



16th April 2025
ITMP Seminar Series

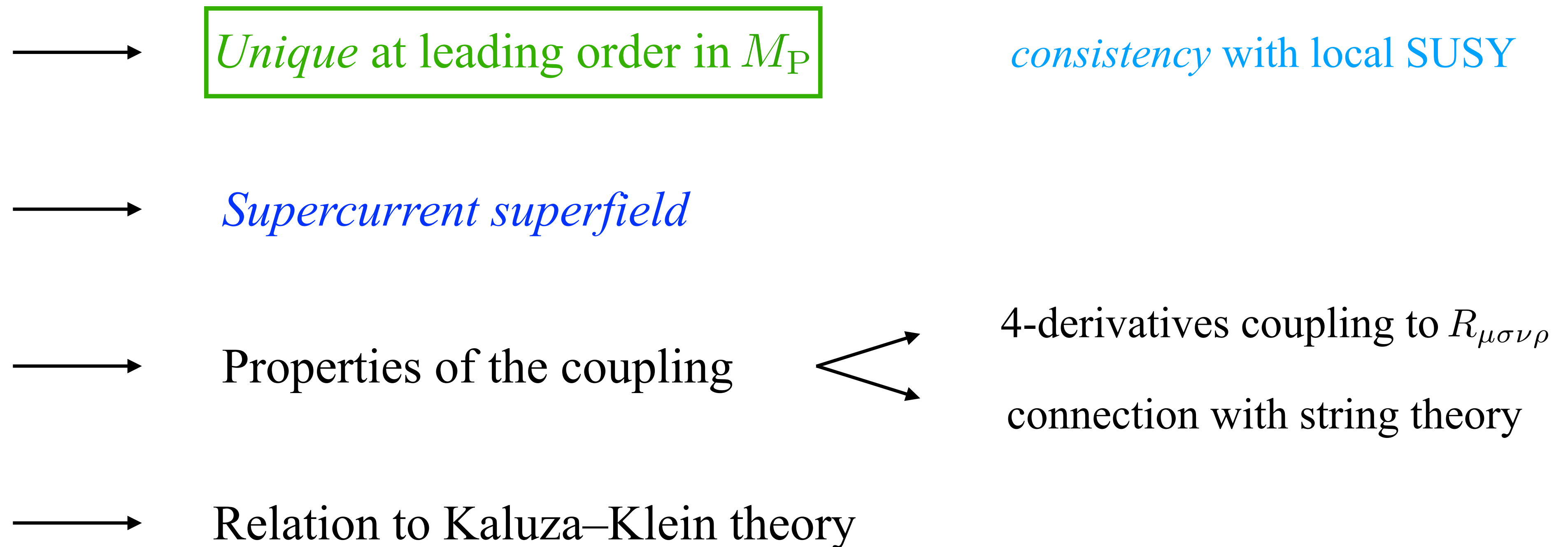
A unique coupling of the massive spin-2 field to supergravity

GABRIELE CASAGRANDE

based on **2502.09599** with G. Bossard, E. Dudas and A. Loty

Summary & outline

- massive spin-2 supermultiplet + $D = 4, \mathcal{N} = 1$ SUGRA



The massive spin-2 field & supersymmetry

The massive spin-2 field

- Fierz-Pauli lagrangian:

$$\mathcal{L}_{\text{FP}} = \frac{1}{4} \partial_\mu h_{\nu\rho} \partial^\nu h^{\mu\rho} - \frac{1}{8} \partial_\rho h_{\mu\nu} \partial^\rho h^{\mu\nu} - \frac{1}{4} \partial_\mu h^{\mu\nu} \partial_\nu h + \frac{1}{8} \partial_\mu h \partial^\mu h - \frac{m^2}{8} (h^{\mu\nu} h_{\mu\nu} - h^2)$$

ghost-free

- Equations of motion & constraints:

$$\square h_{\mu\nu} - 2\partial_\rho \partial_{(\mu} h^{\rho}_{\nu)} + \eta_{\mu\nu} \partial_\rho \partial_\sigma h^{\rho\sigma} + \partial_\mu \partial_\nu h - \eta_{\mu\nu} \square h - m^2 (h_{\mu\nu} - \eta_{\mu\nu} h) = 0$$

The massive spin-2 field

- Fierz-Pauli lagrangian:

$$\mathcal{L}_{\text{FP}} = \frac{1}{4} \partial_\mu h_{\nu\rho} \partial^\nu h^{\mu\rho} - \frac{1}{8} \partial_\rho h_{\mu\nu} \partial^\rho h^{\mu\nu} - \frac{1}{4} \partial_\mu h^{\mu\nu} \partial_\nu h + \frac{1}{8} \partial_\mu h \partial^\mu h - \frac{m^2}{8} (h^{\mu\nu} h_{\mu\nu} - h^2)$$

ghost-free

- Equations of motion & constraints:

$$\square h_{\mu\nu} = m^2 h_{\mu\nu} \qquad \partial^\mu h_{\mu\nu} = 0 \qquad h^\mu{}_\mu \equiv h = 0$$

