

# Closed Strings and Weak Gravity Condition from Higher-Spin Causality

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# Based on:

[arXiv.org](#) > [hep-th](#) > [arXiv:2008.05477](#)

High Energy Physics – Theory

*[Submitted on 12 Aug 2020]*

## **Closed Strings and Weak Gravity from Higher-Spin Causality**

[Jared Kaplan](#), [Sandipan Kundu](#)

[arXiv.org](#) > [hep-th](#) > [arXiv:2009.08460](#)

High Energy Physics – Theory

*[Submitted on 17 Sep 2020]*

## **Causality Constraints in Large $N$ QCD Coupled to Gravity**

[Jared Kaplan](#), [Sandipan Kundu](#)

# Main Idea

General  
Principles

+

Symmetries



Space of Consistent QFTs

# Higher spin particles in nature

Quantum  
Mechanics

+

Special  
Relativity



Massless particles

Spin

$0, \frac{1}{2}, 1, \frac{3}{2}, 2$

There is a discrete difference between massive and massless particles of the same spin.

Massive higher spin particles in general are NOT ruled out!

However, there is a conflict between massive higher spin particles and causality.

**Camanho, Edelstein, Maldacena, Zhiboedov (2014)**

**Caron-Huot, Komargodski, Sever, Zhiboedov (2016)**

**Afkhami-Jeddi, SK, Tajdini (2018)**

*Quantum  
gravity*

QFT

*Higher spin  
particles*



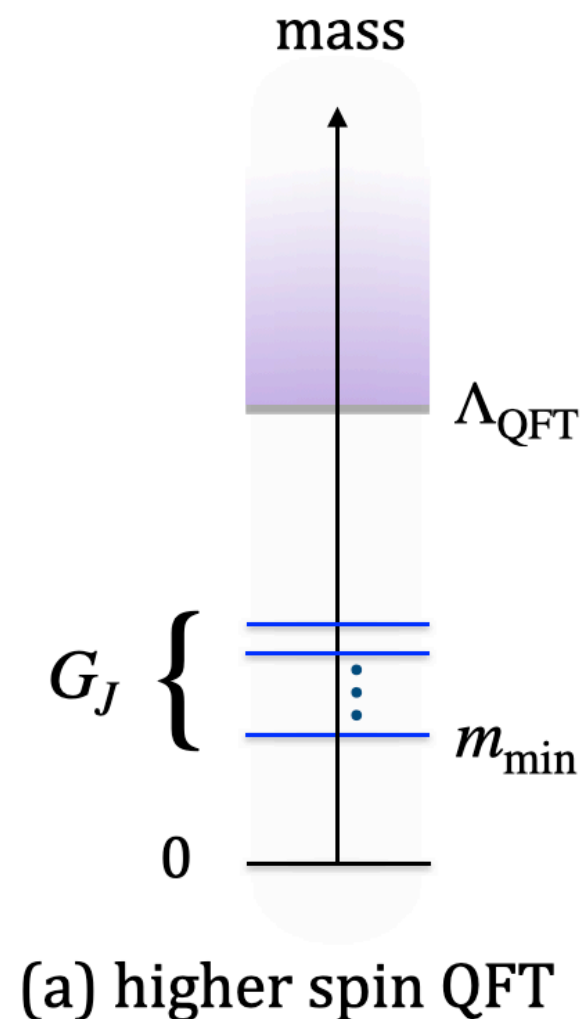
Theories of stable or metastable higher spin particles can be coupled to gravity while preserving causality only when the gravity sector has many of the properties of fundamental strings!

**Kaplan, SK (2020)**

What about large N QCD?



# Theories of metastable higher spin particles



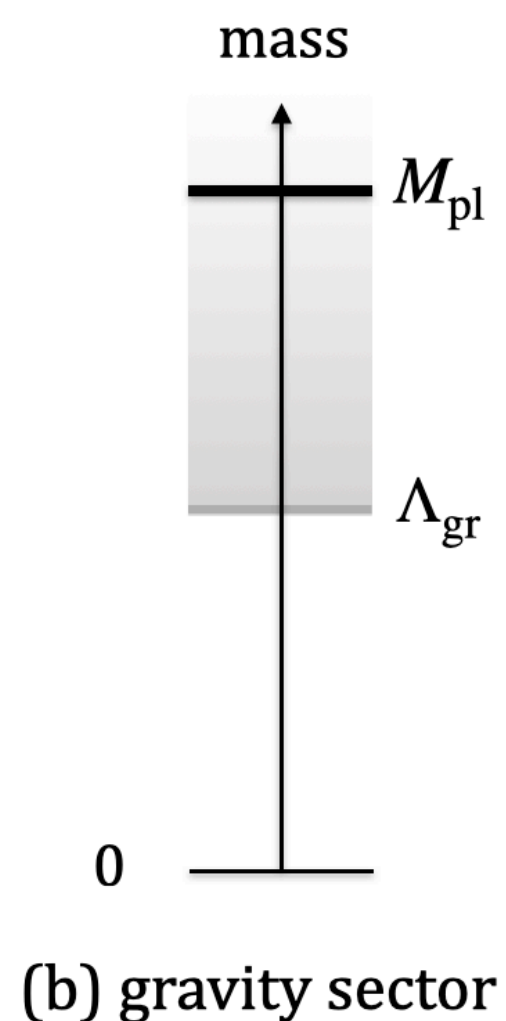
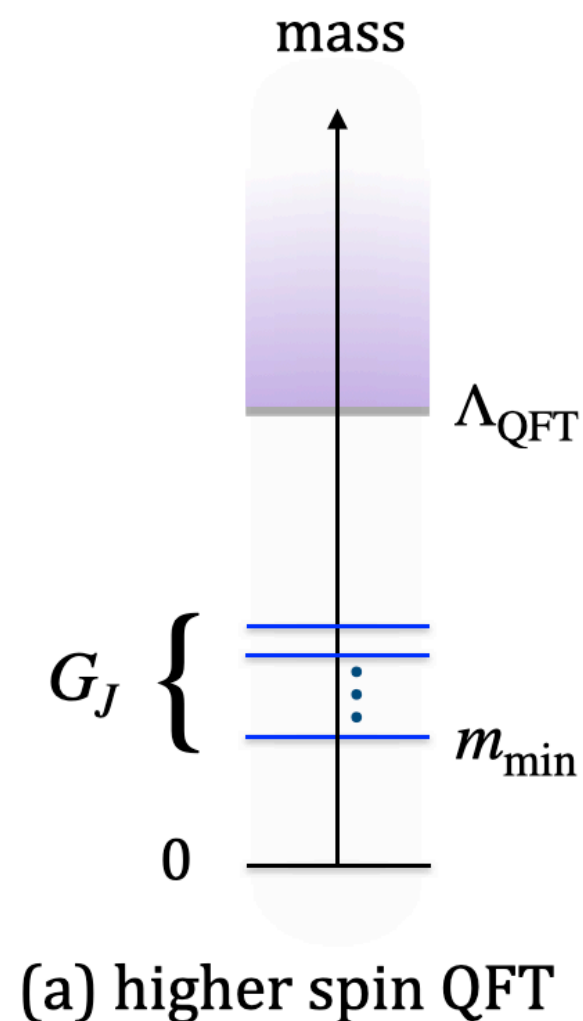
(i)  $\{G_J\}$  = all degrees of freedom of the theory below the cut-off scale  $\Lambda_{\text{QFT}}$ .

(ii) Metastable: All effective low energy couplings are small

$$\langle G_J G_{J'} G_{J''} \rangle \sim \lambda \quad |\lambda| \ll 1$$

(iii) Consistent S-matrix.

# Theories of metastable higher spin particles



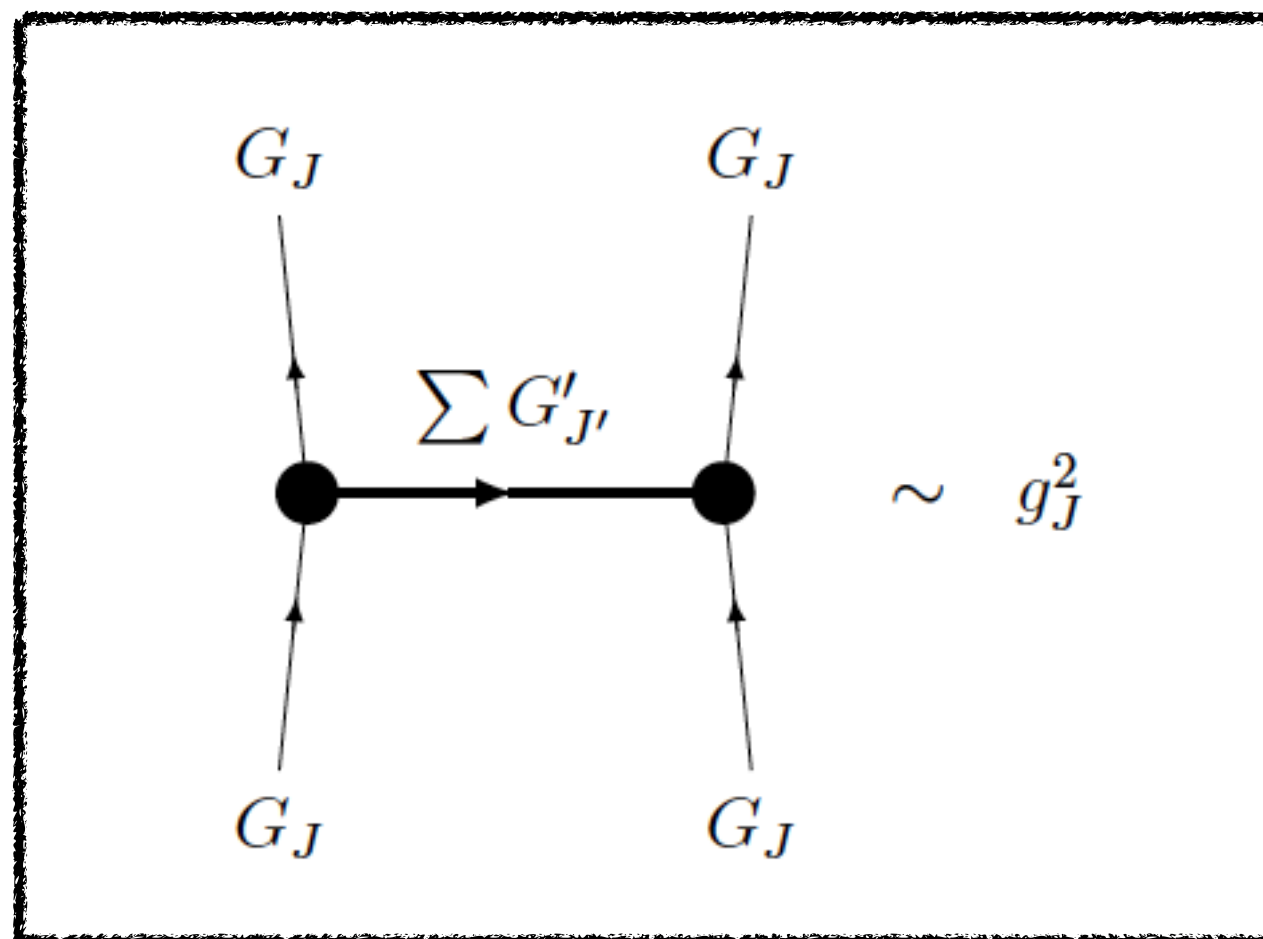
(iv) Gravitationally metastable:  $\langle G_J h_{\mu_1 \nu_1} h_{\mu_2 \nu_2} \rangle \sim \frac{\lambda_G}{M_{\text{pl}}^2}$ ,  $|\lambda_G| \lesssim O(1)$

# Theories of metastable higher spin particles

$$G_J G_J \rightarrow G_J G_J$$

$$M_{\text{pl}} \rightarrow \infty$$

The tree-level scattering amplitude is a meromorphic function with simple poles only at the location of  $\{G_J\}$  particles.



