Schwarzschild BH entropies on a nice slice

J. A. Rosabal

Asia Pacific Center for Theoretical Physics, APCTP

July 22

THERE IS PLEASURE IN RECOGNISING OLD THINGS FROM A NEW VIEWPOINT.

Table of Contents

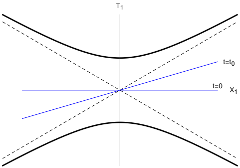
Motivation

- Hawking's calculation. If a black hole starts in a pure quantum state, why the final state is thermal (mixed state)?
- Information paradox.
- Hawking's derivation involves field modes that, near the black hole horizon, have arbitrarily high frequencies.
- Trans-Planckian problem.
- Over the years there have been several "solutions" and more problems and paradoxes.

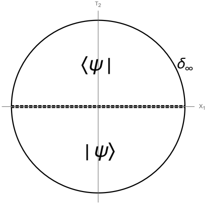
Hawking's calculation

$$Z = \int \mathrm{D}g\mathrm{D}\phi \exp[\mathrm{i}\mathrm{I}[g,\phi]]$$

$$\mathrm{I} = (16\pi)^{-1}\int_M R\sqrt{-g}\mathrm{d}^4x + (8\pi)^{-1}\int_{\delta_\infty}[\mathrm{K}]\sqrt{-h}\mathrm{d}^3\sigma + \mathrm{I_m}[g,\phi]$$



$t \to -\mathrm{i}\tau$



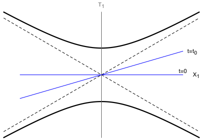
Gibbons and Hawking "Action Integrals and Partition Functions in Quantum Gravity."

Hawking's calculation

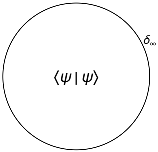
$$Z = \exp[\mathrm{i}I[g,\phi]] \quad ; \quad I = (8\pi)^{-1} \int_{\delta_\infty} [K] \sqrt{-h} d^3\sigma$$

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2$$

$$ds^2 = (1 - \frac{2M}{r})d\tau^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2$$



$t \to -\mathrm{i}\tau$



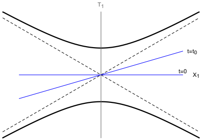
Gibbons and Hawking "Action Integrals and Partition Functions in Quantum Gravity."

Hawking's calculation

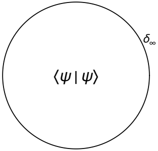
$$Z = \exp[iI[g,\phi]] \quad ; \quad I = 4\pi i M^2$$

$$ds^2 = -(1-\frac{2M}{r})dt^2 + \frac{dr^2}{(1-\frac{2M}{r})} + r^2 d\Omega^2$$

$$ds^2 = (1-\frac{2M}{r})d\tau^2 + \frac{dr^2}{(1-\frac{2M}{r})} + r^2 d\Omega^2$$



$t \to -i\tau$



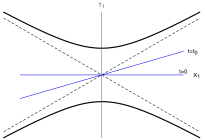
Gibbons and Hawking "Action Integrals and Partition Functions in Quantum Gravity."

Hawking's calculation

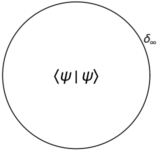
$$S = \frac{A}{4} + S_m$$

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2$$

$$ds^2 = (1 - \frac{2M}{r})d\tau^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2$$



$t \to -i\tau$



Gibbons and Hawking "Action Integrals and Partition Functions in Quantum Gravity."

Hawking's calculation

$$\mathrm{S} = \frac{A}{4} + S_{\mathrm{m}}$$

INFORMATION PARADOX

TRANS-PLANCKIAN PROBLEM

Gibbons and Hawking "Action Integrals and Partition Functions in Quantum Gravity."

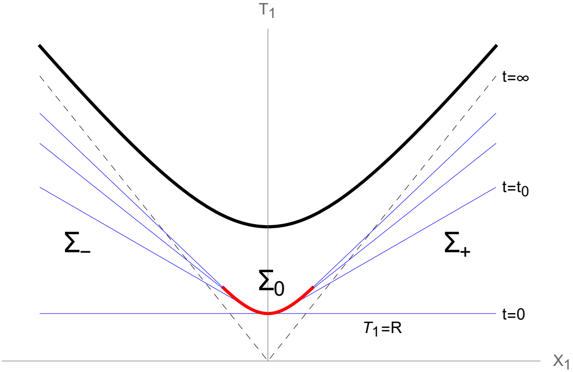
Hawking, "Breakdown of Predictability in Gravitational Collapse."

Table of Contents

Nice slice

- The nice slices are a set of Cauchy surfaces which foliate spacetime.
- The surfaces should avoid regions of strong spacetime curvature but, cut through the infalling matter and the outgoing Hawking radiation.
- Infalling matter and the outgoing Hawking radiation both should have low energy in the local coordinates on each slice.
- The slices must be everywhere smooth, with small extrinsic curvature compared to any microscopic scale.
- We ensure that the effectice QFT description does not break down, and we can follow the evaporation of a black hole until a very late time using QFT.

