Black holes on a conifold with fluxes

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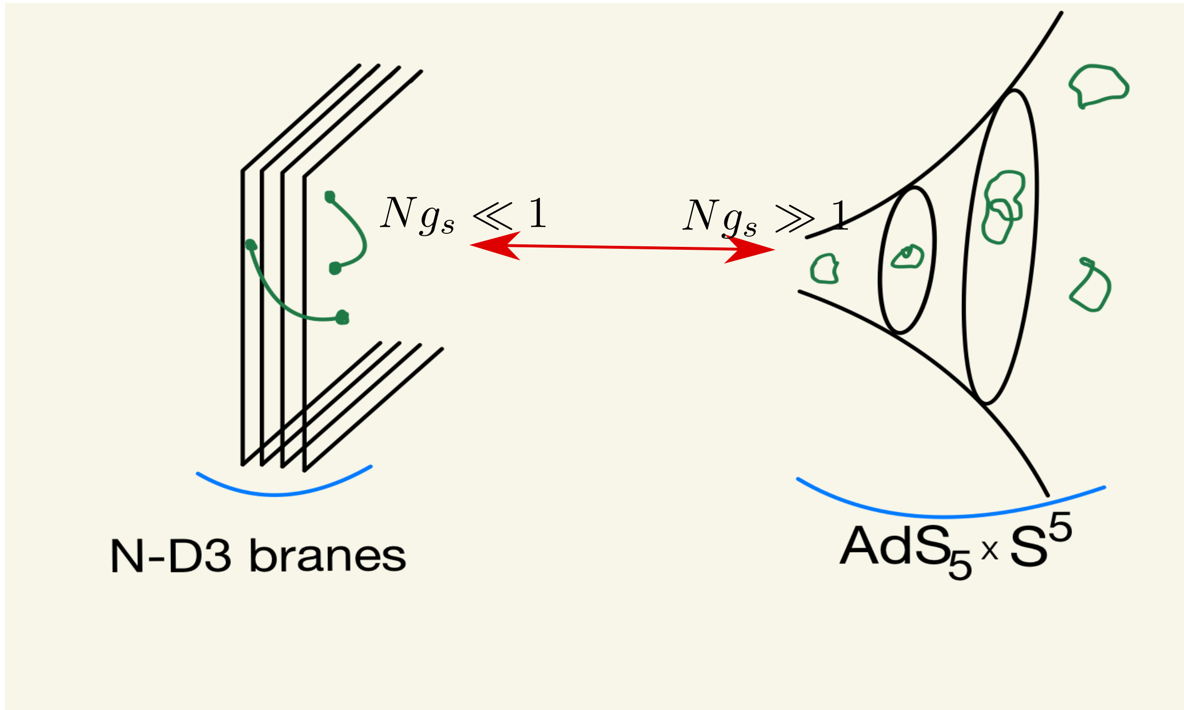
Outline

- Geometrization of thermal phase transitions in gauge/gravity correspondence
- (review) KT/KS cascading gauge theory and its holographic dual
- (review) Type IIB supergravity on warped deformed conifold with fluxes $\equiv$ WDCF
- (warm-up) Vacua and black holes in $AdS_5 \times T^{1,1}$
- S^3 vacua of WDCF
- Black holes and black branes of WDCF
- Conclusions and open questions

Basic AdS/CFT correspondence in the planar limit



- $Ng_s \ll 1$: weakly coupled open strings, ending on D3 branes in Type IIB SUGRA on $\mathbb{R}^{9,1} \Longleftrightarrow \mathcal{N} = 4\ SU(N)$ SYM
- $Ng_s \gg 1$: weakly coupled closed strings in Type IIB SUGRA on $AdS_5 \times S^5$

$\Longrightarrow$ Consider thermal state of $\mathcal{N} = 4$ $SU(N)$ SYM on S^3 of size L_3

- two scales:

$$\underbrace{T}_{\text{temperature}} \qquad \text{and} \qquad \underbrace{\mu = \frac{1}{L_3}}_{S^3 \text{ compactification scale}}$$

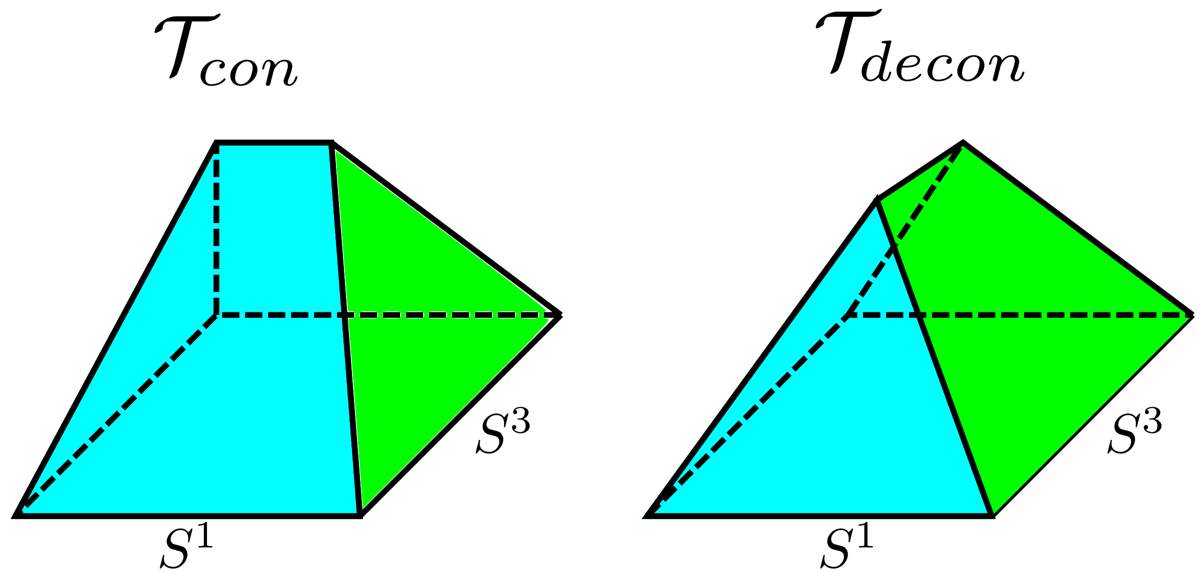
- two distinct phases of the theory:
  - the confined phase $\mathcal{T}_{con}$
  - the deconfined phase $\mathcal{T}_{decon}$

- sharp distinction between the entropy densities of two phases at large-N:

$$\underbrace{s \propto O(N^0)}_{\mathcal{T}_{con}} \qquad \text{and} \qquad \underbrace{s \propto O(N^2)}_{\mathcal{T}_{decon}}$$

- $\mathcal{T}_{decon}$ is a dominant (preferred) phase at sufficiently high $\frac{T}{\mu}$

$\Longrightarrow$ Holographic duality geometrizes $\mathcal{T}_{con}$ and $\mathcal{T}_{decon}$ phases as topologically distinct asymptotically $EAdS_5$ spaces:



- $\mathcal{T}_{con}$: global AdS_5 with periodically identified Euclidean time direction

$$t_E \sim t_E + \frac{1}{T} \qquad\qquad (\mathrm{non-contractable}\ S^1)$$

- $\mathcal{T}_{decon}$: Euclidean black hole in global AdS_5 (contactable S^1)

-

$$\underbrace{\mathrm{Confinement/deconfinement}}_{\text{open strings}} \qquad \Longleftrightarrow \qquad \underbrace{\mathrm{Hawking-Page}}_{\text{closed strings}}$$

$\Longrightarrow$ In this talk we discuss similar transitions in so-called KT/KS gauge theory:

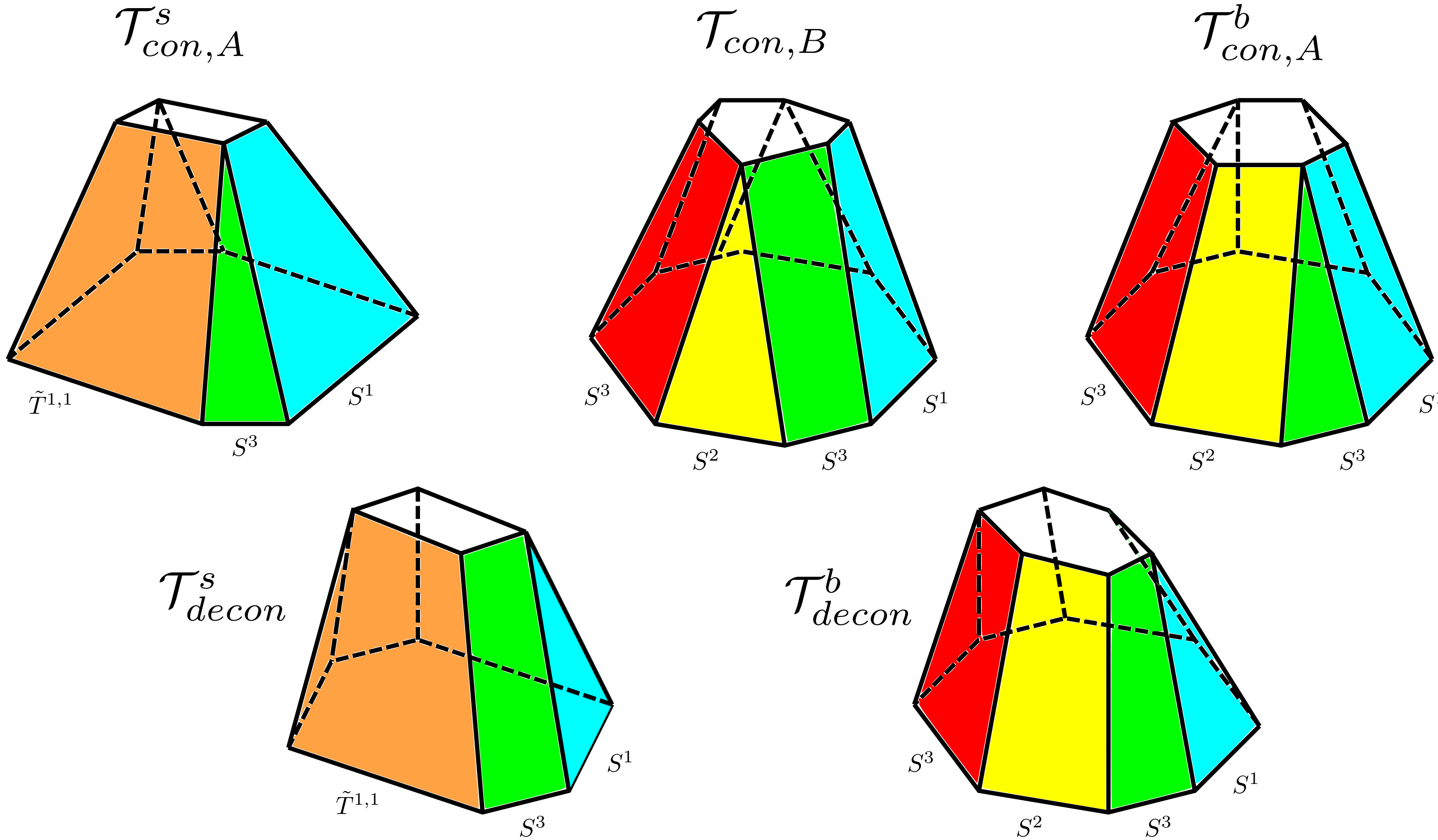
Unlike $\mathcal{N} = 4$ SYM above:

- confines in $\mathbb{R}^{3,1}$
- at low-energies $\mathcal{N} = 1$ $SU(M)$ Yang-Mills theory
- undergoes spontaneous chiral symmetry breaking

$$U(1)_R \to \mathbb{Z}_2$$

- study interplay of confinement/deconfinement and chiral symmetry breaking

<u>What are the phases of theory?</u>



$\Longrightarrow$ Purpose of the talk: to explain all these phases and transitions between them

$\Longrightarrow$ Building KT/KS holographic correspondence in $\mathbb{R}^{3,1}$

- start with *original* $\mathcal{N}=4$ $SU(N)$ - $AdS_5\times S^5$ duality

$$ds^2_{10}=\frac{r^2}{L^2}\Big(\underbrace{-dt^2+dx_i^2}_{\mathbb{R}^{3,1}}\Big)+\frac{L^2}{r^2}dr^2+L^2\left(dS^5\right)^2$$

- there is also a constant dilaton $e^{\Phi}=g_s$, and a quantized self-dual R-R five form $F_5=\mathcal{F}_5+\star\mathcal{F}_5$

$$\mathcal{F}_5=16\pi(\alpha')^2\ N\ \mathrm{vol}(S^5)\,,\qquad \frac{1}{(4\pi^2\alpha')^2}\int_{S^5}\mathcal{F}_5=N\in\mathbb{Z}$$

- dictionary:

$$g^2_{YM}=4\pi g_s\,,\qquad L^4=4\pi g_s N(\alpha')^2$$

- We consider planar limit $N\to\infty$ with $g^2_{YM}N$ kept fixed; and SUGRA approximation:

$$L^4\gg(\alpha')^2\equiv\ell_s^4\qquad\Longrightarrow\qquad g^2_{YM}N\gg1$$

- $\mathcal{N}=4$ SYM is a conformal theory with central charges $c=a$

$$c=a\Big|_{\mathcal{N}=4\ SYM}=\frac{N^2}{4}$$

$$\langle T^\mu{}_\mu\rangle=\frac{1}{(4\pi)^2}\underbrace{\left(cI_4-aE_4+b\ \Box R\right)}_{\text{from SYM boundary metric}}$$

$$E_4=R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}-4R_{\mu\nu}R^{\mu\nu}+R^2\,,\quad I_4=R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}-2R_{\mu\nu}R^{\mu\nu}+\frac{1}{3}R^2$$

- b is renormalization scheme dependent, shifted with finite counterterm

$$S_{ct}^{finite}=-\frac{b}{12(4\pi)^2}\int_{\mathcal{M}_4}\mathrm{vol}_{\mathcal{M}_4}\ R^2$$

- $\Longrightarrow$ Casimir energy on $R\times S^3$ is ambiguous, even for $\mathcal{N}=4$ SYM:

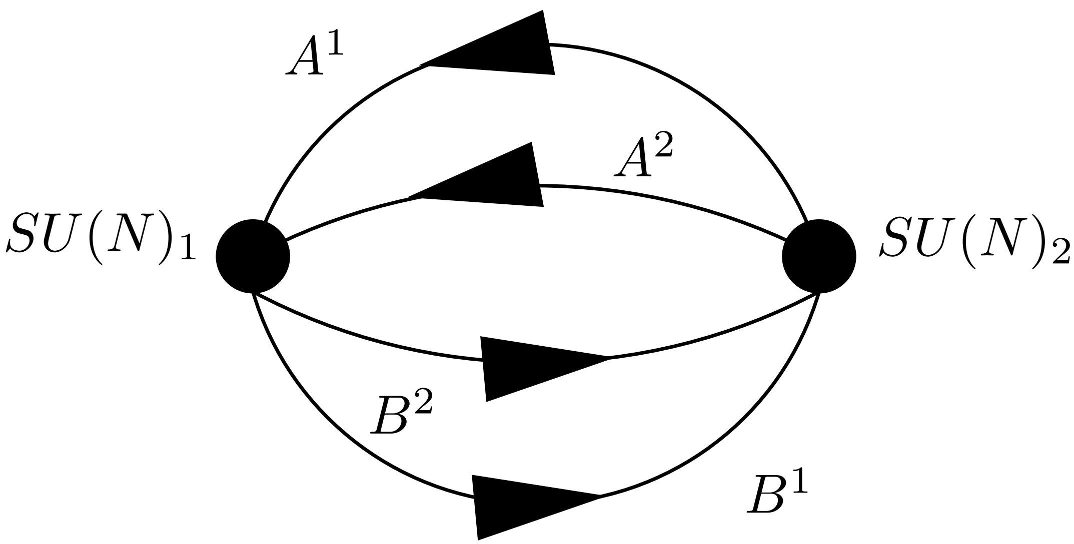
$$\delta E_0=\frac{3b}{8}\ \mu=\frac{3b}{8}\ \frac{1}{L_3}$$

Phase transitions depend on differences of energy/free energy, and so are unambiguous

$\Longrightarrow$ Do the orbifolding:

$$\frac{\mathcal{N}=4\ SU(N)}{\mathbb{Z}_2} \qquad \equiv \qquad \mathcal{N}=2\ SU(N)\times SU(N)$$

- the resulting quiver gauge theory is:



	$SU(N)_1 \times SU(N)_2$
A^i :	$(\bar{N}, N)$
B^i :	$(N, \bar{N})$

$$\mathcal{W}_{\mathcal{N}=2} = g_{YM}\,\mathrm{Tr}\ \Phi\,(A_1B_1 + A_2B_2) + g_{YM}\,\mathrm{Tr}\,\tilde{\Phi}\,(B_1A_1 + B_2A_2)$$

