Supersymmetric Grey Galaxies, Dual Dressed Black Holes and the Superconformal Index

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arXiv: 2501.17217

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Background

- One of the most remarkable achievements in the last 25-30 years in the field of quantum gravity has been Holographic Principle.

Gravitational theory in higher dimensions  QFT in lower dimensions

Type IIB String Theory on $AdS_5 \times S^5$  4d $\mathcal{N} = 4$ SYM with $SU(N)$ gauge group

Dictionary

$$Z_{\text{bulk}}(\phi)\,|_{\phi_{\text{bdry}}=\phi_0} = \langle e^{\int d^d x \phi_0(x)\mathcal{O}(x)} \rangle_{\text{CFT}}$$

$$G_N \quad \Longleftrightarrow \quad \frac{1}{N^2}$$

Let us consider Black hole solutions in the bulk. We know black holes have large entropy given by

$$S_{BH} = \frac{A}{4G_N}$$

Hence when there is a black hole in the bulk, the bulk partition function scales as

$$e^S = e^{\frac{A}{4G_N}} + ..$$

Therefore, in the dual CFT, we expect large number of states ($\mathcal{O}(e^{N^2})$) at these charges.

In this work, we try to resolve a puzzle related to the number of supersymmetric states in the bulk.

We find new supersymmetric black hole solutions and give some implications for the CFT.

SUSY Black holes

- States in $AdS_5 \times S^5$ are labeled by 6 parameters Energy (E), two $SO(4) \subset AdS_5$ charges (J_L, J_R) and three $SO(6) \subset S^5$ R-Charges (Q_1, Q_2, Q_3).
- We consider $\frac{1}{16}^{th}$ -BPS black holes in $AdS_5 \times S^5$ (i.e. the BH solutions that are annihilated by one supercharge ($\mathbb{Q}^{\frac{1}{2},\frac{1}{2},\frac{1}{2}}_{-\frac{1}{2},0}$) and it's complex conjugate).
- BPS condition implies:

$$E = Q_1 + Q_2 + Q_3 + 2J_L$$

The Puzzle

Known SUSY Black holes exist only on co-dimension one slice of the allowed parameter regime (i.e. have only 4 set of parameters).

Explanation of this co-dimension one slice from SYM. Why $O(e^{N^2})$ states exist only for these charges, not for others?

OR

Are we missing some SUSY solutions?

Allowed values of charges

By looking at the charges of $\frac{1}{16}^{th}$ -BPS states in the free SYM, we find that the charges must obey following inequalities

$$\zeta_i + \zeta_j \geq 0 \qquad \forall i, j$$

where $\zeta_i \in \{Q_1, Q_2, Q_3, J_1, J_2\}$ where $J_1 = J_L + J_R$ and $J_2 = J_L - J_R$.

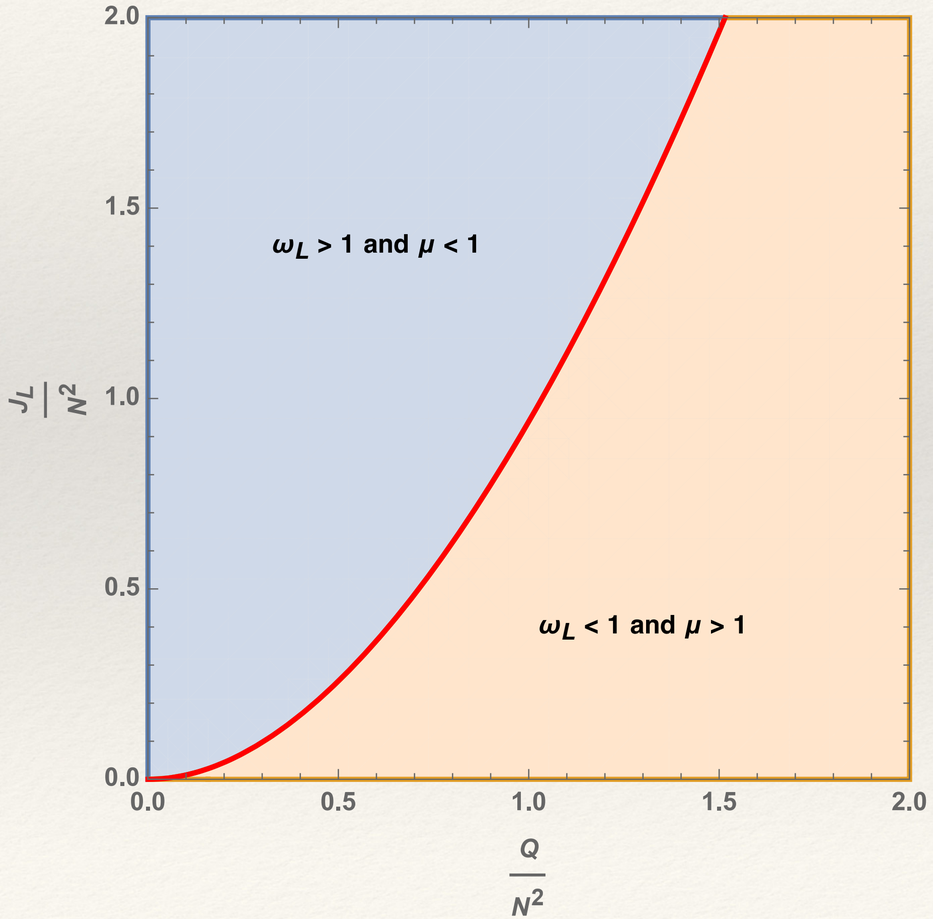
Bosonic Cone: With only bosons states, the allowed region is $\zeta_i \geq 0$.