# Theories of metastable higher spin particles

$$G_J G_J \rightarrow G_J G_J$$

$$M_{\text{pl}} = \text{finite}$$

**New poles!**



graviton

+

other particles in the gravity sector

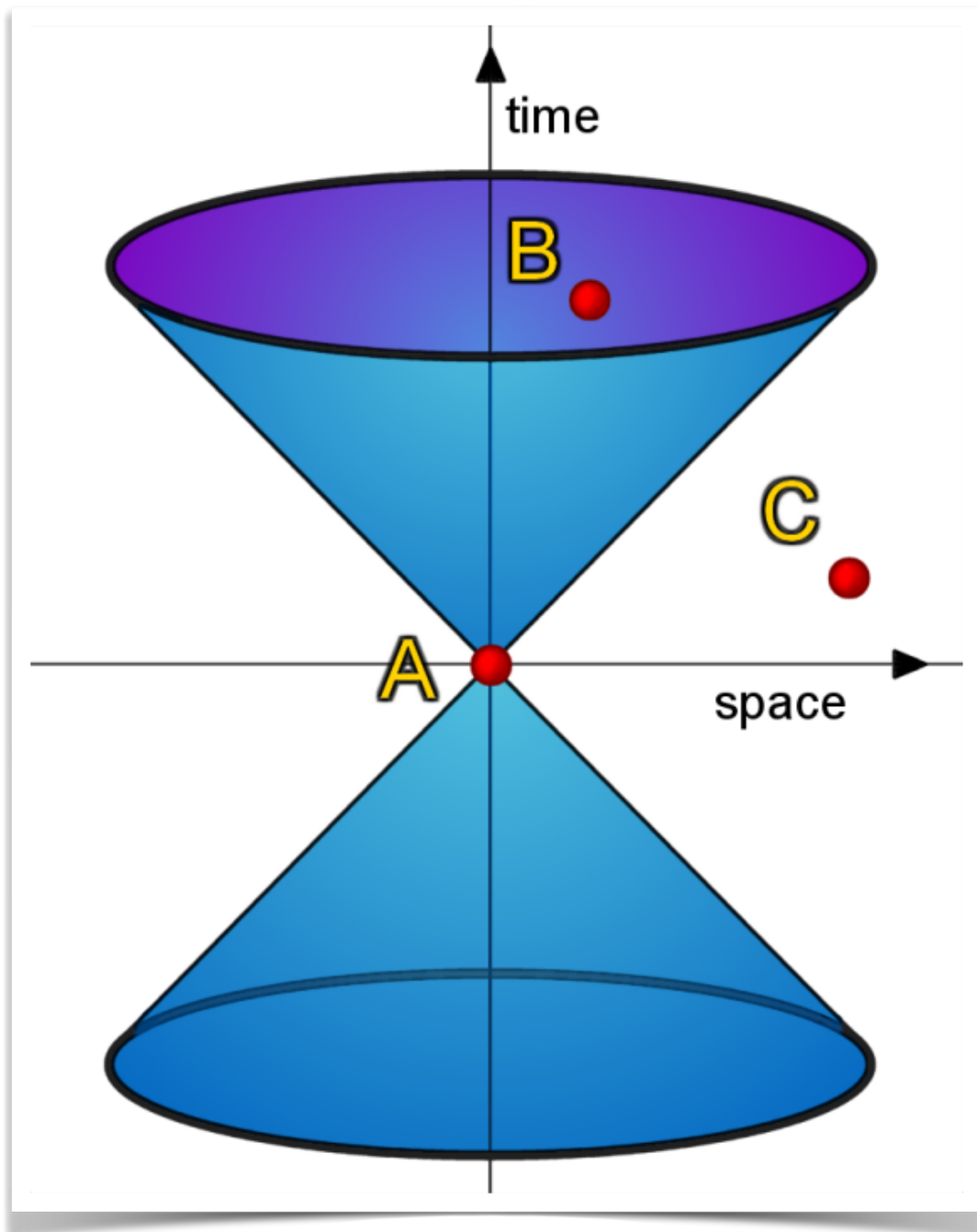
These additional gravitational poles are essential to the  
preservation of causality!

# Outline

- A. Causality in QFT
- B. A weak gravity condition from causality
- C. Emergence of closed strings
- D. Large  $N$  QCD
- E. Conclusions

# Causality in QFT

Causality requires commutators of local operators in Minkowski space to vanish outside the lightcone.



$$[O_A, O_C] = 0$$

A non-trivial  
operator statement!

# Causality in QFT

- Requiring a QFT to be causal in every state of the theory does impose **non-trivial constraints on QFTs**.
- QFTs that appear to be causal in flat spacetime can violate **causality when coupled to gravity**.

# Causality of position space correlators

ANEC

T. Hartman, SK, A. Tajdini (2016)

QNEC

Balakrishnan, Faulkner,  
Khandker, Wang (2017)

Constrain space  
of CFTs

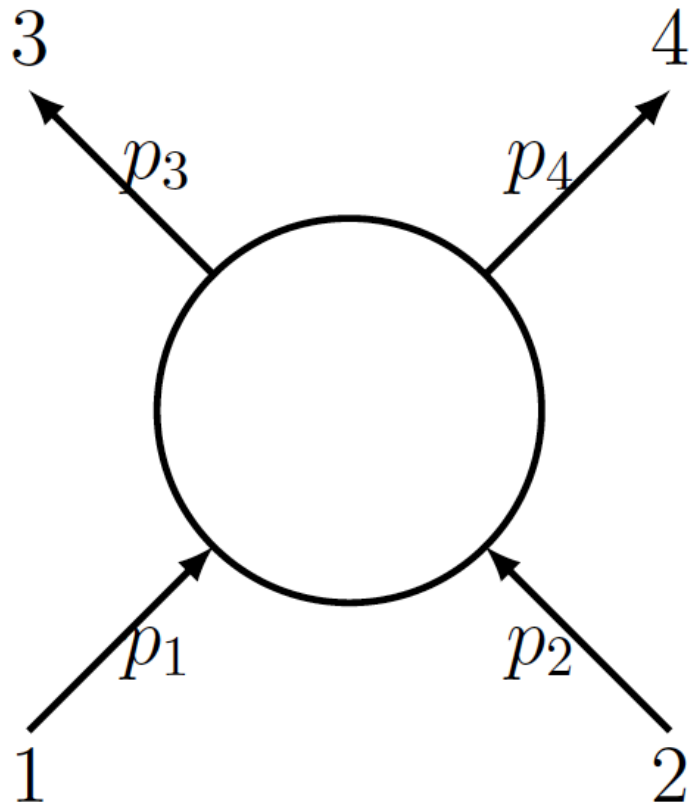
Hartman, Jain, SK (2015,2016); Hofman, Li,  
Meltzer, Poland, Rejon-Barrera (2016);  
Afkhami-Jeddi, Colville, Hartman, Maloney,  
Perlmutter (2017); Dutta Chowdhury, David,  
Prakash (2017); Cordova, Maldacena, Turiaci  
(2017); Cordova, Diab (2017); SK (2020)

Gravity from CFT

N. Afkhami-Jeddi, T. Hartman, SK and A. Tajdini (2016, 2017)  
N. Afkhami-Jeddi, SK and A. Tajdini (2018)



# Scattering amplitudes in flat space



$$s = -(p_1 + p_2)^2$$

$$t = -(p_1 - p_3)^2$$

$$u = -(p_1 - p_4)^2$$

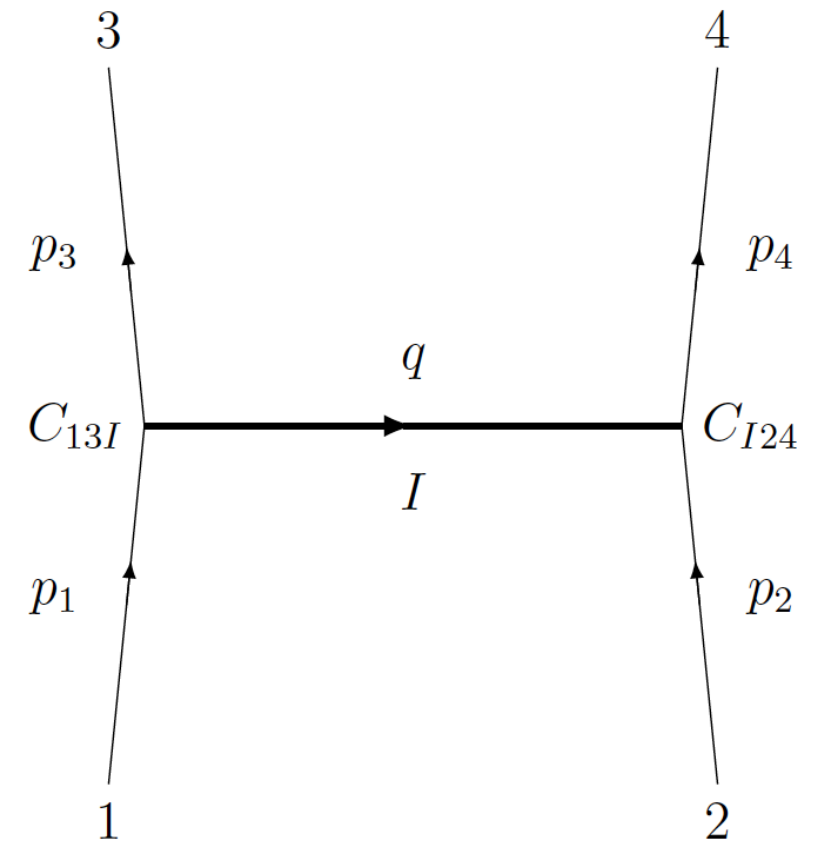
$$|s| \gg |t|, m_i^2$$

# Eikonal scattering

The impact parameter space:

$$\delta(s, \vec{b}) = \frac{1}{2s} \int \frac{d^2 \vec{q}}{(2\pi)^2} e^{i \vec{q} \cdot \vec{b}} M(s, \vec{q})$$

$$= \sum_I \frac{1}{2s} \int \frac{d^2 \vec{q}}{(2\pi)^2} e^{i \vec{q} \cdot \vec{b}}$$



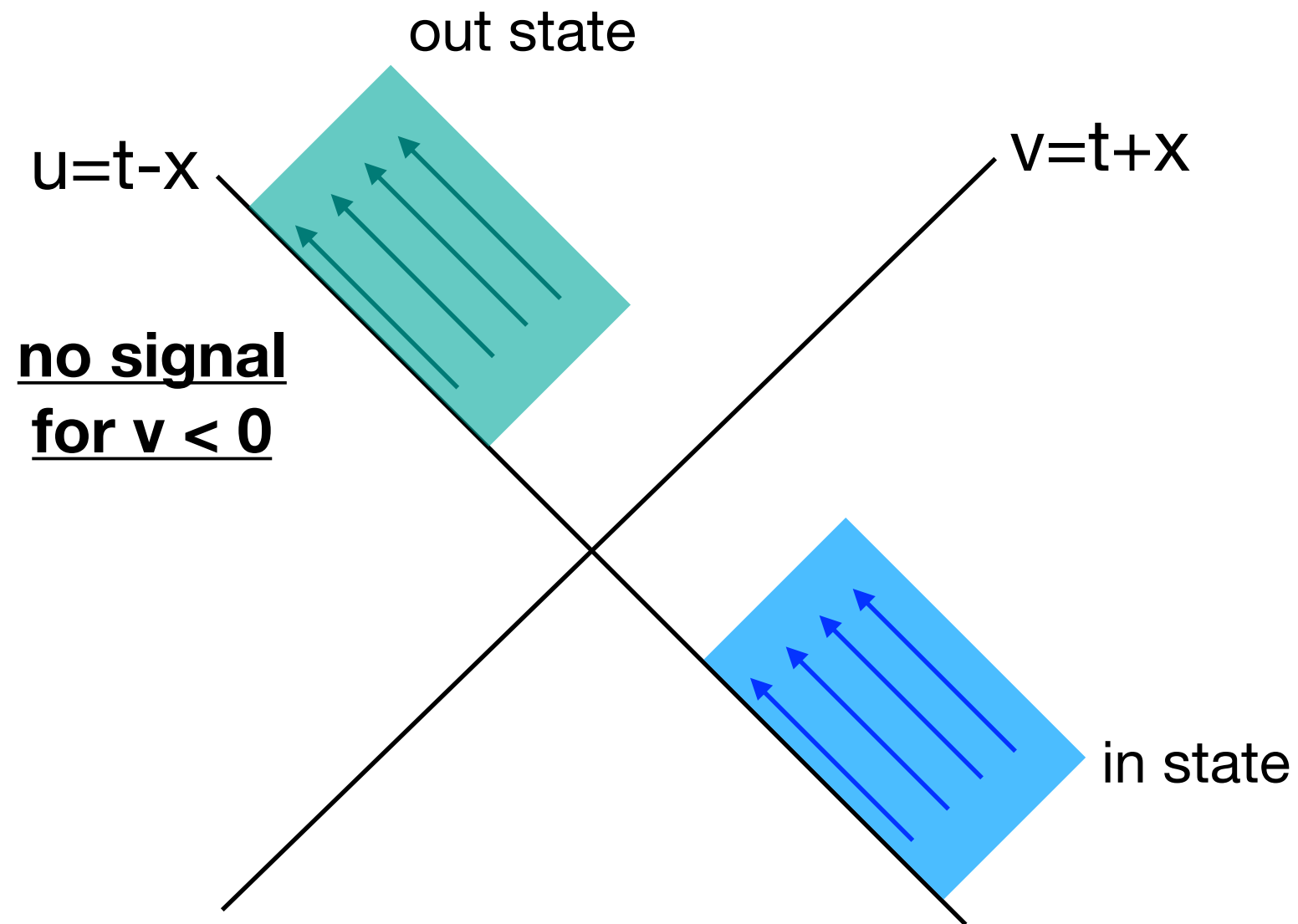
# Causality constraints on the phase-shift

$$\delta(s, \vec{b}) \sim \text{Shapiro time-delay} \geq 0$$

**Camanho, Edelstein,  
Maldacena, Zhiboedov (2014)**

# Causality constraints on the phase-shift

For  $s \gg \frac{1}{b^2}, m_i^2$  an eikonal scattering can be mapped to a signal transmission problem



