Probing boundary states at finite coupling

Edoardo Vescovi

1906.07733, 1907.11242, 21xx.xxxxx with Y. Jiang, S. Komatsu

2101.05117, 21xx.xxxxx

ITMP MSU, 19.05.2021

Plan

$\mathcal{N} = 4$ super Yang Mills in 4d

- Feynman-diagram techniques $\longrightarrow$ small coupling
- AdS/CFT correspondence $\longrightarrow$ large coupling
- High symmetry content $\longrightarrow$ finite coupling

Theo lab for technological **transfer to QCD**, accessing **quantum gravity**, **defining QFT** operatively, …

"**Baryon-baryon-meson** coupling"

$$\langle \det(Z)\ \det(\bar{Z})\, \mathcal{O} \rangle \sim \langle B \,|\, \mathcal{O} \rangle_{2d}$$

Probe **non-local** operators?

Wilson loops
('t Hooft, surface, higher-dim operators)

Other models?

Determinants in fishnet theory
(chiral theories, tensor/matrix models, SYK, …)

Probe **heavy sector** further?

$$\langle \mathcal{D}_1\, \mathcal{D}_2\, \mathcal{D}_3\, \mathcal{D}_4 \rangle$$

Probing Wilson loops

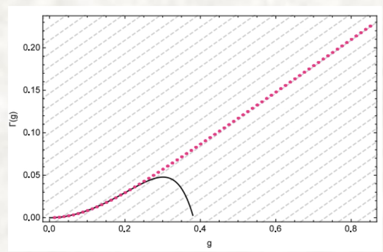
Determinants in SYM

Determinants in fishnet theory

Conclusion

Wilson loops

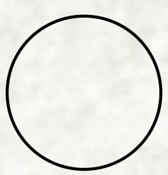
integrability
with operator insertions on $\mathscr{W}$



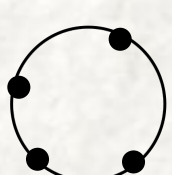
integrable spin chain [Drukker, Kawamoto 06]
TBA [Correa, Maldacena, Sever 12] [Drukker 12]
QSC [Gromov, Levkovich-Maslyuk 15]
3-pt via hexagon [Kim, Kiryu, Komatsu 17-18]

SUSY localization
if $\delta_{\mathrm{SUSY}}\mathscr{W} = 0$

$\langle \mathscr{W} \rangle = \text{exact function in } \lambda, N$



[Erickson, Semenoff, Zarembo 00]
[Drukker, Gross 00] [Pestun 07]
$\sim$ Bremsstrahlung radiation, …

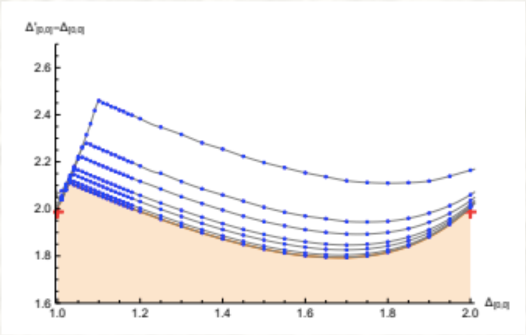


[Giombi, Roiban, Tseytlin 18]
[Giombi, Komatsu 18]

Wilson loops

conformal bootstrap
if $\delta_{\mathrm{SO}(2,1)}\mathscr{W} = 0$

[Liendo, Meneghelli, Mitev 18]

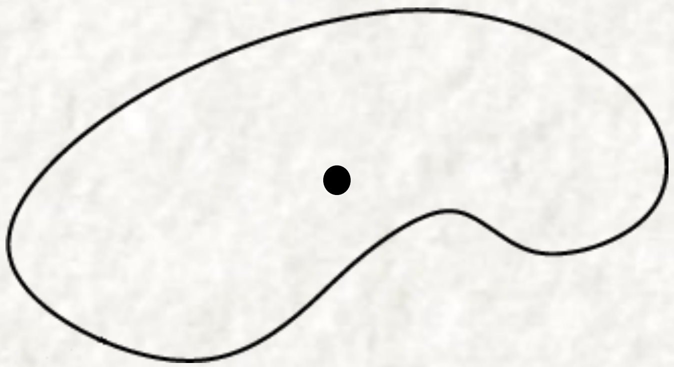


Goal

$\langle \mathscr{W}\,\mathscr{O} \rangle = \,?$

finite λ

via integrable bootstrap



SUSY Wilson loop

$$\mathscr{W} = \mathrm{tr}_{\mathscr{R}} \mathscr{P} \left[e^{\int_0^1 \left(i A_\mu \dot{x}^\mu + |\dot{x}| \theta^I \Phi_I \right) d\tau} \right]$$

$SL(2)$ operator $\leftrightarrow$ spin-chain state

$$\mathscr{O} = \mathrm{tr} \left(D_+^{S_1} Z \ldots D_+^{S_L} Z \right) \quad \leftrightarrow \quad | S_1 \ldots S_L \rangle$$

$$Y^I \Phi_I = \Phi^1 + i \Phi^2$$

$$\hat{D}\,, \quad \Delta(\lambda) \quad \leftrightarrow \quad \text{integrable} \quad \hat{H}, \quad E(\lambda)$$

[Minahan, Zarembo 02]

Wilson loop $\sim$ IR, local probe $\sim$ UV.

