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# Losing the trace to discover dynamical Newton or Planck constants

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**(day 399 of the new era)**

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# Losing the trace to find dynamical Newton or Planck constants

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# Message to take home

- There are *local* field theories where the dynamical degrees of freedom are *global* e.g. given by integrals over the whole Cauchy hypersurface.
- These degrees of freedom can mimic free parameters or "fundamental constants".
- $\Lambda$, $G_N$ and even $\hbar$ can be such global degrees of freedom. Now they are subject to quantum fluctuations!
- The origin of the values of these global degrees of freedom should be in quantum cosmology - they are remnants of the BIG BANG
- Ideal "Landscape" for poor people

# What a Strange Theory!

$$S = \frac{1}{8\pi G_N} \int d^4x \sqrt{-g} \left( -\frac{R}{2} + \Lambda \right)$$

$$\Lambda = 1.7 \times 10^{-66}\, \mathrm{eV}^2$$

$$G_N = \frac{1}{M_{pl}^2} = 0.7 \times 10^{-56}\, \mathrm{eV}^{-2}$$

$$G_N \Lambda \sim 10^{-122}$$

# The Enigma of Gravity

Which mechanism is behind $\Lambda$?

Which mechanism is behind $G_N$?

# Calculating vacuum energy we are in trouble...

$$\varepsilon_\Lambda = \frac{1}{V}\sum_k \frac{\hbar\omega_k}{2} + \mathcal{O}\left(\lambda_i\right) = \hbar\int dk\, k^2\sqrt{k^2+m^2} + \mathcal{O}\left(\lambda_i\right)$$

$$T^{vac}_{\mu\nu} = \varepsilon_\Lambda g_{\mu\nu}$$

mass of the particle $m$

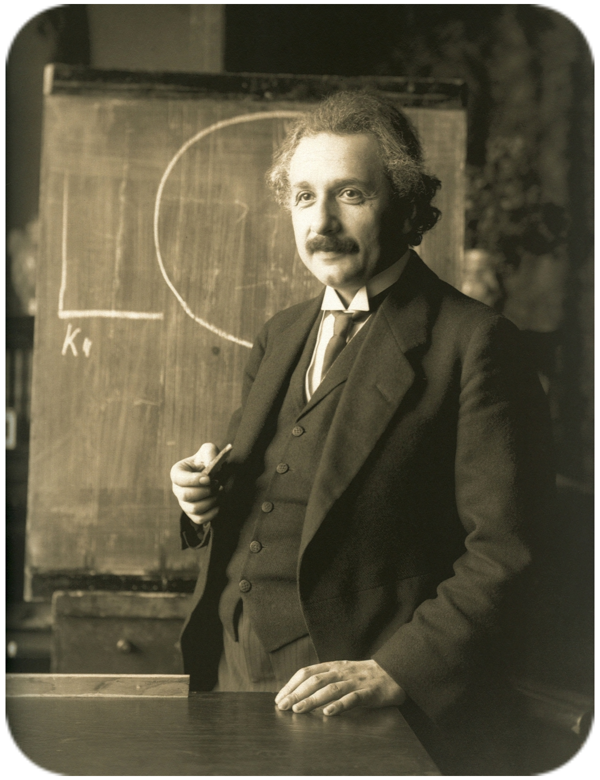
cutoff scale $M$

$$m^4 \log\left(m/M\right) \quad \text{or} \quad M^4 \quad \text{or maybe} \quad \lambda M^4$$

observations $\varepsilon_\Lambda = \frac{\Lambda}{8\pi G_N} \simeq 10^{-29}\,\frac{g}{cm^3}$
$$\begin{cases} \left(10^{-3}\mathrm{eV}\right)^4 \\ \left(10^{-9} m_e\right)^4 \\ \left(10^{-15} E_{\mathrm{LHC}}\right)^4 \end{cases}$$

# Should vacuum really weigh?

# First way to lose the trace



# Traceless Einstein Equations?

$$\sout{G_{\mu\nu} - T_{\mu\nu} = 0}$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

Einstein: Gravitationsfelder im Aufbau der materiellen Elementarteilchen 349

In: *Königlich Preußische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1919): 349–356.