- $SU(2)\times SU(2)$ global flavor symmetry rotates of A_k and B_ℓ

$\Longrightarrow$ Induce the RG flow of the UV quiver with

$$\mathcal{W}_{\mathcal{N}=2} \to \mathcal{W}_{\mathcal{N}=2} + \underbrace{\frac{m}{2}\left(\mathrm{Tr}\,\Phi^2 - \mathrm{Tr}\,\tilde{\Phi}^2\right)}_{\delta\mathcal{W}}$$

$\delta\mathcal{W}$:

- breaks scale invariance, but not $SU(2) \times SU(2)$ global symmetry!
- partially breaks supersymmetry $\mathcal{N} = 2 \;\to\; \mathcal{N} = 1$
- at low energies, $i.e.$, $\ll m^2$, we can integrate out Φ and $\tilde{\Phi}$:

$$\mathcal{W}_{KW} = \frac{g_{YM}^2}{2m}\;\left[\mathrm{Tr}\,(A_1B_1A_2B_2) - \mathrm{Tr}\,(B_1A_1B_2A_2)\right]$$

- classically $\dim[\mathrm{Tr}\,(A_1B_1A_2B_2)] = 4 \Longrightarrow$ relevant deformation
- Klebanov-Witten argued in deep IR anomalous dimensions of the chiral superfields

$$\gamma_{A_k} = \gamma_{B_\ell} = -\frac{1}{4} \qquad \Longrightarrow \qquad \mathcal{W}^{IR}_{KW} = \frac{\lambda}{2}\;\epsilon^{ij}\epsilon^{k\ell}\mathrm{Tr}A_iB_kA_jB_\ell\;,$$

$i.e.$, λ is exactly marginal coupling, and the theory is $\mathcal{N} = 1$ SCFT

- $\mathcal{N}=1$ superconformal algebra is powerful enough to conclude

$$\frac{a\big|_{KW}}{a\big|_{S^5/\mathbb{Z}_2 \text{ orbifold}}} = \frac{c\big|_{KW}}{c\big|_{S^5/\mathbb{Z}_2 \text{ orbifold}}} = \frac{27}{32} \qquad \Longrightarrow \qquad c = a\Big|_{KW} = \frac{27N^2}{64}$$

- on a gravitational side, the end point of the RG flow due to $\delta\mathcal{W}$ is:

$$ds_{10}^2 = \frac{r^2}{L^2}\left(-dt^2 + dx_i^2\right) + \frac{L^2}{r^2}dr^2 + L^2 ds_{T^{1,1}}^2$$

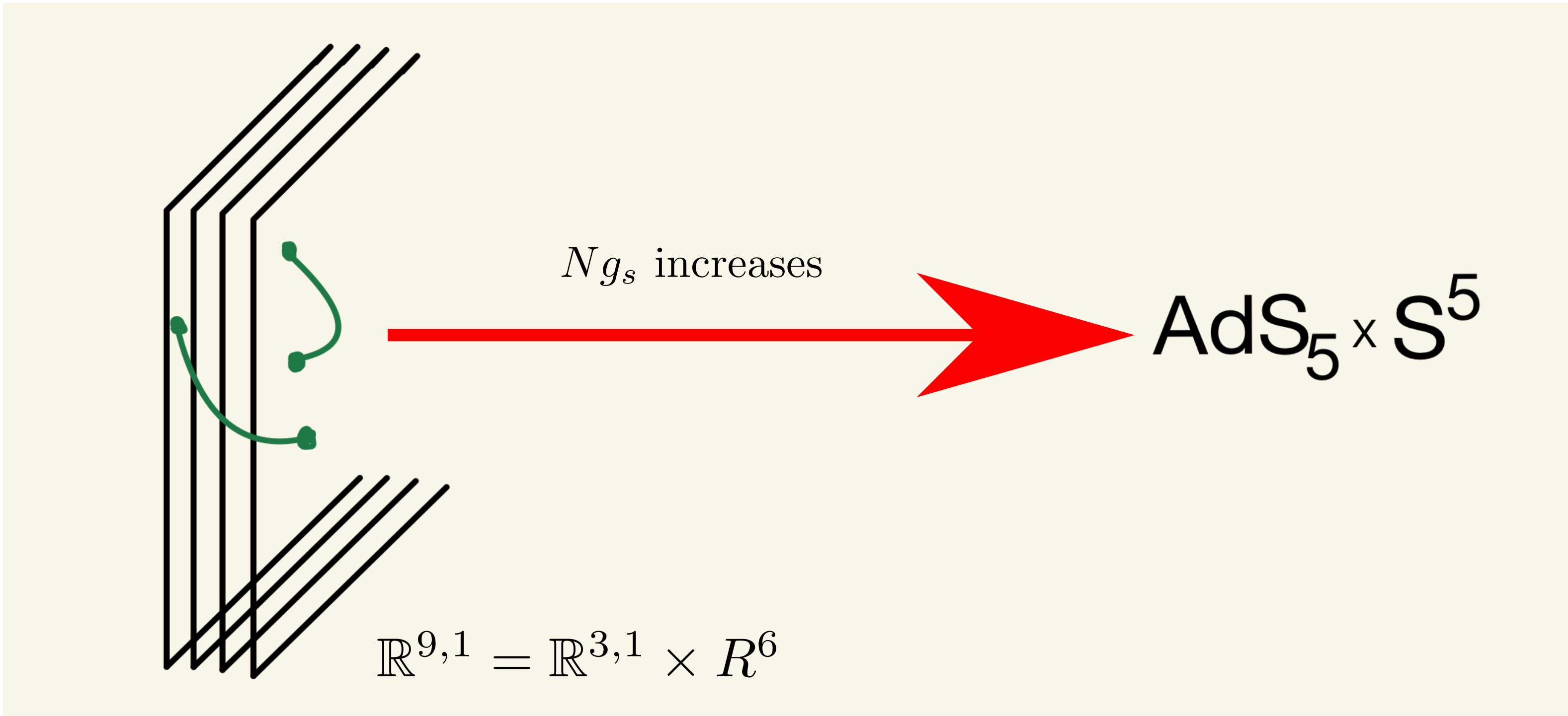
where

$$ds_{T^{1,1}}^2 = \frac{1}{9}\left(d\psi + \sum_{i=1}^{2}\cos\theta_i d\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i d\phi_i^2\right)$$

is the metric on $T^{1,1} \equiv (SU(2)\times SU(2))/U(1)$ coset (the base of the conifold)

$$\mathcal{F}_5 = 27\pi(\alpha')^2\ N\ \mathrm{vol}(T^{1,1})\,, \qquad \frac{1}{(4\pi^2\alpha')^2}\int_{T^{1,1}}\mathcal{F}_5 = N \in \mathbb{Z}$$

$$L^4 = 4\pi g_s N(\alpha')^2\ \frac{\mathrm{vol}(S^5)}{\mathrm{vol}(T^{1,1})} = \frac{27}{4}\pi g_s N(\alpha')^2$$

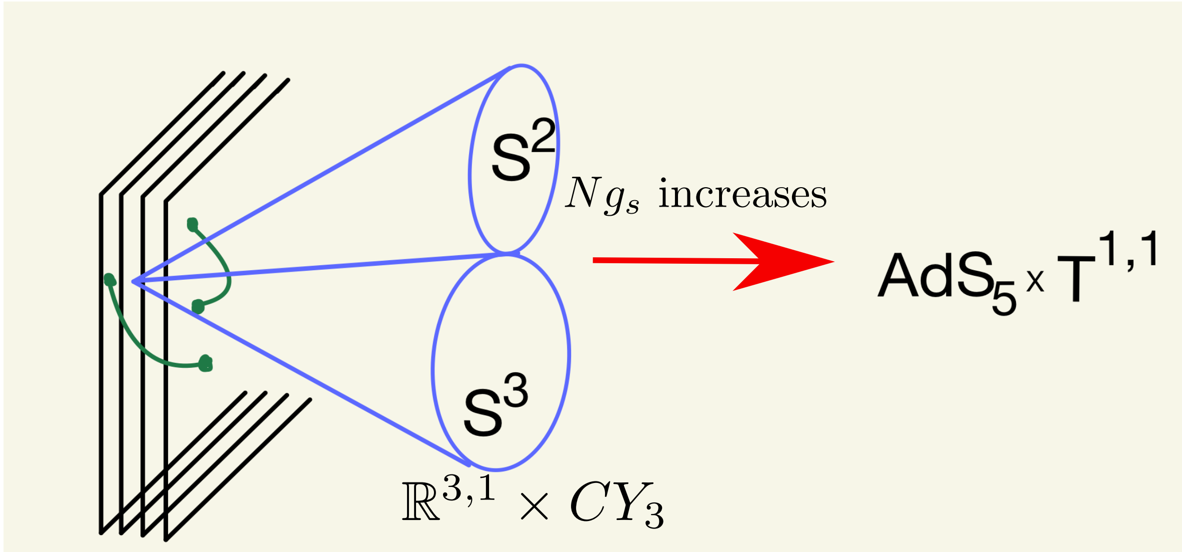


- "conical form" of $\mathbb{R}^6$ metric

$$\mathbb{R}^6 : \qquad ds^2_{\mathbb{R}^6} = dr^2 + r^2 \, \left(dS^5\right)^2$$