Natural object to connect WL with integrability corpus for local operators.

Towards MPS in free theory

Question 1: find evidence that WL “preserves” integrability.

Reformulate $\langle \mathscr{W}\, \mathscr{O} \rangle \sim \langle \text{Matrix Product State} \,|\, \mathscr{O} \rangle$ and check selection rules

- Use **generating function** of $\mathscr{W}$ in rank-r antisymmetric repr.  (r boxes).

$$G\left(a_0\right) = \det\left(\mathbb{I} + e^{ia_0} \mathscr{P}\left[e^{\int_0^1 \left(iA\cdot\dot{x} + |\dot{x}|\,\theta\cdot\Phi \right) d\tau} \right] \right) = 1 + \mathscr{W}_{\square}\, e^{ia_0} + \mathscr{W}_{\boxminus}\, e^{2ia_0} + \dots$$

[Hartnoll, Kumar 06]

- Integrating in 1d fermions $\chi^a(\tau)$ [Gomis, Passerini 06] [Hoyos 18]

$$\langle G\mathscr{O} \rangle = \int \mathscr{D}A_\mu \mathscr{D}\Phi^I \mathscr{D}\chi \mathscr{D}\bar{\chi}\, \mathscr{O}\, e^{-\frac{1}{g^2_{\mathrm{YM}}}\int d^4x\, \mathrm{tr}\left(\partial_\mu\Phi_I\partial_\mu\Phi_I + \partial_\mu A_\nu \partial_\mu A_\nu \right) + i\int_0^1 d\tau \left[\bar{\chi}_a \left(i\partial_\tau + A\cdot\dot{x} - i|\dot{x}|\,\theta\cdot\Phi \right)^a{}_b \chi^b + a_0 \bar{\chi}_a \chi^a \right]}$$

- Integrating out A_μ and Φ^I and integrating in boson $\rho(\tau, \tau')$

Towards MPS in free theory

- Effective theory

$$\boxed{\langle G\mathcal{O}\rangle = \int \mathcal{D}\rho \, \left\langle \mathcal{O}^{(\chi)}[\rho] \right\rangle_{\chi} \, e^{N\, S_{\text{eff}}[\rho]}}$$

$$S_{\text{eff}}\left[\rho\right] = -\frac{1}{\lambda}\int_0^1 d\tau\, d\tau' \rho\left(\tau,\tau'\right)\rho\left(\tau',\tau\right) + \text{Tr}\log\left[\delta\left(\tau-\tau'\right)\left(\partial_{\tau'} - ia_0\right) - i\sqrt{2f\left(\tau,\tau'\right)}\rho\left(\tau,\tau'\right)\right]$$

$$\mathcal{O} \quad \rightarrow \quad \mathcal{O}^{(\chi)} = \mathcal{O}^{(\chi)}[\rho; x, \theta; Y^I, \{S_i\}]$$

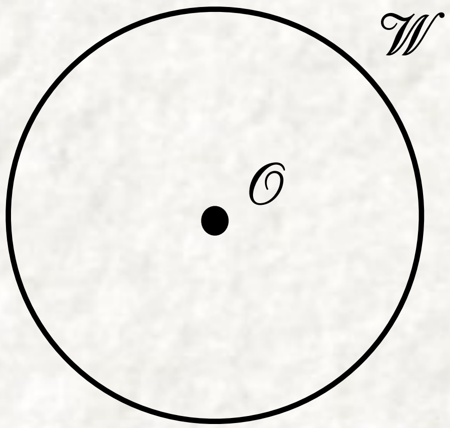
- Saddle-pt approximation

$$\langle G\mathcal{O}\rangle \overset{N\gg 1}{=} \text{Tr}\left(M^{(S_1)}\ldots M^{(S_L)}\right) \qquad \text{w/} \quad \left(M^{(S_i)}\right)_{\tau_i\tau_{i+1}} \sim \text{ function(al) of } x\,,\theta\,,Y\,,S_i$$

$$= \langle \text{MPS}\,|\,\mathcal{O}\rangle = \left[\sum_{L'=0}^{\infty}\sum_{S'_1,\ldots,S'_{L'}=1}^{\infty} \text{Tr}\left(M^{(S'_1)}\ldots M^{(S'_{L'})}\right)\langle S'_1\ldots S'_{L'}|\right]|S_1\ldots S_L\rangle$$

MPS has infinite bond dimension ($\sim$ index τ) due to WL.