At nonzero coupling, we expect some of these states to get lifted and no longer remain supersymmetric but the cones described above give us a maximal space where we can have non-zero number of SUSY states.

Black holes in $AdS_5 \times S^5$

Let us focus on a BH's with $Q_1 = Q_2 = Q_3 = Q$ and $J_R = 0$

$$E_{BPS} = 3Q + 2J_L$$



[Gutowski, Reall '04]
[Cvetic, Lu, Pope '04]

Endpoint of Instability of CLP BH's

The Black holes above the GR curve have $\omega > 1$, so have super-radiant instability.

Grey-galaxies or RBH [Kim et.al '23]

Grey Galaxies: A core black hole in thermal equilibrium with gas modes that carry very large angular momentum ($O(N^2)$). Since the gas carries large angular momentum, these modes are localised very far from the BH and hence the black and the gas are effectively non-interacting.

Revolving BH: A wavefunction of superpositions of BH revolving around the centre of AdS.

Endpoint of Instability of CLP BH's

The Black holes below the GR curve have $\mu > 1$, so have charged super-radiant instability.

BH's with dual giants. [Choi et.al '24]

These solutions are made up of a core BH surrounded by a dual giant graviton (D3 brane that wraps $S^3 \subset AdS_5$ and rotating in S^5) carrying large charge ($O(N^2)$) . This giant graviton has radius (in AdS_5) $r \sim \sqrt{Q}$.

Since it carries large charge, it is stabilised far away from BH and again core BH and dual giant are approx. non-interacting.

Can we lower the energy of Grey Galaxy solutions, RBHs and DDBH all the way down to E_{BPS} and make these solutions supersymmetric?

Can such solutions be Supersymmetric?

Grey Galaxies: If the core BH and the gas surrounding it are both SUSY.

Which modes can make up SUSY gas??

e.g. gravitons $\left(\frac{1}{2}\right.$ BPS in AdS_5, satisfy $\left.E = J_L\right)$. At the probe approximation, these modes remain SUSY even in BH background as long as they have large angular momentum. [Kim et.al.]

There is also direct evidence for gravitons dressing black holes from the direct evaluation of the SUSY Cohomology at SU(2) and SU(3).

RBH: Exactly Supersymmetric - In the dual SYM, these solutions can be obtained by applying supersymmetric derivatives on an ensemble of primaries.

Dual Giants

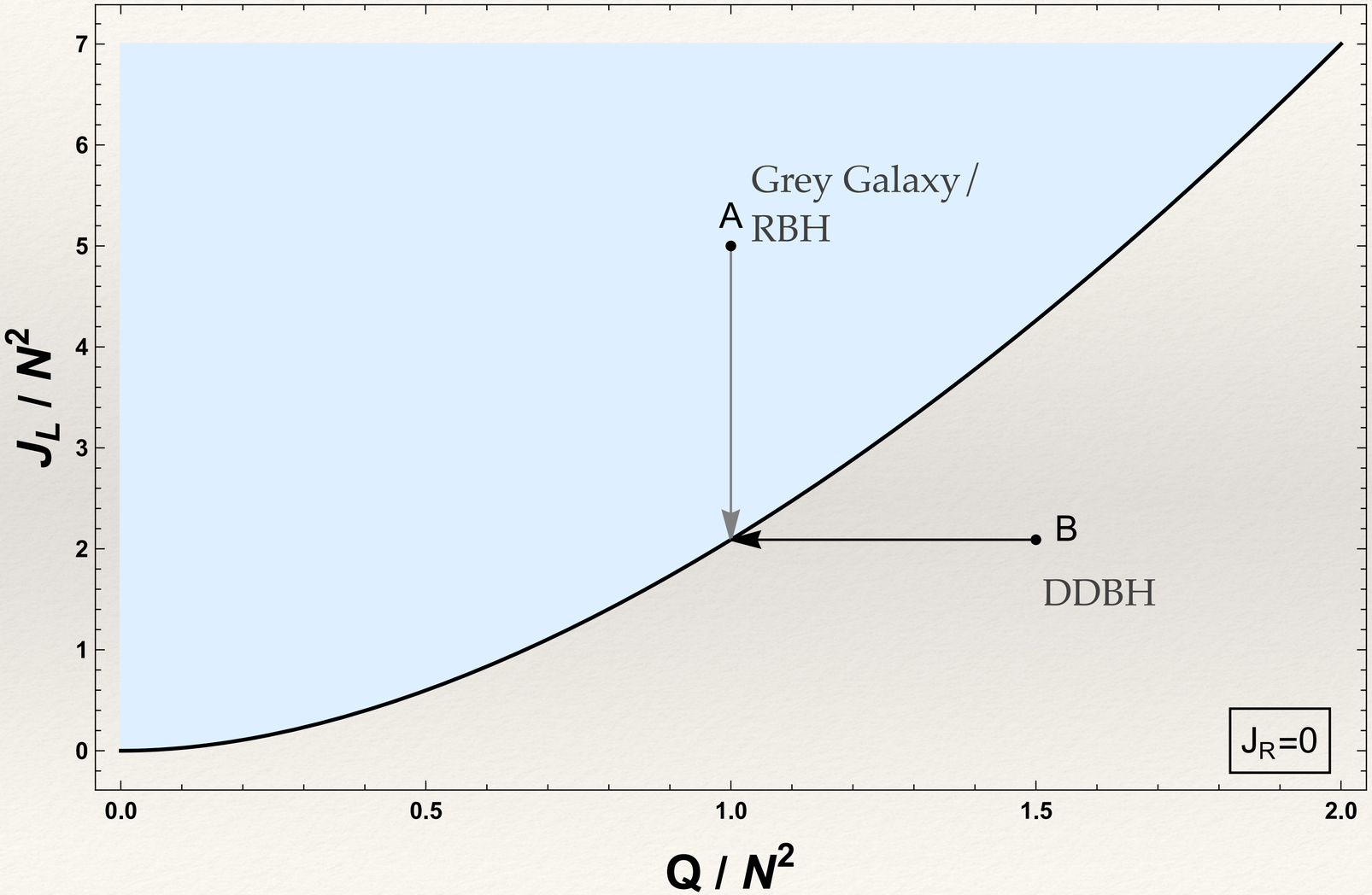
- These solutions preserve SUSY in the probe approximation.
- We checked that the dual giants preserve kappa-symmetry equations in the presence of black hole background.
- The giant gravitons carry energy $E = Q$ in the black hole background.
- Back-reaction of the D3 brane on the BH has been ignored.