# Causality constraints on the phase-shift

For  $s \gg \frac{1}{b^2}, m_i^2$

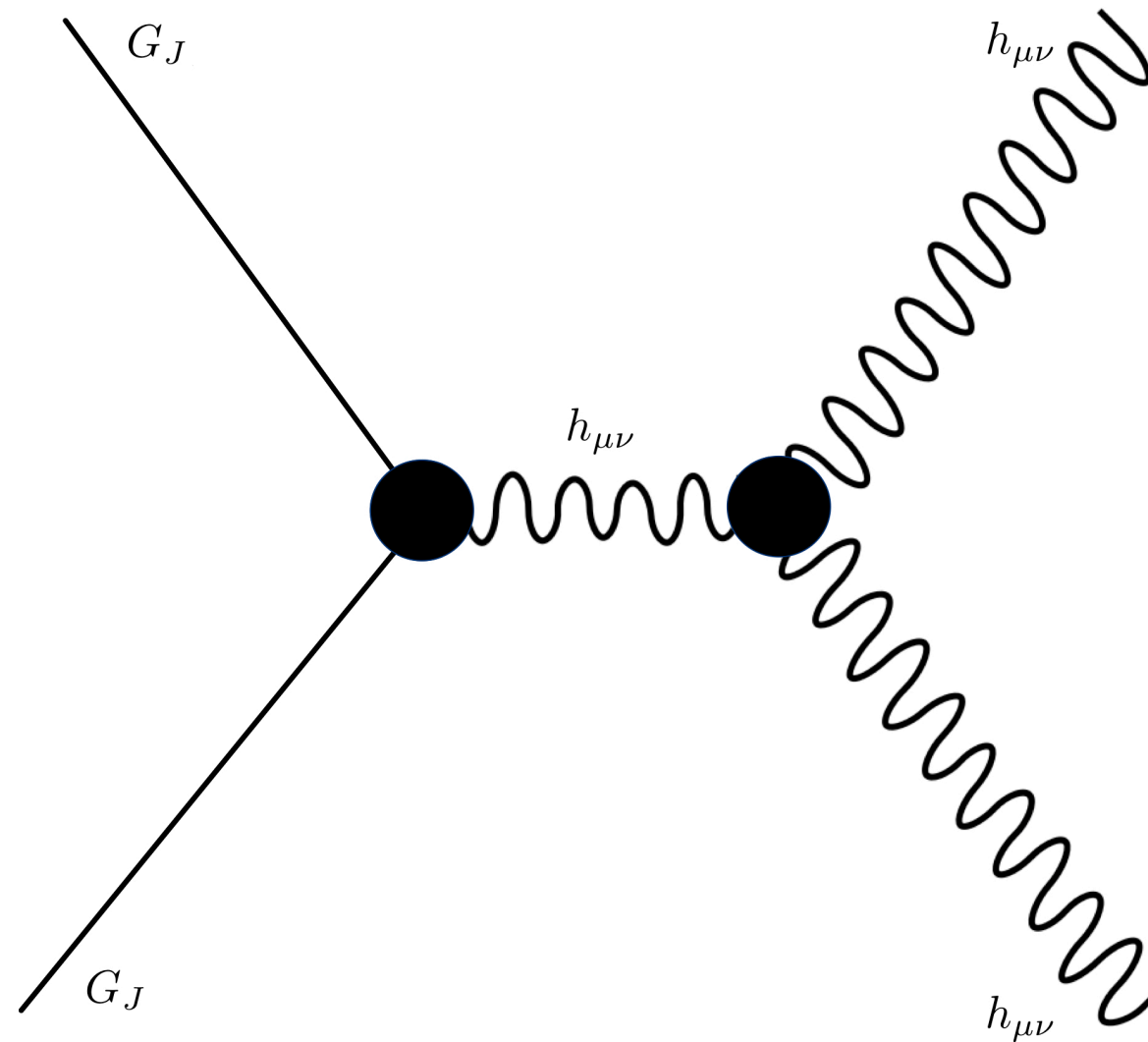
- $\delta(s, \vec{b})$  does not grow faster than  $s$ .
- $\delta(s, \vec{b}) \geq 0$  when it grows linearly with  $s$ .

In AdS, this is related to the chaos bound in the dual CFT.

# Theories of metastable higher-spin particles coupled to gravity in 4d

*Three thought experiments!*

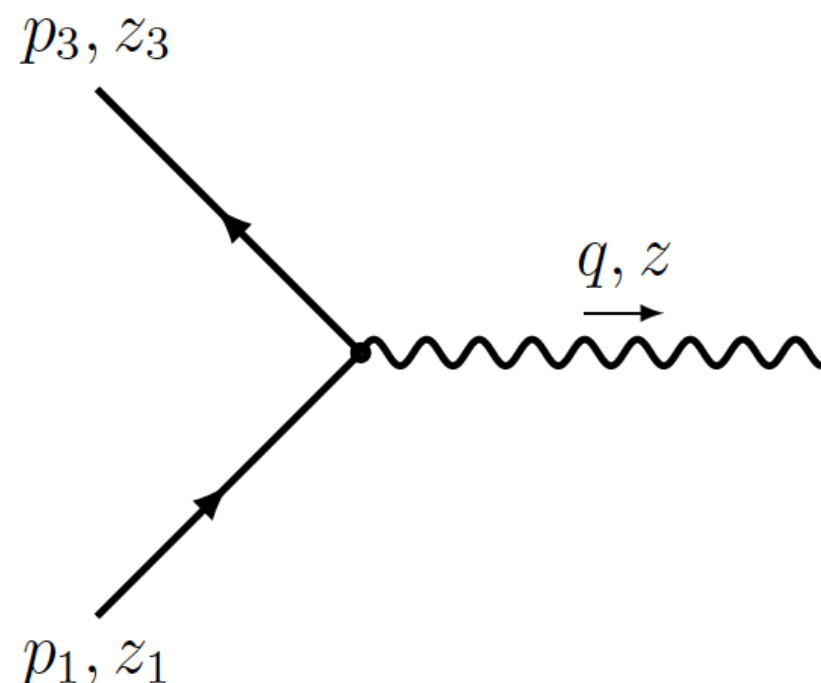
# First thought experiment



$$\delta(s, \vec{b}) = \frac{1}{4\pi s} \Gamma_{hhh}(-i\vec{\partial}_b) \Gamma_{JJh}(-i\vec{\partial}_b) \log\left(\frac{L_{\text{IR}}}{b}\right)$$

on-shell three-point amplitudes

On-shell three-point amplitudes are completely fixed by Lorentz invariance!



$$\Gamma_{JJh} = \frac{2}{M_{\text{pl}}} \sum_{i=1}^{2J+1} a_i \mathcal{A}_i(\vec{q})$$

$$\mathcal{A}_1 = (z \cdot p_3)^2 (z_1 \cdot z_3)^J ,$$

$$\mathcal{A}_2 = (z \cdot p_3)^2 (z_1 \cdot z_3)^{J-1} (z_3 \cdot q)(z_1 \cdot q) ,$$

$\vdots$

$$\mathcal{A}_{J+1} = (z \cdot p_3)^2 (z_3 \cdot q)^J (z_1 \cdot q)^J .$$

$$\mathcal{A}_{J+2} = (z \cdot p_3) ((z \cdot z_3)(z_1 \cdot q) - (z \cdot z_1)(z_3 \cdot q)) (z_1 \cdot z_3)^{J-1} ,$$

$$\mathcal{A}_{J+3} = (z \cdot p_3) ((z \cdot z_3)(z_1 \cdot q) - (z \cdot z_1)(z_3 \cdot q)) (z_1 \cdot z_3)^{J-2} (z_3 \cdot q)(z_1 \cdot q) ,$$

$\vdots$

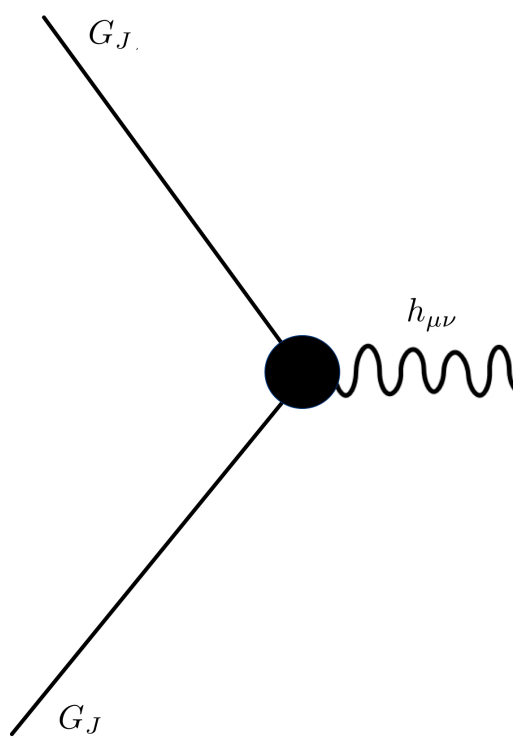
$$\mathcal{A}_{2J+1} = (z \cdot p_3) ((z \cdot z_3)(z_1 \cdot q) - (z \cdot z_1)(z_3 \cdot q)) (z_3 \cdot q)^{J-1} (z_1 \cdot q)^{J-1} .$$

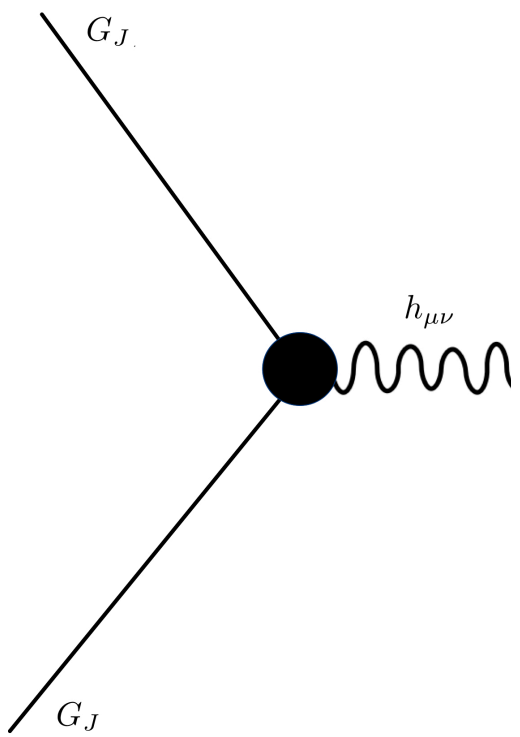


High energy, small impact parameter  $\left(s \gg \frac{1}{b^2} \gg m_J^2\right)$

$$\delta(s, \vec{b}) \geq 0$$




$$= \text{completely fixed} + \frac{1}{M_{\text{pl}}} \mathcal{O}\left(\frac{m_J}{\Lambda_{\text{gr}}}\right)$$



$$= \text{completely fixed} + \frac{1}{M_{\text{pl}}} \mathcal{O} \left( \frac{m_J}{\Lambda_{\text{gr}}} \right)$$

- The unique structure matches with **universal coupling of gravitons to Kerr black holes** in the classical limit.

