Love and Naturalness

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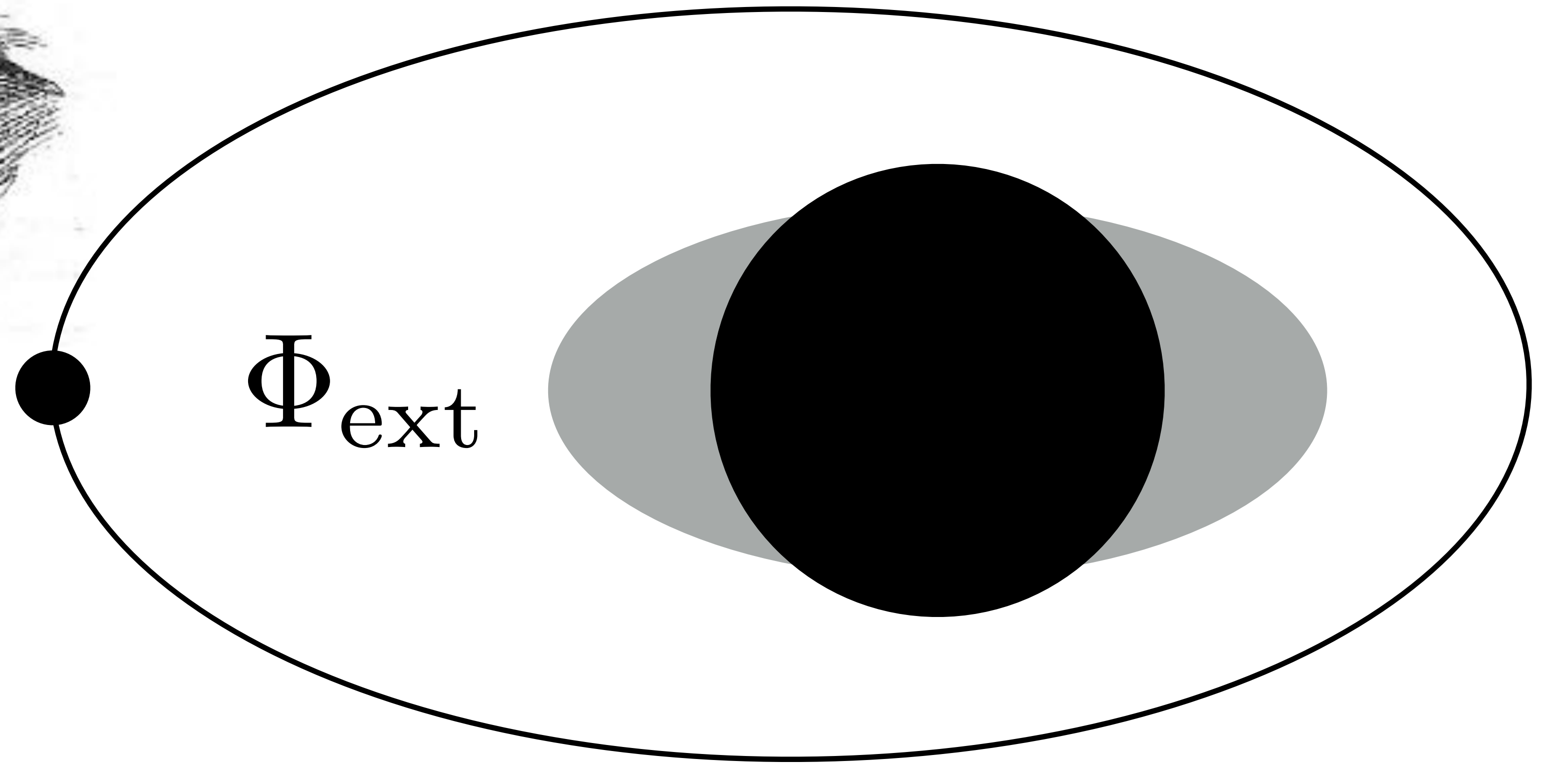
Outline

- Tidal Love numbers in Newton's gravity
- Worldline post-Newtonian EFT
- Naturalness in PN EFT
- Black hole perturbation theory, Teukolksy master eqn
- Love symmetry

Love numbers



Newtonian limit:

