Ricci-flat spacetimes from AdS and its boundary Carrollian dynamics

David Rivera-Betancour

ITMP Seminars
Moscow State University

Based on 2309.15182 and 2205.09142
in collaboration with A. Campoleoni, A. Delfante, S. Pekar, P. M. Petropoulos, M. Vilatte, A. Petkou and K. Siampos

Preface

Contractions of the Poincaré group

Poincaré group: Lorentz transformations and translations (isometry group of Minkowski spacetime).

Consider the Lorentz transformations $(ct, x) \longrightarrow (ct', x')$ (c is the speed of light)

$$ct' = \gamma \left(ct - \frac{v}{c} x \right), \quad x' = \gamma \left(x - vt \right)$$

The Galilean group is reached at the $c \to \infty$ limit

$$t' = t \quad \text{(absolute time)} \quad x' = x - vt$$

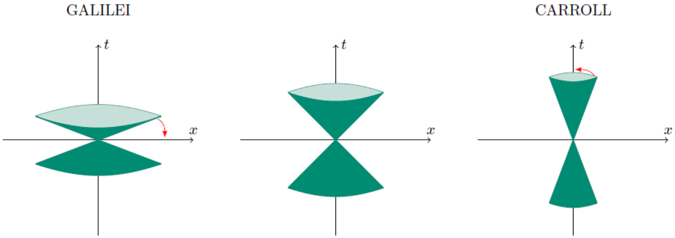
The Carroll group appears at the $c \to 0$ limit [Lévy-Leblond (65), Sen Gupta (66)]

$$t' = t - x\beta \quad x' = x \quad \text{(absolute space)}, \quad \text{with} \quad v = c^2 \beta$$

Carroll geometry from Minkowski

[Lévy-Leblond (65), Sen Gupta (66)]

- Light cone shrinks into a single line in the time axis (absolute space)
- All motion is forbidden
- Carroll group is the isometry group of flat Carroll spacetime



Motivation

Carrollian manifolds

- $c \to 0$ limit of pseudo-Riemannian manifolds
- The geometry of null hypersurfaces: null infinity and black hole horizons [L. Donnay and C. Marteau (19)]

Carrollian approach to flat holography?

This is based on two observations:

- $\text{BMS}_4 \equiv \text{CCarr}(3)$ [C. Duval *et al.* (14)]
- Null infinity possess a Carrollian structure [L.Ciemabelli *et al.* (19)]

In the Carrollian approach, the dual theory of Ricci-flat spacetimes should be Carrollian conformal invariant (BMS invariant) and hosted at null infinity.

Motivation

Carrollian physics through the years

- Development of Carrollian fluid dynamics [J. de Boer *et al.* (17)] [L. Ciambelli *et al.* (18)] [A. Petkou *et al.* (18)]
- Fluid/gravity correspondence for Ricci-flat spacetimes [L. Ciambelli *et al.* (18)]
- Reconstruction of asymptotically flat spacetimes from Carrollian data [A. Campoleoni *et al.* (23)]
- Study of asymptotic symmetries, charges and duality relations [A. Campoleoni *et al.* (22)] [N. Mittal *et al.* (22)] [N. Mittal *et al.* (22)]
- Bridge between Carroll and Celestial holography [L. Donnay *et al.* (22)]
- Etc

Outline

Carrollian fluid dynamics

- Dynamics: (i) Carrollian diffeomorphisms and Weyl invariance, and (ii) expansion of relativistic dynamics around small c
- Isometries and (non-)conservation laws

Ricci-flat spacetimes and its Carrollian dynamics

- Reconstruction of Ricci-flat spacetimes: flat from AdS
- Algebraically special subclass

Carrollian dynamics

Relativistic (fluid) dynamics

Here the dynamics is expressed as conservation laws

$$\nabla_\mu T^{\mu\nu} = 0\,.$$

Decomposition of $T^{\mu\nu}$

$$T^{\mu\nu} = (\varepsilon + p)\frac{u^\mu u^\nu}{c^2} + p g^{\mu\nu} + \tau^{\mu\nu} + \frac{u^\mu q^\nu}{c^2} + \frac{u^\nu q^\mu}{c^2}\,.$$