Nice slice



$$\Sigma_- : \quad \cosh\left(\frac{t}{4M}\right) T_1 + \sinh\left(\frac{t}{4M}\right) X_1 = R \qquad ; \quad X_1 < -R\sinh\left(\frac{t}{4M}\right)$$

$$\Sigma_0 : \quad X_1^2 - T_1^2 = -R^2 \qquad ; \quad -R\sinh\left(\frac{t}{4M}\right) < X_1 < -R\sinh\left(\frac{t}{4M}\right)$$

$$\Sigma_+ : \quad \cosh\left(\frac{t}{4M}\right) T_1 - \sinh\left(\frac{t}{4M}\right) X_1 = R \qquad ; \quad X_1 > R\sinh\left(\frac{t}{4M}\right)$$

Nice slice

- The constant R is assumed to be large by comparison with any microscopic scale, but small enough to keep the slices far from the singularity.
- Σ_0 only grows as we evolve forward in Schwarzschild (REAL) time but it is fixed at a constant r $< 2M$.
- Can we define ground state at $t = 0$, on $\Sigma_- \cup \Sigma_+$?
- Can we define ground state at $t = t_0$, on $\Sigma_- \cup \Sigma_0 \cup \Sigma_+$?

Lowe, Polchinski, Susskind, Uglum, "BH complementarity versus locality."

Hartle and Hawking, "Wave function of the Universe."

Metric and complex extension

$$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} \left(-dT_1^2 + dX_1^2 \right) + r^2 d\Omega^2$$

$$r = 2M\left(1 + W_0\left(\frac{X_1^2 - T_1^2}{e}\right)\right)$$

Metric and complex extension

$$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} \big(- dT_1^2 + dX_1^2 \big) + r^2 d\Omega^2$$

$$r = 2M\big(1 + W_0\big(\frac{X_1^2 - T_1^2}{e}\big)\big)$$

Σ_- :

$$X_1^2 - T_1^2 = \big(\frac{r}{2M} - 1\big) e^{\frac{r}{2M}}$$

$$\cosh\big(\frac{t}{4M}\big)\ T_1 + \sinh\big(\frac{t}{4M}\big)\ X_1 = R$$

Metric and complex extension

$$X_1^2 - T_1^2 = (\frac{r}{2M} - 1)e^{\frac{r}{2M}}$$

$$\cosh(\frac{t}{4M})\ T_1 + \sinh(\frac{t}{4M})\ X_1 = R$$

Σ_- :

$$\begin{aligned} T_1 &= \rho \sinh(\frac{t}{4M}) + R \cosh(\frac{t}{4M}) \\ X_1 &= -\rho \cosh(\frac{t}{4M}) - R \sinh(\frac{t}{4M}) \end{aligned}$$

$$\rho = \sqrt{(\frac{r}{2M} - 1)e^{\frac{r}{2M}} + R^2}$$

Metric and complex extension

$$ds^2 = -(1-\frac{2M}{r})dt^2 + \frac{2R}{\sqrt{(\frac{r}{2M}-1)e^{\frac{r}{2M}} + R^2}}dtdr + \frac{dr^2}{1-\frac{2M}{r}(1-R^2e^{-\frac{r}{2M}})}$$

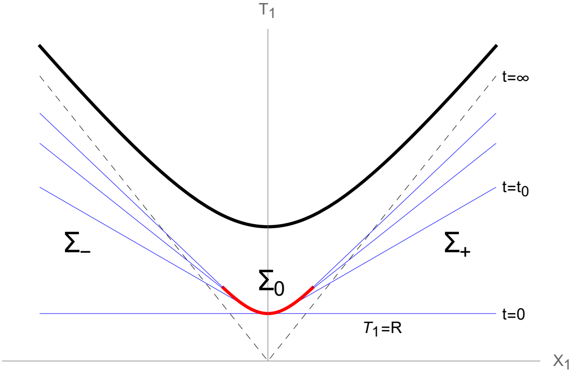
$$ds^2 = -N^2dt^2 + h_{ab}(dx^a - V^a dt)(dx^b - V^b dt)$$

$$N^2 = 1 - \frac{2M}{r}(1 - R^2e^{-\frac{r}{2M}}) > 0$$

$0 \leq t < \infty$, and $r_0 < r < \infty$

$r_0 < 2M$

Nice slice



$$\Sigma_- : \quad \cosh\left(\frac{t}{4M}\right) T_1 + \sinh\left(\frac{t}{4M}\right) X_1 = R \qquad ; \quad X_1 < -R \sinh\left(\frac{t}{4M}\right)$$

$$\Sigma_0 : \quad X_1^2 - T_1^2 = -R^2 \qquad ; \quad -R \sinh\left(\frac{t}{4M}\right) < X_1 < -R \sinh\left(\frac{t}{4M}\right)$$

$$\Sigma_+ : \quad \cosh\left(\frac{t}{4M}\right) T_1 - \sinh\left(\frac{t}{4M}\right) X_1 = R \qquad ; \quad X_1 > R \sinh\left(\frac{t}{4M}\right)$$

Metric and complex extension

- In order to keep track of a particular slice parameterized by a fixed t after a complex extension we find it convenient to shift t in such a way that the "new" $t = 0$ slice is at some "old" $t = t_0$.
- $t \to t + t_0$
- $t \to -\mathrm{i}\tau$, $\mathrm{T}_1 \to \mathrm{T}$, $\mathrm{X}_1 \to \mathrm{X}$

$$\begin{aligned} \mathrm{T} &= \rho \sinh\left(\frac{t_0 - \mathrm{i}\tau}{4\mathrm{M}}\right) + \mathrm{R} \cosh\left(\frac{t_0 - \mathrm{i}\tau}{4\mathrm{M}}\right) \\ \mathrm{X} &= -\rho \cosh\left(\frac{t_0 - \mathrm{i}\tau}{4\mathrm{M}}\right) - \mathrm{R} \sinh\left(\frac{t_0 - \mathrm{i}\tau}{4\mathrm{M}}\right) \quad ; \quad \tau \sim \tau + 8\pi\mathrm{M} \end{aligned}$$