**Spielen Gravitationsfelder im Aufbau der materiellen Elementarteilchen eine wesentliche Rolle?**

Von A. Einstein.

Weder die Newtonsche noch die relativistische Gravitationstheorie hat bisher der Theorie von der Konstitution der Materie einen Fortschritt gebracht. Demgegenüber soll im folgenden gezeigt werden, daß Anhaltspunkte für die Auffassung vorhanden sind, daß die die Bausteine der Atome bildenden elektrischen Elementargebilde durch Gravitationskräfte zusammengehalten werden.

§ 1. Mängel der gegenwärtigen Auffassung.

Die Theoretiker haben sich viel bemüht, eine Theorie zu ersinnen, welche von dem Gleichgewicht der das Elektron konstituierenden Elektrizität Rechenschaft gibt. Insbesondere G. Mie hat dieser Frage tiefgehende Unternehmungen gewidmet. Seine Theorie, welche bei den Fachgenossen vielfach Zustimmung gefunden hat, beruht im wesentlichen darauf, daß außer den Energietermen der Maxwell-Lorentzschen Theorie des elektromagnetischen Feldes von den Komponenten des elektrodyna-

§ 2. Die skalarfreien Feldgleichungen.

Die dargelegten Schwierigkeiten werden dadurch beseitigt, daß man an die Stelle der Feldgleichungen (1) die Feldgleichungen

$$R_{i\kappa} - \frac{1}{4} g_{i\kappa} R = -\varkappa T_{i\kappa} \qquad (1a)$$

setzt, wobei $(T_{i\kappa})$ den durch (3) gegebenen Energietensor des elektromagnetischen Feldes bedeutet.

# Bianchi identity + energy-momentum conservation

$$\nabla_\mu G^{\mu\nu} = 0 \quad + \quad \nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\mu (G - T) = 0$$

$$\Downarrow$$

$$G_{\mu\nu} - T_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$

neue universelle Konstante $\lambda$ eingeführt werden mußte, die zu der Gesamtmasse der Welt (bzw. zu der Gleichgewichtsdichte der Materie) in fester Beziehung steht. Hierin liegt ein besonders schwerwiegender Schönheitsfehler der Theorie.

$\Lambda$ is merely an integration constant!

# Decoupling vacuum energy from spacetime curvature

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

invariant under *vacuum shifts* of energy-momentum tensor

$$T_{\mu\nu} \to T_{\mu\nu} + \Lambda g_{\mu\nu}$$

known under the name "unimodular" gravity

# D'Alembert's principle & Lagrange equation of the first kind

$$\left(m_i \boldsymbol{a}_i - \boldsymbol{F}_i\right) \cdot \delta \boldsymbol{r}^i = 0$$

$$\boldsymbol{C}_i\left(\boldsymbol{r}^k, \boldsymbol{v}^k, t\right) \cdot \delta \boldsymbol{r}^i = 0$$

$$\Rightarrow \quad m_i \boldsymbol{a}_i - \boldsymbol{F}_i = \lambda \boldsymbol{C}_i$$

we can exclude $\lambda$ by multiplying with $\boldsymbol{C}_i$ so that

$$m_i \boldsymbol{a}_i - \boldsymbol{F}_i = \frac{\boldsymbol{C}_k\left(m_k \boldsymbol{a}_k - \boldsymbol{F}_k\right)}{\boldsymbol{C}_n \boldsymbol{C}_n} \cdot \boldsymbol{C}_i$$

---

cf.

$$\left(G_{\mu\nu} - T_{\mu\nu}\right) \cdot \delta g^{\mu\nu} = 0$$

$$g_{\mu\nu} \cdot \delta g^{\mu\nu} = 0$$

$$\Rightarrow \quad G_{\mu\nu} - T_{\mu\nu} = \Lambda g_{\mu\nu}$$

we can exclude $\Lambda$ by multiplying with $g^{\mu\nu}$ !