The massive spin-2 multiplet

- Massive spin-2 *supermultiplet*: $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$ [ZINOVIEV '02]
[BUCHBINDER, GATES, LINCH, PHILLIPS '02]

$$\delta A_\mu = \frac{1}{2} \bar{\epsilon} \lambda_\mu$$

$$\delta \lambda_\mu = -\frac{i}{4} \left(m h_{\mu\rho} + \tilde{F}_{\mu\rho} + 2i \partial_\rho A_\mu \gamma_5 \right) \gamma_5 \gamma^\rho \epsilon$$

$$\delta h_{\mu\nu} = \frac{i}{m} \bar{\epsilon} \gamma_5 \left(\partial_{(\mu} \lambda_{\nu)} - m \gamma_{(\mu} \chi_{\nu)} \right)$$

$$\delta \chi_\mu = \frac{i}{4} \partial_\rho h_{\mu\sigma} \gamma^{\rho\sigma} \gamma_5 \epsilon + \frac{1}{8m} \partial_\mu F_{\rho\sigma} \gamma^{\rho\sigma} \epsilon + \frac{m}{4} A_\rho \left(2\delta_\mu^\rho + \gamma^\rho{}_\mu \right) \epsilon$$

The massive spin-2 multiplet

- Massive spin-2 *supermultiplet*: $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$ [ZINOVIEV '02]
[BUCHBINDER, GATES, LINCH, PHILLIPS '02]

- Supersymmetric Fierz–Pauli lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{SFP}} = & \frac{1}{4}\partial_\mu h_{\nu\rho}\partial^\nu h^{\mu\rho} - \frac{1}{8}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} - \frac{1}{4}\partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{8}\partial_\mu h\partial^\mu h - \frac{m^2}{8}(h^{\mu\nu}h_{\mu\nu} - h^2) \\ & - \frac{3}{8}F^{\mu\nu}F_{\mu\nu} - \frac{3}{4}m^2 A^\mu A_\mu - \frac{1}{2}\bar{\lambda}_\mu\gamma^{\mu\nu\rho}\partial_\nu\lambda_\rho - \frac{1}{2}\bar{\chi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\chi_\rho - m\bar{\lambda}_\mu\gamma^{\mu\nu}\chi_\nu\end{aligned}$$

The massive spin-2 multiplet

- Massive spin-2 *supermultiplet*: $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$ [ZINOVIEV '02]
[BUCHBINDER, GATES, LINCH, PHILLIPS '02]

- Coupling to supergravity:

$$\{g_{\mu\nu}, \psi_\mu\} + \{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\} \longleftrightarrow T^{\mu\nu} \text{ \& \; } \Xi^\mu$$

$$\longrightarrow \boxed{\textit{Supercurrent superfield } S^\mu}$$

The supercurrent superfield

The supercurrent superfield

- Multiplets of currents: $\mathcal{S}^\mu = \{J^\mu, \Xi^\mu, T^{\mu\nu}, X\}$ [FERRARA, ZUMINO '74] [SOHNUS, WEST '81]
[KOMARGODSKI, SEIBERG '10]

$$\mathcal{S}^\mu = \bar{\mathcal{S}}^\mu \quad \& \quad D^2 \mathcal{S}^\mu = 0 \qquad \bar{D}^{\dot{\alpha}} \mathcal{S}_{\alpha\dot{\alpha}} = C_\alpha \sim \gamma^\mu \Xi_\mu, \quad T^\mu{}_\mu, \quad X$$

linear multiplet

conservation-like equation

The supercurrent superfield

- Multiplets of currents: $\mathcal{S}^\mu = \{J^\mu, \Xi^\mu, T^{\mu\nu}, X\}$ [FERRARA, ZUMINO '74] [SOHNUS, WEST '81]
[KOMARGODSKI, SEIBERG '10]

- Local SUSY $\longrightarrow$ $\mathcal{S}^\mu$ is the current of the SUGRA gauge fields:

$$\mathcal{L} \sim -\frac{1}{2}(g_{\mu\nu} - \eta_{\mu\nu})T^{\mu\nu} - \frac{1}{2}\bar{\psi}_\mu\Xi^\mu - \frac{1}{2}V_\mu J^\mu + C_\mu X^\mu$$

off-shell SUGRA

[FERRARA, VAN NIEUWENHUIZEN '78] [STELLE, WEST '78]
[SIEGEL, GATES '79] [SOHNUS, WEST '81]

The supercurrent superfield

- Currents supermultiplets & off-shell SUGRA formulations:

(1) Ferrara–Zumino multiplet: $\mathcal{S}_{\text{FZ}}^\mu = \{J^\mu, \Xi^\mu, T^{\mu\nu}, X\} \longleftrightarrow \textit{old-minimal off-shell SUGRA}$

[FERRARA, ZUMINO '74]

$\partial_\mu J^\mu \neq 0$ complex scalar

[FERRARA, VAN NIEUWENHUIZEN '78]
[STELLE, WEST '78]

(2) R-symmetry multiplet: $\mathcal{S}_{\text{R}}^\mu = \{J^\mu, \Xi^\mu, T^{\mu\nu}, X^{\mu\nu}\} \longleftrightarrow \textit{new-minimal off-shell SUGRA}$

[SOHNIUS, WEST '81]

$\partial_\mu J_{\text{R}}^\mu = 0$ antisymmetric field

[SOHNIUS, WEST '81]

(3) Non-minimal multiplet: $\overline{D}^{\dot{\alpha}} \mathcal{S}_{\alpha\dot{\alpha}}^\mu = D_\alpha \mathcal{U}^{(\text{FZ})} + C_\alpha^{(\text{R})} \longleftrightarrow \textit{non-minimal off-shell SUGRA}$

[KOMARGODSKI, SEIBERG '10]

interpolating between (1) and (2)

[SIEGEL, GATES '79]