Metric and complex extension

$$
\begin{aligned}
T &= \sqrt{(\frac{r}{2M}-1)e^{\frac{r}{2M}}+R^2}\,\sinh(\frac{t_0-i\tau}{4M})+R\cosh(\frac{t_0-i\tau}{4M}) \\
X &= -\sqrt{(\frac{r}{2M}-1)e^{\frac{r}{2M}}+R^2}\,\cosh(\frac{t_0-i\tau}{4M})-R\sinh(\frac{t_0-i\tau}{4M})
\end{aligned}
$$

$$
\begin{aligned}
T_2 &= \sqrt{(\frac{r}{2M}-1)}e^{\frac{r}{4M}}\,\sin(\frac{\tau}{4M}) \\
X_1 &= -\sqrt{(\frac{r}{2M}-1)}e^{\frac{r}{4M}}\,\cos(\frac{\tau}{4M})
\end{aligned}
$$

- There is a clear difference between these sections. The state defined on the slice $t = t_0$, is not the HH state.

Metric and complex extension

$$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} \big(- dT^2 + dX^2 \big) + r^2 d\Omega^2$$

$$ds^2 = N^2 d\tau^2 + h_{ab}(dx^a + iV^a d\tau)(dx^b + iV^b d\tau)$$

$$X^2 - T^2 = (\frac{r}{2M} - 1)e^{\frac{r}{2M}} \in \mathbb{R}$$

$$r = 2M\big(1 + W_0(\frac{X^2 - T^2}{e})\big)$$

Topology of the complex sections

δ_ρ^- :

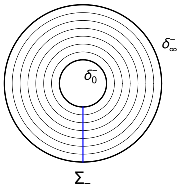
$$
\begin{aligned}
\mathrm{T} &= \rho \sinh\left(\frac{t_0 - \mathrm{i}\tau}{4\mathrm{M}}\right) + \mathrm{R} \cosh\left(\frac{t_0 - \mathrm{i}\tau}{4\mathrm{M}}\right) \\
\mathrm{X} &= -\rho \cosh\left(\frac{t_0 - \mathrm{i}\tau}{4\mathrm{M}}\right) - \mathrm{R} \sinh\left(\frac{t_0 - \mathrm{i}\tau}{4\mathrm{M}}\right) \quad ; \quad \tau \sim \tau + 8\pi\mathrm{M}
\end{aligned}
$$

Topology of the complex sections

δ_ρ^- :

$$T = \rho \sinh\left(\frac{t_0 - i\tau}{4M}\right) + R \cosh\left(\frac{t_0 - i\tau}{4M}\right)$$
$$X = -\rho \cosh\left(\frac{t_0 - i\tau}{4M}\right) - R \sinh\left(\frac{t_0 - i\tau}{4M}\right) \quad ; \quad \tau \sim \tau + 8\pi M$$

- ANNULUS
- Two boundaries, δ_0^- and δ_∞^-



Topology of the complex sections

$$\delta_\rho^- : \{T = T_1 + iT_2 \text{ , } X = X_1 + iX_2\}\Big/$$

$$\begin{aligned}
T_1 &= +R_1(\rho)\cos(\frac{\tau}{4M}) \\
T_2 &= -R_2(\rho)\sin(\frac{\tau}{4M}) \\
X_1 &= -R_2(\rho)\cos(\frac{\tau}{4M}) \\
X_2 &= +R_1(\rho)\sin(\frac{\tau}{4M})
\end{aligned}$$

Topology of the complex sections

Clifford torus construcction for the complex extension of Σ_-:

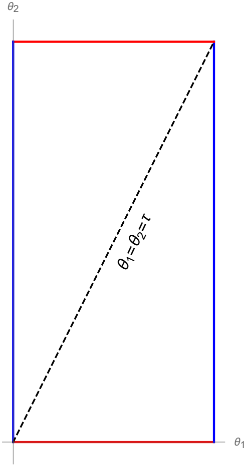
- In $\mathbb{R}^4$ we can define the torus

$$\mathbf{T}^2 = \mathrm{S}^1 \times \mathrm{S}^1 : \; (\mathrm{T}_1, \mathrm{X}_2, \mathrm{T}_2, \mathrm{X}_1) =$$

$$\left(\mathrm{R}_1(\rho_0)\cos(\frac{\theta_1}{4\mathrm{M}}), \mathrm{R}_1(\rho_0)\sin(\frac{\theta_1}{4\mathrm{M}}), -\mathrm{R}_2(\rho_0)\sin(\frac{\theta_2}{4\mathrm{M}}), -\mathrm{R}_2(\rho_0)\cos(\frac{\theta_2}{4\mathrm{M}})\right)$$

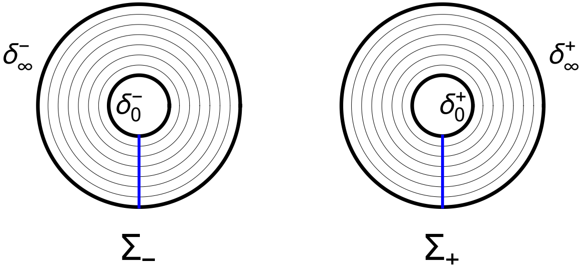
- Now we pick the curve $\delta^-_{\rho_0}$, on the torus, parameterized by τ, $\theta_1 = \theta_2 = \tau$