ε and p are the energy density and pressure

- u^μ is congruence with normalization $u^2 = -c^2$
- u^μ is arbitrary (hydrodynamic-frame invariance)
- q^μ and $\tau^{\mu\nu}$: non perfect part, expressed in term of temperature and velocity gradients (constitutive relations)

Derivation from symmetries

Given the action $S = \frac{1}{c} \int \mathrm{d}^{d+1}x\sqrt{-g}\mathcal{L}$

One has

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}}$$

- diffeomorphism invariance $(\xi = \xi^\mu(t, \mathbf{x})\partial_\mu) \longrightarrow \nabla_\mu T^{\mu\nu} = 0$
- Weyl invariance $\longrightarrow T^\mu{}_\mu = 0$ $(\mathscr{D} \equiv \nabla + \mathrm{A})$
- $T_{\mu\nu} = T_{\nu\mu}$ implies local Lorentz invariance

Isometries generated by Killing fields ξ $(\mathcal{L}_\xi g_{\mu\nu} = 0)$

- Divergence-free current $I^\mu = T^{\mu\nu}\xi_\nu \longrightarrow \nabla_\mu I^\mu = 0$
- $Q_\xi = \int_\Sigma *\mathrm{I}$ is conserved on-shell

Carrollian geometry

$d+1$-dimensional manifold $\mathcal{M} = \mathbb{R} \times \mathcal{S}$ (one-dimensional fiber and d-dimensional base $\mathcal{S}$)

Ingredients

- Degenerate metric: $\mathrm{d}s^2 = 0 \cdot \mathrm{d}t^2 + a_{ij}(t, \mathbf{x})\mathrm{d}x^i \mathrm{d}x^j$
- Kernel generated by: $\upsilon = \frac{1}{\Omega}\partial_t$ such that $\upsilon^\mu g_{\mu\nu} = 0$.
- Dual 1-form: $\mu = -\Omega \mathrm{d}t + b_i \mathrm{d}x^i$ (Ehresmann connection)
- Basis of Carroll vectors: $\{\frac{1}{\Omega}\partial_t, \hat{\partial}_i\}$ with $\hat{\partial}_i = \partial_i + \frac{b_i}{\Omega}\partial_t$
- Basis for 1-forms: $\{\mu, \mathrm{d}x^i\}$

Example: Flat Carroll spacetime ($\upsilon = \partial_t$, $\mu = -\mathrm{d}t + b_i \mathrm{d}x^i$, $\mathrm{d}s^2 = \delta_{ij}\mathrm{d}x^i \mathrm{d}x^j$).

General covariance with respect to Carrollian diffeomorphisms: $t' = t'(t, \mathbf{x})$ and $\mathbf{x}' = \mathbf{x}'(\mathbf{x})$ preserving the time/space splitting

A strong Carroll structure is equipped with an ambiguous connection.

Here we use the Carroll-Levi-Civita Connection (time/space splitting)

$$\hat{\nabla}_i V^j = \hat{\partial}_i V^j + \hat{\gamma}^j_{ik} V^k\,, \quad \text{with} \quad \hat{\gamma}^j_{ik} = \frac{1}{2} a^{jl} \left(\hat{\partial}_i a_{kl} + \hat{\partial}_k a_{il} - \hat{\partial}_l a_{ik} \right)$$

$$\frac{1}{\Omega} \hat{D}_t V^i = \frac{1}{\Omega} \partial_t V^i + \hat{\gamma}^i{}_j V^j\,, \quad \text{with} \quad \hat{\gamma}_{ij} = \frac{1}{2\Omega} \partial_t a_{ij}$$

Some geometric quantities

$$\left[\hat{\nabla}_i, \frac{1}{\Omega} \hat{D}_t \right] \Phi = -\varphi_i \frac{1}{\Omega} \hat{D}_t \Phi\,, \quad \left[\hat{\nabla}_i, \hat{\nabla}_j \right] \Phi = 2\varpi_{ij} \frac{1}{\Omega} \hat{D}_t \Phi$$

$$\left[\hat{\nabla}_k, \hat{\nabla}_l \right] V^i = \hat{R}^i{}_{jkl} V^j + \varpi_{kl} \frac{2}{\Omega} \hat{D}_t V^i$$

