}$$

$\langle J \rangle \to J_\infty = \text{const}$ — stationary flow



Small-amplitude expansion from EFT

- If $\epsilon^2 = m^2 - \omega^2 \ll m^2$, $R \sim O(\epsilon^{-1})$, $I_0 \sim O(\epsilon^2)$, $\varphi \sim O(\epsilon)$

$$\Downarrow$$

both EFT and small-amplitude expansion are applicable.

- Generic scalar potential, $m = 1$:

$$V = \frac{1}{2}\varphi^2 + \frac{g_3}{4}\varphi^4 + \frac{g_5}{6}\varphi^6 + \dots$$

- Action-angle variables — as half-integer series

$$\Phi = \sqrt{2I}\,\cos\theta + (2I)^{3/2}\,\frac{g_3}{32}\,[\cos(3\theta) - 6\cos\theta] + \dots\,,$$

$$h = I + \frac{3}{8}g_3 I^2 + \gamma_3 I^3 + \dots\,, \qquad \gamma_3 \equiv \frac{5}{12}\,g_5 - \frac{17}{64}\,g_3^2.$$

- EFT form factors — as series

$$\frac{1}{\mu_I} = \frac{1}{4I} - \frac{9}{16}\,g_3 + O(I)\,, \qquad\qquad d_3 = \frac{1}{8\omega} + O(I)\,,$$

Small-amplitude expansion from EFT

- Effective Lagrangian (up to second order):

$$F^{(1)} + F^{(2)} = \int d^d\boldsymbol{x} \left\{ \frac{1}{2} (\partial_i \psi)^2 + \frac{\epsilon^2}{2} \psi^2 + \frac{3}{8} g_3 \psi^4 - \frac{9}{8} g_3 \psi^2 (\partial_i \psi)^2 + \frac{\epsilon^4}{8} \psi^2 + \gamma_3 \psi^6 - \frac{1}{8} (\Delta\psi)^2 + O(\epsilon^8) \right\}$$

- Search for solution perturbatively in ϵ:

$$\psi = \frac{\epsilon}{\sqrt{2}} p_1(\boldsymbol{x}) + \frac{\epsilon^3}{4\sqrt{2}} [4 p_3(\boldsymbol{x}) - p_1(\boldsymbol{x})] + O(\epsilon^5)$$

$$\Downarrow$$

$$\begin{cases} \epsilon^{-2} \Delta p_1 - p_1 - \frac{3}{4} g_3 p_1^3 = 0 \, , \\ \epsilon^{-2} \Delta p_3 - p_3 - \frac{9}{4} g_3 p_1^2 p_3 = \left(\frac{5 g_5}{8} + \frac{3 g_3^2}{128} \right) p_1^5 \, . \end{cases}$$

G. Fodor et al. *Phys. Rev. D.* **78**, 025003 (2008) [arXiv:0802.3525]

- Oscillonic field:

$$\varphi(t, \boldsymbol{x}) = (\epsilon p_1 + \epsilon^3 p_3) \cos(\omega t) + \frac{g_3}{32} (\epsilon p_1)^3 \cos(3\omega t) + O(\epsilon^5) \, ,$$

Numerical simulations

- To absorb the outgoing radiation we introduce the sponge $H(r)$:

$$\left(\partial_t^2 + H(r)\partial_t - \Delta\right)\varphi = -V'(\varphi)\ ,$$

$$H(r) = 0 \text{ at } r < R_s,$$
$$H(r) \text{ smoothly increases at } r > R_s$$

$$\Downarrow$$

Outgoing scalar waves fade out exponentially at $r > R_s$.

- Starting configuration: I-balls $\varphi = \Phi(0, I(r))$ with various ω.

Second-order EFT coefficients for $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

$$N_A = \left[2\bar{I}(1-\bar{I})(2-\bar{I})\right]^{-1}, \quad z_2 = \bar{I}/(2-\bar{I})$$

$$c_1 = \frac{\ln(1-z_2^2)\,\partial_{\bar{I}} N_A^2}{\omega \bar{I}(2-\bar{I})}\left\{\frac{2-2\bar{I}}{\bar{I}(2-\bar{I})} - \frac{1}{\omega} - \frac{\partial_{\bar{I}} N_A}{N_A}\right\} + \frac{(\partial_{\bar{I}} N_A)^2}{\omega^2}\,\mathrm{Li}_2(z_2^2)$$
$$+ \frac{N_A^2}{4\omega(1-\bar{I})(2-\bar{I})^2}\left\{\frac{4-3\bar{I}}{(1-\bar{I})(2-\bar{I})} + 6\,\frac{\partial_{\bar{I}} N_A}{N_A} + \frac{1}{\omega}\right\},$$

$$c_2 = \frac{2\ln(1-z_2^2)\,N_A^2}{\omega \bar{I}(2-\bar{I})}\left\{\frac{2-2\bar{I}}{\bar{I}(2-\bar{I})} - \frac{3\partial_{\bar{I}} N_A}{N_A} - \frac{1}{\omega}\right\} + \frac{2N_A^2}{\omega(1-\bar{I})(2-\bar{I})^2} + \frac{\partial_{\bar{I}} N_A^2}{\omega^2}\,\mathrm{Li}_2(z_2^2),$$

$$c_3 = \frac{N_A^2}{\omega}\left\{\frac{1}{\omega}\,\mathrm{Li}_2\left(z_2^2\right) - \frac{4\ln(1-z_2^2)}{\bar{I}(2-\bar{I})}\right\},$$

$$d_1 = 16\psi^4 c_1 + 8\psi^2 c_2 + 4c_3, \quad d_2 = 8(\psi^2 c_2 + c_3), \quad d_3 = 4\psi^2 c_3,$$

Introduction: oscillons