Topology of the complex sections



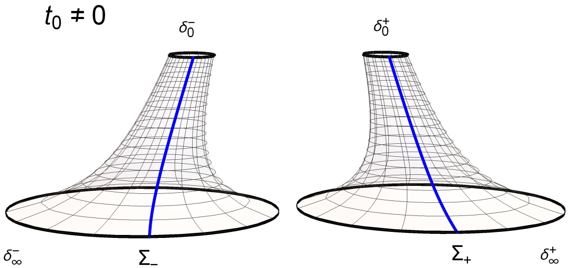
Topology of the complex sections

SO FAR:

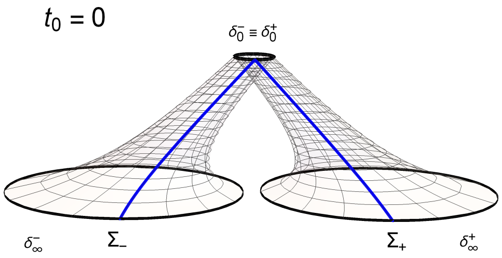


Topology of the complex sections

SO FAR:

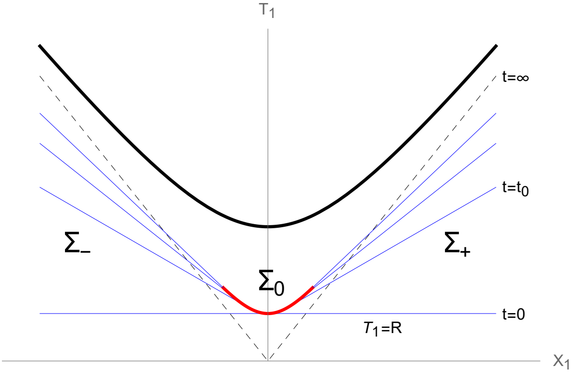


Topology of the complex sections



$$\delta_0^- : \quad \left(R\cos(\frac{\tau}{4M}), +R\sin(\frac{\tau}{4M}), 0, 0\right)$$
$$\delta_0^+ : \quad \left(R\cos(\frac{\tau}{4M}), -R\sin(\frac{\tau}{4M}), 0, 0\right)$$

Topology of the complex sections



$$\Sigma_- : \quad \cosh\left(\frac{t}{4M}\right) T_1 + \sinh\left(\frac{t}{4M}\right) X_1 = R \qquad ; \quad X_1 < -R \sinh\left(\frac{t}{4M}\right)$$

$$\Sigma_0 : \quad X_1^2 - T_1^2 = -R^2 \qquad ; \quad -R \sinh\left(\frac{t}{4M}\right) < X_1 < -R \sinh\left(\frac{t}{4M}\right)$$

$$\Sigma_+ : \quad \cosh\left(\frac{t}{4M}\right) T_1 - \sinh\left(\frac{t}{4M}\right) X_1 = R \qquad ; \quad X_1 > R \sinh\left(\frac{t}{4M}\right)$$

Topology of the complex sections

$$\Sigma_0 = \Sigma_{0-} \cup \Sigma_{0+} : \quad X_1^2 - T_1^2 = -R^2$$

Σ_{0-} :

$$\begin{aligned} T_1 &= +R \cosh\left(\frac{\zeta}{4M}\right) \\ X_1 &= -R \sinh\left(\frac{\zeta}{4M}\right) \end{aligned}$$

Σ_{0+} :

$$\begin{aligned} T_1 &= +R \cosh\left(\frac{\zeta}{4M}\right) \\ X_1 &= +R \sinh\left(\frac{\zeta}{4M}\right) \end{aligned}$$

$$0 \leq \zeta \leq t_0$$

Topology of the complex sections

$$t \to -\mathrm{i}\tau$$

Topology of the complex sections

$$t \to -\mathrm{i}\tau$$

WARNING

Σ_0 does not depend on t

Topology of the complex sections

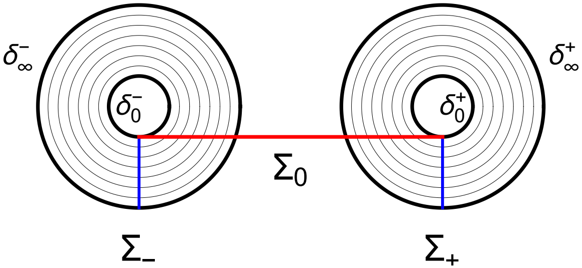
$$t \to -\mathrm{i}\tau$$

WARNING

Σ_0 does not depend on t

How can we extend Σ_0?

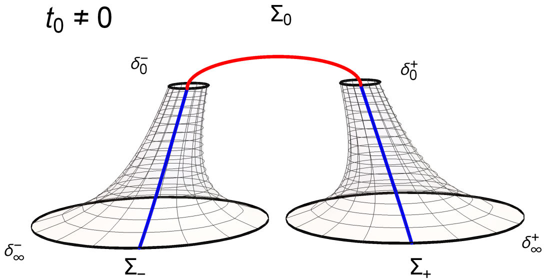
Topology of the complex sections



Topology of the complex sections

$$\Sigma_0 = \Sigma_{0-} \cup \Sigma_{0+} : \quad ds^2 = \frac{2M}{r_0} e^{-\frac{r_0}{2M}} R^2 d\zeta^2 + r_0^2 d\Omega^2$$

$$\partial\Sigma_-^{\text{ext}} = \delta_0^- \text{ and } \partial\Sigma_+^{\text{ext}} = \delta_0^+ : \quad ds^2 = -\frac{2M}{r_0} e^{-\frac{r_0}{2M}} R^2 d\tau^2 + r_0^2 d\Omega^2$$