Terminology

We define rank of a Grey Galaxies or DDBH solutions as 2 (number of charges carried by the gas or by Dual giants).

- GG of Rank 2: The graviton gas carries one of the angular momenta (either J_1 or J_2).
- GG of Rank 4: The graviton gas carries both angular momenta J_1 and J_2.
- Rank 2 DDBH: Carries one dual giant.
- Rank 4 DDBH: Carries two dual giants rotating along two different $S^1 \subset S^5$.
- Rank 6 DDBH: Carries three dual giants rotating along three different $S^1 \subset S^5$.

Final phase diagram for $J_R = 0$



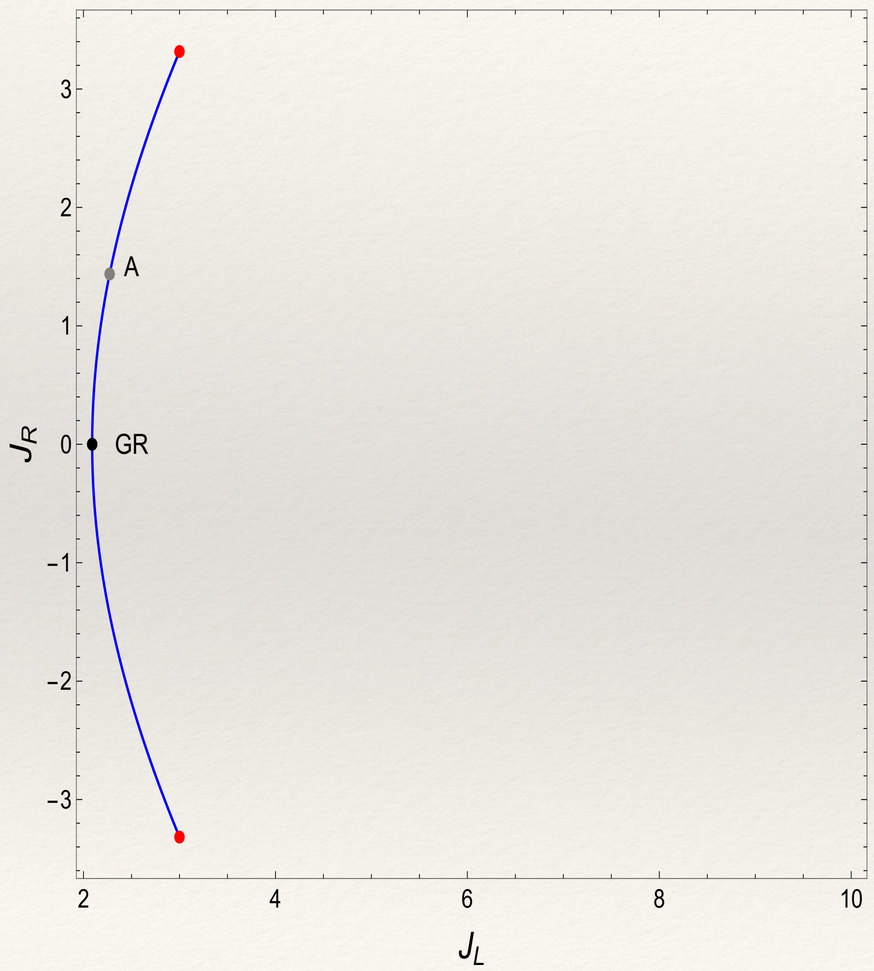