Weyl-Carroll connection

Under Weyl rescaling, it is possible to define Weyl-Carroll covariant derivatives as $\hat{\mathscr{D}}_i = \hat{\nabla}_i + w\varphi_i$ and $\frac{1}{\Omega}\hat{\mathscr{D}}_t = \frac{1}{\Omega}\hat{D}_t + w\theta$ with $\theta = \hat{\gamma}^i{}_i$ the Carroll expansion.

Weyl-Carroll covariant curvature

$$\left[\hat{\mathscr{D}}_k, \hat{\mathscr{D}}_l\right] V^i = \left(\hat{\mathscr{R}}^i{}_{jkl} - 2\xi^i{}_j \varpi_{kl}\right) V^i + \varpi_{kl} \frac{2}{\Omega} \hat{\mathscr{D}}_t V^i + w \left(2\hat{\partial}_{[k}\varphi_{l]} - \varpi_{kl}\theta\right) V^i \,.$$

Carrollian dynamics: from symmetries [arXiv:2205.09142]

Given the action $S = \int \mathrm{d}^d x \mathrm{d}t \Omega \sqrt{a} \mathcal{L}$

We can compute the set of momenta:

$$\Pi^{ij} = \frac{2}{\sqrt{a}\Omega}\frac{\delta S}{\delta a_{ij}}, \quad \Pi^i = \frac{1}{\sqrt{a}\Omega}\frac{\delta S}{\delta b_i}, \quad \Pi = -\frac{1}{\sqrt{a}}\left(\frac{\delta S}{\delta \Omega} + \frac{b_i}{\Omega}\frac{\delta S}{\delta b_i}\right)$$

- Diffeo invariance $\left(\xi = \xi^t(t, \mathbf{x})\partial_t + \xi^i(\mathbf{x})\partial_i\right) \longrightarrow$ Conservation for Π, Π^i, Π^{ij}
- Weyl invariance $\longrightarrow \Pi^i{}_i = \Pi$
- Carroll boost invariant if $\Pi^i = 0$

The resulting conservation equations are

$$\mathcal{E} = \frac{1}{\Omega}\hat{\mathscr{D}}_t \Pi + \hat{\mathscr{D}}_i \Pi^i + \Pi^{ij}\xi_{ij} = 0\,, \quad \mathcal{G}_j = \hat{\mathscr{D}}_i \Pi^i{}_j + 2\Pi^i \varpi_{ij} + \left(\frac{1}{\Omega}\hat{\mathscr{D}}_t \delta^i_j + \xi^i{}_j\right) P_i = 0$$

Isometries and (non-)conservations laws

These are generated by Carrollian Killing fields $\{\xi^t(t,\mathbf{x}), \xi^i(\mathbf{x})\}$ that preserve the Carrollian geometry, namely they satisfy

$$\boxed{\mathcal{L}_\xi a_{ij} = 0\,, \quad \text{and} \quad \mathcal{L}_\xi \nu = 0}$$

We can define Carrollian isometric currents

- Current: $\kappa = \xi^i P_i - \xi^{\hat{t}}\Pi$, $K^i = \xi^j \Pi_j{}^i - \xi^{\hat{t}}\Pi^i$
- It is not guaranteed to be conserved!!

$$\text{Div}(\kappa, K^i) = \frac{1}{\Omega}\hat{\mathscr{D}}_t\kappa + \hat{\mathscr{D}}_j K^j = -\Pi^i\left(\mathscr{L}_\xi \mu\right)_i$$

- Conserved only if: $\Pi^i = 0$ or $\mathcal{L}_\xi \mu = 0$ (strong Killing)
- Charge: $Q = \int_\Sigma \mathrm{d}^2 x\sqrt{a}\left(\kappa + b_i K^i\right)$

Carrollian dynamics: limiting procedure [arXiv:2205.09142]