$c = 1$

$G = 1$

$$\Phi = -\frac{M}{r} + E_{ij}x^i x^j + \frac{Q_{ij}x^i x^j}{r^5}$$

tidal field   mass quadrupole

$$Q_{ij} = C E_{ij} + \ldots$$

$$\Phi = -\frac{M}{r} + E_{ij}x^i x^j \left(1 + \frac{C}{r^5}\right)$$

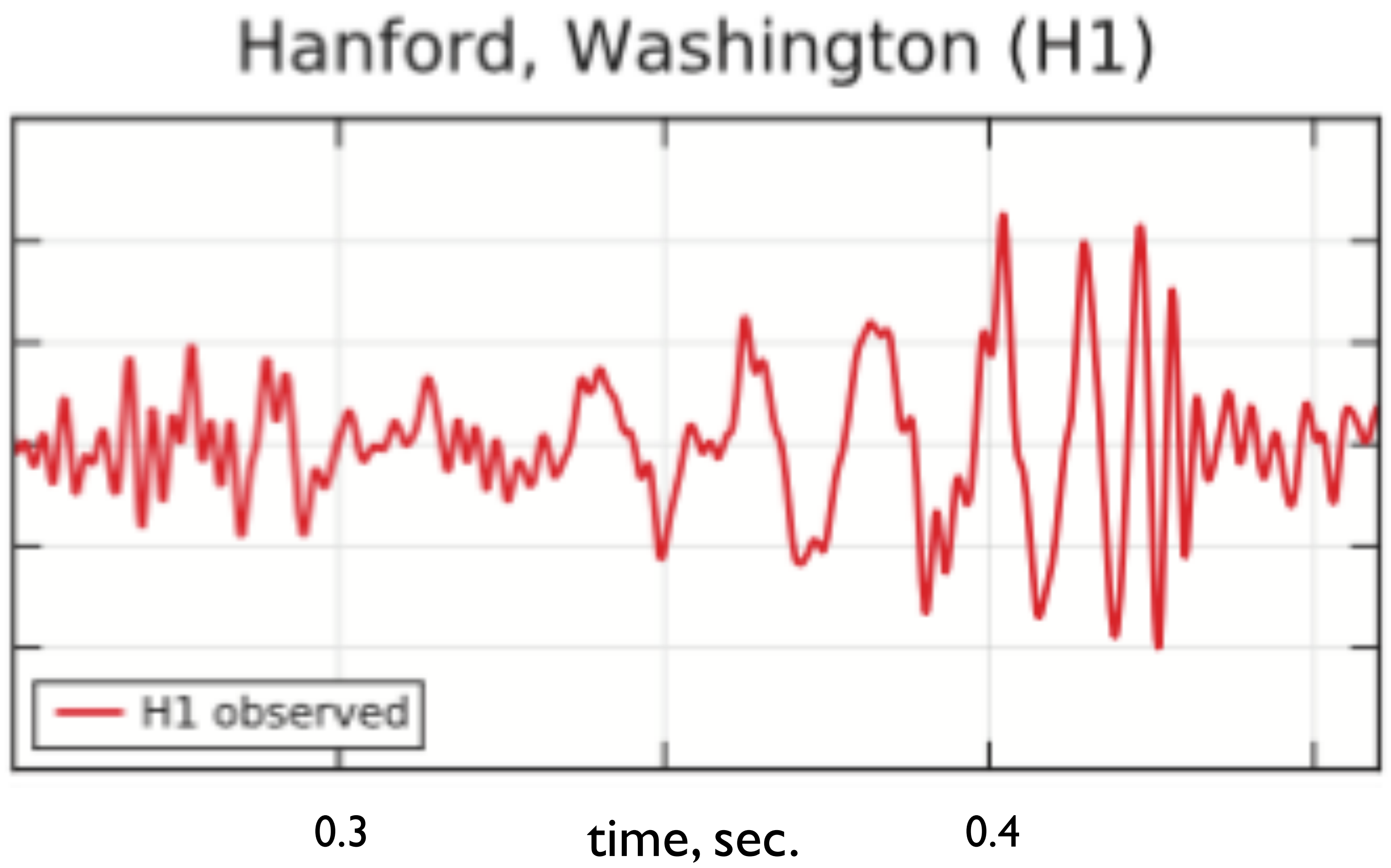
black holes (D=4): $C = 0$

$D > 4: \quad C \neq 0$

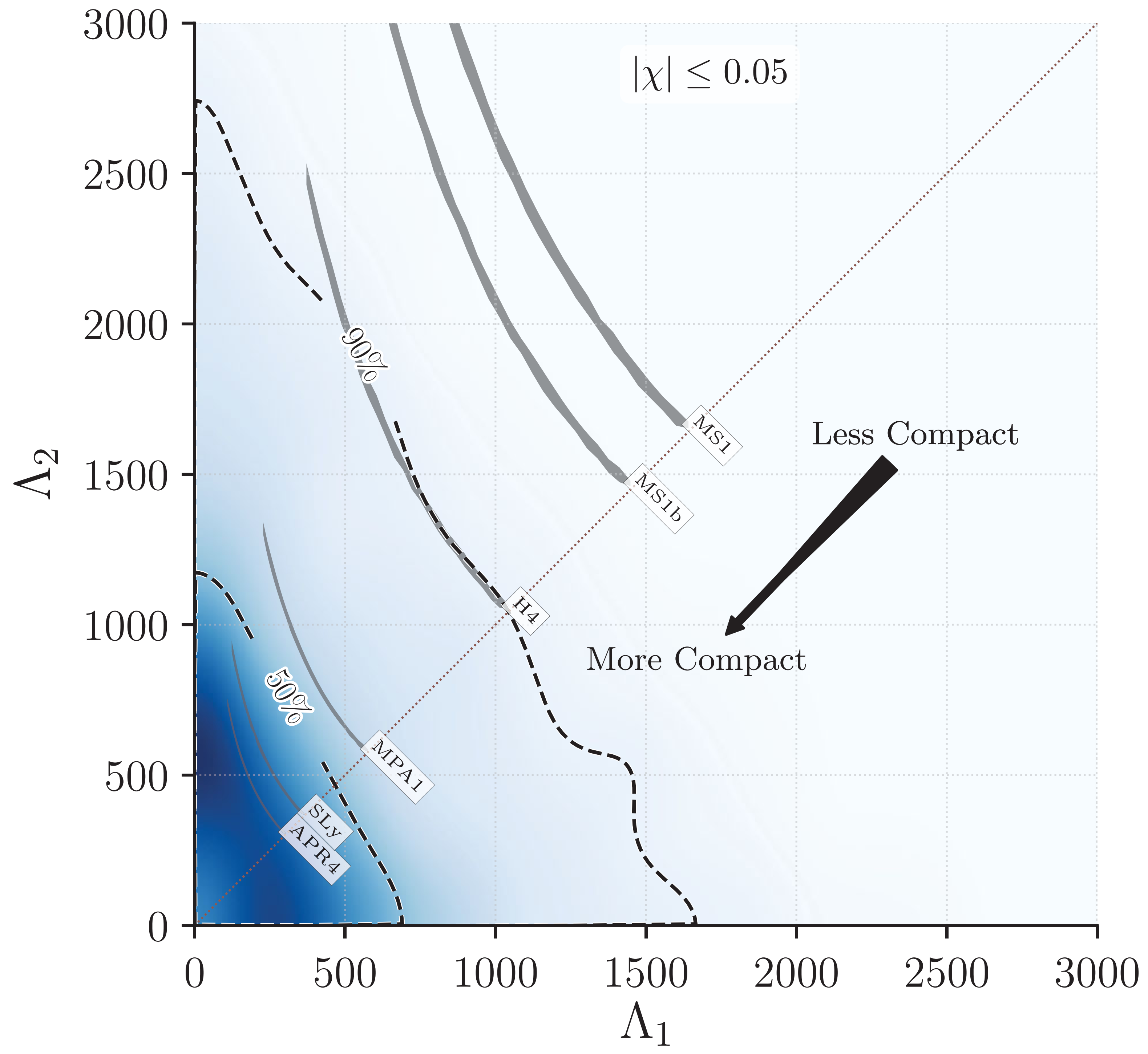
BH are most rigid objects!

Fang & Lovelace'05
Damour & Nagar'09
Binnington & Poisson'09
Kol & Smolkin'11

Love Numbers = Smoking gun of BSM or MG

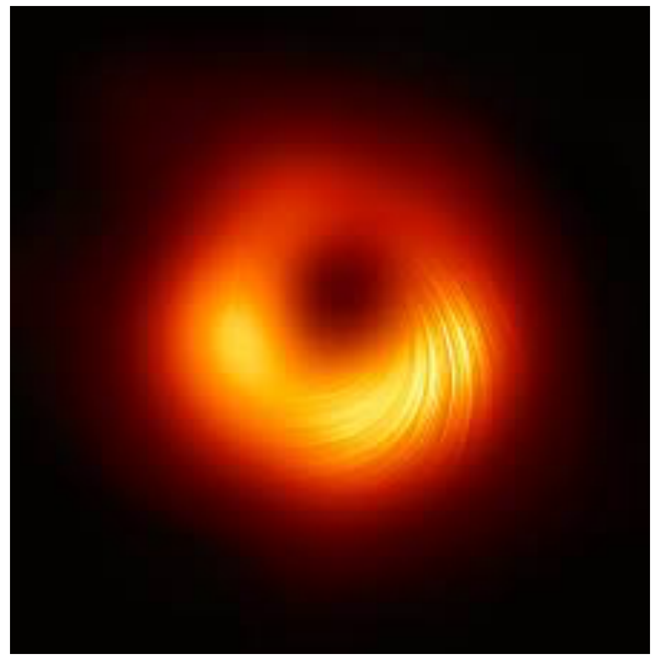


Credit: LIGO collaboration



$$m_{1,2} > 3M_{\odot}$$

Abbott et. al (2017)



$$C = 0$$

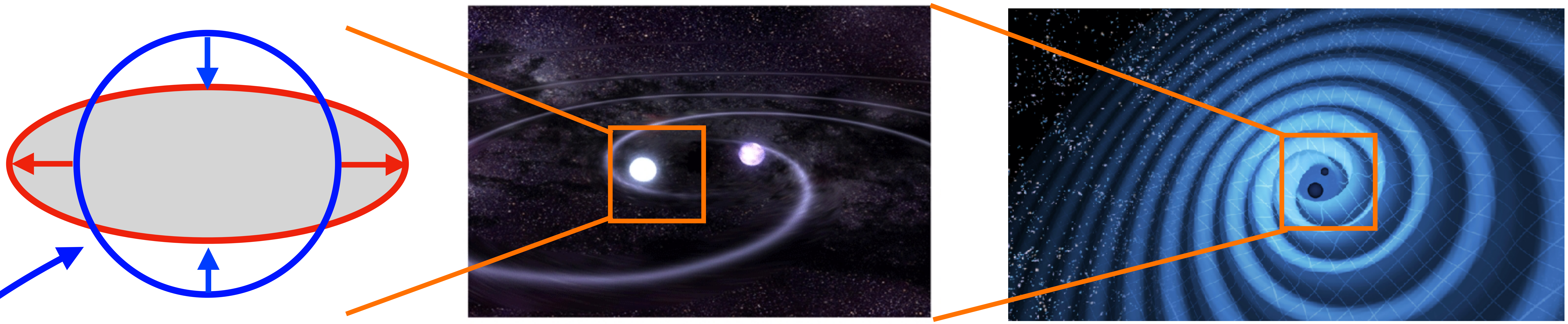
black hole

Credit: EHT

$$C \neq 0$$

something else

Post-Newtonian EFT for GW



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EFT framework: separate effects in a systematic perturbative expansion