$$G_{\mu\nu} - T_{\mu\nu} = \Lambda g_{\mu\nu}$$

multiplying with $g^{\mu\nu}$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

However, if we assume that $G_{\mu\nu}$ is not vanishing (along with $G$) we could equivalently multiply with $G^{\mu\nu}$ and write

$$G_{\mu\nu} - T_{\mu\nu} = \frac{\left(G_{\alpha\beta} G^{\alpha\beta} - T_{\alpha\beta} G^{\alpha\beta}\right)}{G} \cdot g_{\mu\nu}$$

Or if we assume that $T_{\mu\nu}$ is not vanishing (along with $T$)

$$G_{\mu\nu} - T_{\mu\nu} = \frac{\left(G_{\alpha\beta} T^{\alpha\beta} - T_{\alpha\beta} T^{\alpha\beta}\right)}{T} \cdot g_{\mu\nu}$$

All these equations are diff. inv./general covariant, as so was the diff. constraint!

# Or we could use det

$$\frac{G_{\mu\nu} - T_{\mu\nu}}{\left(-\mathrm{det}\left(G_{\alpha\beta} - T_{\alpha\beta}\right)\right)^{1/4}} = \frac{g_{\mu\nu}}{\left(-\mathrm{det} g_{\alpha\beta}\right)^{1/4}}$$

Jiroušek, Shimada, Vikman, Yamaguchi (2020)

$$\Updownarrow$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

# Integrating constraint gives a non-covariant "unimodular" condition

$$\sqrt{-g} = f(x)$$

where $f(x)$ is *arbitrary* unspecified / external non-dynamical function which is often taken $f(x) = 1$

Let's make $f(x)$ internal / dynamical function, which would still be irrelevant, but save general covariance!

# Theory of *all* (a)dS

$$S_{dS}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

Henneaux, Teitelboim(Bunster)(1989)

- Gauge degeneracy $W^\mu \to W^\mu + \epsilon^\mu$ where $\nabla_\mu \epsilon^\mu = 0$
- Fake violation of the Lorenz-symmetry, $\langle W^\mu \rangle \neq 0$
- DE / CC energy density as a Lagrange multiplier to make $\sqrt{-g} = \partial_\mu\left(\sqrt{-g}\,W^\mu\right)$
- Similarly to the Ostrogradsky Hamiltonian, the system is *linear* in canonical momentum $\pi$, i.e. in energy density of DE, $\Lambda$

$$\pi = \frac{\delta L}{\delta \dot{W}^t} = \sqrt{-g}\,\Lambda \qquad \text{However,} \qquad \partial_\mu \Lambda = 0$$

- Scale invariance, as there is no fixed scale in the action

# Henneaux–Teitelboim "unimodular" gravity (1989)

$$S_{dS}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

*global* degree of freedom canonically conjugated to the CC

$$\mathcal{T}(t) = \int d^3\mathbf{x}\sqrt{-g}\,W^t(t,\mathbf{x}) \qquad \text{cf.}\quad S = \int dt\,(p\dot{q} - H)$$

gauge invariance $W^\mu \rightarrow W^\mu + \epsilon^\mu$ generates global shift-symmetry $\mathcal{T} \rightarrow \mathcal{T} + c$

$$\dot{\mathcal{T}} = \int_\Sigma d^3\mathbf{x}\,\partial_t\left(\sqrt{-g}\,W^t(t,\mathbf{x})\right) = \int_\Sigma d^3\mathbf{x}\left[\sqrt{-g} - \partial_i\left(\sqrt{-g}\,W^i(t,\mathbf{x})\right)\right]$$

invariant - four volume of space time $\mathcal{T}(t_2) - \mathcal{T}(t_1) = \int_{t_1}^{t_2} dt \int d^3\mathbf{x}\sqrt{-g}$

shift-symmetry in coordinate

conservation of momentum, $\Lambda = conts$

Four-volume of spacetime is
canonically conjugated
to the cosmological constant

Heisenberg uncertainty relation

$$\delta\Lambda \times \delta\int_{\Omega} d^4x\sqrt{-g} \geq 4\pi\ell_{Pl}^2$$

$W^\mu$ is a rather unusual field,
is there something more familiar,
e.g. a gauge potential / connection $A_\mu$
(e.g. for SU(N))?