Before taking the small-c expansion

- Papapetrou-Randers gauge: $\mathrm{d}s^2 = -c^2\left(\Omega \mathrm{d}t - b_i \mathrm{d}x^i\right)^2 + a_{ij}\mathrm{d}x^i\mathrm{d}x^j$
- Stable under Carrollian diffeomorphisms
- explicit dependence on c
- $c \to 0$ limit leads to a Carroll structure

Expansion of $T^{\mu\nu}$ in powers of c

- $\frac{1}{\Omega^2}T_{00} = \cdots + \Pi + \mathcal{O}(c^2)$, $\frac{c}{\Omega}T_0{}^i = \cdots + \Pi^i + c^2 P^i + \mathcal{O}(c^4)$, $T^{ij} = \cdots + \Pi^{ij} + \mathcal{O}(c^2)$
- $\frac{c}{\Omega}\nabla_\mu T^{\mu 0} = \cdots + \frac{1}{c^2}\mathcal{F} + \mathcal{E} + \mathcal{O}(c^2)$
- $\nabla_\mu T^{\mu i} = \cdots + \frac{1}{c^2}\mathcal{H}^i + \mathcal{G}^i + \mathcal{O}(c^2)$

The Cotton tensor

Cotton tensor [arXiv:2310.19929]

In $1+2$ dimensions, one can define the Cotton tensor as

$$C_{\mu\nu} = \eta_\mu{}^{\rho\sigma} \nabla_\rho \left(R_{\nu\sigma} - \frac{R}{4} g_{\nu\sigma} \right) ,$$

which measures deviation from conformal flatness

Properties

- Two index symmetric tensor
- It is identically traceless: $C^\mu{}_\mu = 0$
- Conserved due to Bianchi identity: $\nabla_\mu C^{\mu\nu} = 0$
- Similar decomposition as the energy-momentum tensor: $C_{\mu\nu} \to c_{\text{den}},\ c_\mu,\ c_{\mu\nu}$

Considering the isometry generated by ξ

- Divergence-free current $I^\mu_{\text{Cott}} = C^{\mu\nu}\xi_\nu \longrightarrow \nabla_\mu I^\mu_{\text{Cott}} = 0$
- $Q^{\text{Cott}}_\xi = \int_\Sigma * \mathsf{I}_{\text{Cott}}$ is conserved (irrespective of any dynamics)

Carroll Cotton descendants

We can also take the small-c expansion of the Cotton tensor

In our decomposition the latter reads

$$\begin{aligned} c_{\text{den}} &= c_{(-1)}c^2 + c_{(0)} + \frac{c_{(1)}}{c^2} + \frac{c_{(2)}}{c^4}, \\ c^i &= c^2\psi^i + \chi^i + \frac{z^i}{c^2}, \\ c^{ij} &= c^2\Psi^{ij} + X^{ij} + \frac{Z^{ij}}{c^2} \end{aligned}$$

Then we can define the isometric current

- Current: $\kappa_{\text{Cott}} = \xi^i P_{\text{Cott}\,i} - \xi^{\hat{t}}\Pi_{\text{Cott}}$, $K^i_{\text{Cott}} = \xi^j \Pi^i{}_{\text{Cott}\,j} - \xi^{\hat{t}}\Pi^i_{\text{Cott}}$
- Conserved only if: $\Pi^i_{\text{Cott}} = 0$ or $\mathcal{L}_\xi \mu = 0$ (strong Killing)
- Charge: $Q_{\text{Cott}} = \int_\Sigma \mathrm{d}^2x\sqrt{a}\left(\kappa_{\text{Cott}} + b_i K^i_{\text{Cott}}\right)$

Bulk from boundary and flat from AdS

Solving Einstein's equations

For $\mathrm{d}s^2_{\text{bulk}} = G_{AB}\mathrm{d}x^A\mathrm{d}x^B$ in D dimensions, this goes as

1. Select a coordinate system plus a gauge conditions (D conditions)
2. Einstein's equations + fall off conditions $\longrightarrow G_{AB}$ in a radial expansion with coefficients $\{f(t, \mathbf{x})\}$
3. Asymptotic symmetries, gravitational charges and their algebras