The massive spin-2 supercurrent & the coupling to supergravity

The massive spin-2 supercurrent

- Supercurrent for the multiplet $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$:

→ R-symmetry: $\lambda_\mu \rightarrow e^{i\alpha\gamma_5} \lambda_\mu$, $\chi_\mu \rightarrow e^{-i\alpha\gamma_5} \chi_\mu$, $\epsilon \rightarrow e^{-i\alpha\gamma_5} \epsilon$

The massive spin-2 supercurrent

- Supercurrent for the multiplet $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$:

→ R-symmetry current: $J_0^\mu = \varepsilon^{\mu\nu\rho\sigma} (\bar{\lambda}_\rho \gamma_\nu \lambda_\sigma - \bar{\chi}_\rho \gamma_\nu \chi_\sigma)$ $\partial_\mu J_0^\mu = 0$

⇒ R-symmetry multiplet: $\delta J_\mu = -i\bar{\epsilon}\gamma_5\Xi_\mu$

[SOHNUS, WEST '81]

$$\delta\Xi_\mu = T_{\mu\nu}\gamma^\nu\epsilon + X_{\mu\nu}\gamma^\nu\epsilon + \frac{1}{8}\varepsilon_{\mu\nu\rho\sigma}\partial^\rho J^\sigma\gamma^\nu\epsilon + \frac{i}{4}\partial_\rho J_\mu\gamma_5\gamma^\rho\epsilon$$

$$\delta T_{\mu\nu} = -\frac{1}{4}\bar{\epsilon}\gamma_{(\mu}{}^\lambda\partial_{\lambda}\Xi_{\nu)}$$

$$\delta X_{\mu\nu} = -\frac{i}{8}\varepsilon_{\mu\nu\rho\sigma}\bar{\epsilon}\gamma_5\gamma^\rho\gamma^\lambda\partial^\sigma\Xi_\lambda$$

new-minimal
off-shell SUGRA

The massive spin-2 supercurrent

- Supercurrent for the multiplet $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$:

$\longrightarrow$ R-symmetry current: $J_0^\mu = \varepsilon^{\mu\nu\rho\sigma} (\bar{\lambda}_\rho \gamma_\nu \lambda_\sigma - \bar{\chi}_\rho \gamma_\nu \chi_\sigma)$
 $\partial_\mu J_0^\mu = 0$

$\xrightarrow{\delta_{\text{SUSY}}}$ stress-energy tensor: $T_0^{\mu\nu} = T_K^{\mu\nu} + \partial_\rho \partial_\sigma G_0^{\mu\sigma\nu\rho} + \partial_\rho B_0^{\rho\mu\nu}$
improvement terms $\longleftrightarrow$
[CALLAN, COLEMAN, JACKIW '69]

$\longleftrightarrow$ non-minimal couplings

$\swarrow$
 $\searrow$

$g_{\mu\nu} \partial_\rho \partial_\sigma G_0^{\mu\sigma\nu\rho} \sim R_{\mu\sigma\nu\rho} G_0^{\mu\sigma\nu\rho}$ diff-invariant
 $g_{\mu\nu} \partial_\rho B_0^{\rho\mu\nu} \sim \Gamma_{\rho\mu\nu} B_0^{\rho\mu\nu}$ not diff-invariant

$\implies$ *$B^{\rho\mu\nu}$ terms do not define consistent couplings to gravity*

The unique consistent coupling

- *Improvements* to the supercurrent: [FERRARA, ZUMINO '74] [KOMARGODSKI, SEIBERG '10]

$$\mathcal{S}_0^\mu \longrightarrow \mathcal{S}^\mu = \mathcal{S}_0^\mu + \Delta\mathcal{S}^\mu \quad \text{multiplet of improvements}$$

$$\longrightarrow \Delta\mathcal{S}^\mu = \{\Delta J^\mu, \Delta\Xi^\mu, \Delta T^{\mu\nu}, \Delta X^{\mu\nu}\} = \frac{1}{m} \partial_\nu \mathcal{L}^{\mu\nu} + \sigma^{\mu\alpha\dot{\beta}} [D_\alpha, \bar{D}_{\dot{\beta}}] \mathcal{U}$$

linear superfield improvement

$$D^2 \mathcal{L}^{\mu\nu} = 0$$

The unique consistent coupling

- *Improvements* to the supercurrent: [FERRARA, ZUMINO '74] [KOMARGODSKI, SEIBERG '10]

$$\mathcal{S}_0^\mu \longrightarrow \mathcal{S}^\mu = \mathcal{S}_0^\mu + \Delta\mathcal{S}^\mu \quad \text{multiplet of improvements}$$

$$\longrightarrow \Delta\mathcal{S}^\mu = \{\Delta J^\mu, \Delta\Xi^\mu, \Delta T^{\mu\nu}, \Delta X^{\mu\nu}\} = \frac{1}{m}\partial_\nu \mathcal{L}^{\mu\nu} + \sigma^{\mu\alpha\dot{\beta}}[D_\alpha, \bar{D}_{\dot{\beta}}]\mathcal{U}$$

non-minimal improvement [KOMARGODSKI, SEIBERG '10]

$$\Delta T_{\mathcal{U}}^{\mu\nu} \sim R U$$

The unique consistent coupling

- *Improvements* to the supercurrent: [FERRARA, ZUMINO '74] [KOMARGODSKI, SEIBERG '10]

$$\mathcal{S}_0^\mu \longrightarrow \mathcal{S}^\mu = \mathcal{S}_0^\mu + \Delta\mathcal{S}^\mu \quad \text{multiplet of improvements}$$