# Use Chern-Simons Current $C^{\mu}$ instead of $W^{\mu}$ !

# Axionic Cosmological Constant

$$S\left[g, A, \Lambda\right] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \Lambda\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta} - 1\right)\right]$$

Hammer, Jiroušek, Vikman arXiv:2001.03169

# Chern-Simons Current

$$C^{\alpha} = \mathrm{Tr}\frac{\varepsilon^{\alpha\beta\gamma\delta}}{\sqrt{-g}}\left(\boldsymbol{F}_{\beta\gamma}\boldsymbol{A}_{\delta} - \frac{2}{3}ig\boldsymbol{A}_{\beta}\boldsymbol{A}_{\gamma}\boldsymbol{A}_{\delta}\right)$$

composite vector variable, yet $C^{t}$ does not depend on $\partial_{t}A_{\mu}$ !

$$\nabla_{\alpha}C^{\alpha} = F_{\alpha\beta}\widetilde{F}^{\alpha\beta} \qquad \widetilde{F}^{\alpha\beta} = \frac{1}{2}\cdot\frac{\epsilon^{\alpha\beta\mu\nu}}{\sqrt{-h}}\cdot F_{\mu\nu}$$

gauge transformations $\quad \boldsymbol{A}_{\mu} \rightarrow U\boldsymbol{A}_{\mu}U^{-1} + \frac{i}{g}\left(\partial_{\mu}U\right)U^{-1}$

introduce the shifts $\quad C^{\mu} \rightarrow C^{\mu} + \epsilon^{\mu} \qquad \nabla_{\mu}\epsilon^{\mu} = 0$

# Axionic Cosmological Constant, Comment I

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(Received 13 January 1971)

**Cosmological Constant and Fundamental Length**

*In usual formulations of general relativity, the cosmological constant Λ appears as an inelegant ambiguity in the fundamental action principle. With a slight reformulation, Λ appears as an unavoidable Lagrange multiplier, belonging to a constraint. The constraint expresses the existence of a fundamental element of space–time hypervolume at every point. The fundamental scale of length in atomic physics provides such a hypervolume element. In this sense, the presence in relativity of an undetermined cosmological length is a direct consequence of the existence of a fundamental atomic length.*

## Maybe this fundamental scale is a confinement scale of a Yang-Mills theory/QCD?

$$S\left[g,A,\Lambda\right]=\int d^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right)+\Lambda\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}-1\right)\right]$$

Hammer, Jiroušek, Vikman arXiv:2001.03169

# Axionic Cosmological Constant, Comment II

$$S\left[g,A,\Lambda\right]=\int d^{4}x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right)+\Lambda\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}-1\right)\right]$$

Hammer, Jiroušek, Vikman arXiv:2001.03169

PHYSICS REPORTS (Review Section of Physics Letters) 104, Nos. 2–4 (1984) 143–157. North-Holland, Amsterdam

**Foundations and Working Pictures in Microphysical Cosmology**

Frank WILCZEK

I would like to briefly mention one idea in this regard, that I am now exploring. It is to do something for the $\Lambda$-parameter very similar to what the axion does for the $\theta$-parameter in QCD, another otherwise mysteriously tiny quantity. The basic idea is to promote these parameters to dynamical variables, and then see if perhaps small values will be chosen dynamically. In the case of the