Extensive Entropy Region

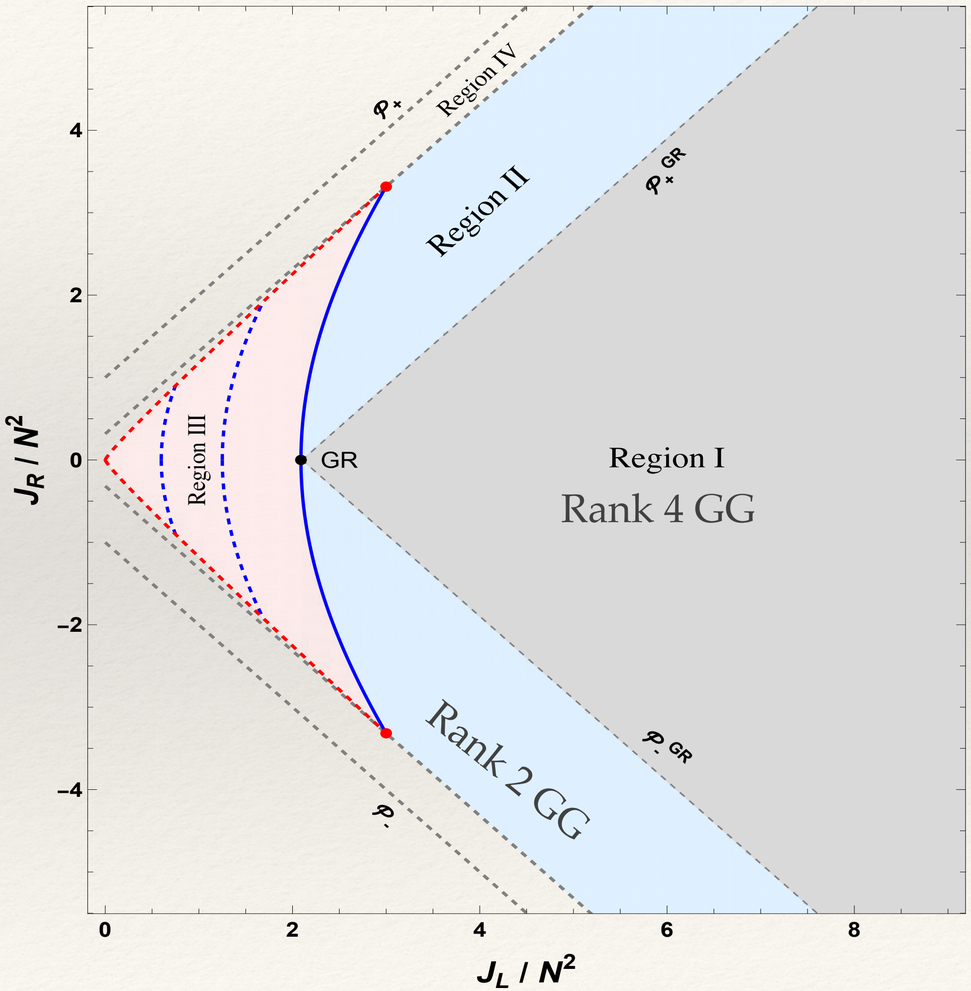
To summarise, the constructions of new solutions work as follows:

- Given a point in the charge space, we look at the black holes on the black hole sheet from where that charge point can be reached by emitting positive charges.
- From these set of black holes, we choose the core black hole to be the one with maximum entropy.

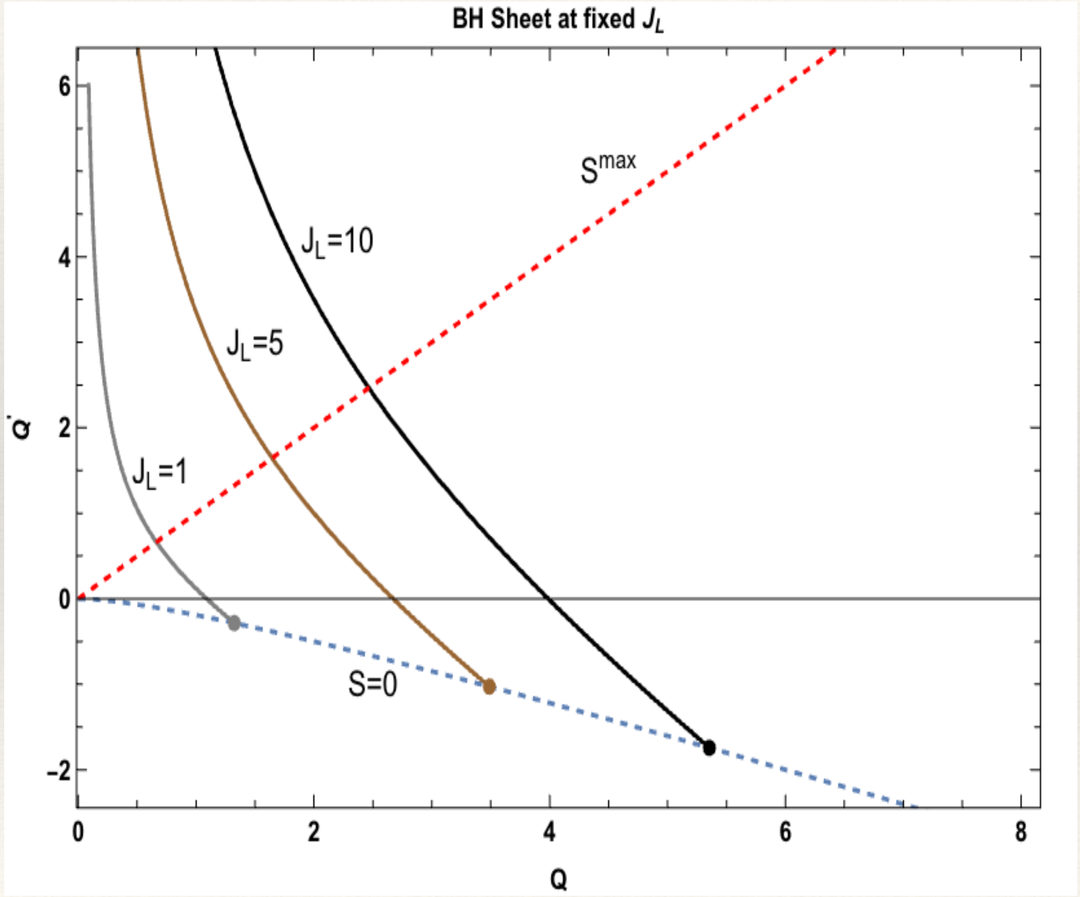
EER: The region in the charge space that can be reached by adding $\zeta_i \geq 0$ charges to the known SUSY black hole (the ones on the black hole sheet).