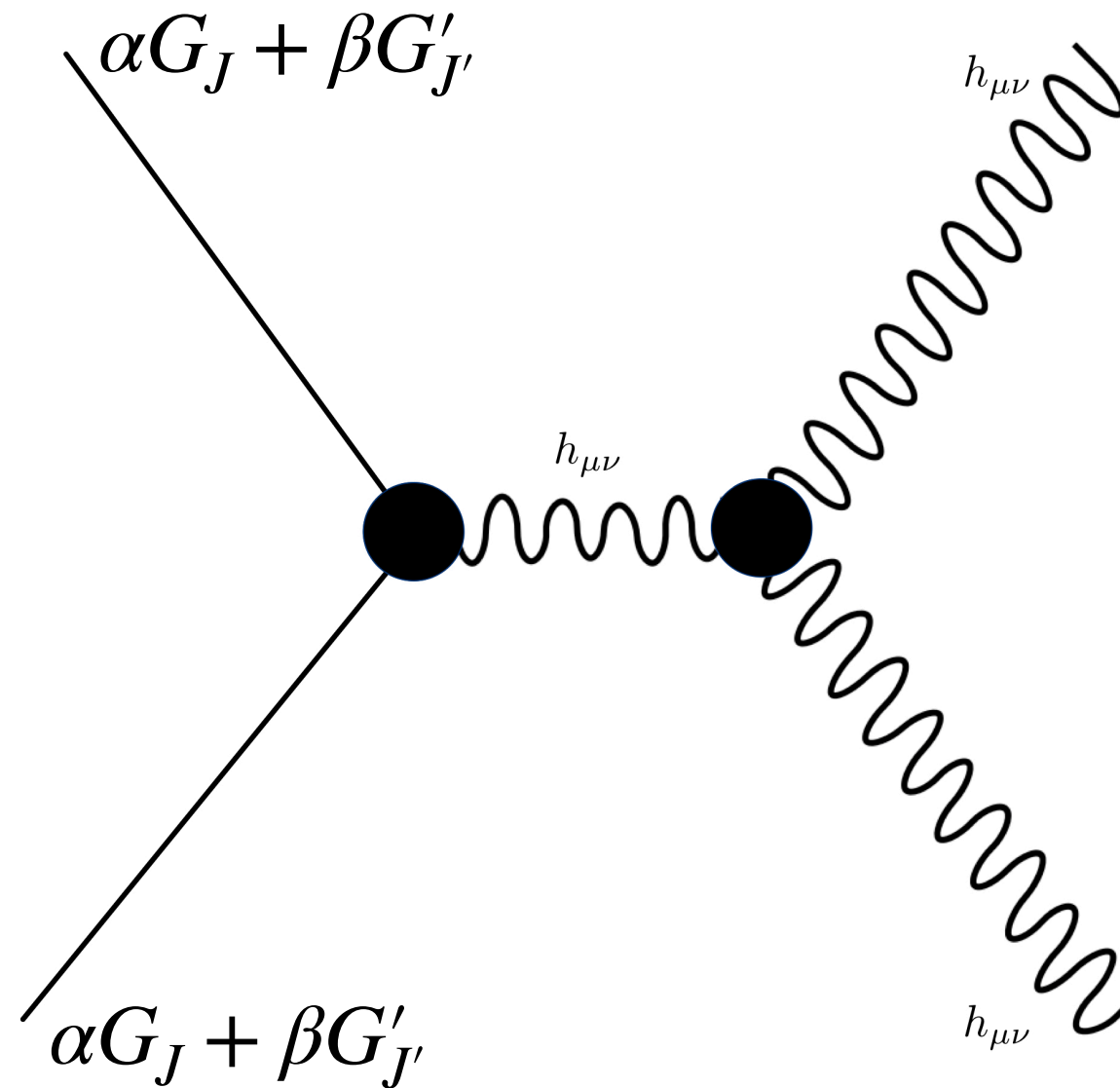
N. Arkani-Hamed, Y.-t. Huang, D. O'Connell (2019)

- $\Lambda_{\text{gr}}$  is the mass of the **lightest higher spin particle in the gravity sector**.
- **Rules out** finite number of elementary higher spin particles.

Afkhami-Jeddi, SK, Tajdini (2018)

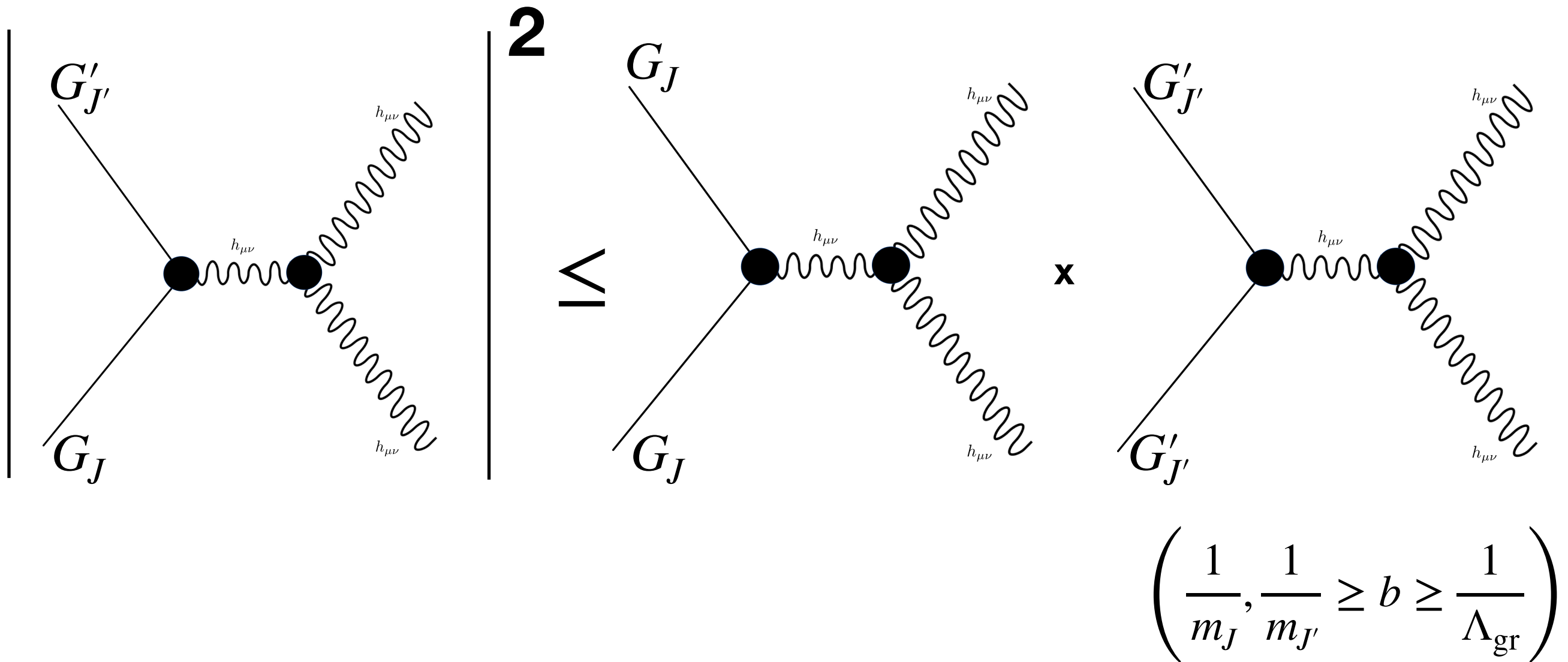
Kaplan, SK (2020)

# Second thought experiment



$$\delta(s, \vec{b}) \geq 0$$

# A bound on the graviton induced mixing



2

$\leq$

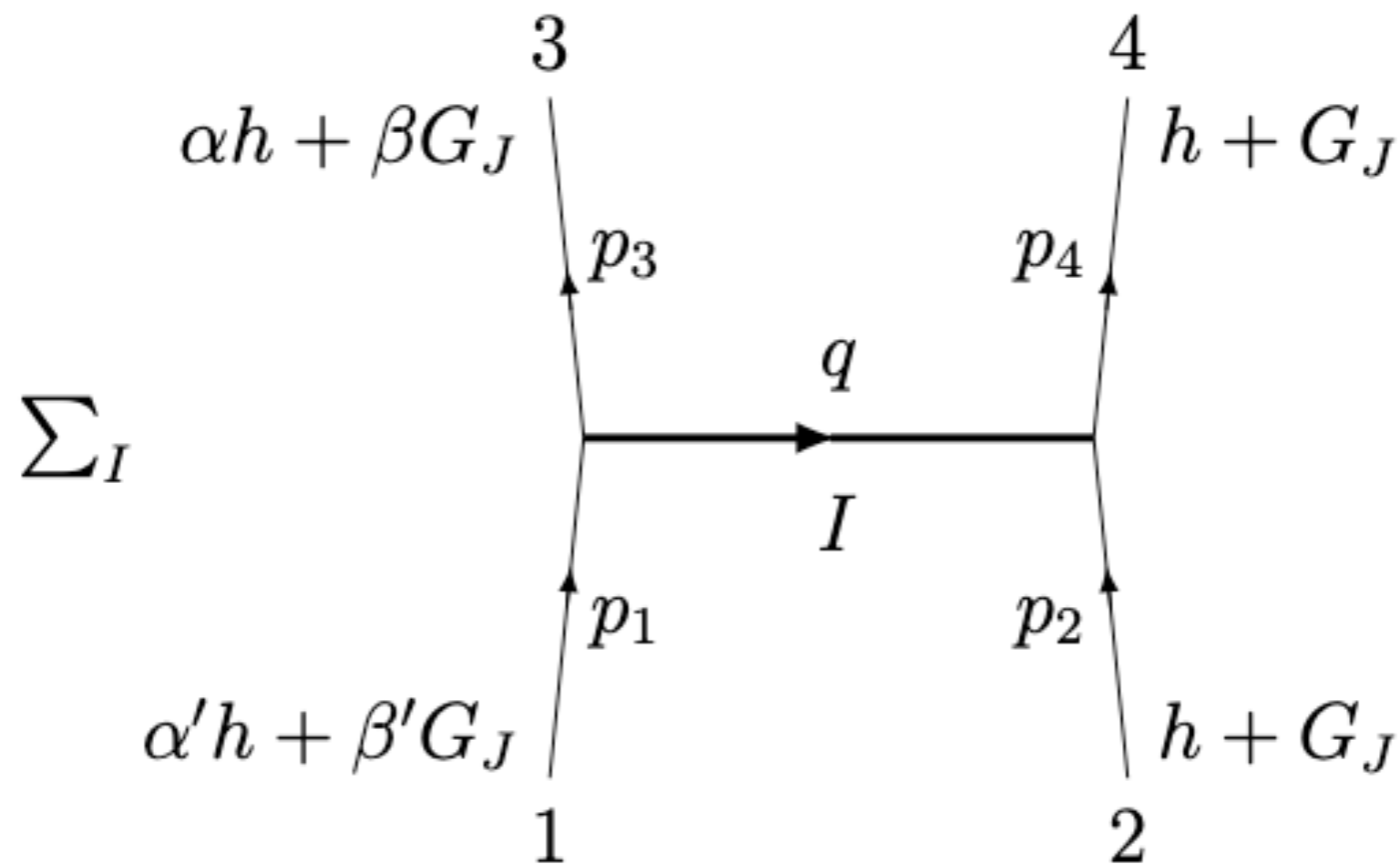
$\times$

$\left( \frac{1}{m_J}, \frac{1}{m_{J'}} \geq b \geq \frac{1}{\Lambda_{\text{gr}}} \right)$