# Almost Normal Axion for $\Lambda$

Canonically normalised $\theta$ instead of $\Lambda$

$$S\left[g, A, \theta\right] = \int d^4x\sqrt{-g}\left(-\frac{R}{2\varkappa} + \frac{1}{2}\left(\partial\theta\right)^2 + \frac{\theta}{f_\Lambda}F_{\alpha\beta}F^{\star\alpha\beta} - V_\lambda\left(\theta\right) \quad - \frac{1}{4\mathbf{g}^2}F_{\alpha\beta}F^{\alpha\beta}\right)$$

formal limit / "confinement" $\mathbf{g} \to \infty$

$$\varkappa = 8\pi G_N$$

New vacuum energy density $V_\lambda\left(\theta\right)$

# Another way of thoughts
# Comment III

## Cleaning up the cosmological constant

2012

**Ian Kimpton and Antonio Padilla**

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We now observe that the vacuum energy coming from particle physics enters the action via a term of the form $-2\Lambda \int d^4x\sqrt{-\tilde{g}}$. This has no effect on the dynamics provided

$$\frac{\delta}{\delta\phi_i}\int d^4x\sqrt{-\tilde{g}} = 0. \tag{2.1}$$

This is only possible when $\tilde{g}_{ab}$ is a composite field, for which $\sqrt{-\tilde{g}}$ is the integrand of a topological invariant, and/or a total derivative. Note that our method is distinct from unimodular gravity in which the metric determinant is *constrained* to be unity [13].

# Second way to lose the trace!

# 1=1 instead of 0=0!

$$\frac{G_{\mu\nu}}{G} = \frac{T_{\mu\nu}}{T}$$

Jiroušek, Shimada, Vikman, Yamaguchi (2020)

Bianchi identity
+energy-momentum conservation
+non-degeneracy of $T_{\mu\nu}$ (say it contains a small $\Lambda$)

$$\partial_\mu \log G/T = 0 \quad \Rightarrow \quad T = M_{pl}^2 G$$

$M_{pl}$ is merely an integration constant!

# D'Alembert's Principle & Constrained Variations

$$\left(G_{\mu\nu} - T_{\mu\nu}\right) \cdot \delta g^{\mu\nu} = 0$$
$$T_{\mu\nu} \cdot \delta g^{\mu\nu} = 0$$

$$\Rightarrow \quad G_{\mu\nu} - T_{\mu\nu} = \lambda(x)\, T_{\mu\nu}$$

we can exclude $\lambda$ by multiplying with $g^{\mu\nu}$ !

$$\Downarrow$$

$$\frac{G_{\mu\nu}}{G} = \frac{T_{\mu\nu}}{T}$$

$G_{\mu\nu} \cdot \delta g^{\mu\nu} = 0$ does the same job!

unitrace

$$\frac{G_{\mu\nu}}{G} = \frac{T_{\mu\nu}}{T}$$

unideterminant

$$\frac{G_{\mu\nu}}{\left(\det G_{\mu\nu}\right)^{1/4}} = \frac{T_{\mu\nu}}{\left(\det T_{\mu\nu}\right)^{1/4}}$$

# Henneaux–Teitelboim analogy for $G_N$

$$S_{G_N}\left[g, C, \alpha\right] = \frac{1}{2}\int d^4x\sqrt{-g}\,\Big(\nabla_\mu C^\mu - R\Big)\,\alpha$$

cf. Kaloper, Padilla, Stefanyszyn, Zahariade (2016)