Topology of the complex sections

$$\Sigma_0 = \Sigma_{0-} \cup \Sigma_{0+} : \quad X_1^2 - T_1^2 = -R^2$$

Σ_{0-} :

$$\begin{aligned} T &= +R(\tau,\zeta)\cosh(\frac{\zeta - i\tau}{4M}) \\ X &= -R(\tau,\zeta)\sinh(\frac{\zeta - i\tau}{4M}) \end{aligned}$$

Σ_{0+} :

$$\begin{aligned} T &= +R(\tau,\zeta)\cosh(\frac{\zeta - i\tau}{4M}) \\ X &= +R(\tau,\zeta)\sinh(\frac{\zeta - i\tau}{4M}) \end{aligned}$$

$$\begin{aligned} 0 \leq &\ \zeta \leq t_0 \\ 0 \leq &\ \tau \leq 8\pi M \end{aligned}$$

Topology of the complex sections

1. $\mathrm{R}(\tau, \zeta) = \mathrm{R}(\tau + 8\pi \mathrm{M}, \zeta) \in \mathbb{R}$
2. $\mathrm{R}(0, \zeta) = \mathrm{R}$
3. $\mathrm{R}(\tau, t_0) = \mathrm{R}$
4. $\partial_\zeta \mathrm{R}(0, \zeta) = 0$
5. $\partial_\tau \mathrm{R}(\tau, t_0) = 0$

Topology of the complex sections

1. $R(\tau, \zeta) = R(\tau + 8\pi M, \zeta) \in \mathbb{R}$
2. $R(0, \zeta) = R$
3. $R(\tau, t_0) = R$
4. $\partial_\zeta R(0, \zeta) = 0$
5. $\partial_\tau R(\tau, t_0) = 0$
6. $\partial_\tau \partial_\tau R(\tau, t_0) = 0$
7. $\partial_\zeta R(\tau, t_0) \leq 0$

Topology of the complex sections

1. $R(\tau,\zeta) = R(\tau+8\pi M,\zeta) \in \mathbb{R}$
2. $R(0,\zeta) = R$
3. $R(\tau,t_0) = R$
4. $\partial_\zeta R(0,\zeta) = 0$
5. $\partial_\tau R(\tau,t_0) = 0$
6. $\partial_\tau \partial_\tau R(\tau,t_0) = 0$
7. $\partial_\zeta R(\tau,t_0) \leq 0$
8. $R(\tau,\zeta) \sim R$

$$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} \left(-dT^2 + dX^2\right) + r^2 d\Omega^2$$

$$r = 2M\left(1 + W_0\left(\frac{X^2 - T^2}{e}\right)\right)$$

Topology of the complex sections

1. $R(\tau,\zeta) = R(\tau + 8\pi M, \zeta) \in \mathbb{R}$
2. $R(0,\zeta) = R$
3. $R(\tau, t_0) = R$
4. $\partial_\zeta R(0,\zeta) = 0$
5. $\partial_\tau R(\tau, t_0) = 0$
6. $\partial_\tau \partial_\tau R(\tau, t_0) = 0$
7. $\partial_\zeta R(\tau, t_0) \leq 0$
8. $R(\tau,\zeta) \sim R$

$$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} \left(-dT^2 + dX^2 \right) + r^2 d\Omega^2$$

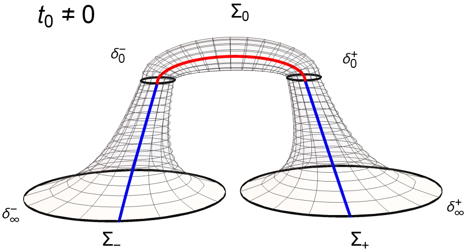
$$r = 2M\left(1 + W_0\left(\frac{X^2 - T^2}{e}\right)\right)$$

$R(\tau,\zeta)$

$$R(\tau,\zeta) = R + \sum_{n=1}^{\infty} a_n(\zeta) \sin\left(\frac{n}{4M}\tau\right) \quad ; \quad a_n(t_0) = 0$$

Topology of the complex sections

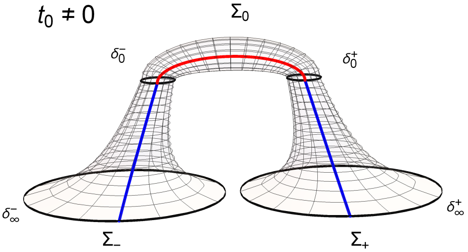
FINALLY:



$$\mathrm{I} = (8\pi)^{-1} \int_{\delta_\infty^-} [\mathrm{K}] \sqrt{-h} \mathrm{d}^3\sigma + (8\pi)^{-1} \int_{\delta_\infty^+} [\mathrm{K}] \sqrt{-h} \mathrm{d}^3\sigma$$

Topology of the complex sections

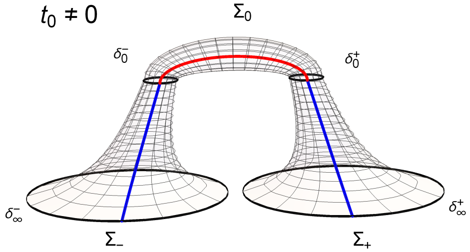
FINALLY:



$$\mathsf{I} = 2 \times (8\pi)^{-1} \int_{\delta_\infty^+} [\mathsf{K}] \sqrt{-h} \mathrm{d}^3\sigma$$