This is consistent with the graviton soft theorem if and only if

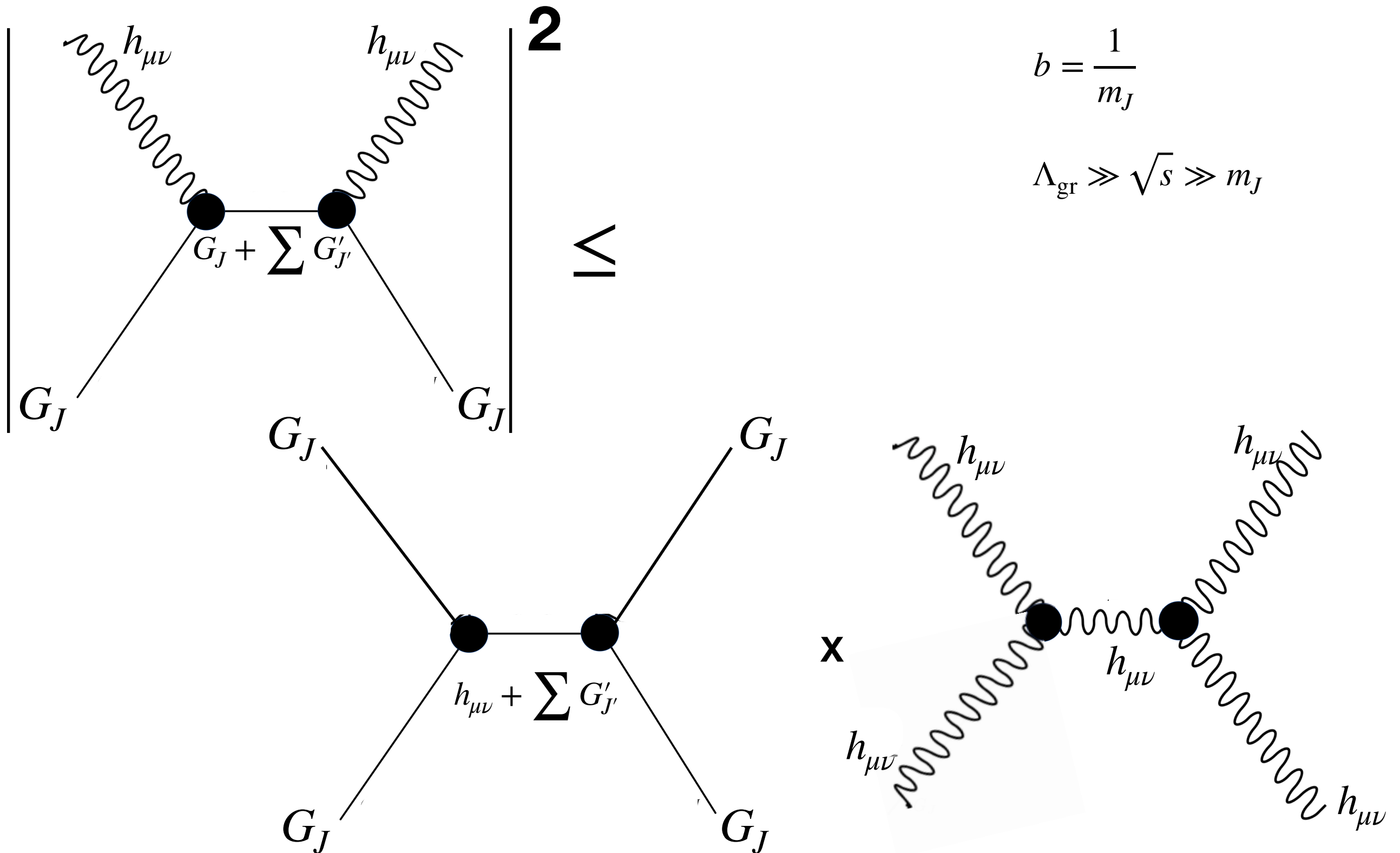
$$|\mathcal{A}(G_J G_{J'} h_{\mu\nu})| \lesssim \frac{1}{M_{\text{pl}}} \frac{\ln(\Lambda_{\text{gr}} L_{\text{IR}})}{\Lambda_{\text{gr}}^n} \quad n \geq 1$$

# Third thought experiment



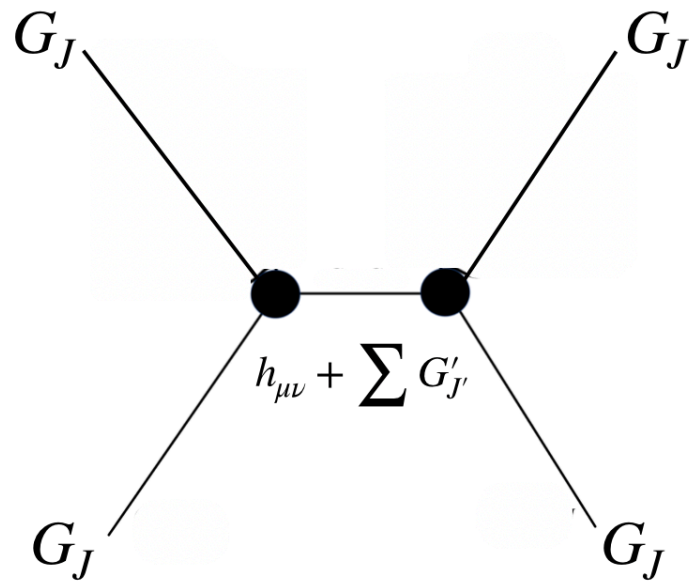
Scattering of coherent states created by superposition of higher spin particles and and gravitons.

# Another bound from causality

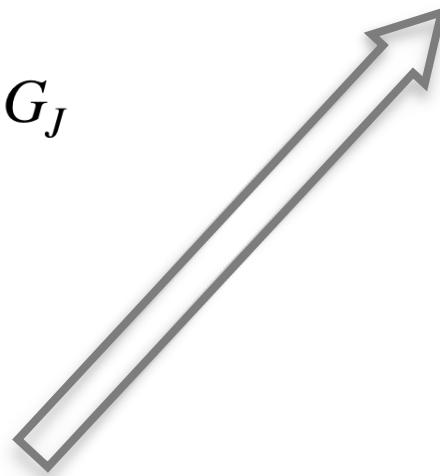


$$b = \frac{1}{m_J}$$

$$\Lambda_{\text{gr}} \gg \sqrt{s} \gg m_J$$



$$\approx \frac{4s}{M_{\text{pl}}^2} \ln \left( \frac{L_{\text{IR}}}{b} \right) + g_J^2 \left( \frac{s}{m_J^2} \right)^{a_J}$$

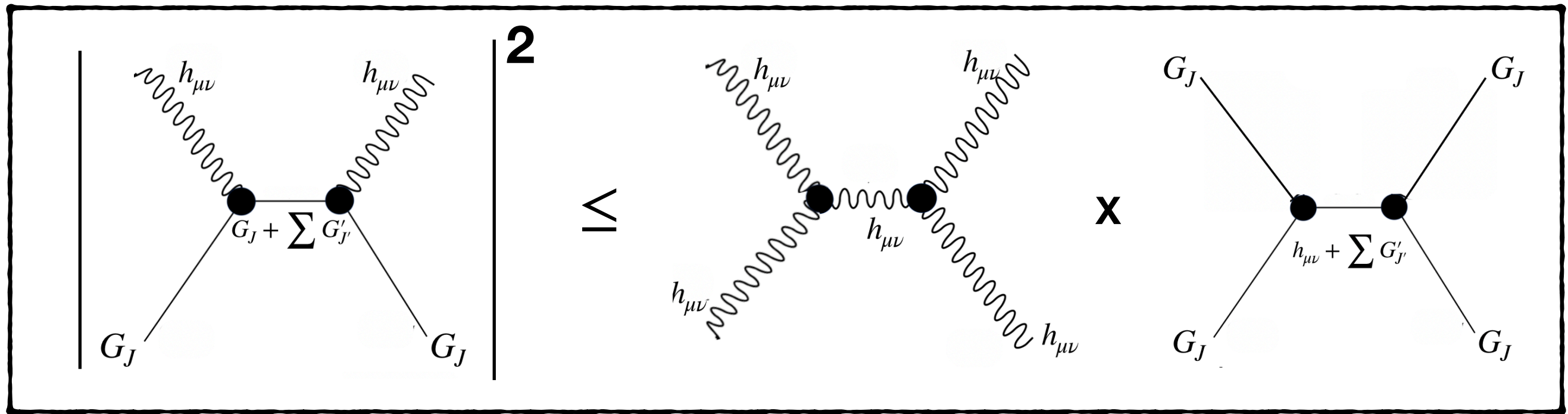


gravitational part  
 $A_{\text{gr}}(G_J G_J \rightarrow G_J G_J)$



non-gravitational part  
 $A_{\text{non-gr}}(G_J G_J \rightarrow G_J G_J)$

# Bounds on the gravity sector



$$J \geq 3$$



$$A_{\text{gr}}(G_J G_J \rightarrow G_J G_J) \gg A_{\text{non-gr}}(G_J G_J \rightarrow G_J G_J)$$