Circle in free theory



- BPS operator $\mathrm{tr}(\underbrace{Z \ldots Z}_{L})$

saddle-pt $$\rho_n = \pi\,(2n+1) - a_0 - \sqrt{\left[\pi\,(2n+1) - a_0\right]^2 + \lambda}$$

residue theorem $$\left\langle G\,\mathrm{tr}(Z^L)\right\rangle = \sum_n (\ldots) = \oint \frac{dx\,(1-x^{-2})}{x^L} \tanh\left(\lambda\,(x+x^{-1}) - ia_0\right)$$

project on fund. $$\left\langle \mathscr{W}_{\square}\,\mathrm{tr}(Z^L)\right\rangle = \oint \frac{dx\,(1-x^{-2})}{x^L} e^{\lambda\,(x+x^{-1})} = L\, I_L\left(\sqrt{\lambda}\right)$$ = localization result

[Semenoff, Zarembo 01]
[Giombi, Pestun 09]

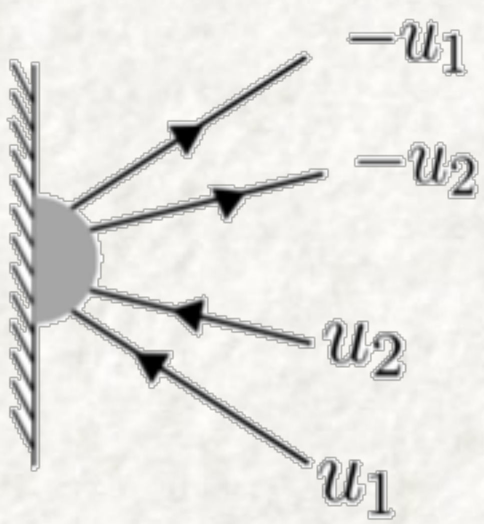
Circle in free theory

- non-BPS operator $\operatorname{tr}\left(D_+^{S_1}Z\ldots D_+^{S_L}Z\right)$

Scan over $\mathcal{O}$ and discover **selection rule**: $\langle G\,\mathcal{O}\rangle \sim \langle \mathrm{MPS}\,|\,u_1,-u_1,u_2,-u_2,\ldots\rangle \neq 0$.

(reminder: Bethe roots u's parametrize $\mathcal{O}$)

Selection rule hints at MPS as **integrable** boundary state [Ghoshal, Zamolodchikov 93] [Pozsgay, Piroli, Vernier 17].



u's $\sim$ momenta

Overlap $\sim$ reflection amplitude **with integrability signature**:

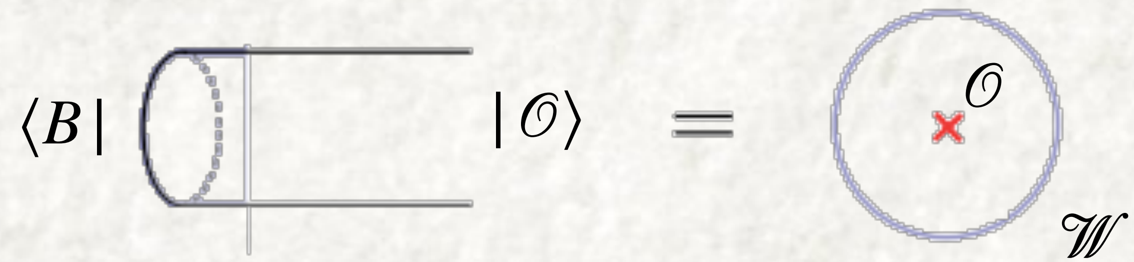
- no particle creation,
- momenta get flipped separately.

Integrable bootstrap

Question 2: uplift result to finite coupling

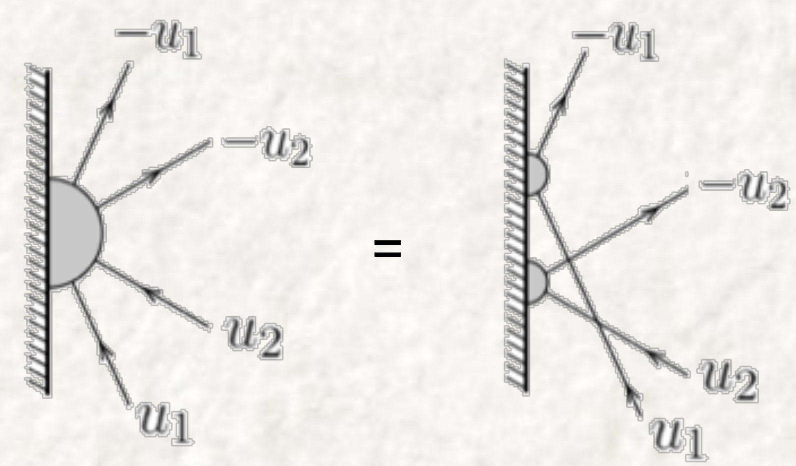
Intuition from AdS/CFT

$$\langle G\,\mathcal{O}\rangle \sim \langle \mathrm{MPS}\,|\,\mathcal{O}\rangle \xrightarrow{\text{finite }\lambda} \langle B\,|\,\mathcal{O}\rangle = ?$$