distance, r

R — Tidal effects

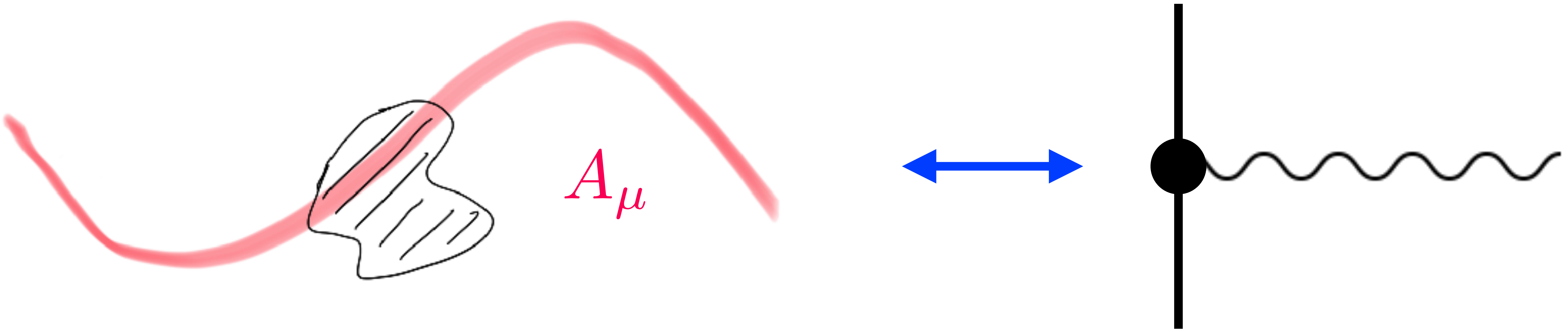
$r_{\rm orbit}$ — Potential modes

λ — Radiation

Post-Newtonian (Point-particle) EFT

Goldberger & Rothstein'05,06 Porto'16

Classical electrodynamics as Worldline EFT



$Q, P_i, Q_{ij}, \ldots$

Long DoFs: center of mass coordinate $x^\mu(\sigma)$, gauge field

Symmetries: gauge, Lorentz, reparams of worldline, rotations

$$u^\mu = \frac{dx^\mu}{d\sigma}$$

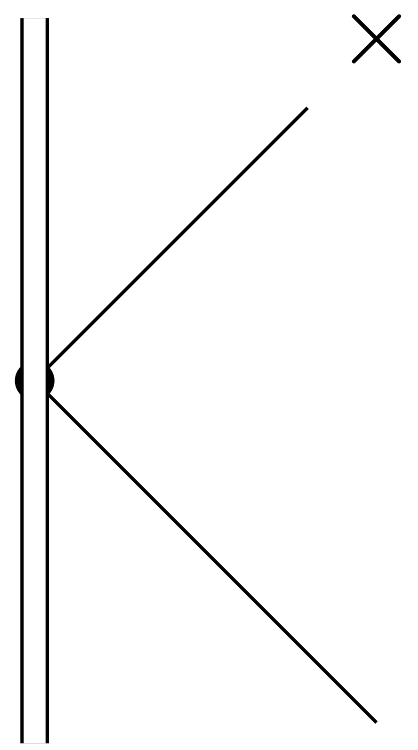
$$S_{\text{eff}} = Q \int d\sigma \; u^\mu A_\mu + \chi \int d\sigma \frac{u^\mu u^\lambda F_{\mu\nu} F^\nu_\lambda}{\sqrt{-u^2}} + \ldots$$

Proper time, body's rest frame:

$$S_{\text{eff}} = Q \int ds \; A_0 + \chi \int ds \; E_i E^i + \ldots$$

point particle　　　　Finite-size effects

Linear Response in Worldline EFT



$$E^i_{\text{tot}} = \bar{E}^i_{\text{source}} + E^i_{\text{response}}$$

$$S_{\text{eff}} = \chi \int ds \; E_i E^i - \frac{1}{4} \int d^4x \; F^2_{\mu\nu}$$

$$P^i = \chi E^i$$

$$E_r = \underbrace{1}_{\text{source}} - \underbrace{\frac{\chi}{r^3}}_{\text{response}}$$

Susceptibility = EFT Wilson coefficient

Electric field
of induced dipole

Including gravity

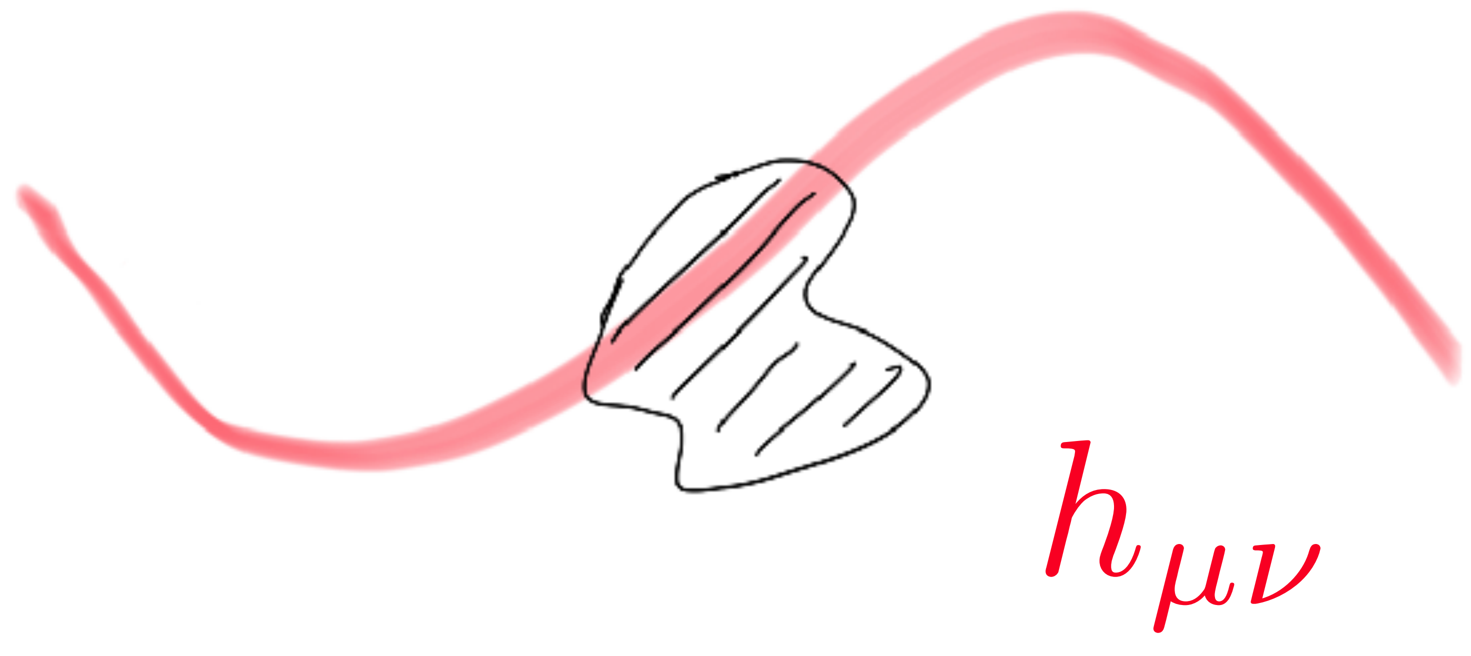
Goldberger & Rothstein’05,06

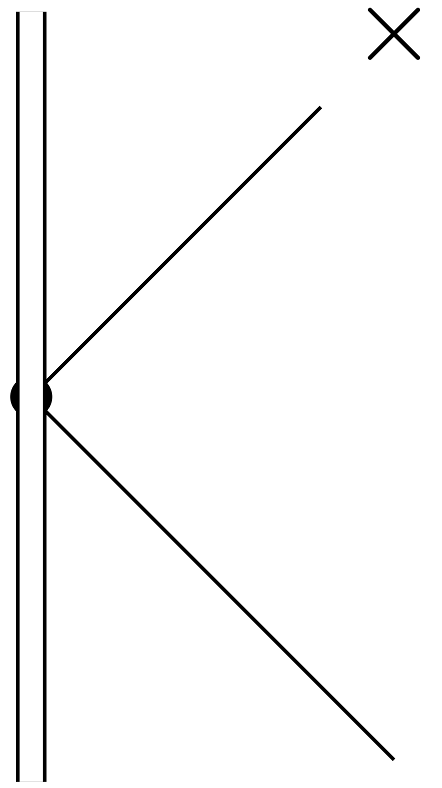
$$S_{\rm eff} = \frac{C_2}{2}\int ds\; E_{ij}E^{ij} - m\int ds$$

$$E_{ij} = R_{0i0j} \quad \leftarrow \quad u^\mu u^\lambda R_{\mu\nu\lambda\rho}$$

$$+\frac{\chi}{2}\int ds\; E_i E^i \quad +\frac{\chi_2}{2}\int ds\; \partial_i E_j \partial^i E^j$$

$$+\int ds\; \left[\frac{\gamma_1}{2}\partial_i\phi\partial^i\phi + \frac{\gamma_2}{2}\partial_i\partial_j\phi\partial^i\partial^j\phi + \ldots\right]$$