$\Longrightarrow$ r direction combines with D3 branes world-volume directions to make up AdS_5

- the symmetry of S^5 is the $SO(6)$ R-symmetry of the $\mathcal{N} = 4$ SYM



- non-compact CY_3 — a singular conifold:
  $\sum_{n=1}^{4} z_n^2 = 0\,,$ where $\{z_n\} \in \mathbb{C}^4$
- Ricci-flat metric on CY_3:

$$ds^2_{CY_3} = dr^2 + r^2\ \left(dT^{1,1}\right)^2$$

- the symmetry of $T^{1,1}$ is

$$\underbrace{SU(2)\times SU(2)}_{\text{flavor}} \qquad \times \qquad \underbrace{U(1)}_{R-\text{symmetry of } KW \text{ SCFT: } \psi\to\psi+\delta}$$

$\Longrightarrow$ The final step: *deform* the $\mathcal{N}=1$ KW SCFT as:

$$SU(N)\times SU(N) \qquad \rightarrow \qquad \underbrace{SU(N+M)}_{\text{shift a rank}} \times SU(N)$$

- A_k and B_ℓ, $k,\ell=1,2$, in $(N+M,\overline{N})$ and $(\overline{N+N},N)$

- when $M\neq 0$, the theory is no longer conformal:

$$\frac{d}{d\ln(\hat{\mu}/\Lambda)}\,\frac{8\pi^2}{g_1^2} = 3(N+M)-2N(1-\gamma) = \boxed{3M} \times \left(1+\mathcal{O}\left(\frac{M}{N}\right)\right)$$

$$\frac{d}{d\ln(\hat{\mu}/\Lambda)}\,\frac{8\pi^2}{g_2^2} = 3N-2(N+M)(1-\gamma) = \boxed{-3M} \times \left(1+\mathcal{O}\left(\frac{M}{N}\right)\right)$$

where g_1 is $SU(N+M)$ coupling and g_2 is $SU(N)$ coupling

- γ is an anomalous dimension:

$$\dim \mathrm{Tr}\, A_i B_j = \underbrace{2}_{classical\ dim} + \gamma\,, \qquad \gamma = -\frac{1}{2}+\mathcal{O}\left(\frac{M}{N}\right)$$

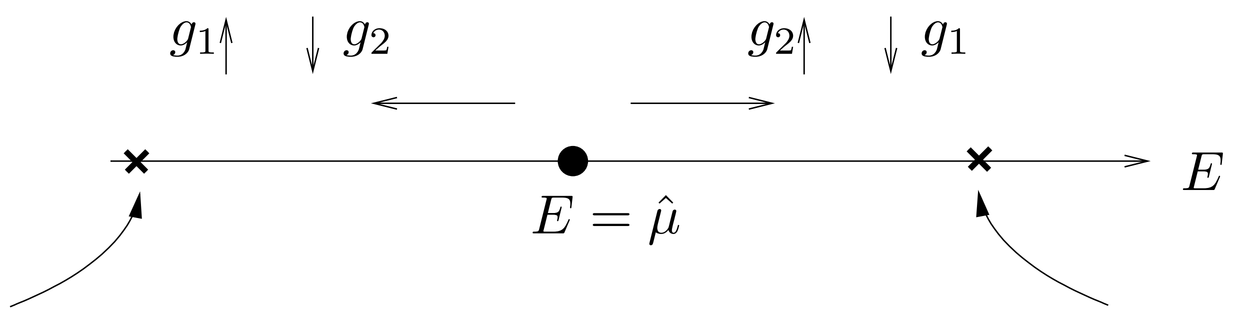
- From 1-loop β-functions:

$$\frac{8\pi^2}{g_1^2} - \frac{8\pi^2}{g_2^2} = 6M \ln \frac{\hat{\mu}}{\Lambda} \;\times\; \Big(1 + \mathcal{O}(M/N)\Big)$$

$$\frac{8\pi^2}{g_1^2} + \frac{8\pi^2}{g_2^2} = \text{const}$$

where Λ is the strong coupling scale of the theory

$\Longrightarrow$



$$\underbrace{\frac{1}{g_1^2} = 0\,,\ SU(N+M) \text{ is strongly coupled}}_{\hat{\mu}=\hat{\mu}_{IR}=\Lambda e^{-\frac{\pi}{3Mg_s}}} \qquad\qquad \underbrace{\frac{1}{g_2^2} = 0\,,\ SU(N) \text{ is strongly coupled}}_{\hat{\mu}=\hat{\mu}_{UV}=\Lambda e^{\frac{\pi}{3Mg_s}}}$$

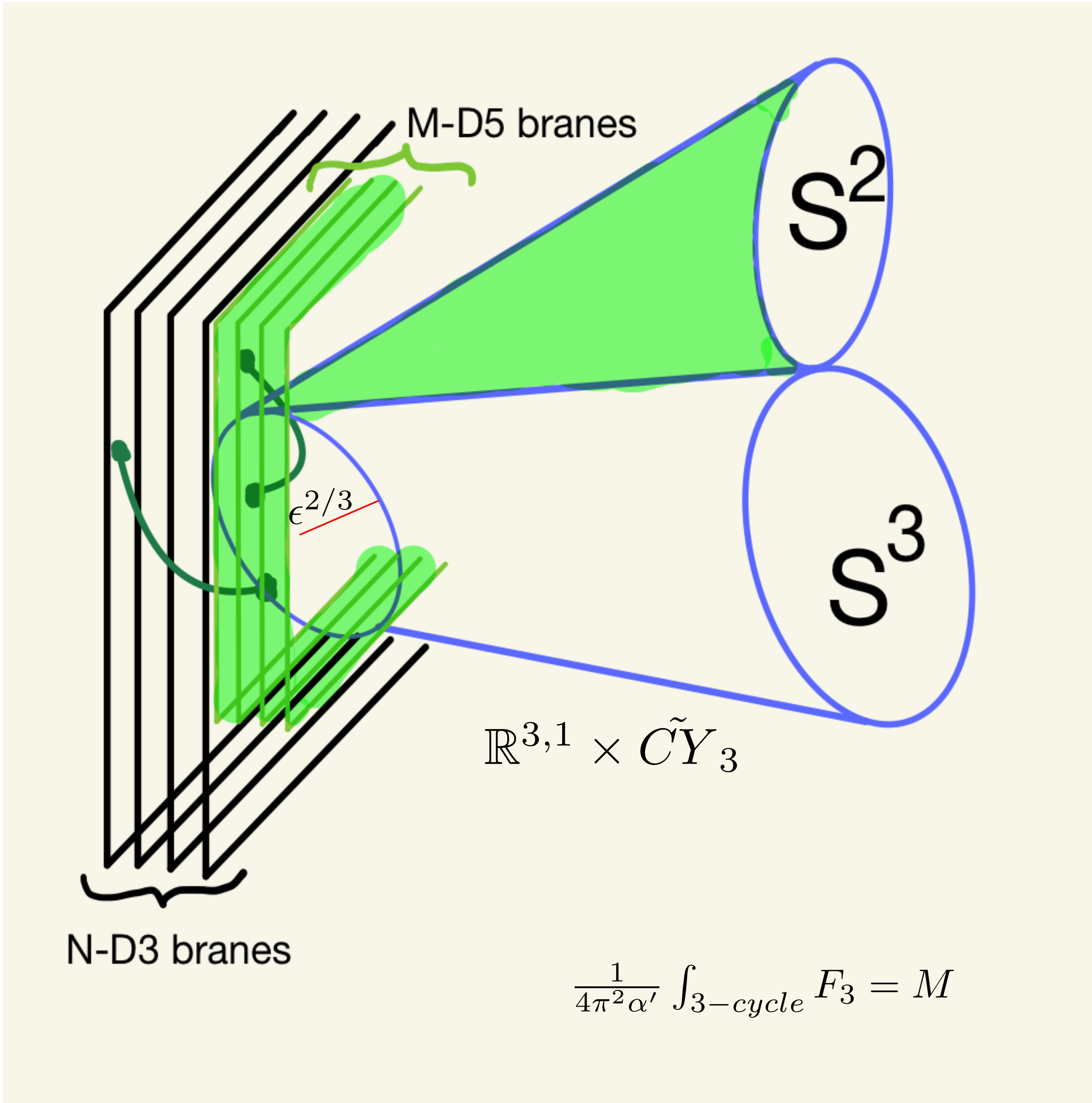
- For RG flow past $\hat{\mu}_{UV}$ or $\hat{\mu}_{IR}$, we must perform $\mathcal{N}=1$ gauge theory Seiberg duality (Klebanov-Strassler):
  - $N \to N+M$ for $\hat{\mu} > \hat{\mu}_{UV}$
  - $N \to N-M$ for $\hat{\mu} < \hat{\mu}_{IR}$