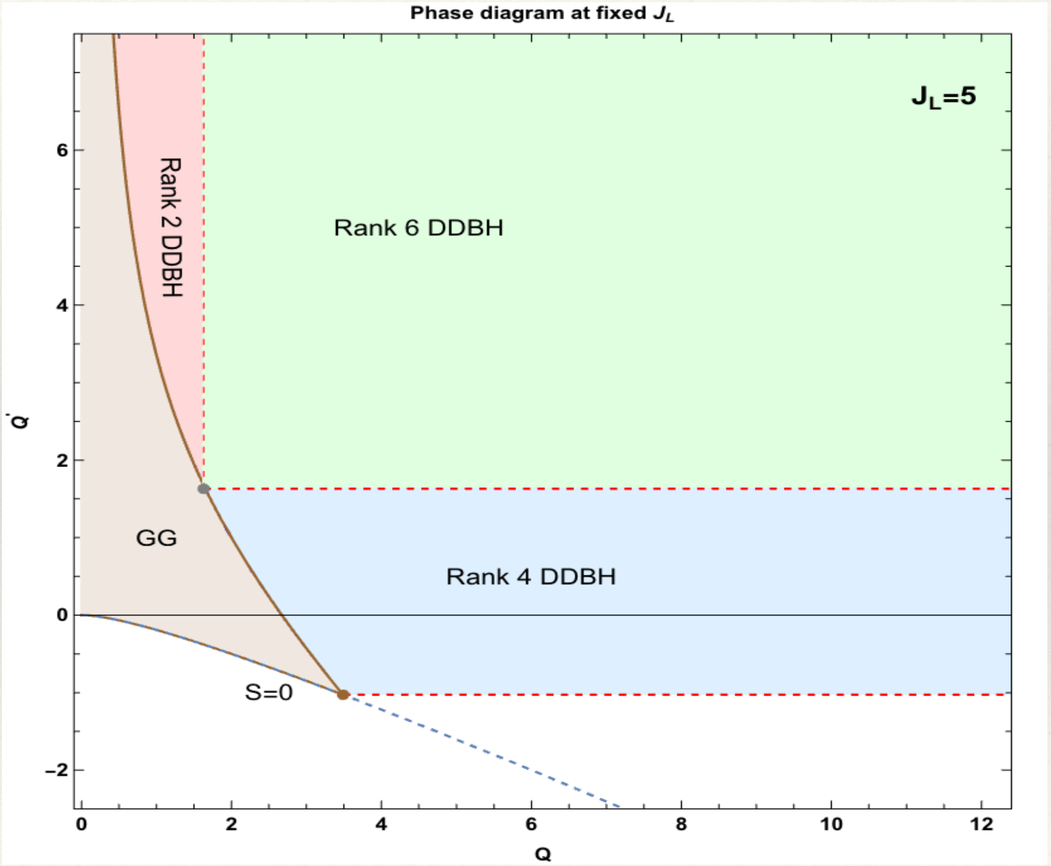
Phase diagram for $J_R \neq 0$





Phase diagram for $Q_1 = Q_2 = Q, Q_3 = Q'$ and $J_R = 0$





SUSY Cohomology

- SUSY Cohomology counts the number of supersymmetric states (modulo exact states) for a given set of charges.
- It is a function of five parameter set of charges.
- Difficult quantity to compute from the field theory.
- A lot of effort has been dedicated to come up with clever methods to compute cohomologies (these has been successful for very small gauge groups).

[Chang et.al., Kim et.al.]

Prediction from bulk

- In our construction, the entropy of the new solutions (GG or DDBH), at leading order in N^2, is given by the entropy of the core black hole.
- Hence we predict $\mathcal{O}(e^{N^2})$ SUSY states for five parameter set of charges lying within EER.
- We give an explicit formula for the number of states as a function of five parameter set of charges.
- SUSY Cohomology computed from field theory should match these predictions.

Superconformal Index

Black hole entropy  Index

The superconformal index in $\mathcal{N} = 4$ SYM is defined as

$$I = \mathrm{Tr}_{\mathrm{BPS}}[(-1)^F e^{-\beta\left(E - \sum_i \mu_i Q_i + \omega_L J_L + \omega_R J_R\right)}]$$
$$= \mathrm{Tr}_{\mathrm{BPS}}[(-1)^F e^{-\sum_i \mu_i^r Q_i - \omega_L^r J_L - \beta\omega_R J_R}]$$

[Kinney, Maldacena, Minwalla, Raju, 2005]

where $\mu_i^r = \beta(1 - \mu_i)$ and $\omega_L^r = \beta(1 - \omega_L)$ are the reduced chemical potentials.