$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

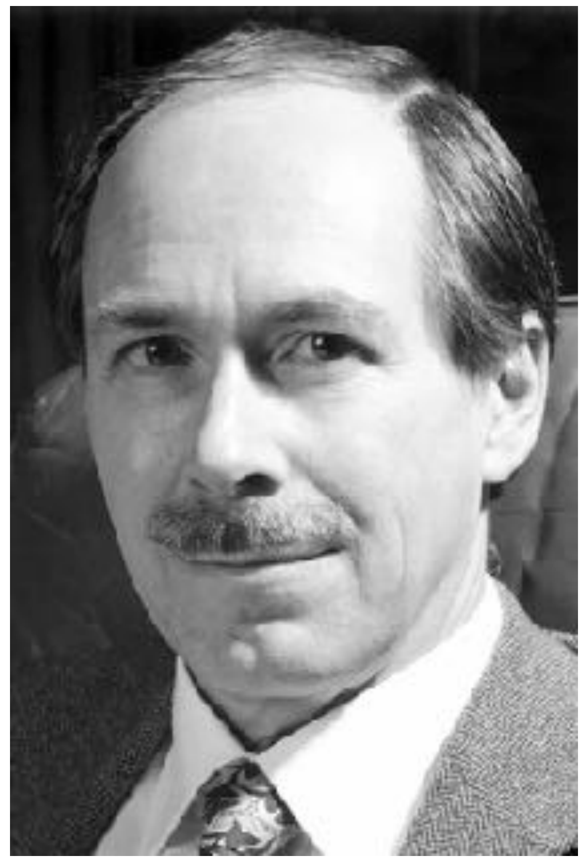
$$h_{00} \propto r^2\left(\underbrace{1}_{\rm source} + \underbrace{\frac{C_2}{r^5}}_{\rm response}\right)$$

$$Q_{ij} = C_2 E_{ij}$$

Tidal Love numbers = EFT Wilson coefficients !

Kol & Smolkin’11, Hui et al.’20

Naturalness



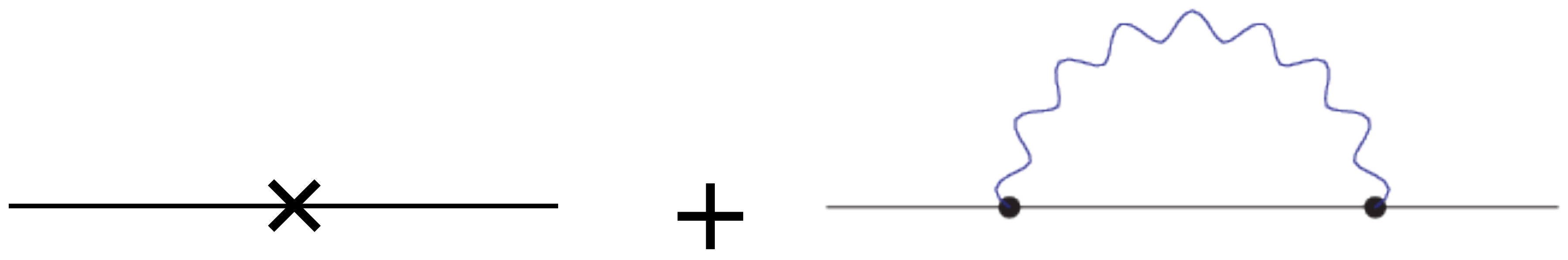
't Hooft

A physical parameter A is allowed to be small
if the replacement A=0 enhances the symmetry of the system

Otherwise "natural" $$\langle \mathcal{O} \rangle = \sum_{a=1} \langle \mathcal{O} \rangle_a \quad \Rightarrow \quad \langle \mathcal{O} \rangle \sim \max \langle \mathcal{O} \rangle_a$$

Ex 1: $$(m_{\pi_+}^2 - m_{\pi_0}^2)_{\text{tree}} + e^2\Lambda^2 = (m_{\pi_+}^2 - m_{\pi_0}^2)_{\text{obs}} \sim e^2 \Lambda_{\text{QCD}}^2$$

Pion mass splitting



Theory: $m_{\pi_+} - m_{\pi_0} \sim 10\ \text{MeV}$

Experiment: $5\ \text{MeV}$

Ex 2: $$(m_H^2)_{\text{tree}} + g^2\Lambda^2 = (m_H^2)_{\text{obs}}$$

Higgs mass

Theory: $m_H > 1\ \text{TeV}$

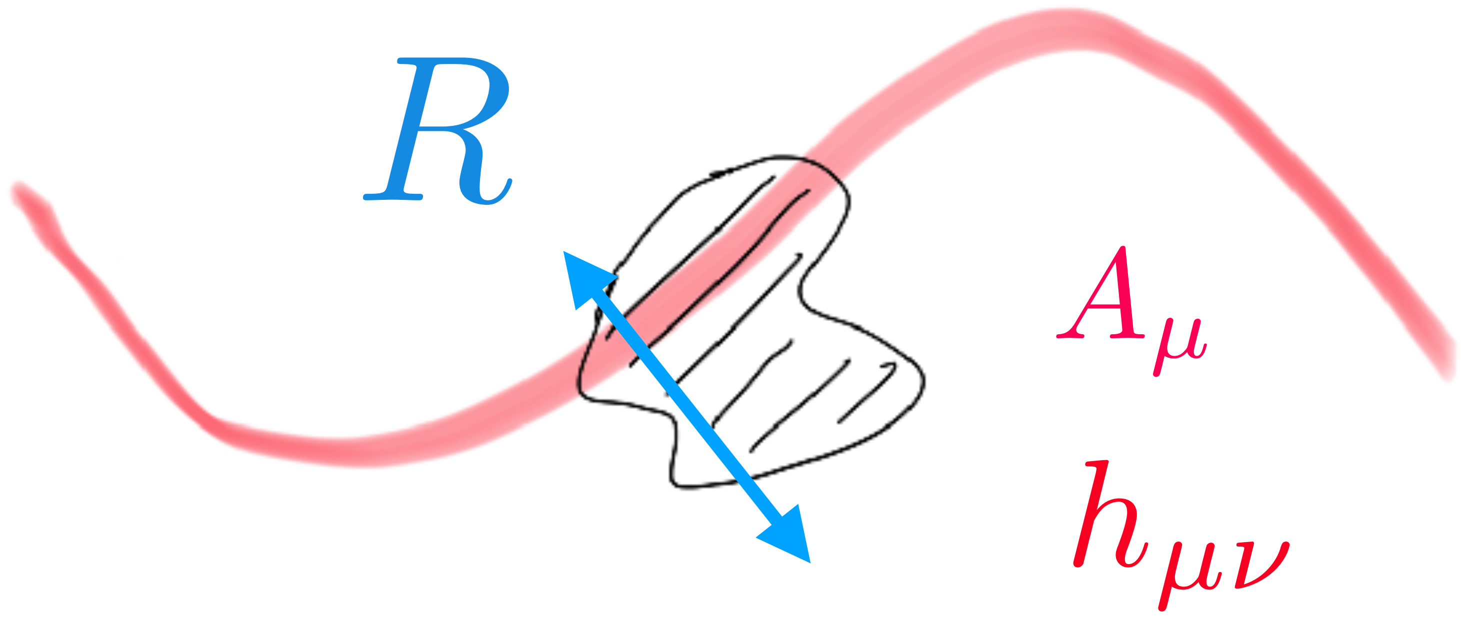
Experiment: $125\ \text{GeV}$

Higgs hierarchy problem

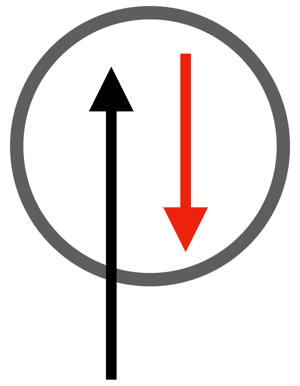
Naturalness in Worldline EFT

$$S_{\text{eff}} = \frac{C_2}{2}\int ds \; E_{ij}E^{ij} + \frac{\chi}{2}\int ds \; E_i E^i$$