Topology of the complex sections

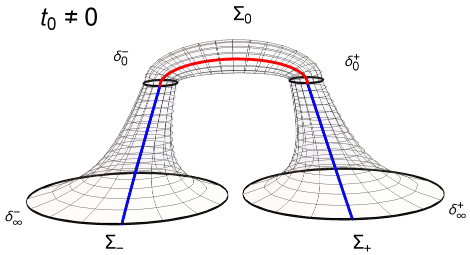
FINALLY:



$$\mathrm{I} = 2 \times (8\pi)^{-1} \int_{\delta_\infty^+} [\mathrm{K}] \sqrt{-h} \mathrm{d}^3\sigma = 2 \times 4\pi \mathrm{i} \mathrm{M}^2$$

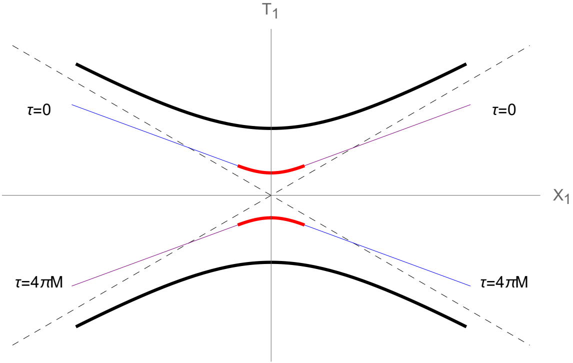
Topology of the complex sections

FINALLY:

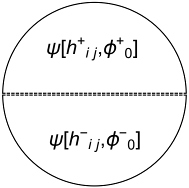


$$S = 2 \times \frac{A}{4}$$

Topology of the complex sections



Density matrix interpretation



Pure state

$$\rho\left[h_{ij}^{-},\phi_0^{-};h_{ij}^{+},\phi_0^{+}\right]=\Psi\left[h_{ij}^{-},\phi_0^{-}\right]\Psi\left[h_{ij}^{+},\phi_0^{+}\right]$$

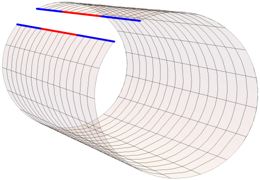
$$\mathrm{P}\left[h_{ij},\phi_0\right]=\Psi\left[h_{ij},\phi_0\right]\Psi\left[h_{ij},\phi_0\right]$$

$$\langle \mathrm{O}\rangle=\mathrm{Tr}\left[\mathrm{O}\rho\right]$$

Hawking, "The Density Matrix of the Universe."

Page, "Density Matrix of the Universe."

Density matrix interpretation



Mixed state

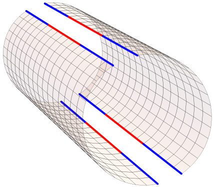
$$\rho\left[h_{ij}^{-}, \phi_0^{-}; h_{ij}^{+}, \phi_0^{+}\right] = \sum_{m,n} C_{mn} \Psi_m\left[h_{ij}^{-}, \phi_0^{-}\right] \Psi_n\left[h_{ij}^{+}, \phi_0^{+}\right]$$

$$\langle \mathrm{O} \rangle = \mathrm{Tr}\left[\mathrm{O}\rho\right]$$

Hawking, "The Density Matrix of the Universe."

Page, "Density Matrix of the Universe."

Density matrix interpretation



Mixed state

$$\rho\left[h_{ij}^{-}, \phi_0^{-}; h_{ij}^{+}, \phi_0^{+}\right] = \int \mathrm{D}h_{ij}^{1}\mathrm{D}\phi_0^{1}\rho_{-}\left[h_{ij}^{-}, \phi_0^{-}; h_{ij}^{1}, \phi_0^{1}\right]\rho_{+}\left[h_{ij}^{1}, \phi_0^{1}; h_{ij}^{+}, \phi_0^{+}\right]$$

$$\langle \mathrm{O} \rangle = \mathrm{Tr}\left[\mathrm{O}\rho\right]$$

Hawking, "The Density Matrix of the Universe."

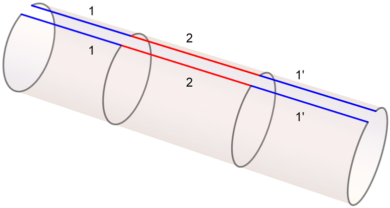
Page, "Density Matrix of the Universe."

Table of Contents

Replica Wormholes and the island rule

- Entanglement Wedge Reconstruction and the Information Paradox, 1905.08255.
- The entropy of bulk quantum fields and the entanglement wedge of an evaporating black hole, 1905.08762.
- The Page curve of Hawking radiation from semiclassical geometry, 1908.10996.
- Simple holographic models of black hole evaporation, 1910.00972.
- Replica wormholes and the black hole interior, 1911.11977.
- Replica Wormholes and the Entropy of Hawking Radiation, 1911.12333.