$$\longrightarrow \Delta\mathcal{S}^\mu = \{\Delta J^\mu, \Delta\Xi^\mu, \Delta T^{\mu\nu}, \Delta X^{\mu\nu}\} = \frac{1}{m} \partial_\nu \mathcal{L}^{\mu\nu} + \sigma^{\mu\alpha\dot{\beta}} [D_\alpha, \bar{D}_{\dot{\beta}}] \mathcal{U}$$

$$\longrightarrow L^{\mu\nu} = c_B \left(2h^{[\mu}{}_{\lambda} \partial^\lambda A^{\nu]} + \frac{1}{m} \varepsilon^{\mu\nu\rho\sigma} \partial_\lambda A_\rho \partial_\sigma A^\lambda \right) + c_{F1} \varepsilon^{\mu\nu\rho\sigma} \bar{\chi}_\rho \psi_\sigma + i c_{F2} \bar{\chi}^{[\mu} \gamma_5 \psi^{\nu]}$$

parity + linear multiplet + 4-derivatives

The unique consistent coupling

- *Improvements* to the supercurrent: [FERRARA, ZUMINO '74] [KOMARGODSKI, SEIBERG '10]

$$\mathcal{S}_0^\mu \longrightarrow \mathcal{S}^\mu = \mathcal{S}_0^\mu + \Delta\mathcal{S}^\mu \quad \text{multiplet of improvements}$$

$$\longrightarrow \Delta\mathcal{S}^\mu = \{\Delta J^\mu, \Delta\Xi^\mu, \Delta T^{\mu\nu}, \Delta X^{\mu\nu}\} = \frac{1}{m} \partial_\nu \mathcal{L}^{\mu\nu} + \sigma^{\mu\alpha\dot{\beta}} [D_\alpha, \bar{D}_{\dot{\beta}}] \mathcal{U}$$

$$\longrightarrow L^{\mu\nu} = c_B \left(2h^{[\mu}{}_{\lambda} \partial^\lambda A^{\nu]} + \frac{1}{m} \varepsilon^{\mu\nu\rho\sigma} \partial_\lambda A_\rho \partial_\sigma A^\lambda \right) + c_{F1} \varepsilon^{\mu\nu\rho\sigma} \bar{\chi}_\rho \psi_\sigma + i c_{F2} \bar{\chi}^{[\mu} \gamma_5 \psi^{\nu]}$$

fix $\{c_B, c_{F1}, c_{F2}\}$ to cancel $\partial_\rho B_0^{\rho\mu\nu}$

The unique consistent coupling

- Resulting improvements to the stress-energy tensor:

$$\text{B} \rightarrow B_{\text{B}}^{\rho\mu\nu} \quad \text{F1} \rightarrow B_{\text{F1}}^{\rho\mu\nu} \quad \text{F2} \rightarrow B_{\text{F2}}^{\rho\mu\nu} = -\frac{1}{4}B_0^{\rho\mu\nu}$$

$$\text{new bad improvements} \implies c_{\text{B}} = c_{\text{F1}} = 0 \quad \implies c_{\text{F2}} = 4$$

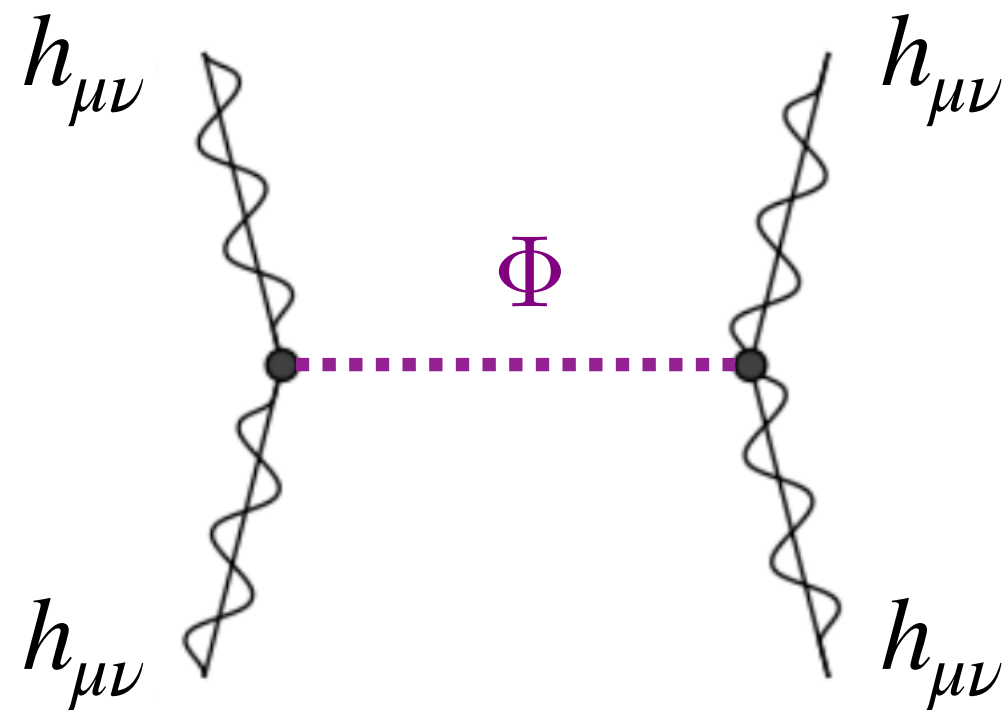
$$\implies J^\mu = J_0^\mu + \frac{4i}{m} \partial_\nu \left(\bar{\chi}^{[\mu} \gamma_5 \lambda^{\nu]} \right) \implies \boxed{T^{\mu\nu} = T_{\text{K}}^{\mu\nu} + \partial_\rho \partial_\sigma \mathcal{G}^{\mu\sigma\nu\rho}}$$