$$C_2 \sim R^5 \qquad \chi \sim R^3$$



Ex 1: **Metal sphere**

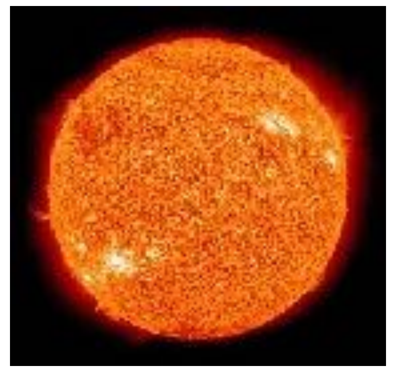


$$\text{IR}: \quad E_r = 1 - \frac{\chi}{r^3}$$

$$\text{UV}: \quad E_r = 1 - \frac{R^3}{r^3}$$

Matching: $\chi = R^3$

Ex 2: **Fluid Star**



$$h_{00} = r^2\left(1 - \frac{C_2}{r^5}\right)$$

$$h_{00} = r^2\left(1 - \bar{c}\frac{R^5}{r^5}\right)$$

$$\bar{c} = \mathcal{O}(1)$$

$$C_2 = \bar{c} \times R^5$$

Poisson, Will ‘textbook

Ex 3: **Black hole**

$\dots$

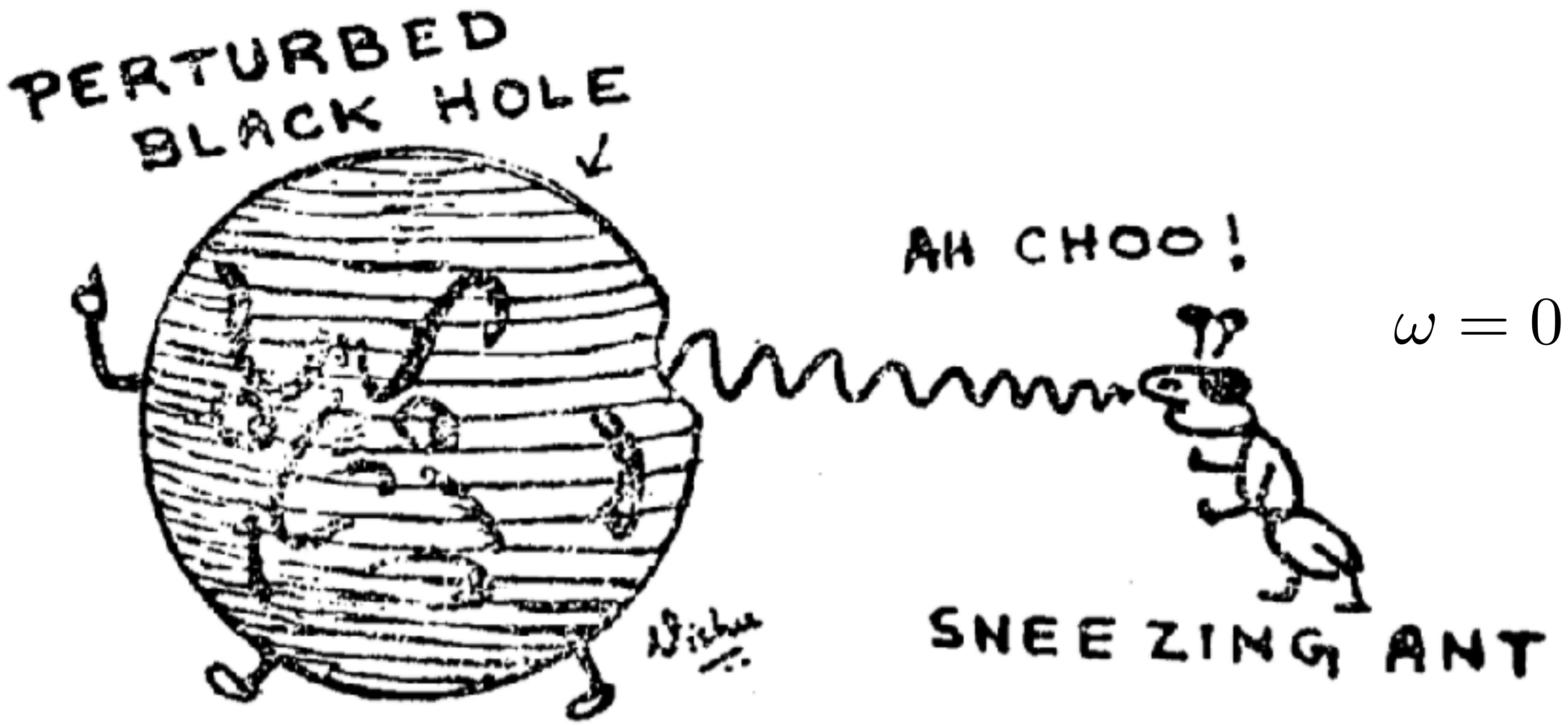
$$E_r = 1 \qquad h_{00} = r^2$$

$$\chi = 0 \ll R^3 \qquad C_2 = 0 \ll R^5$$

Binnington & Possion’09
Damour & Nagar’09

Black hole perturbations

$$ds^2_D = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2_{\mathbb{S}^{D-2}} \qquad f(r) = 1 - \left(\frac{r_s}{r}\right)^{D-3}$$



credit: C. V. Vishveshwara

$$\omega = 0$$

$$\Box\phi = 0$$

static $\partial_t \phi = 0$

with source boundary conditions at infinity

multipolar index

$$\phi = r^{\ell} \cdot {}_2F_1\left(-\hat{\ell}, -\hat{\ell}, 1; 1 - \left(\frac{r_s}{r}\right)^{D-3}\right)$$

$$\hat{\ell} = \frac{\ell}{D-3}$$

Kol & Smolkin'11, Hui et al.'20

Black hole perturbations

$$\phi = r^{\ell} \cdot {}_2F_1\left(-\hat{\ell}, -\hat{\ell}, 1; 1 - \left(\frac{r_s}{r}\right)^{D-3}\right) \qquad \hat{\ell} = \frac{\ell}{D-3}$$

at infinity:

$$\phi = r^{\ell}\left(1 + \ldots + k\left(\frac{r_s}{r}\right)^{2\ell+D-3}\right)$$

$$k = \frac{1}{2^{2+4\hat{\ell}}} \frac{\Gamma^2(\hat{\ell}+1)}{\Gamma\left(\hat{\ell}+\frac{1}{2}\right)\Gamma\left(\hat{\ell}+\frac{3}{2}\right)} \tan(\pi\hat{\ell})$$

special cases:

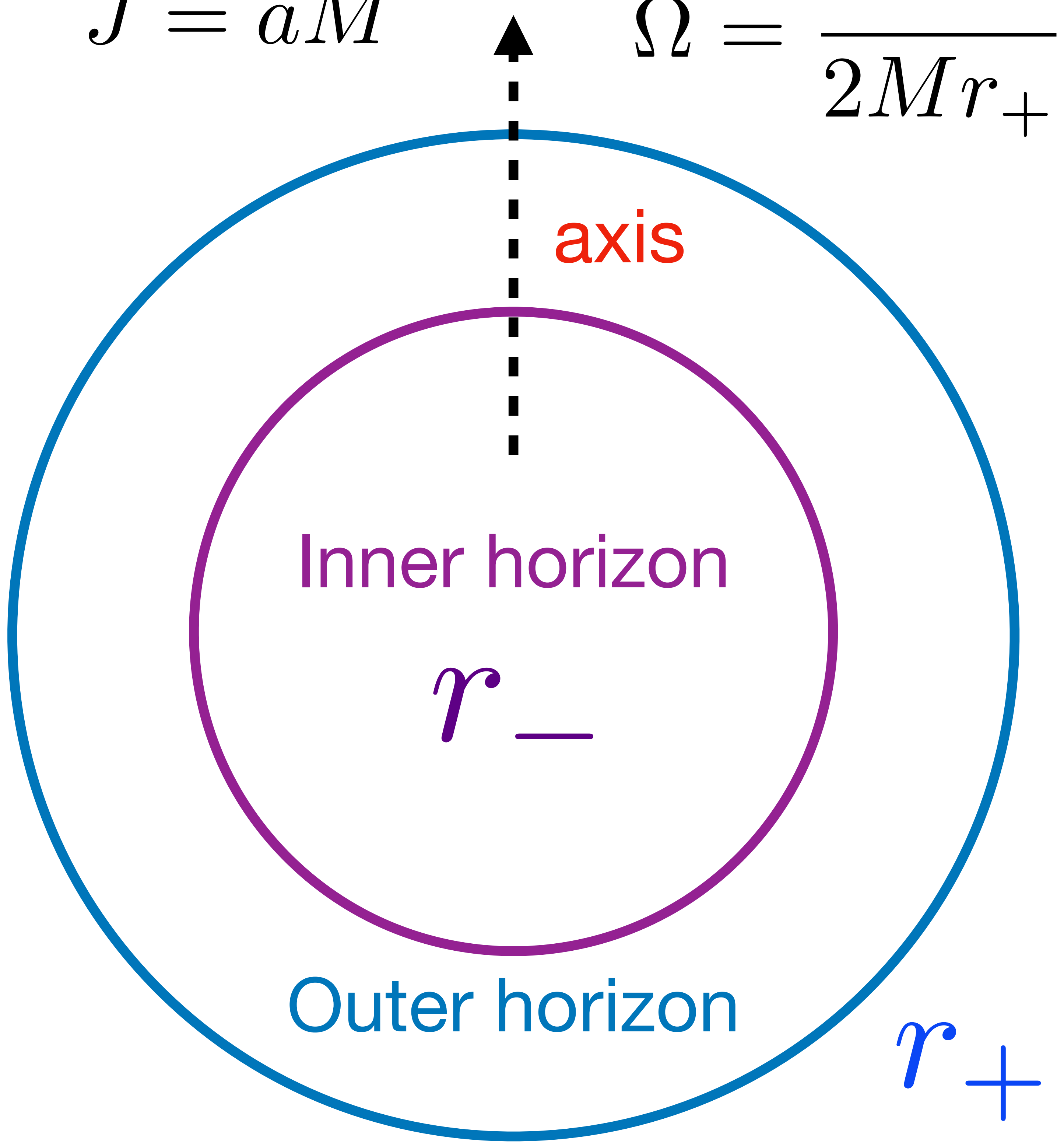
$\hat{\ell} = 1, 2, 3, \ldots$	$k = 0$	$\phi = r^{\ell} + \ldots + 1$
$\hat{\ell} = \frac{1}{2}, \frac{3}{2}, \ldots$	$k = \ln(r/r_s)$	Classical RG