Strategy:

- ascertain $\langle B\,|$ is integrable boundary (basically done)
- $\langle B\,|\,\mathcal{O}\rangle_{L\gg 1}$ decomposes into $\langle B\,|\,u,-u\rangle$
- $SU(1\,|\,2)^2$ symmetry + integrability axioms
  $\rightarrow$ fix $\langle B\,|\,u,-u\rangle$ at finite λ



- $\langle B\,|\,u,-u\rangle$ is building block for $\langle B\,|\,\mathcal{O}\rangle_{L\gg 1}$
- $\langle B\,|\,u,-u\rangle$ is input for thermodynamic Bethe ansatz for finite L (not today)

Result

$$\langle B \,|\, u, -u\rangle \sim \frac{(u^2+1/4)^2}{(u^2-(a+i/2)^2)(u^2-(a-i/2)^2)} \times \dots$$

poles indicate **bound state** $\langle B' |$
(boundary state - bulk particle) [Ghoshal, Zamolodchikov 93] [Komatsu, Wang 20]

axioms determine also $\langle B' \,|\, u, -u\rangle$

repeat and find infinite tower of bound states

$$\langle G\,\mathcal{O}\rangle \sim \sum_{n\in\mathbb{Z}} \langle B^{(n)} \,|\, u, -u\rangle = \oint \frac{dx\,(1-x^{-2})}{x^L} \tanh\left(\lambda\,(x+x^{-1}) - ia_0\right) \frac{Q(i/2)\,Q(-i/2)}{Q(u+i/2)\,Q(u-i/2)} (\dots) \det G^+$$

residue theorem

$$u = \frac{\sqrt{\lambda}}{4\pi}(x+x^{-1}), \qquad Q(u) = \prod_k (u-u_k)$$

Result

$$\boxed{\begin{gathered}\langle G\,\mathcal{O}\rangle \sim \sum_{n\in\mathbb{Z}} \langle B^{(n)}|u,-u\rangle = \oint \frac{dx\,(1-x^{-2})}{x^L}\tanh\left(\lambda\,(x+x^{-1})-ia_0\right)\frac{Q(i/2)\,Q(-i/2)}{Q(u+i/2)\,Q(u-i/2)}(\dots)\det G^+ \\ u=\frac{\sqrt{\lambda}}{4\pi}(x+x^{-1}), \qquad Q(u)=\prod_k (u-u_k)\end{gathered}}$$

- hybrid of integrability/localization

$\det G^+$ = Gaudin-like det $\qquad$ $Q(u)$ = Baxter Q-functions

- bond dimension of MPS $(\lambda \ll 1)$ = number of bound states $\langle B^{(n)}|$ (finite λ)

$\langle G\,\mathcal{O}\rangle$	$\langle \det Z\ \det \bar{Z}\,\mathcal{O}\rangle$	$\langle (\text{defect})\,\mathcal{O}\rangle$
# = ∞	# = 2	# = k
(here)	[Jiang, Komatsu, EV 19]	[de Leeuw, Kristjansen, Zarembo 15] [Komatsu, Wang 20]

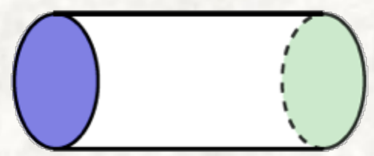
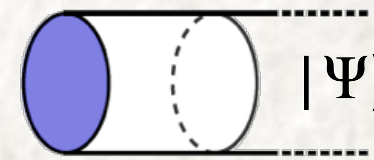
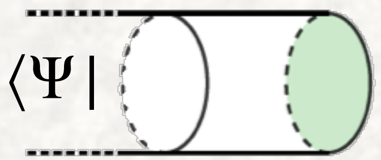
Probing Wilson loops