- Effectively, we get a *cascading* gauge theory with rank N running with the energy scale

$$N = N(\hat{\mu}) \;\propto\; +M^2 \;\ln\frac{\hat{\mu}}{\Lambda}\,.$$

- BUT: $N(\hat{\mu}) > 0$, so the cascade must stop in the IR:
  - for $N \propto M$, the end point is $\mathcal{N}=1$ $SU(M)$ YM theory
  - confines
  - has a spontaneous breaking of the $U(1)_R$ chiral symmetry:

$$U(1)_R \to \mathbb{Z}_2$$



- ϵ is a conifold c.s. deformation: $\sum_{n=1}^{4} z_n^2 = \epsilon^2$

- KW $SU(N) \times SU(N)$ SCFT parameters:

$$\{N\,,\ g_s\} \qquad \text{with} \qquad \frac{1}{(4\pi^2\alpha')^2}\int_{T^{1,1}} \mathcal{F}_5 = N$$

- KS $SU(N+M) \times SU(N)$ cascading gauge theory parameters:

$$\{M\,,\ g_s\,, \epsilon\} \qquad \text{with} \qquad \frac{1}{4\pi^2\alpha'}\int_{3-cycle} F_3 = M$$

and

$$\Lambda = \frac{2^{5/12} e^{1/3}}{3^{3/2}}\ \frac{\epsilon^{2/3}}{M g_s^{1/2}\alpha'}$$

- Note in KS theory $N = N(\hat{\mu})$ is an effective parameter, determined by $\{M\,,\ g_s\,, \epsilon\}$

$\Longrightarrow$ The KS Type IIB SUGRA dual to cascading gauge theory:

- fully determined by $\{M\ ,\ g_s\ ,\epsilon\}$
- the metric

$$ds_{10}^2 = H_{KS}^{-1/2}\left(-dt^2 + dx_i^2\right) + \underbrace{\Omega_1^2\left(d\tau^2 + g_5^2\right) + \Omega_2^2\left(g_3^2 + g_4^2\right) + \Omega_3^2\left(g_1^2 + g_2^2\right)}_{\text{warped deformed conifold}}$$

- g_i are 1-forms on $T^{1,1}$; τ is holographic (radial) direction
- Ω_i warping and deformation factors of the conifold
- $\{F_3, H_3, F_5\} \neq 0$, the dilaton is constant, due to

$$\frac{8\pi^2}{g_1^2} + \frac{8\pi^2}{g_2^2} \equiv \frac{2\pi}{g_s} = \text{const}$$

- KS solution is known analytically

$\Longrightarrow$ Our goal is to study cascading gauge theory

- on arbitrary $\mathcal{M}_4$, not just $\mathbb{R}^{3,1}$ as in KS solution
- at finite temperature T
- dynamical/non-equilibrium processes in KS plasma
- KS as a model of confinement/deconfinement and χSB in de Sitter

$\Longrightarrow$ Need effective action WDCF

$\Longrightarrow$ We can derive and effective 5d action by consistently truncating 10d Type IIB SUGRA on deformed $\tilde{T}^{1,1}$:

$$S_5\left[g_{\mu\nu}, \Omega_{i=1\cdots 3}, \Phi, h_{i=1\cdots 3}\,,\ \{M, \epsilon, g_s\}\right]$$

- $g_{\mu\nu}$ — 5d metric, asymptote at the boundary to an arbitrary metric on $\mathcal{M}_4$
- $\Omega_{i=1\cdots 3}$ — 5d scalars reconstructing upon 10d uplift warped-deformed conifold
- Φ — the nontrivial dilaton, asymptoting to the string coupling $\ln g_s$
- $h_{i=1\cdots 3}$ — 5d scalars reconstructing the fluxes on the conifold

$\Longrightarrow$ Gauge/gravity correspondence dictionary for the conifold is well-understood:

- the theory is holographically renormalized on arbitrary $\mathcal{M}_4$ (stationary or time dependent)

$$ds^2_{\mathcal{M}_4} = G_{ij}(y)\ dy^i dy^j$$

-

$$G_{ij} \qquad \Longleftrightarrow \qquad \underbrace{T_{ij}}_{\rm stress-energy\ tensor}$$

- the scalars dual to operators $\mathcal{O}_\Delta$ of the cascading theory:

$$\underbrace{\{\Omega_i, h_i, \Phi\}}_{3+3+1=7\ \rm in\ total} \qquad \Longleftrightarrow \quad \underbrace{\{\mathcal{O}_4^{\beta=\{1,2\}}, \mathcal{O}_6, \mathcal{O}_8\}}_{4\ \rm chirally\ symmetric}\ \&\ \underbrace{\{\mathcal{O}_3^{\alpha=1,2}, \mathcal{O}_7\}}_{3\ \chi\rm SB}$$

- In phases with unbroken chiral symmetry

$$\langle \mathcal{O}_3^\alpha \rangle = 0\,, \qquad \langle \mathcal{O}_7 \rangle = 0$$

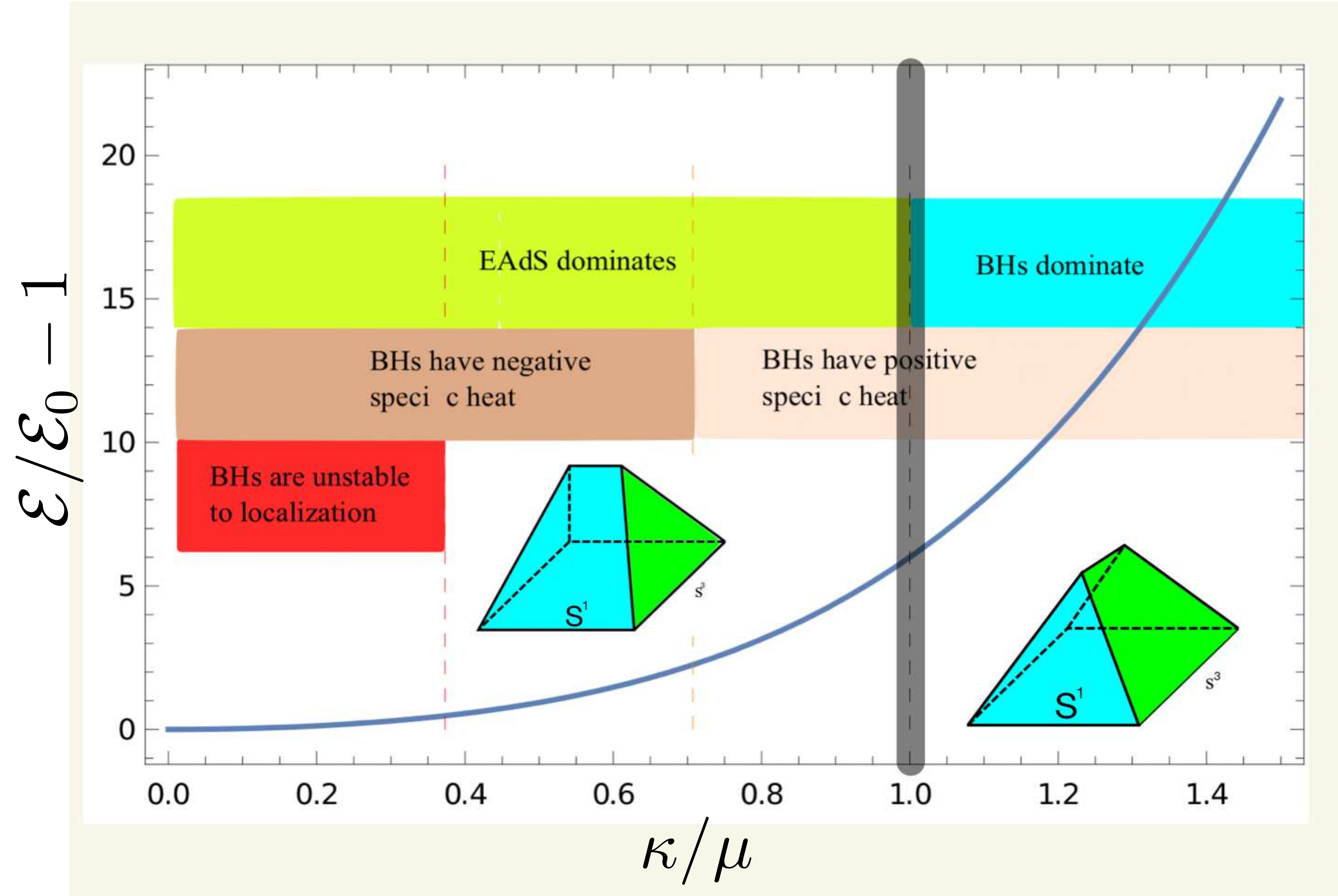
$\Longrightarrow$ $AdS_5 \times T^{1,1}$, *i.e.*, KW model on S^3, can be studied within discussed effective action:

$$M = 0\,, \qquad \epsilon = 0\,, \qquad \mathcal{M}_4 = \mathbb{R} \times S^3$$

- vacuum/BHs can be studied analytically
- $$T_{ij}^{vacuum} = \text{diag}\biggl\{\mathcal{E}_0, \mathcal{P}, \mathcal{P}, \mathcal{P}\biggr\}\,, \qquad \mathcal{E}_0 = 3\mathcal{P}\,, \qquad \mathcal{E}_0 \propto c\ \mu^4 = \frac{c}{L_3^4}$$

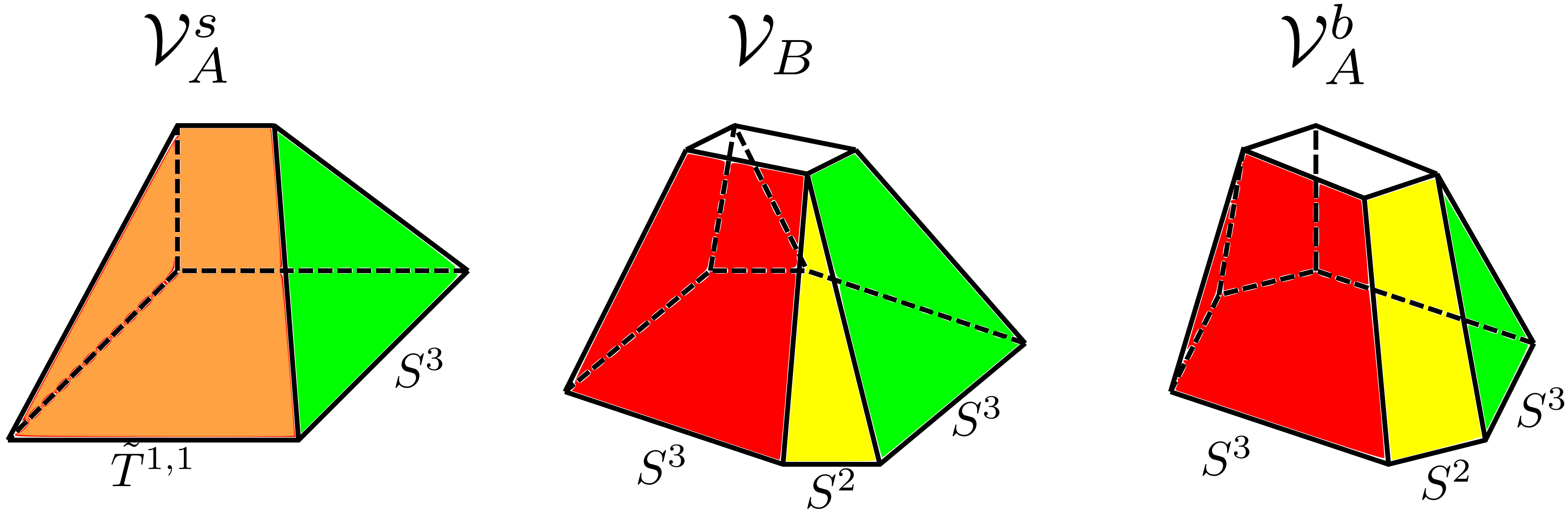
  The precise value of the Casimir energy density $\mathcal{E}_0$ is renorm-scheme ambiguous due to finite counterterms
- In vacuum or at finite T:

$$\langle \mathcal{O}_\Delta \rangle = 0\,, \qquad \Delta = \{3, 4, 6, 7, 8\}$$

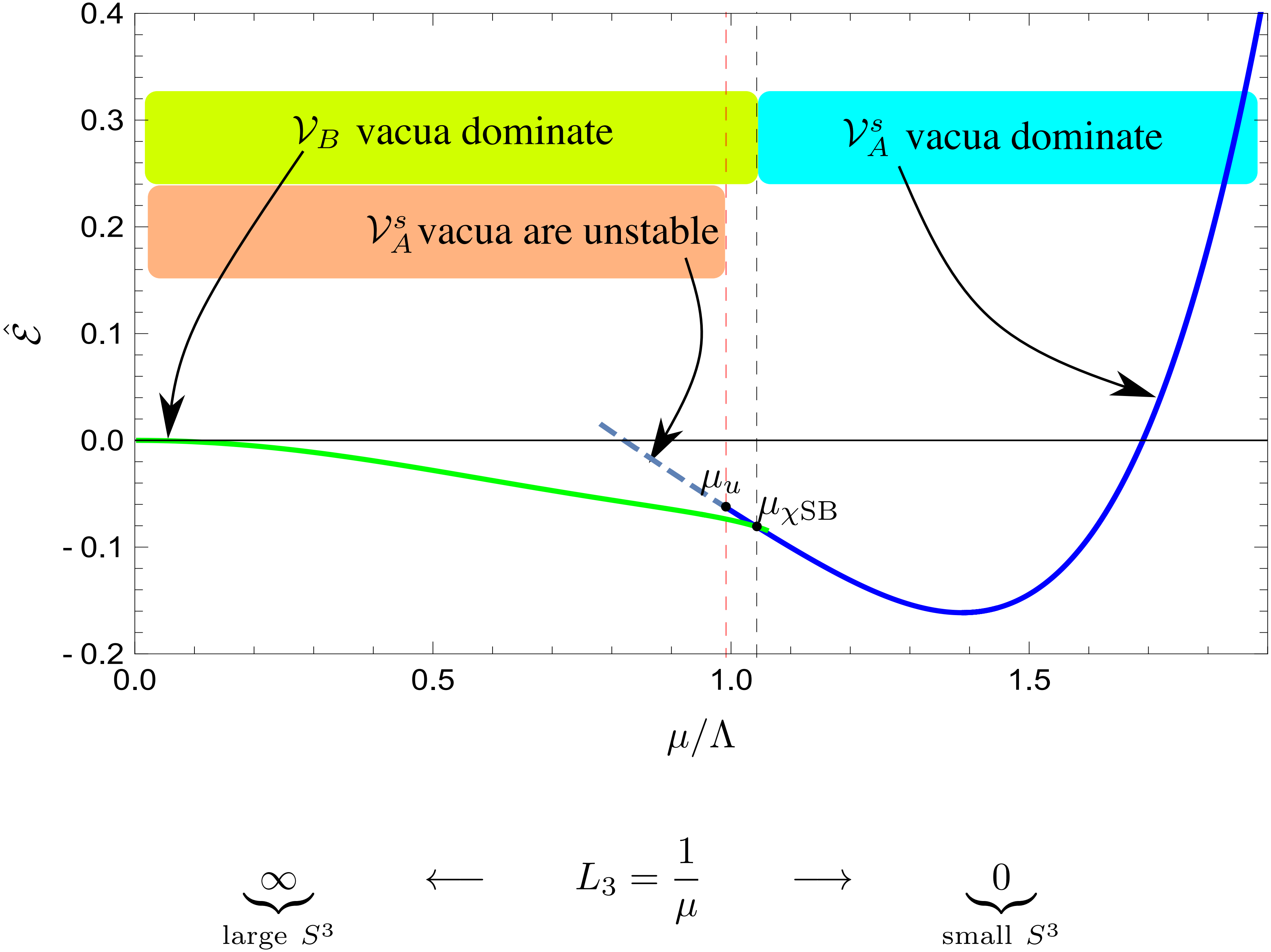


$$T = (\mu^2 + 2\kappa^2)/(2\pi\kappa)$$

$\Longrightarrow$ KS on S^3 vacua:

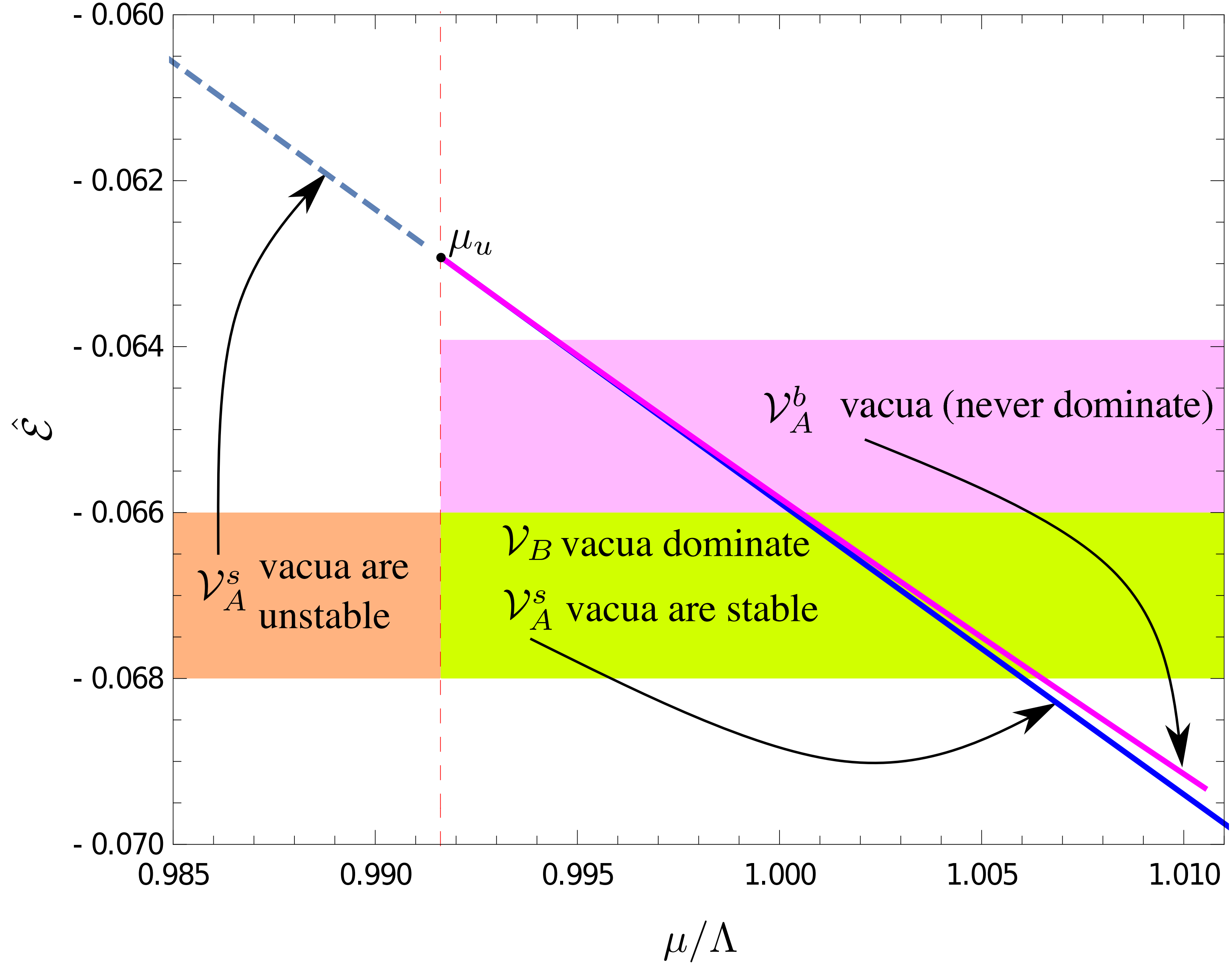


- $\mathcal{V}_A^s$ — chirally symmetric, similar to $\mathcal{N} = 4$ confined
- $\mathcal{V}_A^b$ — χSB, similar to $\mathcal{N} = 4$ confined
- $\mathcal{V}_B$ — χSB, novel, can not exist when $\mu \gg \Lambda$

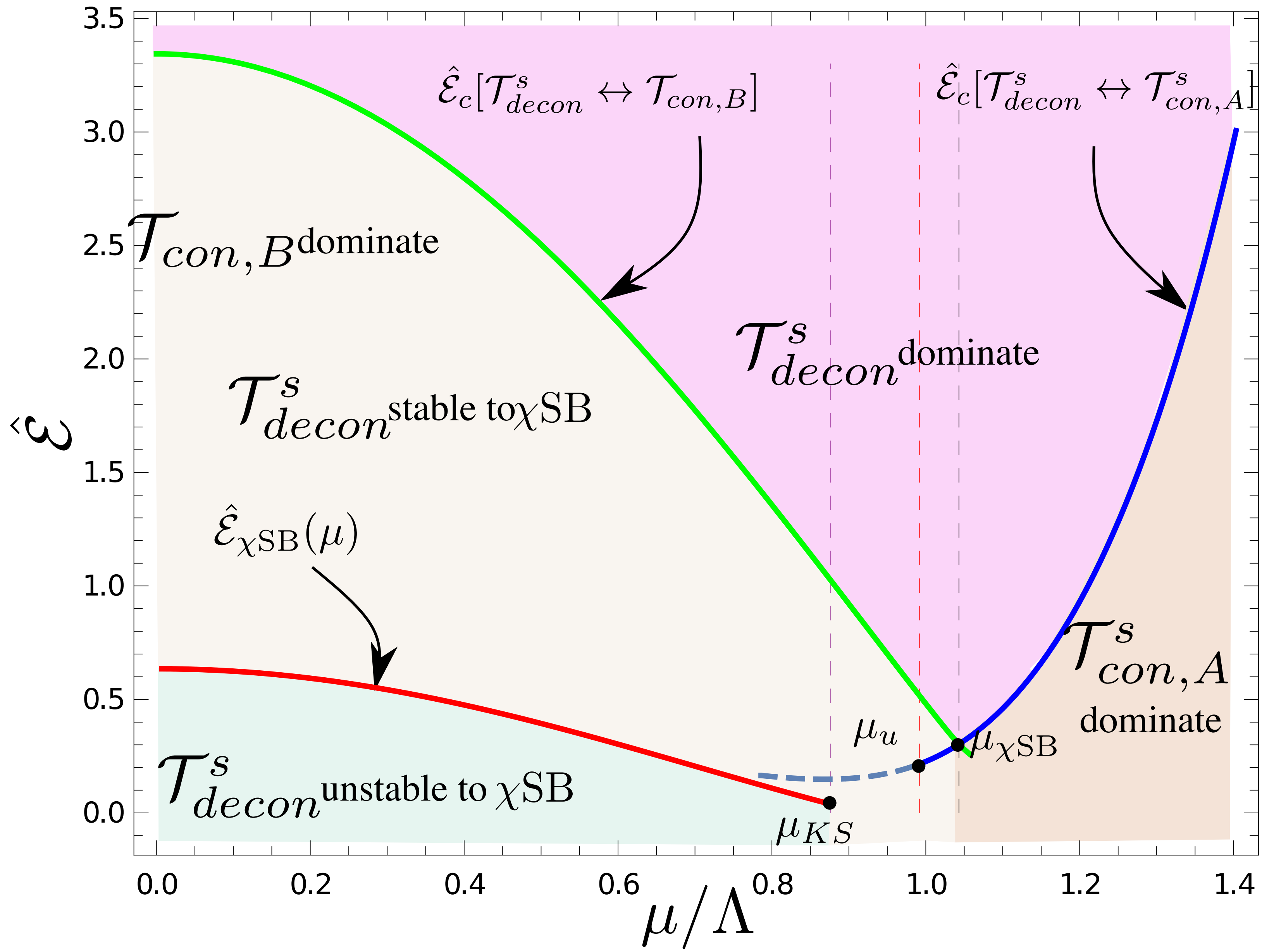


$\mathcal{V}_B$ vacua dominate
$\mathcal{V}_A^s$ vacua dominate
$\mathcal{V}_A^s$ vacua are unstable
$\hat{\mathcal{E}}$
μ_u
$\mu_{\chi\mathrm{SB}}$
0.4
0.3
0.2
0.1
0.0
- 0.1
- 0.2
0.0
0.5
1.0
1.5
μ/Λ
$\underbrace{\infty}_{\text{large } S^3} \longleftarrow L_3 = \frac{1}{\mu} \longrightarrow \underbrace{0}_{\text{small } S^3}$

$\Longrightarrow$ Where are $\mathcal{V}_A^b$ vacua?