N.B. same behaviors for higher spins

Kol & Smolkin'11, Hui et al.'20

Black hole perturbations in Kerr

$J = aM$ $\qquad \Omega = \dfrac{a}{2Mr_+}$



$\Delta = (r - r_-)(r - r_+)$

$l^\mu, n^\nu, m^\lambda, \bar{m}^\rho$ Null tetrades — Newman, Penrose (1962)

$s = 2 \qquad \psi_0 = C_{\mu\nu\lambda\rho} l^\mu m^\nu l^\lambda m^\rho, \; \psi_2, \; ..., \; \psi_4$

$s = 1 \qquad \phi_0 = F_{\mu\nu} l^\mu m^\nu, \; \phi_1, \phi_2$

$s = 0 \qquad \varphi$

Teukolsky (1972)

Consider first the vacuum case ($T = 0$). Then the master equation (4.7) can be separated by writing

$$\psi = e^{-i\omega t} e^{im\varphi} S(\theta) R(r) \,. \tag{4.8}$$

The equations for R and S are

$$\Delta^{-s} \frac{d}{dr}\left(\Delta^{s+1} \frac{dR}{dr}\right) + \left(\frac{K^2 - 2is(r-M)K}{\Delta} + 4is\omega r - \lambda\right) R = 0 \,, \tag{4.9}$$

$$\frac{1}{\sin\theta} \frac{d}{d\theta}\left(\sin\theta \frac{dS}{d\theta}\right) + \left(a^2\omega^2 \cos^2\theta - \frac{m^2}{\sin^2\theta} - 2a\omega s \cos\theta - \frac{2ms\cos\theta}{\sin^2\theta} - s^2 \cot^2\theta + s + A\right) S = 0 \,,$$

Kerr in EFT

$$S_{\rm eff} = \chi \int ds\; E_i E^i$$

$$S_{\rm eff} = \frac{C_2}{2} \int ds\; E_{ij} E^{ij}$$

Anisotropy:

$$\cancel{SO}(3) \supset SO(2)$$

$\longrightarrow$

$$S_{\rm eff} = \chi_{ij} \int ds\; E^i E^j$$

$$S_{\rm eff} = \frac{1}{2} \int ds\; \Lambda_{ij,kl} E^{ij} E^{kl}$$

Dissipation: $P^i = \bar{\chi}^i_j(\omega) E^j$

$$\bar{\chi}(\omega) = \chi + i\chi_1 \omega + \ldots$$

Absorption coefficient can't be obtained from local action!

$$Q^{ij} = \bar{\Lambda}^{ij}_{kl}(\omega') E^{kl}$$

body's rest frame

$$\bar{\Lambda}(\omega') = \Lambda + i\Lambda_1 \omega' + \ldots$$

$$\bar{\Lambda}(\omega) = \Lambda + i\Lambda_1 (\omega - m\Omega) + \ldots$$

observer's frame

Dissipation even in static regime!

Superradiance = tidal locking

For experts: $\int ds\; E_i \mathcal{O}_i(X) \leftrightarrow \int_{\partial \mathrm{AdS}_5} \phi(z \to 0, x) \mathcal{O}_{\rm CFT}(x)$

Love Numbers in EFT

IR : $$S_{\text{eff}} = \frac{1}{2}\int ds\ \Lambda_{ij,kl} E^{ij} E^{kl}$$

$$\Lambda_{ij,kl} = k \cdot \mathcal{E}_{ij,kl}$$

STF basis tensors

Weyl scalar (l=2 sector)

$$\psi_0 \propto \underbrace{1}_{\text{source}} + \underbrace{\frac{k}{r^5}}_{\text{response}}$$

UV : Teukolsky eq. on ψ_0

$$\psi_0 \propto r^{-2}(r^2 + r + 1)$$

Fixing Wilson coefficients via matching to BH perturbation theory

Charalambous, Dubovsky, MI'21,
Le Tiec, Casals'20, Chia'20,
Poisson'20, Goldberger et al'20

$\longrightarrow$ $\Lambda_{ij,kl} = 0$

Kerr Naturalness Paradox

Near zone

Teukolsky (1972)

$$\varphi = \Phi(t, r, \phi) S(\theta) = R(r) S(\theta) e^{-i\omega t + im\phi}$$

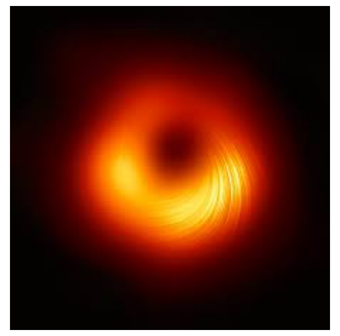
$$\partial_r(\Delta \partial_r R) + (V_0 + V_1) R = \ell(\ell+1) R$$

$$V_0 = \frac{(2Mr_+)^2}{\Delta}\left((\omega - \Omega m)^2 - 4\omega\Omega m \frac{r - r_+}{r_+ - r_-}\right)$$

$$V_1 = \frac{2M(\omega a m + 4M^2\omega^2 r_+ \beta)}{r_+ \beta (r - r_-)} + \omega^2(r^2 + 2Mr + 4M^2)$$

$$\Omega = \frac{a}{2Mr_+} \qquad \beta = \frac{r_+ - r_-}{4Mr_+} = 2\pi T_H$$

$$\Delta = (r - r_-)(r - r_+)$$



V_0

V_1

``Near zone”

$$(r - r_s)\omega \ll 1$$

$$\omega M \ll 1$$

$$V_1 = 0$$

``Far zone”

$$r \gg r_s$$

$$V_0 = 0$$

Starobinsky (1965), Page (1975)

Chia (2020)

SL(2,R) Love symmetry

$$L_0 = -\beta^{-1}\partial_t\,,$$

$$L_{\pm 1} = e^{\pm\beta t}\left(\mp\Delta^{1/2}\partial_r + \beta^{-1}\partial_r(\Delta^{1/2})\partial_t + \frac{a}{\Delta^{1/2}}\partial_\phi\right)$$

$$[L_n, L_m] = (n-m)L_{n+m}\,, \quad n, m = -1, 0, 1\,.$$

$$\beta = \frac{r_+ - r_-}{4Mr_+} = 2\pi T_H$$

Casimir $\mathcal{C}_2 \equiv L_0^2 - \frac{1}{2}(L_{-1}L_1 + L_1 L_{-1})$

$$\mathcal{C}_2\Phi = \partial_r(\Delta\partial_r\Phi) + V_0\Phi = \ell(\ell+1)\Phi$$

$$L_0\Phi = i(2\pi T_H)^{-1}\omega\Phi \equiv h\Phi$$

BH perturbations form reps of SL(2,R)