Determinants in SYM

Determinants in fishnet theory

Conclusion

4-pt determinants

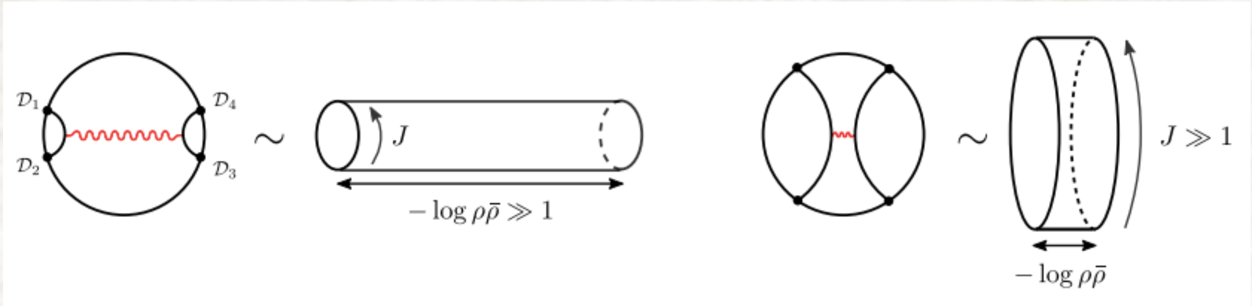
$\langle \mathscr{D}_1 \mathscr{D}_2 \, \mathrm{tr}(\ldots) \rangle = \langle B | \Psi \rangle = \sqrt{\quad}$  $= \sqrt{\langle B | \quad}$  $| \Psi \rangle \times \langle \Psi | \quad$  $| B \rangle$

$\langle \mathscr{D}_1 \mathscr{D}_2 \mathscr{D}_3 \mathscr{D}_4 \rangle_{\text{conn.}} \sim$ 

finite cross-ratios → finite cylinder length → TBA would be hard

Z_{cylinder} should exhibit phase transition at large N

- closed-string ch.: Hagedorn growth of states
- open-string ch.: tachyons in spectrum



cf. torus partition function at $T = T_{\text{Hagedorn}}$ [Atick, Witten 88] [Harmack, Wilhelm 17-18]

Result

Question 1: pert. data + dependence on cross-ratios

- include 1-loop interactions
- integrate out all fields
- ρ = Hermitian matrix
- eases combinatorics of diagrams

$$\langle \mathscr{D}_1 \mathscr{D}_2 \mathscr{D}_3 \mathscr{D}_4 \rangle = \int d\rho \left(1 + \lambda \left\langle \mathscr{O}^{(\chi)}_{1-\text{loop}}[\rho] \right\rangle_\chi + O(\lambda^2) \right) e^{-N S_{\text{eff}}[\rho]}$$

$$= \underbrace{f(u_1, u_2)}_{\text{saddle-pt I}} + \underbrace{f\left(\frac{1}{u_1}, \frac{u_2}{u_1}\right) u_1^N}_{\text{II}} + \underbrace{f(u_2, u_1) \left(\frac{u_1}{u_2}\right)^N}_{\text{III}}$$

$$f(u_1, u_2) = \left[\frac{u_2}{(1-u_1)(u_2-u_1)} - \lambda \frac{\alpha^2 \bar{\alpha}^2 (z-\alpha)(z-\bar{\alpha})(\bar{z}-\alpha)(\bar{z}-\bar{\alpha})}{8\pi^2 (z\bar{z} - \alpha\bar{\alpha})^2} \frac{z\bar{z}(1-z)(1-\bar{z}) F^{(1)}(z,\bar{z})}{\left[z\bar{z}(\alpha+\bar{\alpha}-1) - \alpha\bar{\alpha}(z+\bar{z}-1) \right]^2} + O(\lambda^2) \right] + O\left(\frac{1}{N}\right)$$

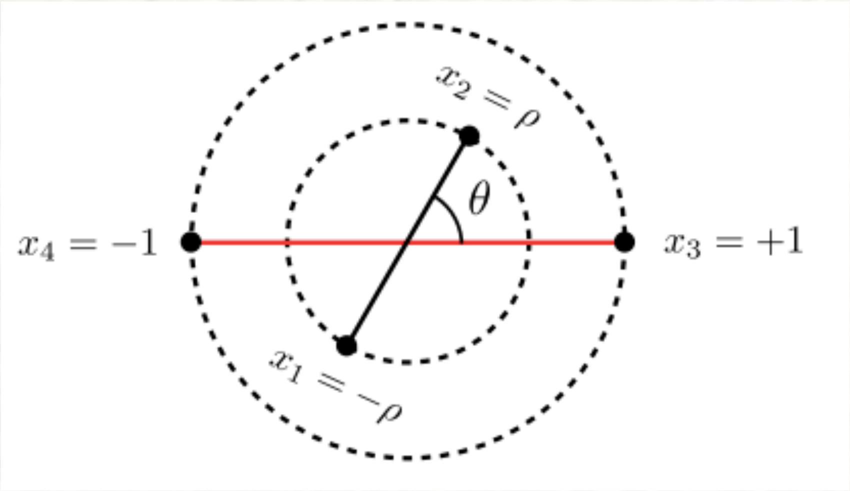
$F^{(1)}$ = box master integral [Usyukina, Davydychev 93]