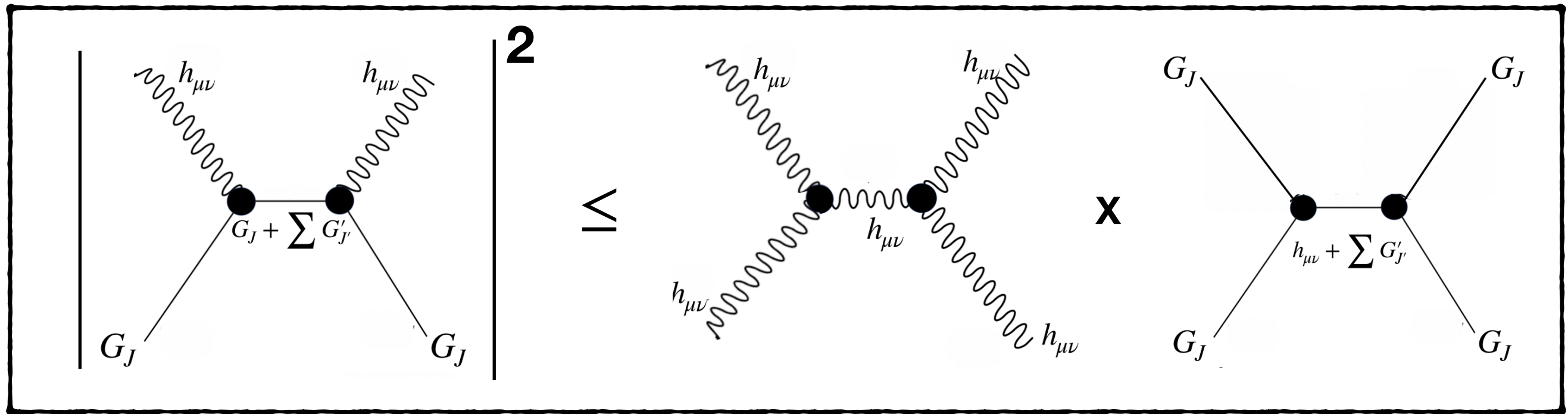
$$\Lambda_{\text{gr}} \lesssim m_J$$

gravity sector

**No QFT description!**



# Bounds on the gravity sector



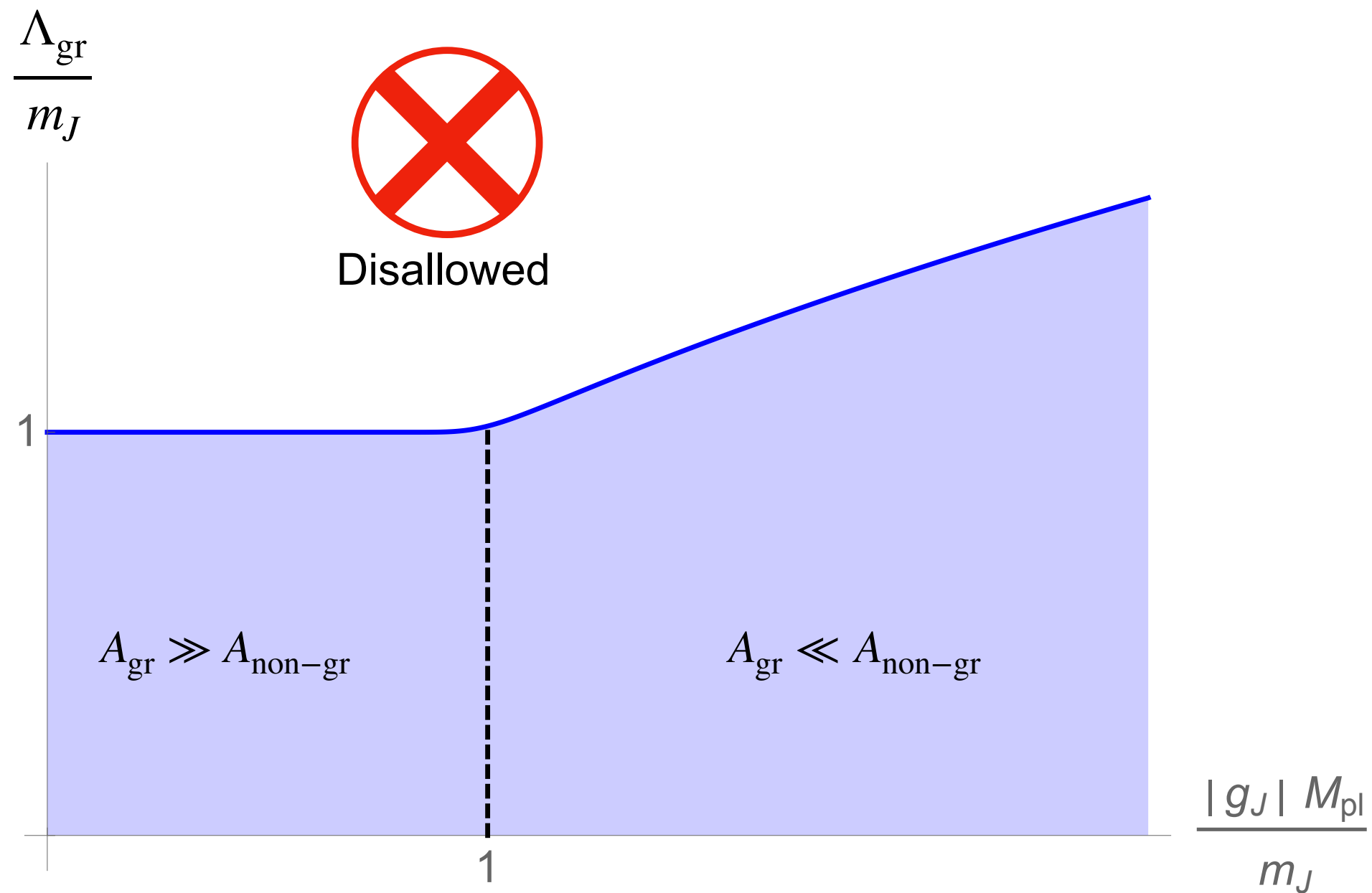
$$J \geq 3$$



$$A_{\text{gr}}(G_J G_J \rightarrow G_J G_J) \ll A_{\text{non-gr}}(G_J G_J \rightarrow G_J G_J)$$

$$\Lambda_{\text{gr}} \lesssim m_J \left( \frac{|g_J| M_{\text{pl}}}{m_J} \right)^{\frac{1}{2(J-2)}} \ll M_{\text{pl}}$$

# Bound on $\Lambda_{\text{gr}}$



# Summary

Higher spin ( $J > 2$ ) metastable particles cannot couple to gravity while preserving causality unless there exist higher spin states in the gravity sector much below the Planck scale.

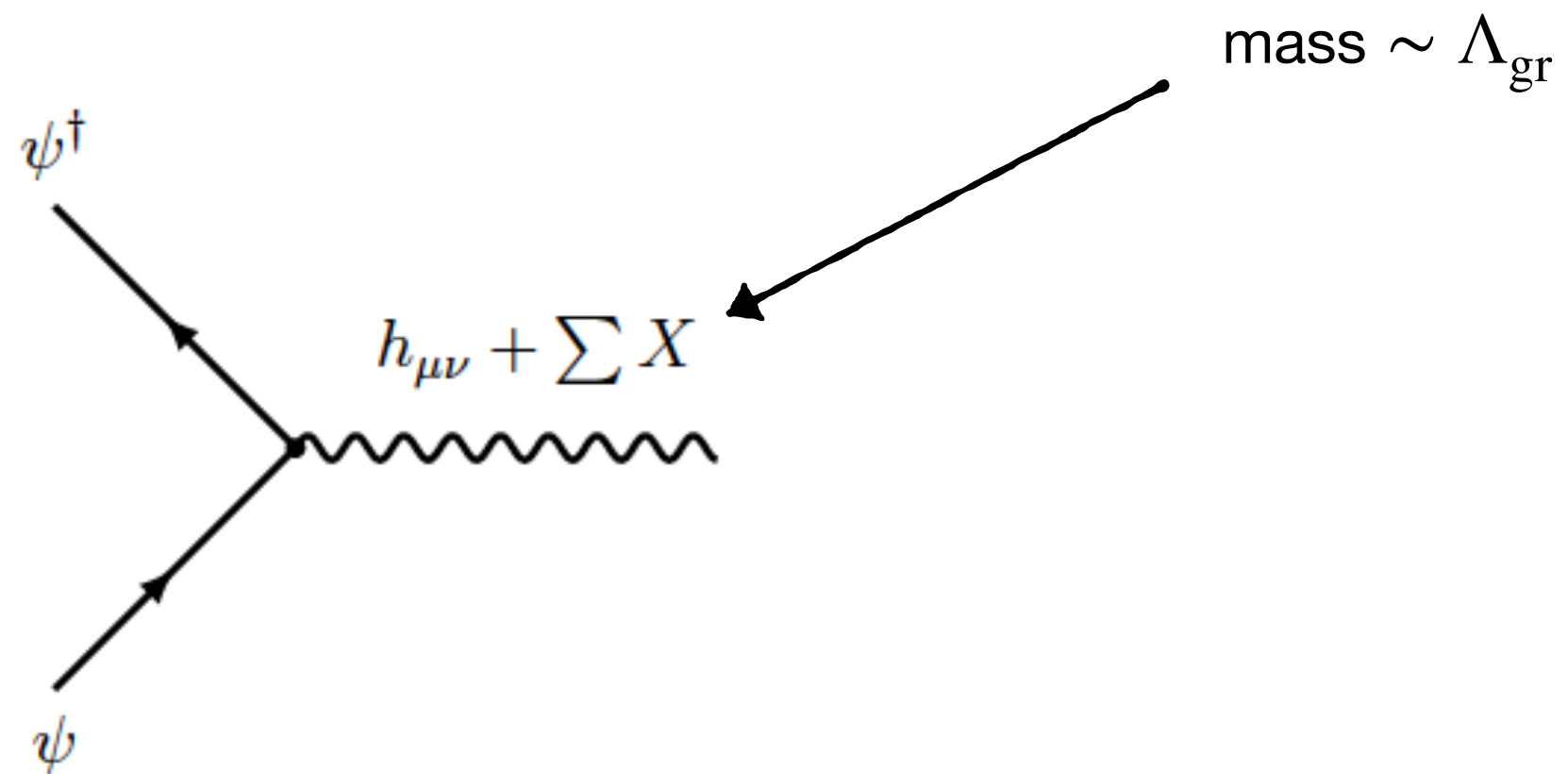
**Underlying UV complete theory**



**?**

**Metastable higher spin particles  
coupled to gravity**

# Gravitational scattering in the full theory



# CKSZ uniqueness theorem

arXiv.org > hep-th > arXiv:1607.04253

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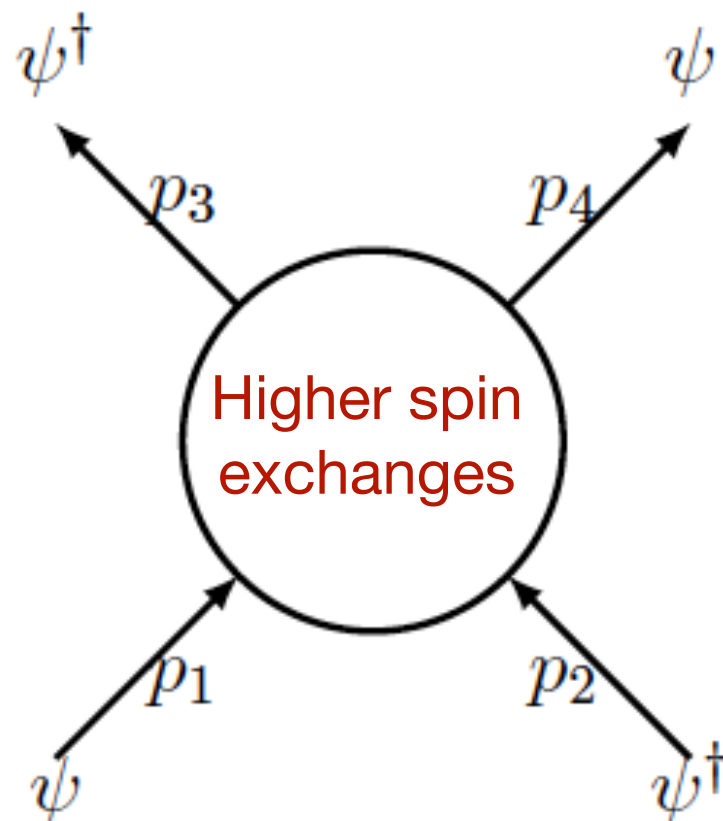
High Energy Physics – Theory

[Submitted on 14 Jul 2016]

## Strings from Massive Higher Spins: The Asymptotic Uniqueness of the Veneziano Amplitude

Simon Caron-Huot, Zohar Komargodski, Amit Sever, Alexander Zhiboedov

We consider weakly-coupled theories of massive higher-spin particles. This class of models includes, for instance, tree-level String Theory and Large- $N$  Yang-Mills theory. The  $S$ -matrix in such theories is a meromorphic function obeying unitarity and crossing symmetry. We discuss the (unphysical) regime  $s, t \gg 1$ , in which we expect the amplitude to be universal and exponentially large. We develop methods to study this regime and show that the amplitude necessarily coincides with the Veneziano amplitude there. In particular, this implies that the leading Regge trajectory,  $j(t)$ , is asymptotically linear in Yang-Mills theory. Further, our analysis shows that any such theory of higher-spin particles has stringy excitations and infinitely many asymptotically parallel subleading trajectories. More generally, we argue that, under some assumptions, any theory with at least one higher-spin particle must have strings.



**Asymptotic uniqueness**

- Must be an infinite tower of higher spin states
- Amplitude is unique for  $t, s \gg 1$
- Coincide with the tree-level string theory

# Conditions on the S-matrix of the gravity sector

1. **Weak coupling** —  $A_{\text{gravity}}(s, t)$  is a meromorphic function with simple poles
2. **Unitarity** — Positive expansion
3. **Crossing symmetry**
4. **Regge behavior** —  $A_{\text{gravity}}(s, t) = F(t)(-s)^{j(t)}$  holds for  $|s|, |t| \gg m^2$
5. **No accumulation point in the spectrum**

# Emergence Closed Strings

Gravity sector of the full theory must have the following properties:

**Asymptotically Linear leading Regge trajectory**

$$j(t \rightarrow \infty) = \frac{\alpha'}{2}t$$

$$\alpha' \sim \frac{1}{\Lambda_{\text{gr}}^2}$$



# Emergence Closed Strings

Gravity sector of the full theory must have the following properties:

**Large impact parameter scattering**

$$\text{Im } A_{\text{gravity}}(b, s) \approx e^{-\frac{b^2}{2\alpha' \log(\alpha' s/2)}} \quad \alpha' \sim \frac{1}{\Lambda_{\text{gr}}^2}$$

Existence of **extended objects** of size  $\sqrt{\alpha' \log(\alpha' s)}$  !