The index commutes with the supercharge $\mathbb{Q}^{+++}_{-\frac{1}{2},0}$ iff

$$\sum \mu_i^r = \omega_L^r$$

$$I(\mu_i^r, \omega_R^r) = \mathrm{Tr}_{\mathrm{BPS}}[(-1)^F e^{-\sum_i \mu_i^r (Q_i + J_L) - \beta \omega_R^r J_R}]$$

Hence the index is a function of 4-parameters.

Specialising to the case when $Q_1 = Q_2 = Q_3 = Q$, we get

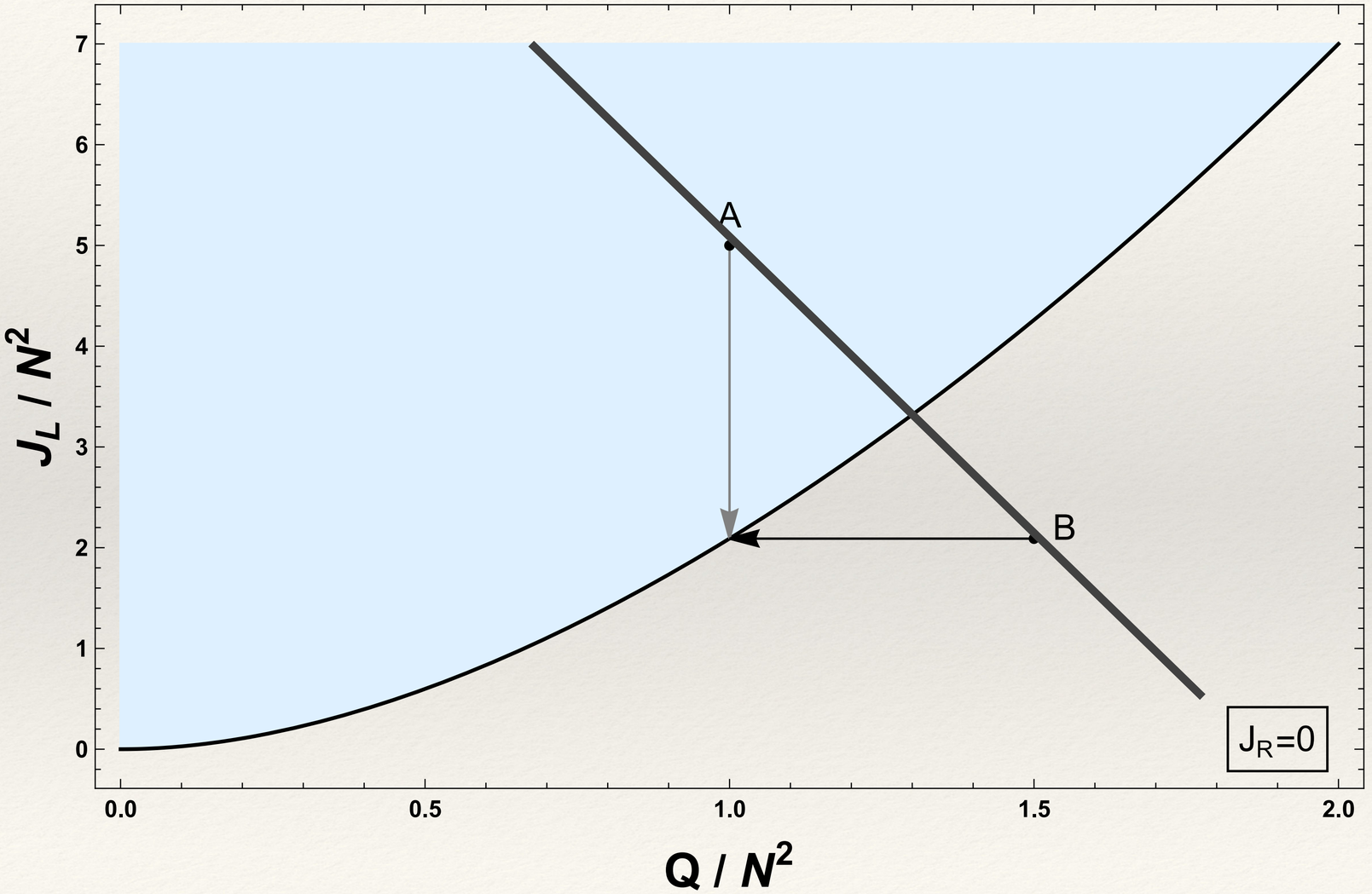
$$I(\mu^r, \omega_R^r) = \mathrm{Tr}_{\mathrm{BPS}}[(-1)^F e^{-\mu^r (Q + J_L) - \omega_R^r J_R}]$$

It counts difference between number of fermions and bosons along lines of constant $Q + J_L$.

Recents calculations show an agreement between the Index and the black hole entropy for the Gutowski Reall Black holes ($J_R = 0, Q_1 = Q_2 = Q_3$).