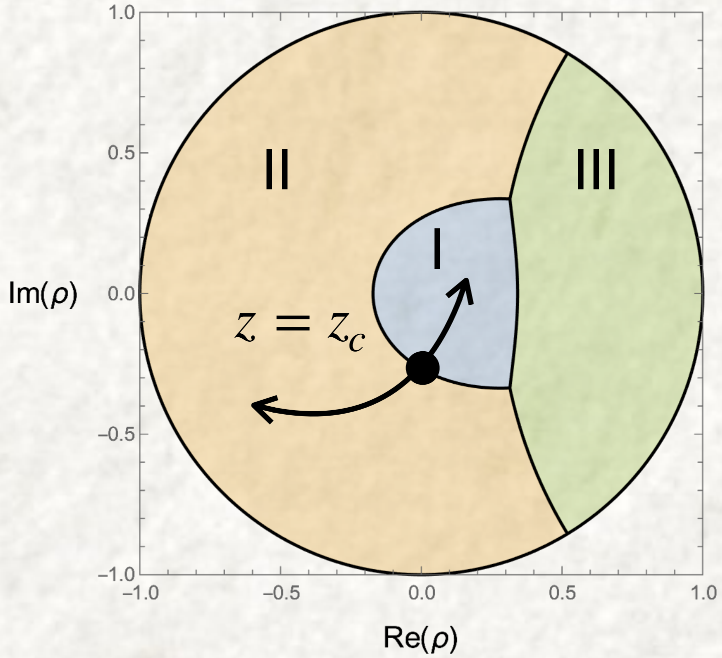
- Dominance of saddle-pts depends on $u_1 = \dfrac{z\bar{z}}{\alpha\bar{\alpha}}$, $u_2 = \dfrac{(1-z)(1-\bar{z})}{(1-\alpha)(1-\bar{\alpha})}$.

- Cannot project dets onto single traces $\longrightarrow$ not derivable from 4-pt of 1/2-BPS operators of weight k
  [Arutyunov, Sokatchev 03]
  [Chicherin, Drummond, Heslop, Sokatchev 15]

Phase transition(s)

Question 2: while TBA is still missing, ET can detect phase transitions.





Possible phase transitions

- $z = z_c$: 2 saddle-pts have equal weight
- $z = z_H(\lambda)$: a fluctuation around saddle-pt becomes massless

(Partial) result

$O(\lambda^0)$: $z_H(\lambda) = z_c$ $\rightarrow$ 2nd order
$O(\lambda^1)$: $z_H(\lambda) \stackrel{?}{=} z_c$ $\rightarrow$ 1st/2nd order?
ρ-integrand and masses depend on regularisation choice.
Alternative formulation of effective theory?

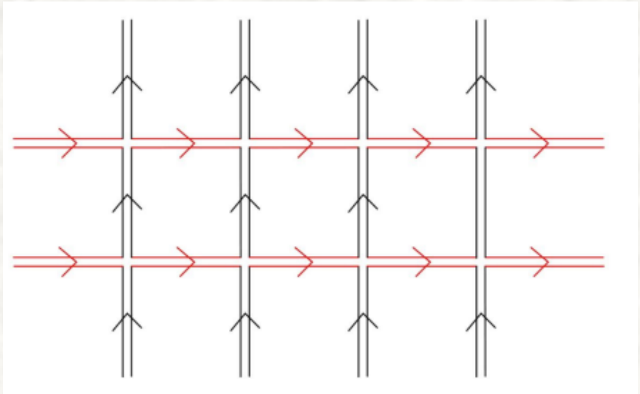
Probing Wilson loops

Determinants in SYM

Determinants in fishnet theory

Conclusion

Goal and motivation



Fishnet theory = double-scaling limit of
γ-deformed SYM [Gurdogan, Kazakov 15]

$$\mathscr{L}_{\text{int}} \sim \xi^2\, \mathrm{tr}(\bar{X}\bar{Z}XZ) + \underbrace{\mathrm{tr}(\ldots)\mathrm{tr}(\ldots)}_{\text{counterterms}}$$

No SUSY nor unitarity
Rare example of conformal, integrable in $d > 2$

Goal:

$$\Big\langle \det(Z)\,\det(\bar{Z})\,\mathrm{tr}(\ldots)\Big\rangle \quad\longrightarrow\quad \int d\rho\ \langle \mathrm{tr}(\ldots)\rangle_\chi\ e^{-NS_{\text{eff}}[\rho]} \quad\longrightarrow\quad \text{exact analytic } \{\Delta_i, C_{ijk}\}$$

- All-loop diagrams, also with length-2 traces from dets.
- Conformal symmetry + graphs' iterative pattern, cf. [Grabner, Gromov, Kazakov, Korchemski 17] [Gromov, Kazakov, Korchemski 18] [Kazakov, Olivucci, Preti 19]

Motivation:
- Exact "baryons-mesons" coupling.
- Next candidates for QSC? Lesson for SYM?
- Strings and D-branes in holographic dual [Basso, Zhong 18] [Gromov, Sever 19]?