# Emergence Closed Strings

Gravity sector of the full theory must have the following properties:

**Amplitude in the regime  $t, s \gg 1$**

$$\lim_{s, t \gg 1} A_{\text{gravity}}(s, t) = A_0 \exp \left( \frac{\alpha'}{2} ((s + t) \ln(s + t) - s \ln s - t \ln t) \right)$$

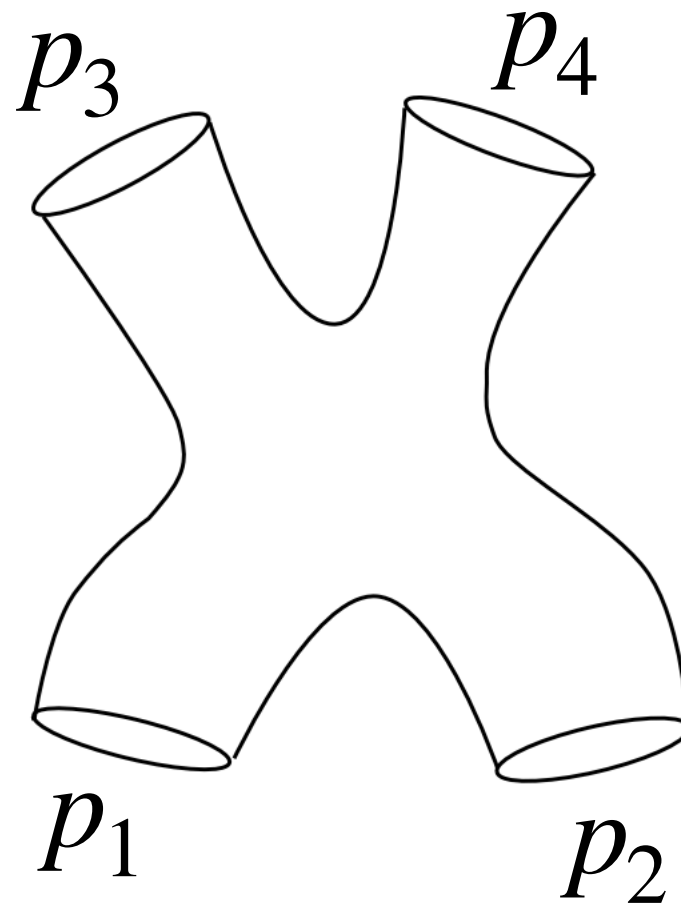
# Emergence Closed Strings

Gravity sector of the full theory must have the following properties:

Amplitude in the regime  $t, s \gg 1$

$$\lim_{s, t \gg 1} A_{\text{gravity}}(s, t) =$$

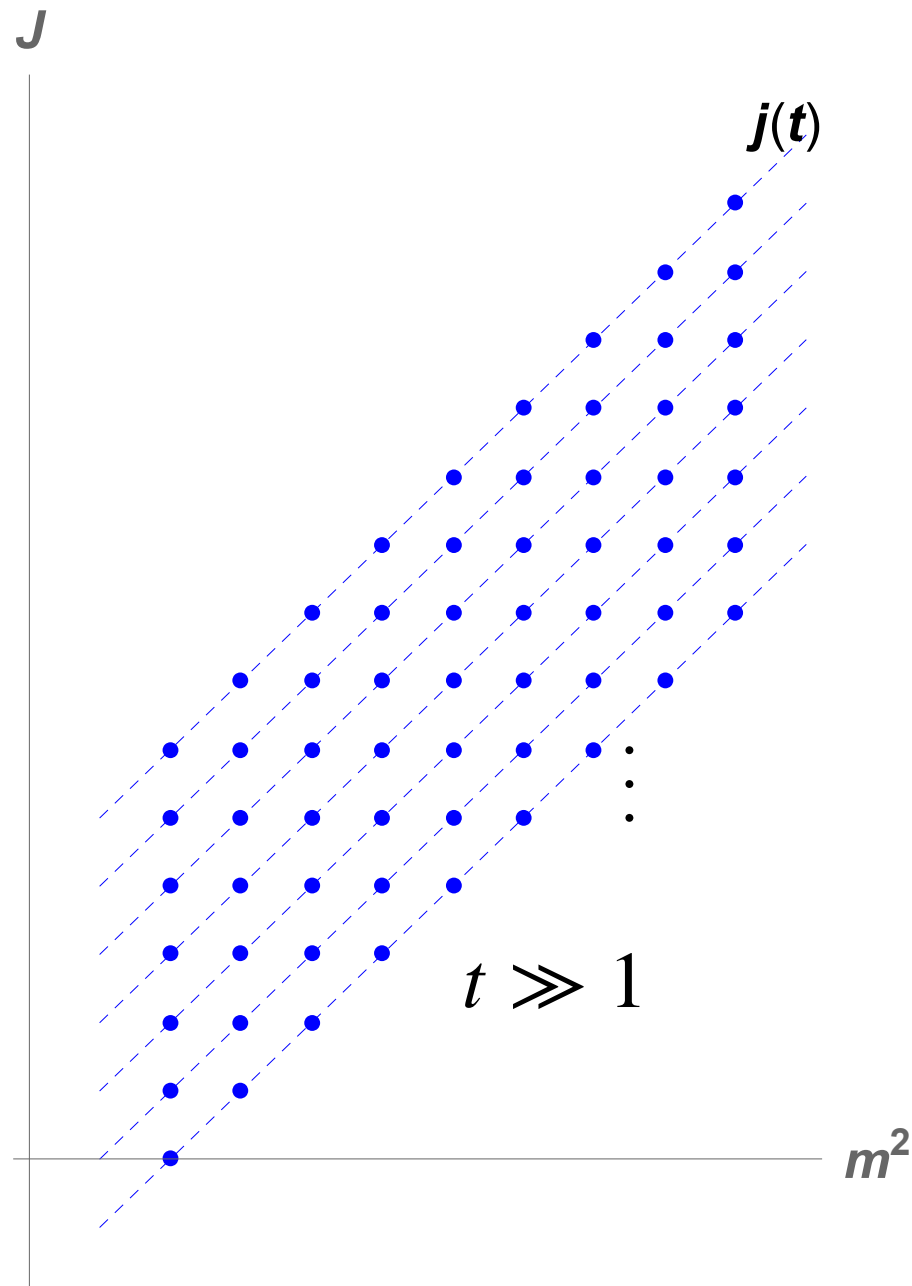
$$|t|, |s| \rightarrow \infty$$



Tree-level Gross-Mende string amplitude! Gross, Mende (1987)

# Emergence Closed Strings

Gravity sector of the full theory must have the following properties:

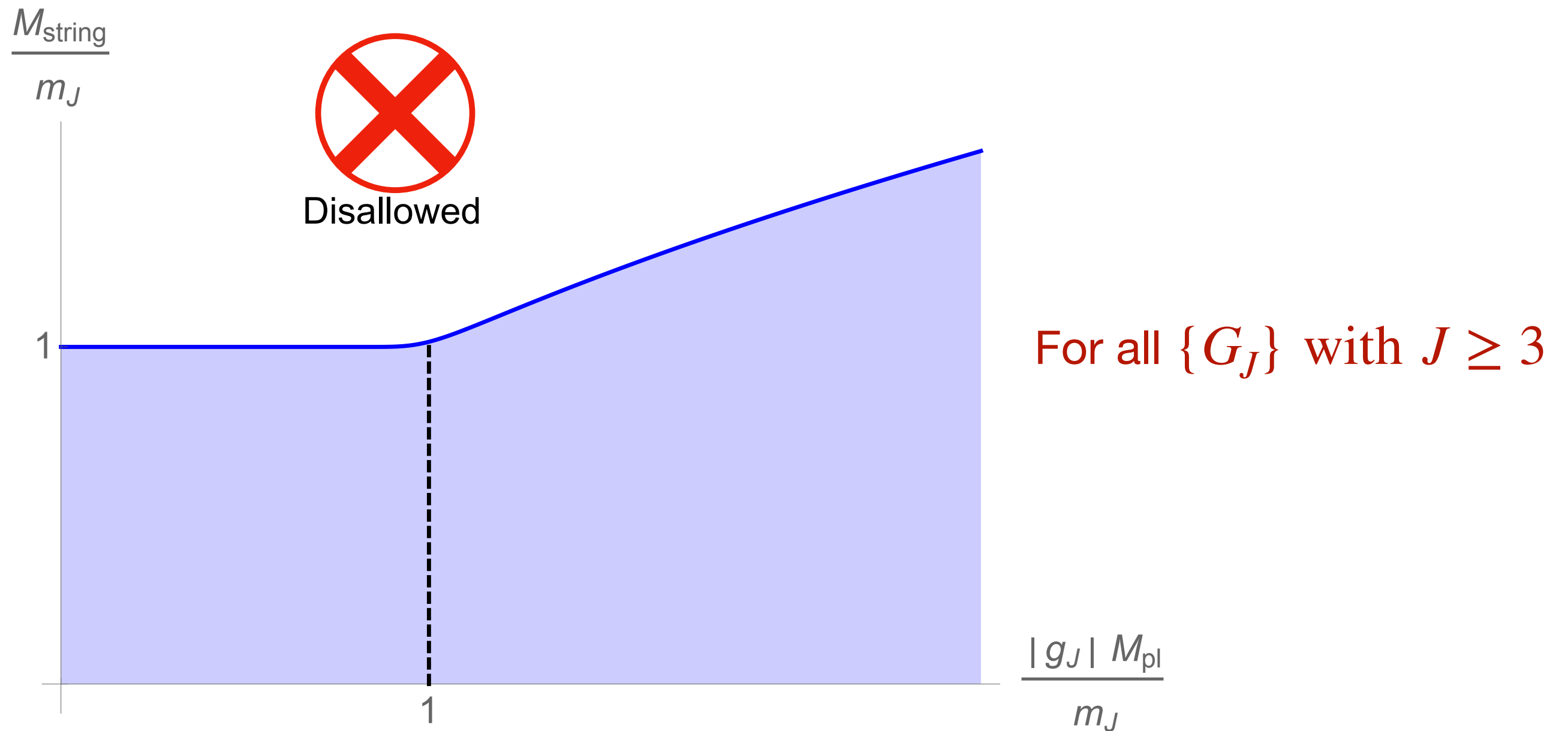


Infinite towers of **Regge trajectories**

Asymptotically coincide with tree-level closed string theory

# A bound on $M_{\text{string}}$

$$\Lambda_{\text{gr}} \approx M_{\text{string}}$$



# Theories of metastable higher spin particles coupled to gravity

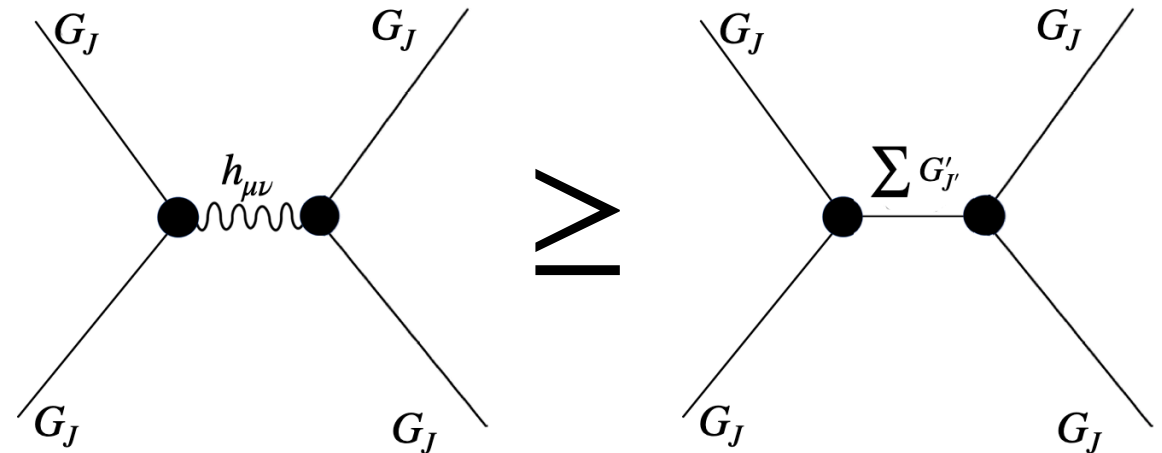
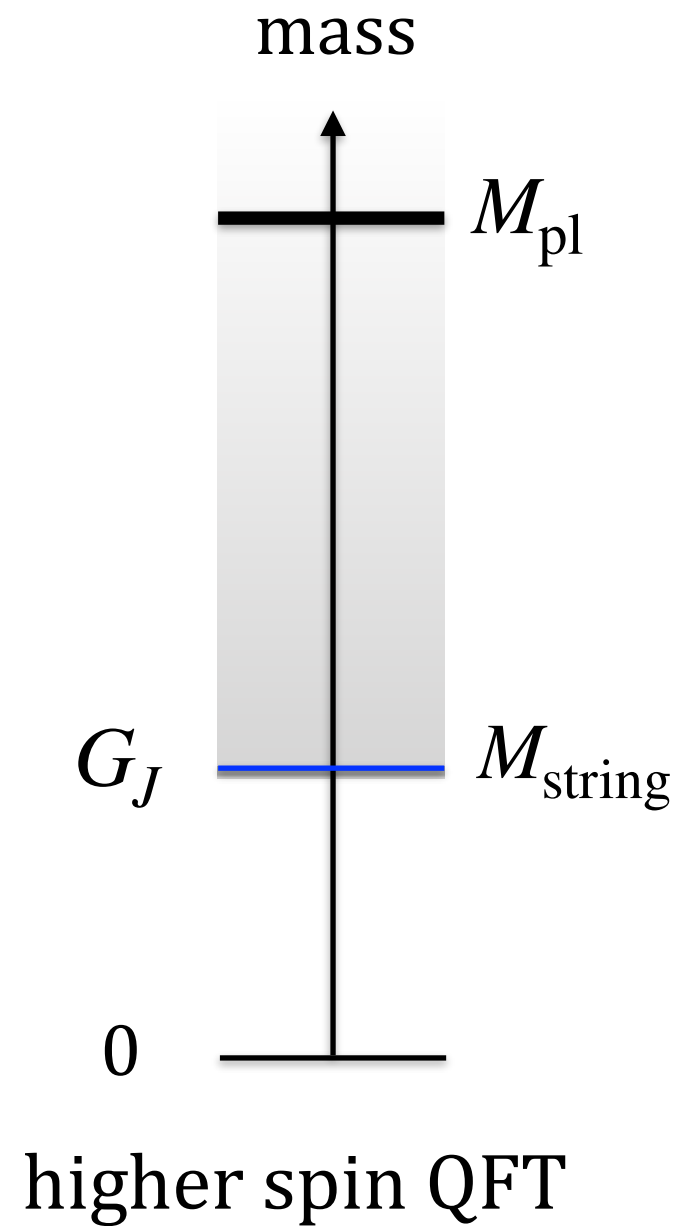
Additional gravitational poles are essential to the preservation of causality.



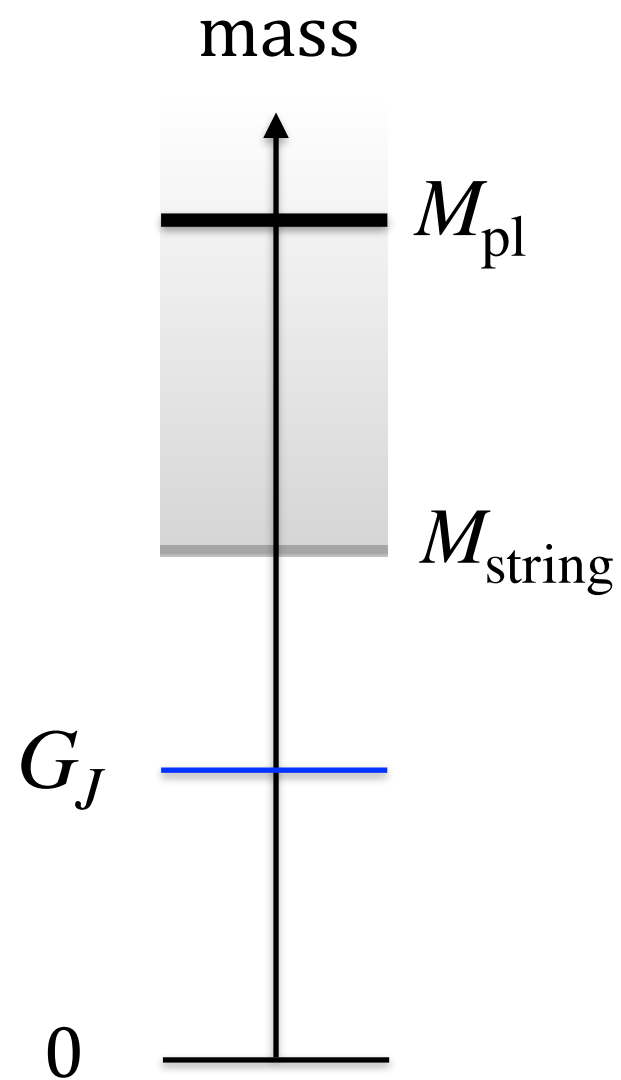
***CKSZ theorem***

Any weakly coupled UV completion of such a theory must have a **stringy gravity sector**.

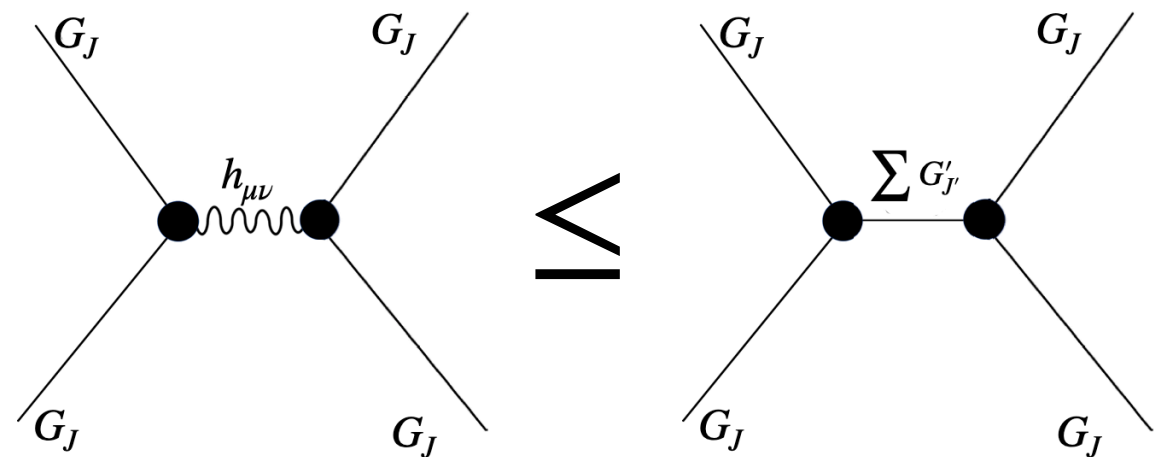
# A weak gravity condition in 4d



# A weak gravity condition in 4d



higher spin QFT



**Necessary condition!**

Not true in  $d > 4$ !



# What about **large N QCD**?

Large N confining gauge theories in 4d

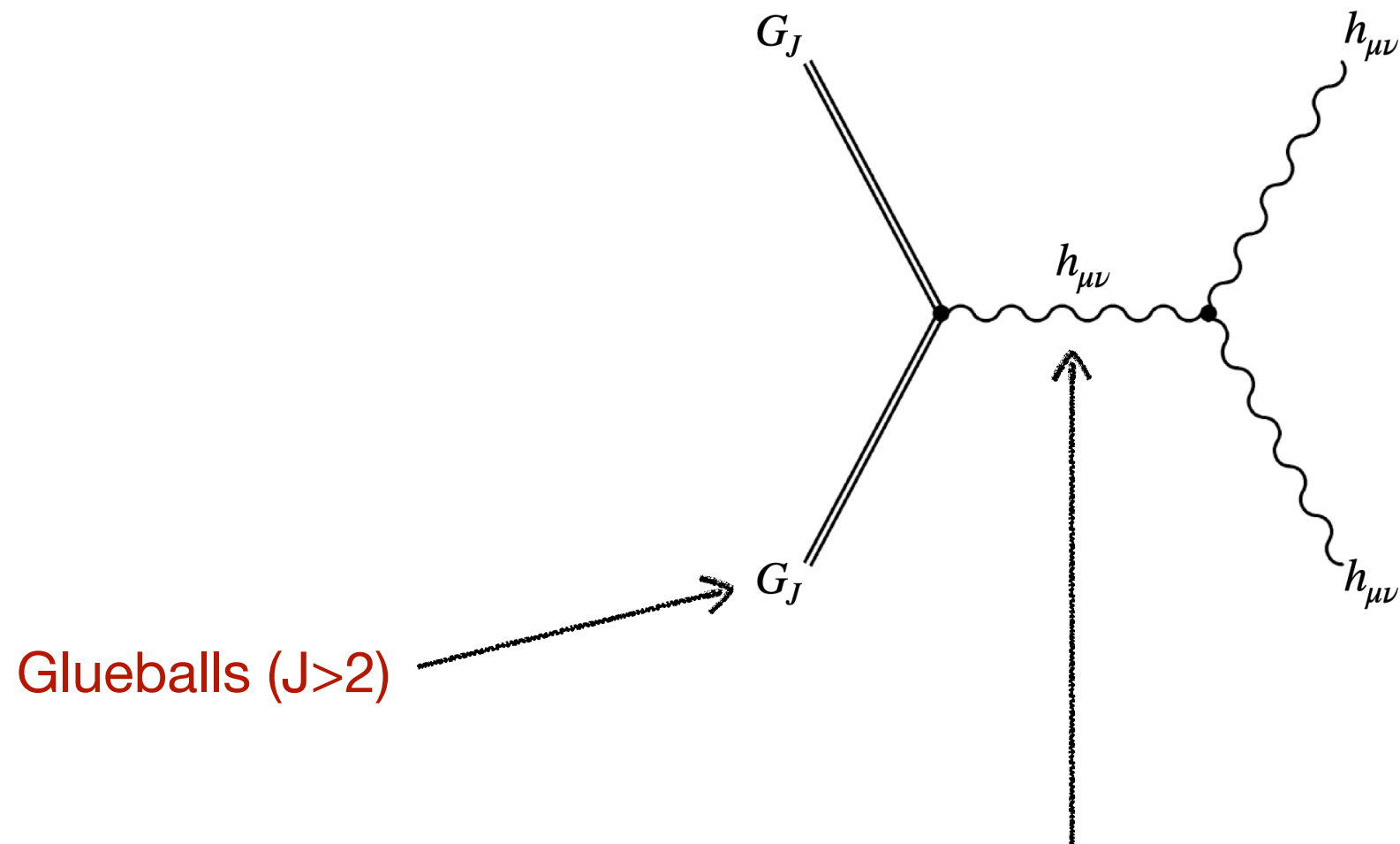


Metastable glueballs and mesons of arbitrarily large spin

Large N QCD coupled to gravity is constrained!

# Causality Constraints in Large $N$ QCD Coupled to Gravity

Jared Kaplan<sup>1,\*</sup> and Sandipan Kundu<sup>1,†</sup>



Additional t-channel poles are essential to the preservation of causality!

# Causality Constraints in Large $N$ QCD Coupled to Gravity

Jared Kaplan<sup>1,\*</sup> and Sandipan Kundu<sup>1,†</sup>

**gauge sector**

$$\langle G_J h_{\mu\nu} \rangle \sim \frac{N}{M_{\text{pl}}} , \quad \langle G_J h_{\mu_1\nu_1} h_{\mu_2\nu_2} \rangle \sim \frac{N}{M_{\text{pl}}^2} ,$$

**gravity sector**

$$M_{\text{string}} \lesssim \sqrt{\frac{M_{\text{pl}} \Lambda_{\text{QCD}}}{N}}$$

# Large N confining gauge theories coupled to gravity

For weakly coupled gravity, both scenarios imply that the theory has a stringy description above

$$N \gtrsim \frac{M_{\text{pl}}}{\Lambda_{\text{QCD}}}$$

# Conclusions

- QFTs that appear to be causal in flat spacetime can violate **causality when coupled to gravity**.
- Some features of **quantum gravity emerges** from QFT.
- Theories of metastable higher spin particles can be coupled to gravity while preserving causality only when the gravity sector has many of the **properties of closed strings**.
- Metastable higher spin particles in 4d with masses below the string scale **must satisfy a weak gravity condition**.
- These bounds also have surprising implications for **large N QCD coupled to gravity**.

*Thank You!*