Charalambous, Dubovsky, MI'21

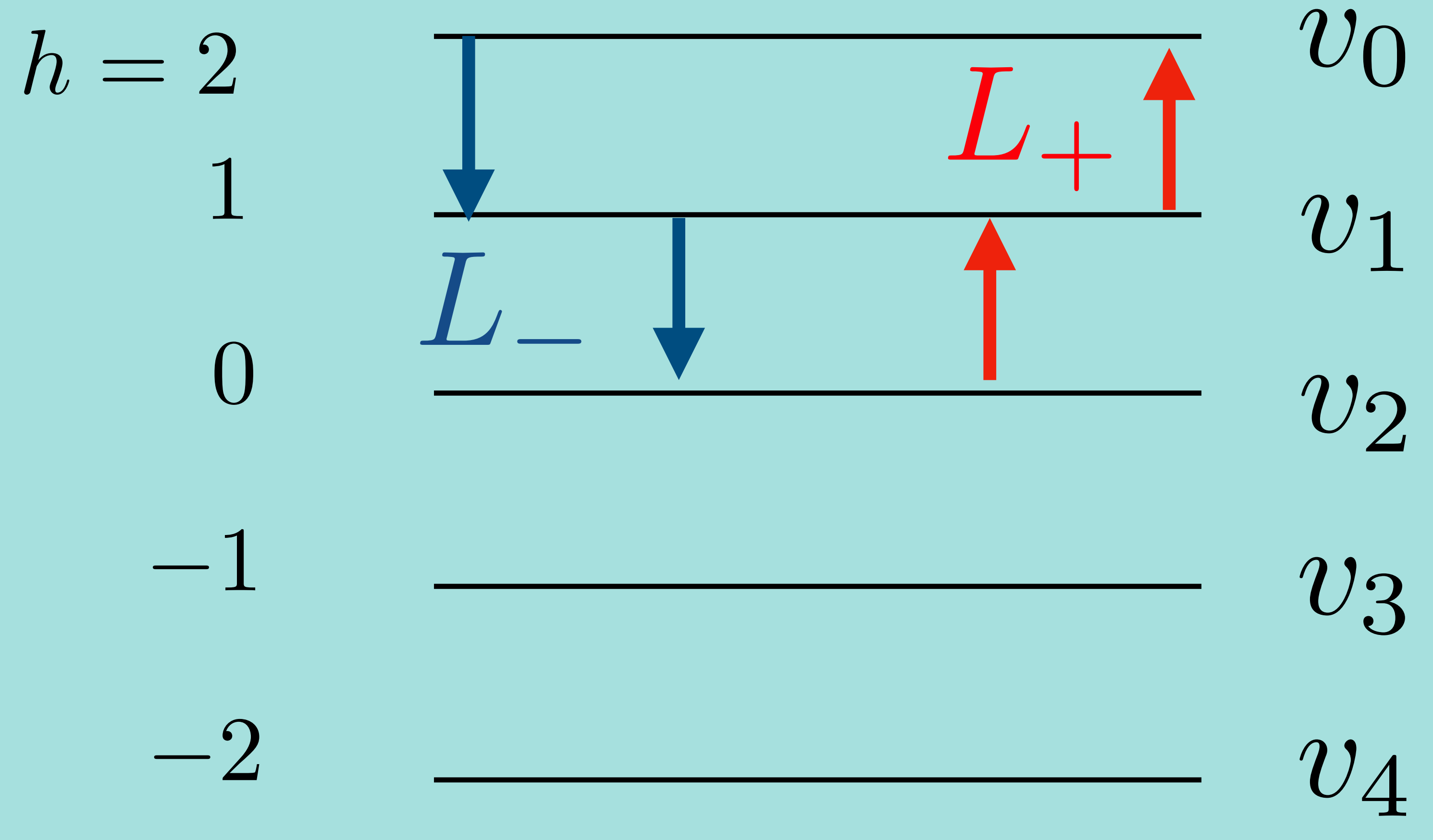
Highest weight banishes Love

$$L^2 = \ell(\ell+1)$$
$$L_0 v = -hv$$

ℓ integer

$|h|_{\min} = 0$

highest weight rep



$$L_+ v_0 = 0$$
$$v_1 = L_- v_0$$
$$\cdots$$

$$L_0 = -\beta^{-1}\partial_t$$
$$h_{\max} = \ell = 2$$

$$[L_+, L_-] = 2L_0$$
$$[L_\pm, L_0] = \pm L_\pm$$

Schwarzshild: finite-dim rep

Kerr: infinite-dim rep (“Verma module”)

$$L_+^3 v_2 = \partial_r^3 v_2 = 0 \qquad v_2 = r^2 + r + 1$$

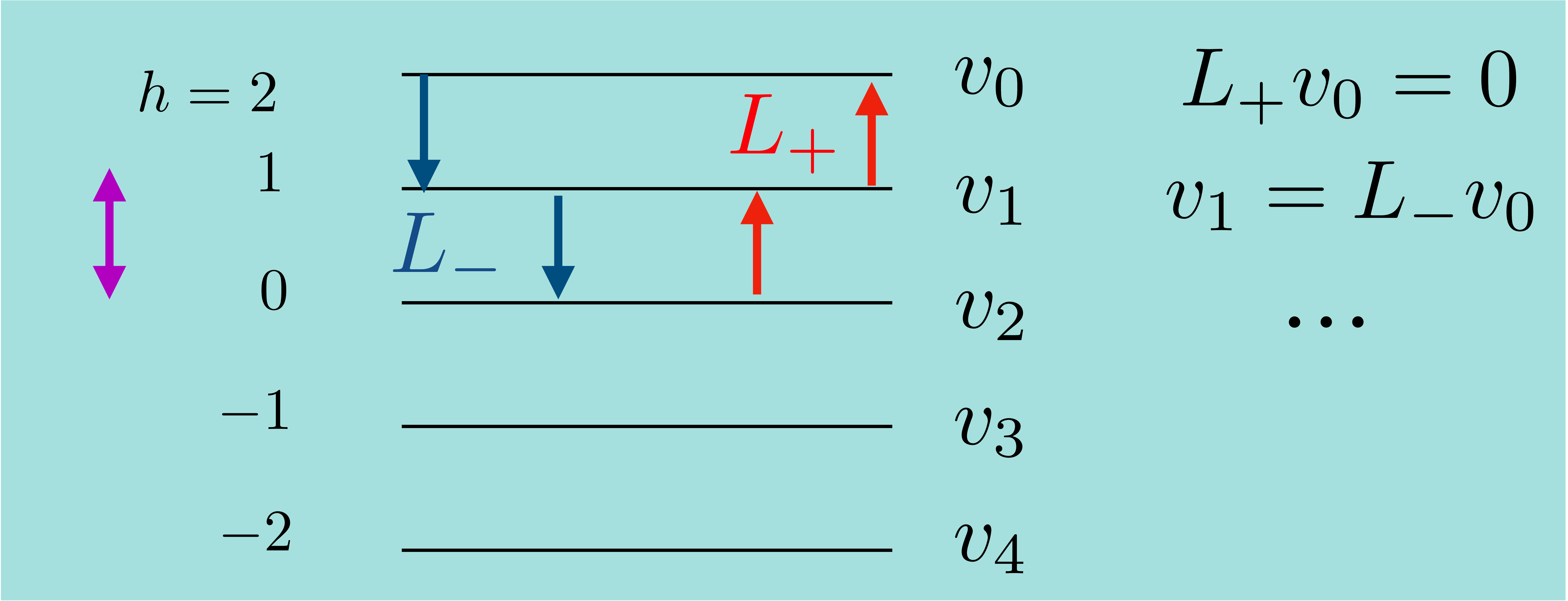
v_2 Static solution describing Love

Some comments

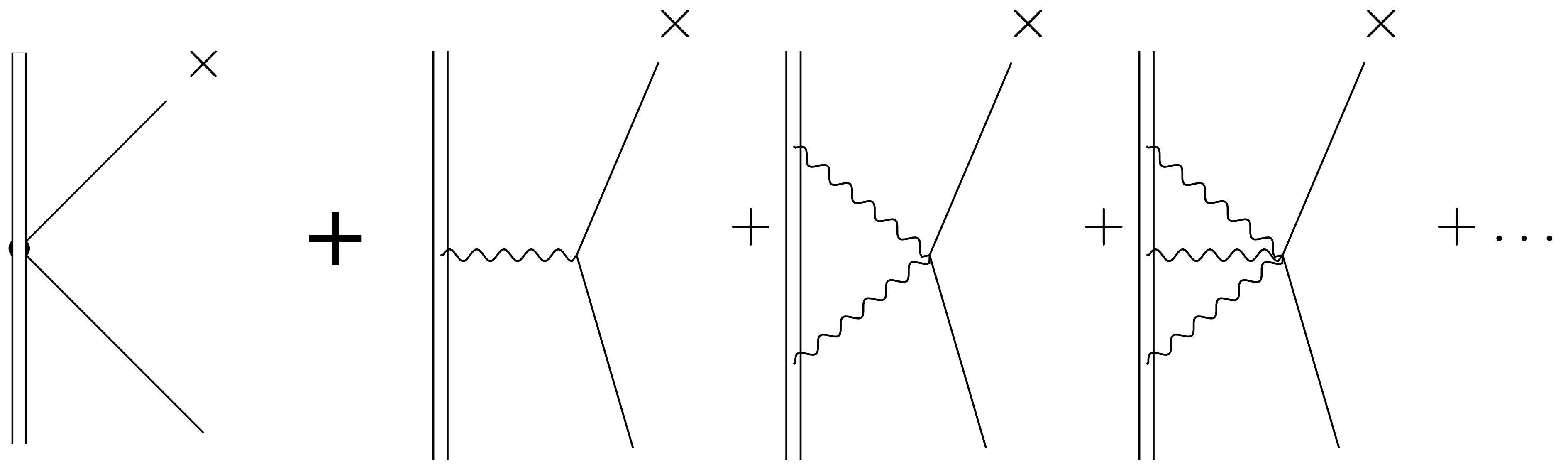
- Spacing matches highly damped QNMs

Berti, Kokkotas (2003)

$$\Delta\omega = -i(2\pi T_H)$$



- Explains higher-dimensional fine-tunings (absence of logs)