2-point functions

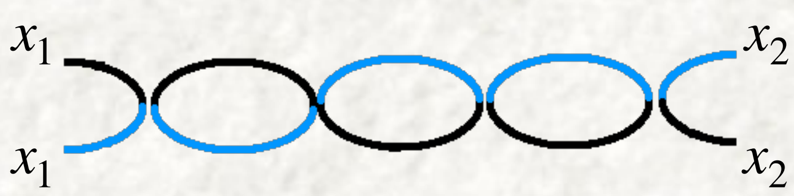
$$\left\langle \det \Phi(x) \ \det \Phi^{\dagger}(x) \right\rangle \sim \frac{1}{x^{2\Delta}}$$

- Protected

$$\Phi = Z \qquad \rightarrow \qquad \Delta = N$$

- Non-protected

$\Phi = X + Z,\ \bar{X} + Z \qquad \rightarrow \qquad \Delta(\xi) \neq 0$ & in progress; $\sim$ exact renormalised mass of baryons in QCD



+ other diagrams

cf. [Grabner, Gromov, Kazakov 17]

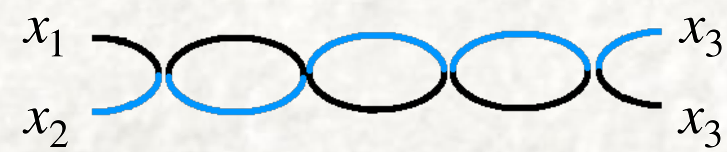
3-point functions

- Protected

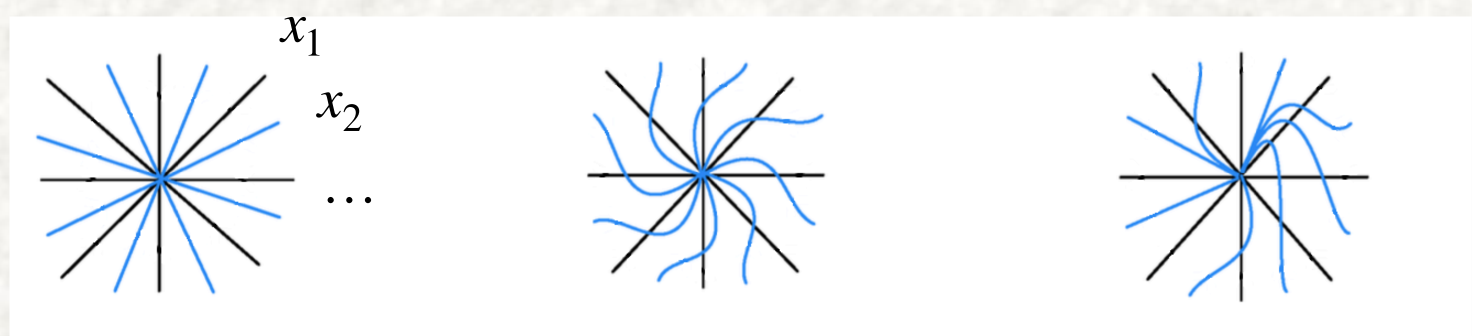
$$\left\langle \det Z(x_1)\ \det \bar{Z}(x_2)\ \mathrm{tr}(Z(x_3)\bar{Z}(x_3))^L \right\rangle$$

- Non-protected

$$\left\langle \det(\bar{X}+Z)(x_1)\ \det \bar{Z}(x_2)\ \mathrm{tr}(XZ)(x_3) \right\rangle$$

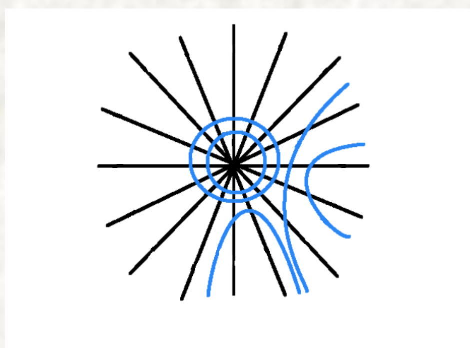


$$\left\langle \det(\bar{X}+Z)(x_1)\ \det \bar{Z}(x_2)\ \mathrm{tr}(\ldots)(x_3) \right\rangle$$

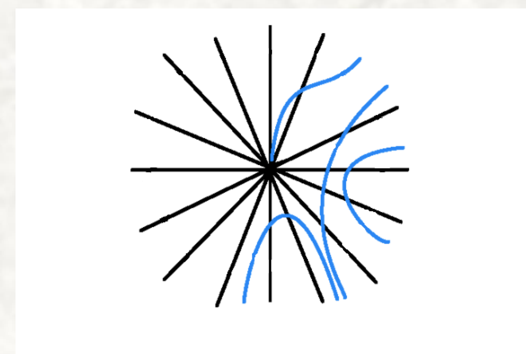


tree level for tr $(XZ)^8$ spiderweb for tr $(XZ)^8$ spiderweb for tr $(Z^4X^3ZX^2ZXZ^2X^2)$

$$\left\langle \det(\bar{X}+Z)(x_1)\ \det(\bar{X}+\bar{Z})(x_2)\ \mathrm{tr}(X^{L-M}Z^M)(x_3) \right\rangle$$



wheels and arcs for tr (X^{16})