unique coupling to gravity at 4-derivatives

$$\longrightarrow \mathcal{G}^{\mu\sigma\nu\rho} = \mathcal{G}_{hh}^{\mu\sigma\nu\rho} + \mathcal{G}_{AA}^{\mu\sigma\nu\rho} + \mathcal{G}_{Ah}^{\mu\sigma\nu\rho} + \mathcal{G}_{\text{F}}^{\mu\sigma\nu\rho} \quad \text{non-trivial coupling to the Riemann tensor}$$

$2 \rightarrow 2$ massive spin-2 scattering amplitude

(1) High-energy limit: $s \rightarrow \infty$, θ fixed



$\Phi = \text{spin-2, spin-1, spin-0}$

$$\mathcal{A}(s) \sim s^5$$

[ARKANI-HAMED, GEORGI, SCHWARTZ '03]

&

$$\mathcal{A}(s) \geq s^3$$

[BONIFACIO, HINTERBICHLER '18]

[BONIFACIO, HINTERBICHLER, ROSEN '19]

$$\mathcal{A}(s) = s^3 \longleftrightarrow$$

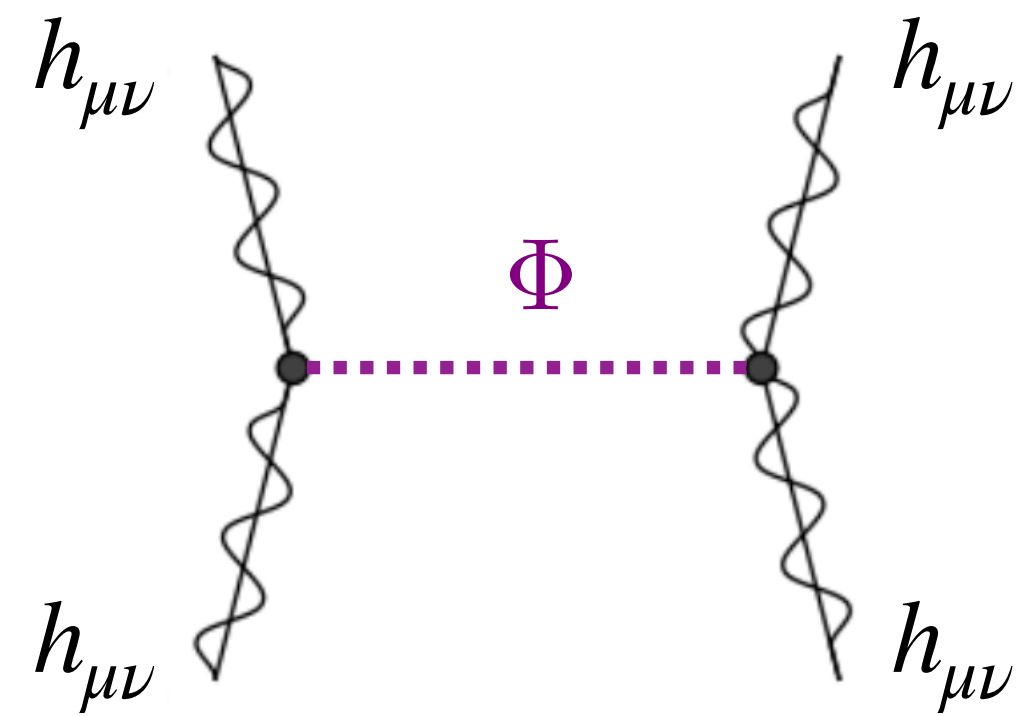
dRGT massive gravity & Hassan-Rosen bigravity

[DE RHAM, GABADADZE, TOLLEY '11]

[HASSAN, ROSEN '12]

$2 \rightarrow 2$ massive spin-2 scattering amplitude

(1) High-energy limit: $s \rightarrow \infty, \theta$ fixed



$\Phi = \text{spin-2, spin-1, spin-0}$

$$\mathcal{A}(s) \sim s^5 \quad \& \quad \mathcal{A}(s) \geq s^3$$

[ARKANI-HAMED, GEORGI, SCHWARTZ '03]

[BONIFACIO, HINTERBICHLER '18]
[BONIFACIO, HINTERBICHLER, ROSEN '19]