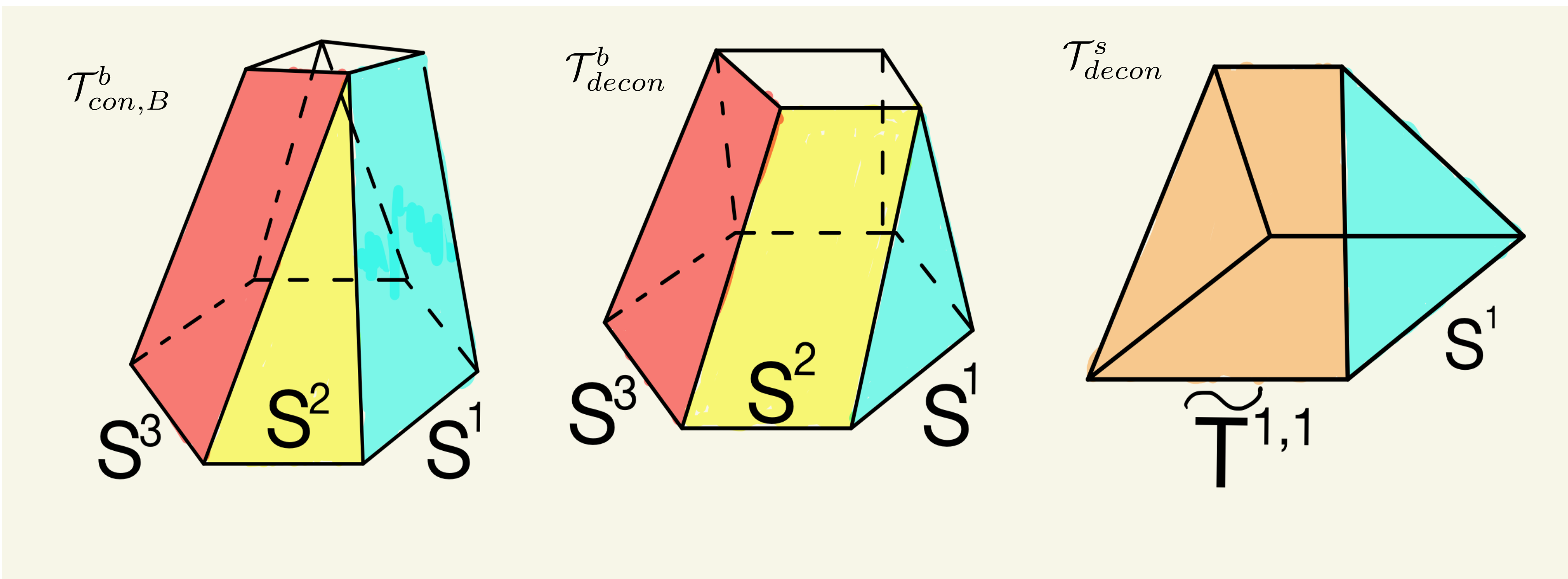


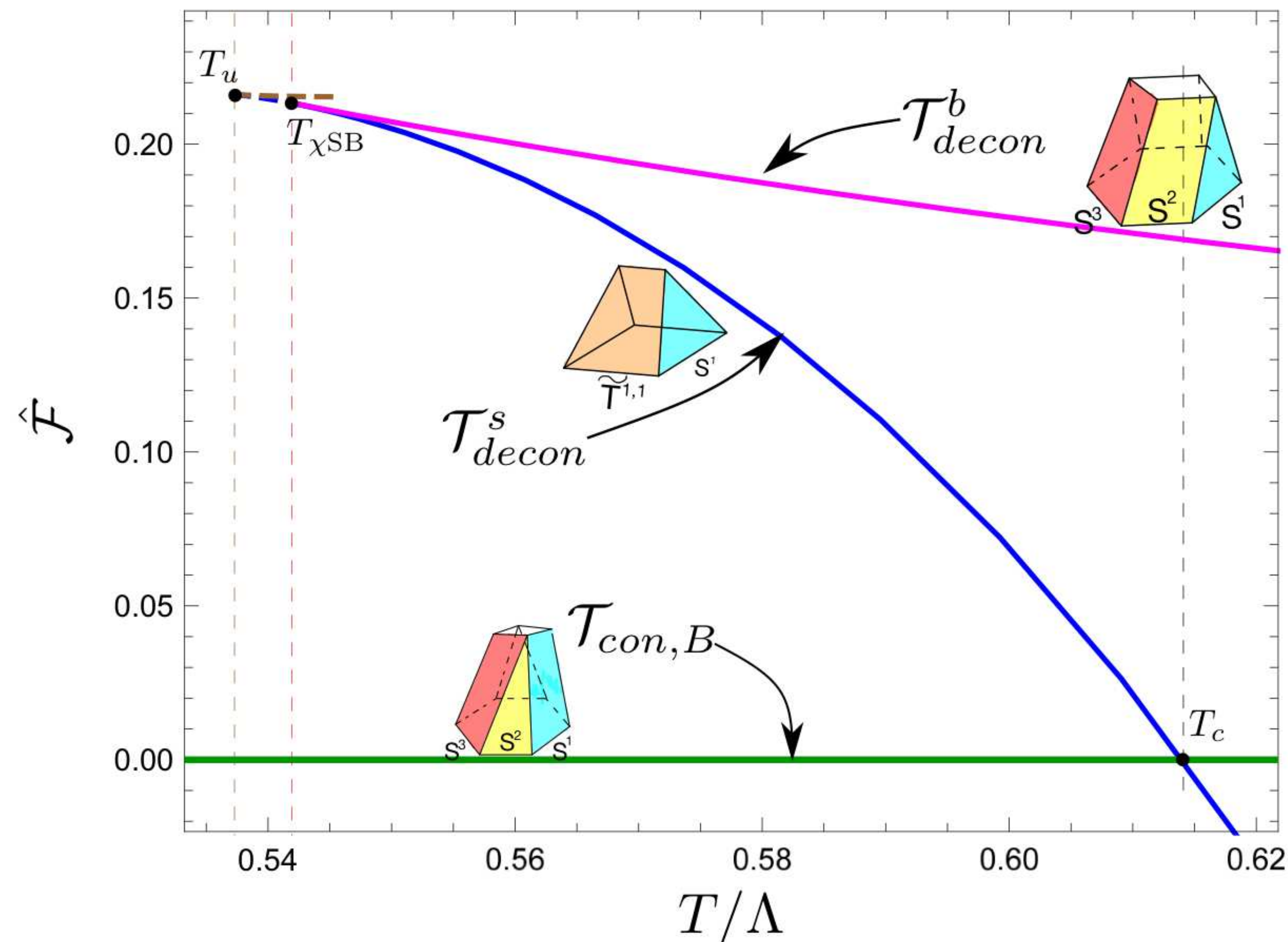
$\Longrightarrow$ KS BHs complicated (5 different phases!); here are critical energies of the various phase transitions:



$\Longrightarrow$ Some cool feature (also present (and is richer!) for black holes):

- To simplify discussion, decompactify S^3, *i.e.*, $L_3 \to \infty$
- We are dealing with black branes on the conifold
- the only relevant scales are T and Λ
- the phases are:





T_u
$T_{\chi\mathrm{SB}}$
$\mathcal{T}^b_{decon}$
$\mathcal{T}^s_{decon}$
$\mathcal{T}_{con,B}$
T_c
S^3
S^2
S^1
$\widetilde{T}^{1,1}$
$\hat{\mathcal{F}}$
T/Λ
0.20
0.15
0.10
0.05
0.00
0.54
0.56
0.58
0.60
0.62

- T_c : confinement/deconfinement (Hawking-Page) first-order phase transition in
$$\underbrace{\mathbb{R}^{3,1}}_{\text{the only example known in the literature}}$$

- $T_{\chi\mathrm{SB}} < T_c$: (perturbative) chiral symmetry breaking transition

- deconfined phase with χSB (the KS black brane) is *exotic*: it exists at high-T, rather than a low-T typical for phases with spontaneously broken symmetries

- This exotic phase does not dominate in a canonical ensemble (changing T), but is more entropic (dominates) in the microcanonical ensemble (changing $\mathcal{E}$)

Conclusions and open questions

- Presented a comprehensive picture of black branes/black holes on the conifold with fluxes, and discussed the plethora of phase transitions (5 BH phases, 3 vacua phases)

- Analysis have implication for string theory landscape/swampland

- Study stability of black holes discussed
  - stability of phases with $SU(2) \times SU(2)$ global symmetry
  - spontaneous breaking of $SU(2) \times SU(2)$ global symmetry — localization of the smeared black branes/holes on the conifold

- Can the exotic phases exist as $T \to \infty$? Conformal order? (scalarization of asymptotically AdS_5 black holes — violation of the no-hair theorem in String Theory)