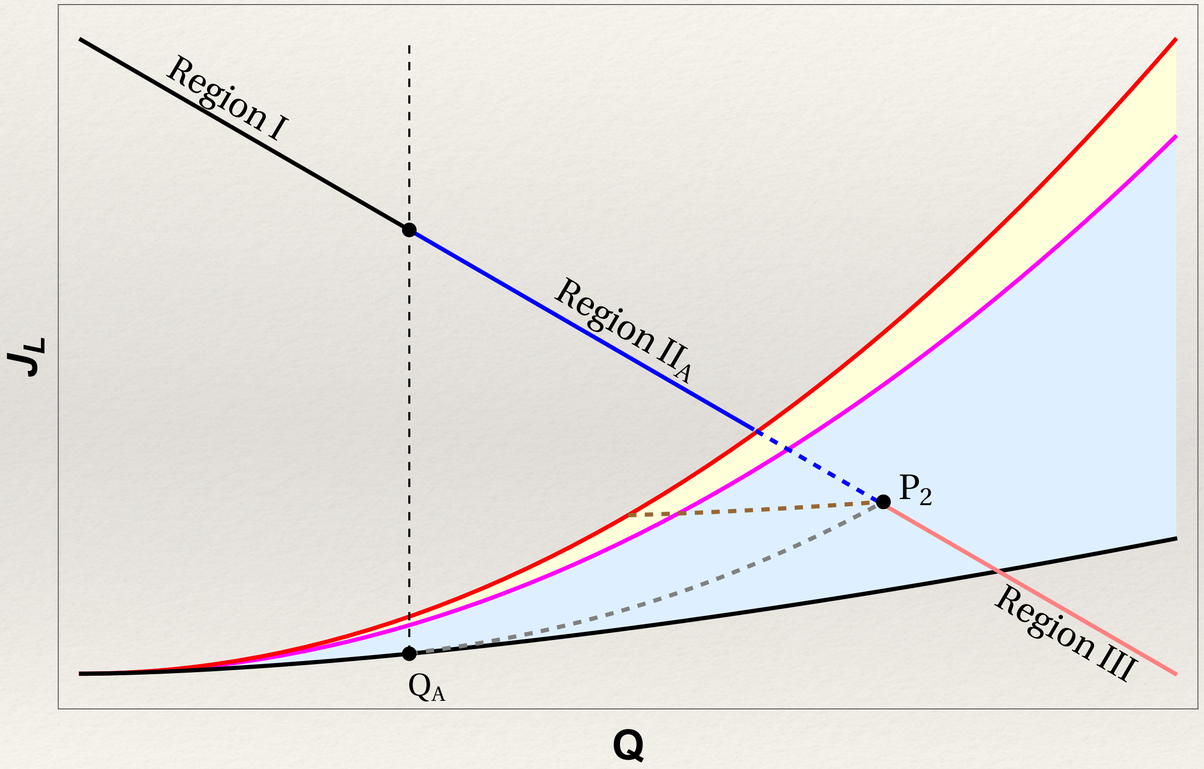
[Cabo-Bizet, Cassani, Martelli,Murthy 2018
Choi, Kim, Kim, Nahmgoong 2018
Benini, Milan 2018]

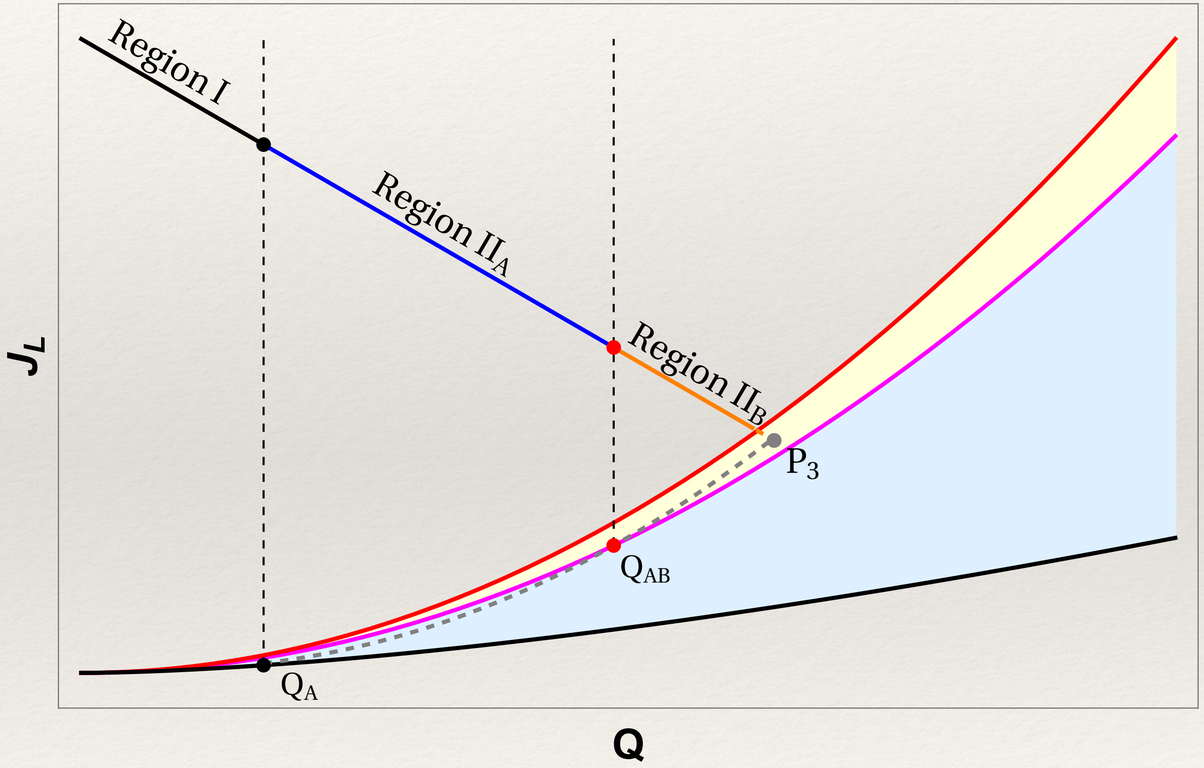
Index $J_R = 0$



Are there regions in the space of indicia charges where new solutions can dominate?