*global* shift-symmetric degree of freedom

canonically conjugated to $\alpha = M_{pl}^2$

$$\mathscr{R}\,(t) = \frac{1}{2}\int_\Sigma d^3\mathbf{x}\sqrt{-g}\;C^t\,(t, \mathbf{x})$$

$$\dot{\mathscr{R}} = \frac{1}{2}\int_\Sigma d^3\mathbf{x}\partial_t\left(\sqrt{-g}\;C^t\,(t, \mathbf{x})\right) = \frac{1}{2}\int_\Sigma d^3\mathbf{x}\left[\sqrt{-g}R - \partial_i\left(\sqrt{-g}\;C^i\,(t, \mathbf{x})\right)\right]$$

$$\mathscr{R}\,\left(t_2\right) - \mathscr{R}\,\left(t_1\right) = \frac{1}{2}\int_{\mathscr{V}} d^4x\sqrt{-g}\;R$$

integrated Ricci scalar

# Einstein Transverse Way

## Heisenberg uncertainty relation

$$\frac{\delta \ell_{Pl}}{\ell_{Pl}} \times \frac{\delta \int_{\mathscr{V}} d^4x \sqrt{-g}\, R}{\ell_{Pl}^2} \geq 4\pi$$

# Another Almost Normal Axion for $G_N$

Canonically normalised $\nu$ instead of $\alpha$

$$S\left[g,\mathscr{A},\nu\right]=\int d^4x\sqrt{-g}\left[-\frac{1}{2}\nu^2R+\frac{1}{2}\left(\partial\nu\right)^2+\frac{\nu}{f_\alpha}\mathscr{F}_{\gamma\sigma}\mathscr{F}^{\star\gamma\sigma}-V_\alpha\left(\nu\right)\qquad-\frac{1}{4\mathbf{g}^2}\mathscr{F}_{\gamma\sigma}\mathscr{F}^{\gamma\sigma}\right]$$

Again formal limit / “confinement” $\mathbf{g}\to\infty$

# Matter Transverse Way

$$S\left[g,\beta,L,\Phi_m\right]=\int d^4x\sqrt{-g}\,\beta\left(\mathscr{L}_m-\nabla_\lambda L^\lambda\right)$$

Momentum rescaling

$$\pi=\beta\pi^{(m)}=\beta\sqrt{-g}\frac{\partial\mathscr{L}_m}{\partial\dot{\phi}}\quad\Rightarrow\quad\left[\phi\left(\mathbf{x}\right),\pi^{(m)}\left(\mathbf{y}\right)\right]=\frac{i\hbar}{\beta}\delta\left(\mathbf{x}-\mathbf{y}\right)$$

$$\bar{\hbar}=\frac{\hbar}{\beta}$$

## Heisenberg uncertainty relation

$$\delta\bar{h} \times \delta\int_{\mathscr{V}} d^4x\sqrt{-g}\,\mathscr{L}_m \geq \frac{1}{2}\bar{h}^2$$

# Last Almost Normal Axion for $\hbar$ and $G_N$

Canonically normalised $\eta$ instead of $\beta$

$$S\left[g,\eta,A,\Phi_m\right] = \int d^4x\sqrt{-g}\left[-\frac{R}{2\varkappa}+\frac{1}{2}\left(\partial\eta\right)^2 - V_\beta\left(\eta\right)+\frac{\eta^2}{M_m^2}\mathscr{L}_m - \frac{\eta}{f_\beta}F_{\mu\nu}F^{\star\mu\nu} - \frac{1}{4\mathbf{g}^2}F_{\alpha\beta}F^{\alpha\beta}\right]$$

formal limit / “confinement”

$\varkappa = 8\pi G_N$

# Unimodular, Unicurvature and Unimatter

for the globally dynamical $\Lambda$

Unimodular $$S\left[g,\Lambda\right] = \int d^4x\,\Lambda\left[1 - \sqrt{-g}\,\right]$$

Henneaux-Teitelboim construction with fixed $W^\mu = \delta^\mu_t \frac{t}{\sqrt{-g}}$

similarly one can write for the globally dynamical $G_N$

Unimatter $$S\left[g,\beta,\Phi_m\right] = \int d^4x\,\beta\left(\sqrt{-g}\mathscr{L}_m - 1\right)$$

Unicurvature $$S\left[g,\alpha\right] = \frac{1}{2}\int d^4x\left(1 - \sqrt{-g}R\right)\alpha$$

*Thanks a lot for attention!*