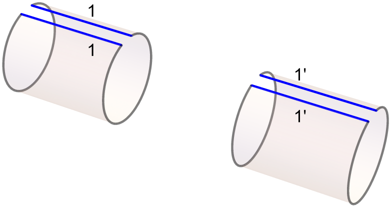
Entanglement entropy



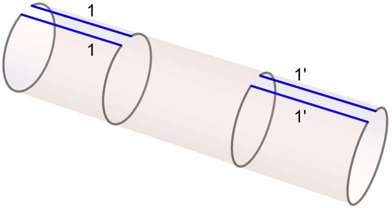
$$\rho\left[h_{ij}^{-}, h_{ij}^{+}\right] = \sum_{mn} C_{mn} \Psi_m\left[h_{ij}^{-}\right] \Psi_n\left[h_{ij}^{+}\right]$$

$$\mathrm{Tr}[\rho] = \int \mathrm{D}h_{ij} \; \rho\left[h_{ij}, h_{ij}\right] = 2 \times \frac{A}{4}$$

Entanglement entropy



Entanglement entropy

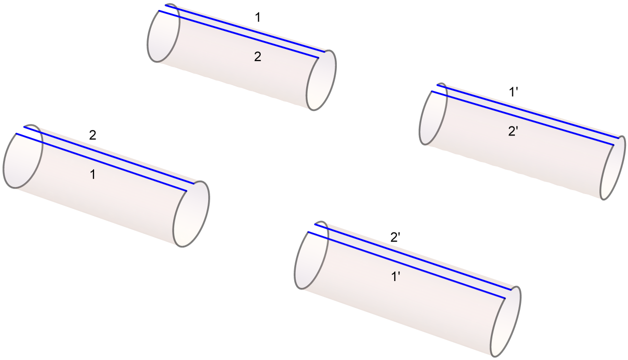


$$\tilde{\rho}\left[h_{ij}^{-}, h_{ij}^{+}\right]$$

$$\mathrm{Tr}[\tilde{\rho}] = \int \mathrm{D}h_{ij}\ \tilde{\rho}\left[h_{ij}, h_{ij}\right] = 2 \times \frac{A}{4}$$

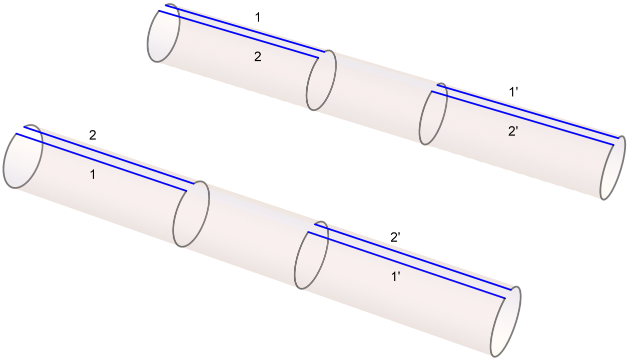
Entanglement entropy

$$\text{Tr}[\tilde{\rho}^2] =$$



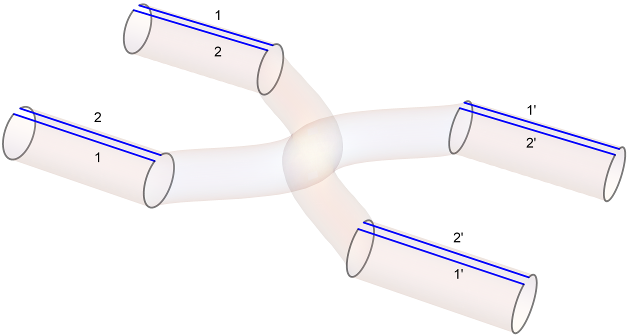
Entanglement entropy

$$\mathrm{Tr}[\tilde{\rho}^2] =$$



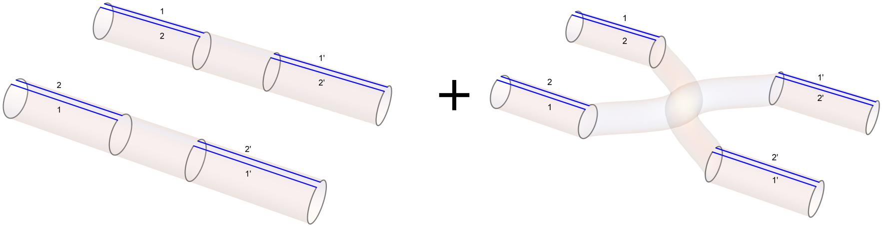
Entanglement entropy

$$\mathrm{Tr}[\tilde{\rho}^2] =$$



Entanglement entropy

$$\mathrm{Tr}[\tilde{\rho}^2] =$$



Entanglement entropy

- $$\begin{aligned}\tilde{\rho} &= \mathrm{A} \\ \tilde{\rho}^2 &= \mathrm{A}^2 + \text{connected}\end{aligned}$$
- leads to the concept of Replica wormholes.
- connected is needed to reproduce the Page curve.
- connected leads to the INfamous factorization problem.
- ensemble average.

$$\begin{aligned}\langle\tilde{\rho}\rangle &= \mathrm{A} + \langle \mathrm{a}\rangle \quad ; \quad \langle a\rangle = 0 \\ \langle\tilde{\rho}^2\rangle &= \mathrm{A}^2 + \langle a^2\rangle\end{aligned}$$

- the universe does not need an ensemble average.

Replica wormholes and the black hole interior, 1911.11977.

Replica Wormholes and the Entropy of Hawking Radiation, 1911.12333.