Index $J_R \neq 0$



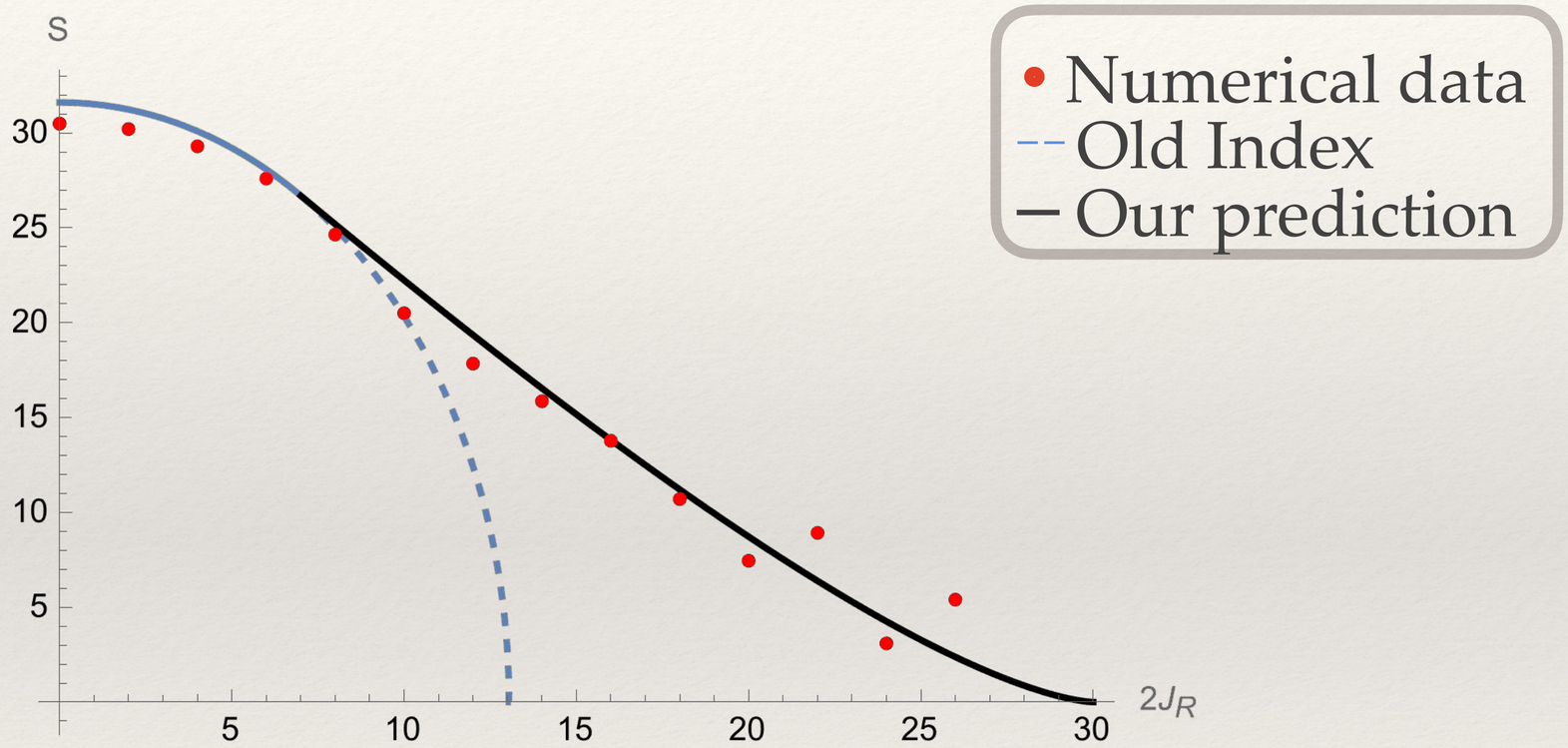


Phase transitions in the Index

To obtain a prediction for the super-conformal index, we look at the SUSY solutions along the whole index line and then find the maxima of entropy among these solutions.

- We find that the new solutions (namely GG or DDBH) dominate the super-conformal index whenever
    - The index line intersects the BH sheet in a region where at least one of the reduced chemical potential is negative.
    - The index line does not intersect the BH sheet but passes via Extensive Entropy region.

Numerical Evidence



Plot of the Indicial entropy for N=10 and $Q + J_L = 90$

What Next!!

- In our recent work, we analysed the full five parameter phase space of AdS_5 black holes and gave predictions for the phase transitions in the superconformal index.
- Can we find explicit SUSY gravity solution?
- Understanding our solutions from Field theory (Can we see them as new saddles in Bethe ansatz or in Matrix Model) .
- Why is the SUSY sheet important from field theory side?

Thanks!!