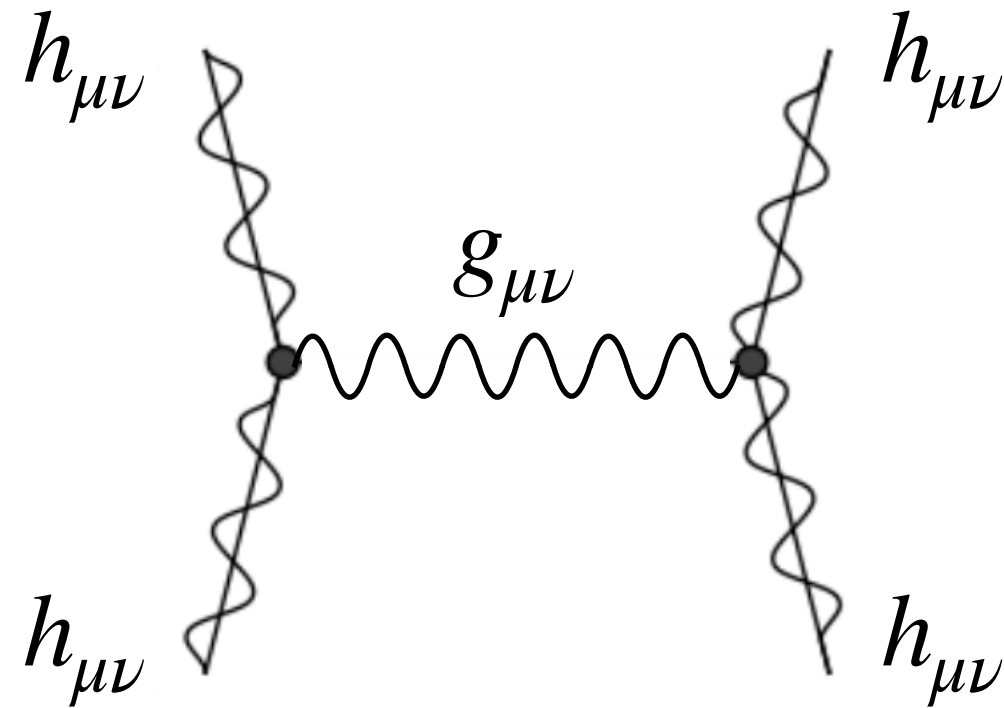
(2) Regge limit: $s \rightarrow \infty, t$ fixed

$$\mathcal{A}(s) \geq s^3$$

[KUNDU, PALTÍ, QUIRANT '23]

$2 \rightarrow 2$ massive spin-2 scattering amplitude

- Massive spin-2 scattering from the unique coupling:



$$T_h^{\mu\nu} = T_{\text{FP}}^{\mu\nu} + \partial_\rho \partial_\sigma \left(-\frac{1}{2} h^{\rho[\mu} h^{\sigma]\nu} + \frac{1}{2} \eta^{\rho[\mu} h^{\sigma]\lambda} h^\lambda{}_\nu \right)$$

$$\Rightarrow \mathcal{A}(s) \sim s^4 \quad \text{both high-energy \& Regge}$$

—→ agreement with [ARKANI-HAMED, GEORGI, SCHWARTZ '03] [BONIFACIO, HINTERBICHLER '18] [BONIFACIO, HINTERBICHLER, ROSEN '19] [KUNDU, PALTI, QUIRANT '23]

—→ better behaviour than the generic s^5

—→ non-minimal improvements worsen the amplitude

The unique coupling & open string theory

- Connection with *superstring theory*:

$$(1) \quad T_h^{\mu\nu} = \frac{1}{4} T_{\text{FP}}^{\mu\nu} + \partial_\rho \partial_\sigma \left(-\frac{1}{2} h^{\rho[\mu} h^{\sigma]\nu} + \frac{1}{2} \eta^{\rho[\mu} h^{\sigma]}{}_\lambda h^{\lambda[\nu} \right) \equiv \text{coupling of a massive } open \text{ string state of mass } m = 1/\sqrt{\alpha'}$$

[LÜST, MARKOU, MAZLOUMI, STIEBERGER '21]

$$(2) \quad R_{\mu\sigma\nu\rho} \mathcal{G}^{\mu\sigma\nu\rho} \supset R_{\mu\sigma\nu\rho} F^{\mu\sigma} F^{\nu\rho} \longleftrightarrow \text{potential violations of causality} \longleftrightarrow \text{tower of higher-spin fields}$$

[CAMANHO, EDELSTEIN, MALDACENA, ZHIBOEDOV '14]

$\implies$ Evidence for the *string lamppost* principle in $D = 4$ and with $\mathcal{N} = 1$ SUSY

[ADAMS, DEWOLFE, TAYLOR '10] [KIM, TARAZI, VAFA '19] [MONTERO, VAFA '20] [BEDROYA, HAMADA, MONTERO, VAFA '21]

The Kaluza–Klein coupling

The Kaluza–Klein coupling

- Kaluza–Klein reduction of *5D pure supergravity on the orbifold* $S^1/\mathbb{Z}_2$:

$$\{E_M^A, \Psi_M^i, \mathcal{A}_M\}$$



$$S^1/\mathbb{Z}_2 \quad \mathcal{N} = 2 \longrightarrow \mathcal{N} = 1$$

$$\{g_{\mu\nu}, \psi_\mu\} \quad + \quad \{T, \zeta\} \quad + \quad \sum_n \{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}^{(n)}$$

SUGRA multiplet

chiral multiplet

tower of massive spin-2
supermultiplets

The Kaluza–Klein coupling

- Kaluza–Klein reduction $\longleftrightarrow$ *Stückelberg organisation of the massive degrees of freedom*

$$h_{\mu\nu}^S = g_{\mu\nu} + \frac{1}{m} \partial_{(\mu} B_{\nu)} - \frac{1}{m^2} \partial_\mu \partial_\nu \phi \quad \longleftrightarrow \quad \delta_S g_{\mu\nu} = \partial_{(\mu} \xi_{\nu)}, \quad \delta_S B_\mu = -m \xi_\mu + \partial_\mu \Lambda, \quad \delta_S \phi = m \Lambda \quad \text{5D gauge invariance}$$

$$\longrightarrow \{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\} \quad \longleftrightarrow \quad \text{unitary gauge:} \quad B_\mu = 0 \quad \& \quad \phi = 0$$

$$\longrightarrow \text{unitary gauge + massive SUSY} \quad \delta_\epsilon B_\mu = 0, \quad \delta_\epsilon \phi = 0 \quad \implies \quad \boxed{\text{deformation of the massless SUSY}}$$