Entanglement entropy

$$\tilde{\rho} = \mathsf{A}$$
$$\tilde{\rho}^2 = \mathsf{A}^2$$

$$\tilde{\rho}(2) = \mathsf{A}^2 + \mathrm{connected}(2)$$

Entanglement entropy

$$\tilde{\rho} = \mathsf{A}$$
$$\tilde{\rho}^2 = \mathsf{A}^2$$

$$\tilde{\rho}(2) = \tilde{\rho}^2 + \text{connected}(2)$$

Entanglement entropy

$$\begin{aligned} \tilde{\rho} &= \mathsf{A} \\ \tilde{\rho}^n &= \mathsf{A}^n \end{aligned}$$

$$\tilde{\rho}(n) = \tilde{\rho}^n + \textcolor{red}{\text{connected}(n)}$$

Entanglement entropy

$$\begin{aligned}\tilde{\rho} &= \mathrm{A} \\ \tilde{\rho}^n &= \mathrm{A}^n\end{aligned}$$

$$\tilde{\rho}(n) = \tilde{\rho}^n + \text{connected}(n)$$

$$S = -\lim_{n\to 1} \partial_n \mathrm{Tr}\left[\tilde{\rho}(n)\right] = -\lim_{n\to 1} \mathrm{Tr}\left[\partial_n\left(\tilde{\rho}^n + \text{connected}(n)\right)\right]$$

Entanglement entropy

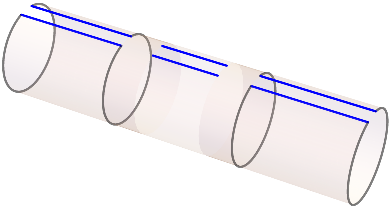
$$\tilde{\rho} = \mathrm{A}$$
$$\tilde{\rho}^n = \mathrm{A}^n$$

$$\tilde{\rho}(n) = \tilde{\rho}^n + \text{connected}(n)$$

$$S = -\mathrm{Tr}\left[\tilde{\rho}\log\tilde{\rho}\right] - \lim_{n\to 1} \mathrm{Tr}\left[\partial_n \text{connected}(n)\right]$$

Entanglement entropy

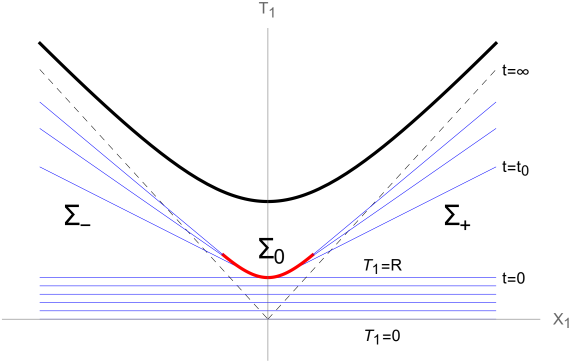
$$S = -\mathrm{Tr}\left[\rho_I \log \rho_I\right]$$



Summary

- We have computed the gravitational thermodynamic entropy of a 4D BH on a nice slice.
- This is a calculation we can trust within the framework of QFT because of the nice slice.
- We have found a new state, a mixed state, and the density matrix interpretation of it.
- When extending to the entanglement entropy we, naturally, arrive at the concepts of Replica Wormholes and Islands.
- We have to include matter.

Discussion



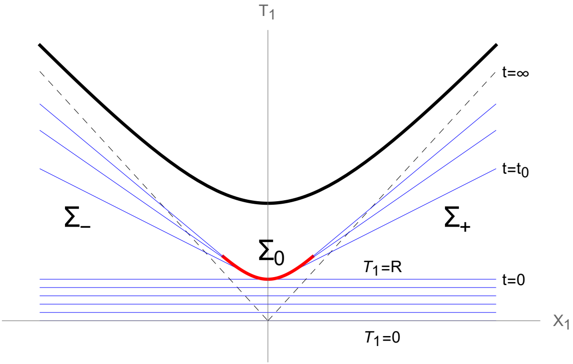
Mixed state

$$\rho\left[h_{ij}^{-},\phi_0^{-};h_{ij}^{+},\phi_0^{+}\right]=\sum_{m,n}C_{mn}\Psi_m\left[h_{ij}^{-},\phi_0^{-}\right]\Psi_n\left[h_{ij}^{+},\phi_0^{+}\right]$$

Pure state HH

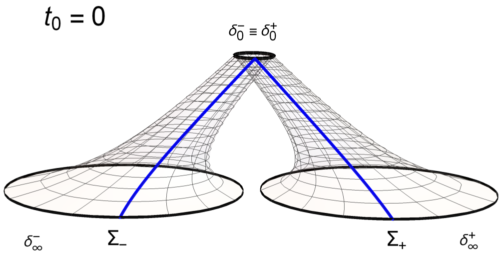
$$\rho\left[h_{ij}^{-},\phi_0^{-};h_{ij}^{+},\phi_0^{+}\right]=\Psi\left[h_{ij}^{-},\phi_0^{-}\right]\Psi\left[h_{ij}^{+},\phi_0^{+}\right]$$

Discussion

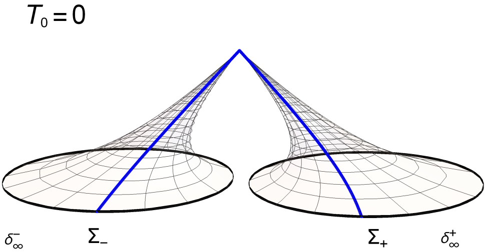


$$\cos\left(\frac{\tau}{4M}\right) T - i \sin\left(\frac{\tau}{4M}\right) X = T_0 \qquad \cos\left(\frac{\tau}{4M}\right) T + i \sin\left(\frac{\tau}{4M}\right) X = T_0$$

Discussion


$t_0 = 0$
$\delta_0^- \equiv \delta_0^+$
δ_∞^-
Σ_-
Σ_+
δ_∞^+

Discussion



Hawking, "The Density Matrix of the Universe."

Page, "Density Matrix of the Universe."

THANKS