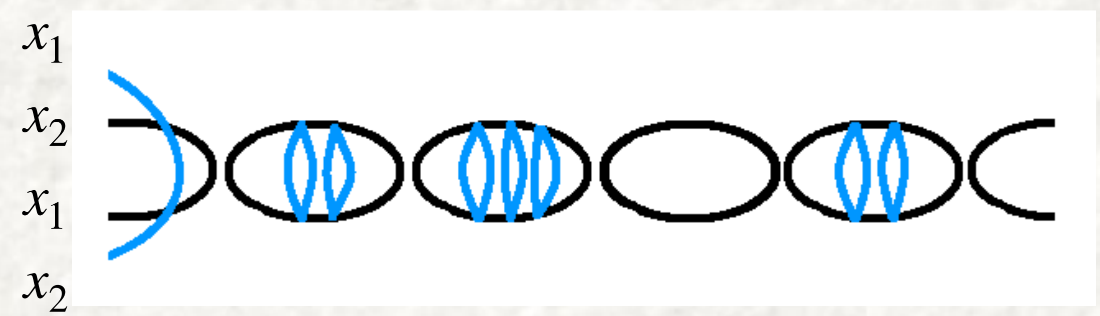


spiderweb and arcs for tr $(X^{17}Z)$

cf. [Caetano, Gurdogan, Kazakov 16]

4-point functions

$$\left\langle \det(\bar{X}+Z)(x_1)\ \det(\bar{X}+\bar{Z})(x_2)\ \mathrm{tr}(X(x_3)X(x_4)) \right\rangle$$



$$\left\langle x_1, x_2 \right| \sum_{n=0}^{\infty} \left(-\alpha_1^2\, \hat{\mathscr{V}} + \xi^4\, \hat{\mathscr{H}}_B\right)^n \hat{\mathscr{H}}_B \left| x_3, x_4 \right\rangle$$

$$\left\langle x_1, x_2 \right|\ \xi^4\, \hat{\mathscr{H}} \sum_{n=0}^{\infty} \left(-\alpha_1^2\, \hat{\mathscr{V}} + \xi^4\, \hat{\mathscr{H}}_B\right)^n \hat{\mathscr{H}}_B \left| x_3, x_4 \right\rangle$$

Graph-building kernels $\hat{\mathscr{V}}$, $\hat{\mathscr{H}}_B$ [Grabner, Gromov, Kazakov 17] and new $\hat{\mathscr{H}}$.

$$\left\langle \det(\bar{X}+Z)(x_1)\ \det(\bar{X}+\bar{Z})(x_2)\ \mathrm{tr}(X(x_3)X(x_4)) \right\rangle \sim \frac{1}{x_{12}^{2N} x_{34}^{2}} \sum_{\Delta,S} C'_{\Delta,S}\ C_{\Delta,S}\ g_{\Delta,S}(z,\bar{z})$$ in progress

Probing Wilson loops

Determinants in SYM

Determinants in fishnet theory

Conclusion

Conclusion

Correlator of (circular) WL and $SL(2)$ operator in $\mathcal{N} = 4$ SYM

$\langle \mathscr{W}\ \mathcal{O} \rangle \sim \langle B \,|\, \mathcal{O} \rangle \quad \rightarrow$ assume *integrable* boundary state

$\rightarrow$ consistency conditions fix $\langle B \,|\, \mathcal{O} \rangle \quad \rightarrow \quad \langle \mathscr{W}\ \mathcal{O} \rangle$ at finite coupling, $L \gg 1$

- Classify WLs $\sim$ integrable boundaries; WLs on S^2 [Drukker, Giombi, Ricci, Trancanelli 07]

- 't Hooft, surface operators, …

- Relation to spectral-parameter deformation of WLs [Ishizeki, Kruczenski, Ziama 11] [Cooke, Dekel, Drukker, Trancanelli, EV 18]

- Classify solutions of axioms $\sim$ WLs

Conclusion

Quantitative remarks and phase transition(s) of 4-pt dets   (2nd order, in free theory)

- ET without regularisation-choice ambiguity (use fermion bilinear in SYK?)

- Lagrangian insertion approach [Chicherin, Drummond, Heslop, Sokatchev 15]

- Kinematical limits

- Classical D-brane configuration

- Integrability in heavy sector

Det correlators in fishnet theory

- ET for heavy operators and WLs in 3d/4d chiral theories [Caetano, Gurdogan, Kazakov 16] [Pittelli, Preti19], tensor/matrix models, SYK, …

- SoV formulation [Cavaglia', Gromov, Levkovich-Maslyuk 21]