$\implies$ the Kaluza–Klein coupling cannot be captured by the undeformed supercurrent superfield setup

Infinite distance paradigm

String oscillator

- long multiplet in $\mathcal{N} \geq 2$ SUSY
- unique coupling to gravity $\mathcal{S}^\mu$ [BOSSARD, GC, DUDAS, LOTY '25]
- higher-spin tower becoming light for $m \rightarrow 0$



tensionless limit of the string

Kaluza–Klein mode

- 1/2-BPS short multiplet in $\mathcal{N} \geq 2$ SUSY
- KK coupling to gravity deformation expected to be unique
- massive spin-2 tower becoming light for $m \rightarrow 0$



decompactification limit

$\implies$ realisation of the infinite distance paradigm of the swampland program

[OOGURI, VAFA '06] [GRIMM, PALTI, VALENZUELA '18] [LEE, LERCHE, WEIGAND '19]

Conclusions & future directions

The coupling of $\{h_{\mu\nu}, \lambda_\mu, \chi_\mu, A_\mu\}$ to *undeformed* $D = 4, \mathcal{N} = 1$ SUGRA is *unique*

→ Future directions:

- extension to a fully non-linear lagrangian $\longleftrightarrow$ propagation of ghosts? causality violation?
- extend the improvement analysis to higher-derivative orders (*e.g.* 6-derivatives)
- prove that the KK is (not) the *only* deformed coupling
- similar analysis for higher-spin supermultiplet $\{h_{(s)}, \lambda_{(s-1)}, \chi_{(s-1)}, A_{(s-2)}\}$

Thank you!

Non diff-invariant improvements

$$\begin{aligned}
 B_0^{\rho\mu\nu} = & \frac{1}{4} \left(h^{(\nu}{}_{\sigma} \partial^{\mu)} h^{\rho\sigma} - \partial^{(\mu} h^{\nu)}{}_{\sigma} h^{\rho\sigma} \right) + \frac{5}{4} \left(A^{(\nu} \partial^{\mu)} A^{\rho} - A^{\rho} \partial^{(\mu} A^{\nu)} \right) \\
 & + \frac{1}{4m} \left(\partial^{(\mu} \tilde{F}^{\rho\sigma} h^{\nu)}{}_{\sigma} - \tilde{F}^{\rho\sigma} \partial^{(\mu} h^{\nu)}{}_{\sigma} + \tilde{F}^{(\nu}{}_{\sigma} \partial^{\mu)} h^{\rho\sigma} - \partial^{(\mu} \tilde{F}^{\nu)}{}_{\sigma} h^{\rho\sigma} \right) \\
 & + \frac{1}{2m} \left(\partial^{(\mu} \bar{\chi}^{\nu)} \lambda^{\rho} - \bar{\chi}^{(\nu} \partial^{\mu)} \lambda^{\rho} + \partial^{(\mu} \bar{\lambda}^{\nu)} \chi^{\rho} - \bar{\lambda}^{(\nu} \partial^{\mu)} \chi^{\rho} \right) ,
 \end{aligned}$$

$$\Delta B_{\text{B}}^{\rho\mu\nu} = \frac{1}{8m^2} \partial_{\sigma} \left(A^{(\mu} \partial_{\sigma} \partial^{\nu)} A^{\rho} - A^{\rho} \partial^{\sigma} \partial^{(\mu} A^{\nu)} \right)$$

$$\begin{aligned}
 \Delta B_{\text{F1}}^{\rho\mu\nu} = & \frac{1}{2m} \left(\partial^{(\mu} \bar{\chi}^{\nu)} \lambda^{\rho} - \bar{\chi}^{(\nu} \partial^{\mu)} \lambda^{\rho} + \partial^{(\mu} \bar{\lambda}^{\nu)} \chi^{\rho} - \bar{\lambda}^{(\nu} \partial^{\mu)} \chi^{\rho} \right) \\
 & + \frac{1}{2m} \left(\partial^{(\mu} \bar{\lambda}_{\sigma} \gamma^{\rho} \gamma^{|\nu)} \chi^{\sigma} - \partial^{\rho} \bar{\lambda}_{\sigma} \gamma^{(\mu} \gamma^{\nu)} \chi^{\sigma} \right)
 \end{aligned}$$