- Without symmetry logs are generic, ex: R^3 gravity
- With symmetry logs require “resonance condition” $\hat{\ell} = \frac{1}{2}, \frac{3}{2}, \dots$

cf. Zamolodchikov’86

Higher spin fields

$$L_0^{(s)} = L_0 + s\,,$$
$$L_{\pm 1}^{(s)} = L_{\pm 1} - s e^{\pm \beta t}(1 \pm 1)\partial_r(\Delta^{1/2})$$

Teukolsky master equation in NZ:

$$\mathcal{C}_2^{(s)}\psi_s = \left(\mathcal{C}_2 + s(\partial_r \Delta)\partial_r + s\frac{2Mr_+(r_+ - r_-)}{\Delta}\partial_t + s\frac{2(r-M)}{\Delta}a\partial_\phi + s^2 + s\right)\psi_s = \ell(\ell+1)\psi_s\,,$$

s spin weight

- highest-weight property dictates vanishing of Love numbers
- geometric meaning: GHP Lie derivatives w.r.t. Love vectors

~(approximate) Killing vectors

Geroch, Held, Penrose'73

$$\text{\L}_\xi = \mathcal{L}_\xi + b\, n_\mu \mathcal{L}_\xi \ell^\mu - s\, \bar{m}_\mu \mathcal{L}_\xi m^\mu$$

b boost weight

Ludwig'99

Triumph of Naturalness?

"UV miracle" Love symmetry mixes IR (static) and UV (QNMs)

Clash with Near Zone validity: ``Near zone" QNMs

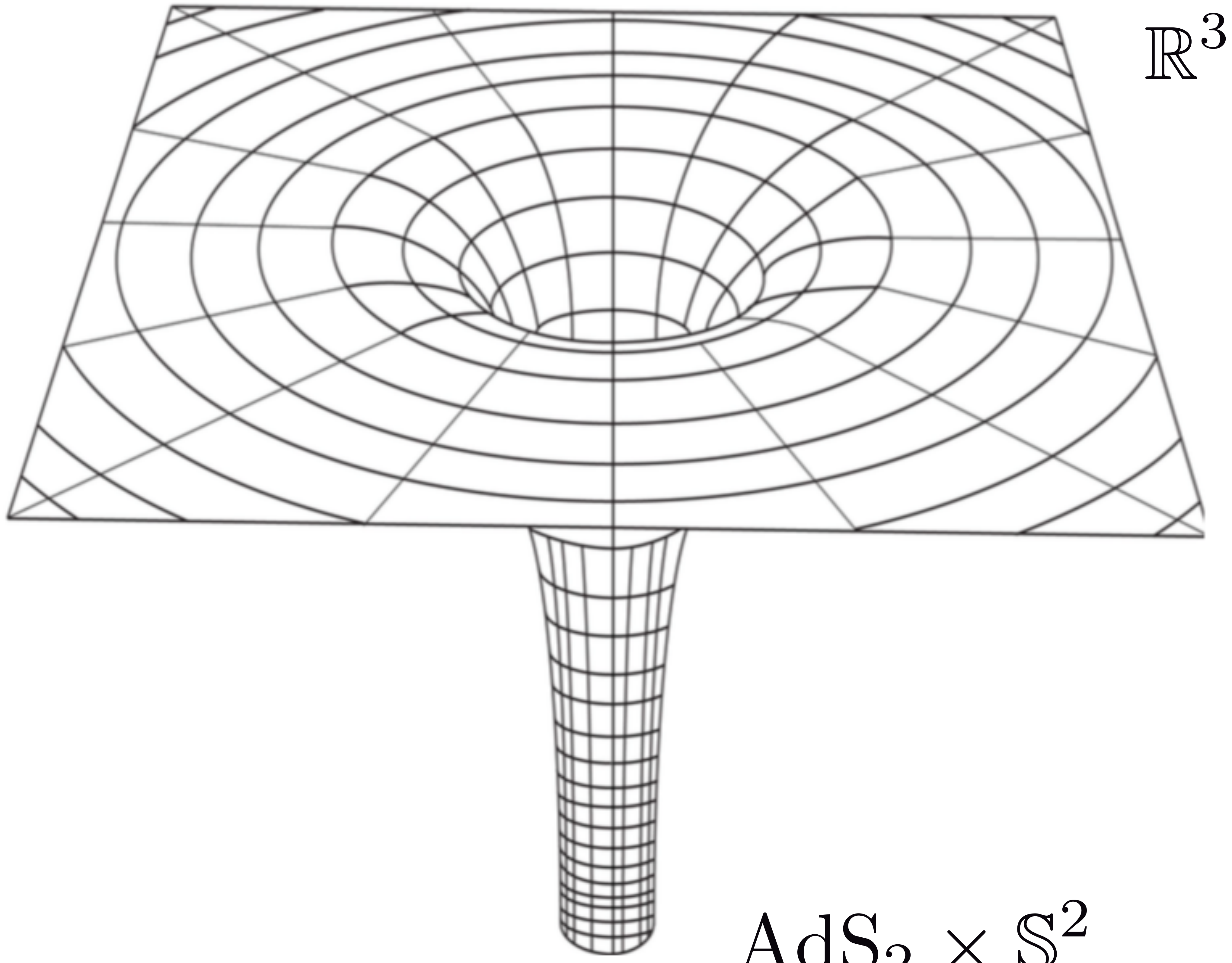
$\omega M \ll 1$ $\omega \sim -iT_H \sim iM^{-1}$

Formally accurate if $J \to M^2$ $T_H \to 0$

Approach: solve the near zone theory exactly and perturb around it

Results exact for static solution

Extremal RN/Kerr black holes



$$\mathrm{AdS}_2 = SL(2,\mathbb{R})$$

Bardeen, Horowitz (1998)

$$Q = M$$

RN: $$\lim_{Q \to M} SL(2,\mathbb{R})_{\text{Love}} = SL(2,\mathbb{R})_{\text{NH}}$$

$$J = M^2$$

Kerr: $$\lim_{a \to M} SL(2,\mathbb{R})_{\text{Love}} \ltimes \hat{U}(1) \supset SL(2,\mathbb{R})_{\text{NH}}$$

$$L_a \to L_a + v_a \partial_\phi \qquad v_a \in SL(2,\mathbb{R})$$

"Infinite-dimensional Love"

Non extremal ~ Spontaneously broken NH isometry

Interpretation

- non-extremal Kerr-CFT realization

Castro, Maloney, Srtominger' 10

$$SL(2,\mathbb{R})_R \times SL(2,\mathbb{R})_L$$

broken by $\phi \to \phi + 2\pi$

- accidental symmetry, cf. Runge-Lentz, lepton flavor, etc.

$$\varphi = \frac{Q}{r} + \frac{P_i x^i}{r^3} + \frac{Q_{ij} x^i x^j}{r^5} + \ldots$$

$$SO(3) \supset SO(2) \supset \varnothing$$

Summary and Outlook

- ✓ Love symmetry resolves the Love naturalness paradox
- ✓ Kerr(AdS)/CFT interpretation - ?
- ✓ Love symmetry ~ Chiral symmetry = > systematic waveform calculations

Series solution to Teukolsky equation exhibits symmetry breaking patterns *Mano et al'96*

Analogs of Gell–Mann–Okubo relations!

Thank you !