Non-minimal coupling to the Riemann tensor

$$\begin{aligned}\mathcal{G}^{\mu\sigma\nu\rho} = & -\frac{1}{2}h^{\rho[\mu}h^{\sigma]\nu} + \frac{1}{2}\eta^{\rho] [\mu}h^{\sigma]}_{\lambda}h^{\lambda[\nu} + 2\eta^{\rho] [\mu}A^{\sigma]}A^{[\nu} - \frac{3}{4m^2}F^{\mu\sigma}F^{\nu\rho} \\ & + \frac{1}{2m^2}\eta^{\rho] [\mu}F^{\sigma]}_{\lambda}F^{\lambda[\nu} + \frac{1}{2m}\eta^{\rho] [\mu}\left(\tilde{F}^{\sigma]}_{\lambda}h^{\lambda[\nu} - h^{\sigma]}_{\lambda}\tilde{F}^{\lambda[\nu}\right) \\ & + \frac{1}{m^2}\left(\eta^{\sigma] [\nu}F^{\rho]}_{\lambda}\partial^{\lambda}A^{[\mu} + \eta^{\rho] [\mu}F^{\sigma]}_{\lambda}\partial^{\lambda}A^{[\nu}\right) \\ & + \frac{1}{2m}\left(\varepsilon^{\mu\sigma\lambda\tau}A_{\lambda}\partial^{[\nu}h^{\rho]}_{\tau} + \varepsilon^{\nu\rho\lambda\tau}A_{\lambda}\partial^{[\mu}h^{\sigma]}_{\tau}\right) \\ & + \frac{1}{m}\left(\bar{\lambda}^{[\sigma}\gamma^{\mu][\rho}\chi^{\nu]} + \bar{\lambda}^{[\rho}\gamma^{\nu][\sigma}\chi^{\mu]}\right) \\ & - \frac{1}{2}RU + \bar{\rho}_{\mu\nu}\gamma^{\mu\nu}\Upsilon\end{aligned}$$

The Kaluza–Klein reduction

- 5D pure supergravity on the orbifold $S^1/\mathbb{Z}_2$:

1) 5D supergravity algebra: [GÜNAYDIN, SIERRA, TOWNSEND '83] [CERESOLE, DALL'AGATA '00]

$$\delta E_M{}^A = \frac{1}{2} \bar{\varepsilon} \gamma^A \Psi_M$$

$$\delta \mathcal{A}_M = -\frac{i}{2} \bar{\varepsilon} \Psi_M$$

$$\delta \Psi_M = D_M \varepsilon + \frac{i}{2} E_M{}^A \mathcal{F}_{BC} \left(\frac{1}{4} \gamma_A{}^{BC} - \delta_A{}^B \gamma^C \right) \varepsilon$$

The Kaluza–Klein reduction

- 5D pure supergravity on the orbifold $S^1/\mathbb{Z}_2$:

1) 5D supergravity algebra [GÜNAYDIN, SIERRA, TOWNSEND '83] [CERESOLE, DALL'AGATA '00]

2) compactification ansatz:

$$E^a = \phi^{-\frac{1}{2}} e^a$$

$$E^4 = \phi (dy + B)$$

$$\mathcal{A} = a (dy + B) + A$$

$$\Psi = \phi^{\frac{5}{4}} \lambda (dy + B) + \phi^{-\frac{1}{4}} \left(\psi - \frac{1}{2} e^a \gamma_a \gamma_5 \lambda \right)$$

The Kaluza–Klein reduction

- 5D pure supergravity on the orbifold $S^1/\mathbb{Z}_2$:
 - 1) 5D supergravity algebra [GÜNAYDIN, SIERRA, TOWNSEND '83] [CERESOLE, DALL'AGATA '00]
 - 2) compactification ansatz
 - 3) mode decomposition of the fields along $S^1/\mathbb{Z}_2$:

$$A_\mu(y) = \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \sin\left(\frac{ny}{R}\right) \qquad a(y) = a_0 + \sum_{n=1}^{\infty} a_n(x) \cos\left(\frac{ny}{R}\right)$$

The Kaluza–Klein reduction

- 5D pure supergravity on the orbifold $S^1/\mathbb{Z}_2$:
 - 1) 5D supergravity algebra [GÜNAYDIN, SIERRA, TOWNSEND '83] [CERESOLE, DALL'AGATA '00]
 - 2) compactification ansatz
 - 3) mode decomposition of the fields along $S^1/\mathbb{Z}_2$
 - 4) *Stückelberg* organisation of the massive degrees of freedom:

$$A_\mu^S = A_\mu - \frac{1}{m} \partial_\mu a \quad \longleftrightarrow \quad \delta_s A_\mu = \partial_\mu \lambda \quad \& \quad \delta_s a = m \lambda$$

Stückelberg combinations

$$A_{\mu}^{(n)S} = \phi^{(0)\frac{1}{2}} A_{\mu}^{(n)} - \frac{\phi^{(0)-1}}{m_n} \partial_{\mu} a^{(n)}$$

$$\lambda_{\mu}^{(n)S} = \left(\psi_{1\mu L}^{(n)} + \psi_{2\mu R}^{(n)} \right) - \frac{1}{2} \gamma_{\mu} \left(\zeta_{1R}^{(n)} - \zeta_{2L}^{(n)} \right) + \frac{i}{m_n} \partial_{\mu} \left(\zeta_{1L}^{(n)} + \zeta_{2R}^{(n)} \right)$$

$$\chi_{\mu}^{(n)S} = -i \left[\left(\psi_{1\mu R}^{(n)} - \psi_{2\mu L}^{(n)} \right) - \frac{1}{2} \gamma_{\mu} \left(\zeta_{1L}^{(n)} + \zeta_{2R}^{(n)} \right) + \frac{i}{m_n} \partial_{\mu} \left(\zeta_{1R}^{(n)} - \zeta_{2L}^{(n)} \right) \right]$$

$$g_{\mu\nu}^{(n)S} = 2e_{(\mu}^{(0)a} e_{\nu)}^{(n)a} - \phi^{(n)} g_{\mu\nu}^{(0)} + \frac{\phi^{(0)\frac{3}{2}}}{m_n} \left(\partial_{\mu} B_{\nu}^{(n)} + \partial_{\nu} B_{\mu}^{(n)} \right) - \frac{2}{m_n^2} \partial_{\mu} \partial_{\nu} \phi^{(n)}$$