Solution space $\equiv$ set of data $\{f(t, \mathbf{x})\}$

- They satisfy a set of dynamical equations
- Desirable feature: covariance with respect to the conformal boundary

Standard gauges

Fefferman-Graham gauge

- A(I)AdS spacetimes reconstructed in terms of **boundary** $g_{\mu\nu}$ and $T_{\mu\nu}$
- **No smooth vanishing Λ limit**

Newman-Unti gauge

$$\mathrm{d}s^2_{\text{bulk}} = \frac{V}{r}\mathrm{d}u^2 - \mathrm{d}u\mathrm{d}r + G_{ij}\left(\mathrm{d}x^i - U^i\mathrm{d}u\right)\left(\mathrm{d}x^j - U^j\mathrm{d}u\right),$$

where V, U_i and G_{ij} are functions of *all* coordinates

1. Gauge conditions: $G_{rr} = 0, \quad G_{ri} = 0, \quad G_{ru} = -1$
2. Valid for $\Lambda \neq 0$ and $\Lambda = 0$
3. For Ricci-flat spacetimes: infinite number of independent functions appearing in the power expansion of the metric
4. Not covariant with respect to the $3D$ null boundary structure

Covariant Newman-Unti gauge: A(l)AdS case

This is a relaxation of the Newman-Unti Gauge, where the gauge conditions are given by

$$\boxed{G_{rr} = 0 \quad \text{and} \quad G_{r\mu} = \frac{u_\mu}{k^2}\,,}$$

with $\Lambda = -3k^2$

Boundary data

- Boundary metric: $\mathrm{d}s^2 = -k^2\left(\Omega \mathrm{d}t - b_i \mathrm{d}x^i\right)^2 + a_{ij}\mathrm{d}x^i \mathrm{d}x^j$
- Boundary timelike congruence: $||\mathrm{u}||^2 = -k^2$ with $u^\mu \partial_\mu = \frac{1}{\Omega}\partial_t$, $u_\mu \mathrm{d}x^\mu = -k^2\left(\Omega \mathrm{d}t - b_i \mathrm{d}x^i\right)$
- Weyl connection: A_μ
- Elements of $T^{\mu\nu}$: ε, q_μ, and $\tau_{\mu\nu}$

Reconstruction of A(I)AdS spacetimes

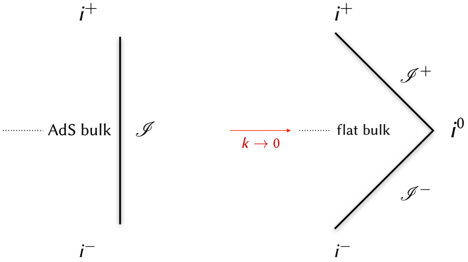
Guided by diffeomorphisms, Weyl covariance and Einstein dynamics in an asymptotic expansion on r

$$ds^2_{\text{bulk}} = \frac{2}{k^2} u_\mu \mathsf{d}x^\mu \mathsf{d}r + \sum_{n\geq 0} r^{2-n} g^{(n)}_{\mu\nu} \mathsf{d}x^\mu \mathsf{d}x^\nu \,.$$

Resolution of Einstein's equations order by order

- Boundary data $\{g^{(0)}_{\mu\nu}, u_\mu, g^{(3)}_{\mu\nu}\}$
- $g^{(1)}_{\mu\nu} = 2\ell^2 u_{(\mu} A_{\nu)} + \mathscr{C}_{\mu\nu}$ with $A_\mu = \frac{1}{k^2}\left(a_\mu - \frac{1}{2}\Theta u_\mu\right)$ and $k^2 \mathscr{C}_{\mu\nu} = -2\sigma_{\mu\nu}$
- $g^{(2)}_{\mu\nu} = 2u_{(\mu}\mathcal{D}_\lambda \left({\sigma_{\nu)}}^\lambda + {\omega_{\nu)}}^\lambda\right) - \frac{1}{2}\mathcal{R} u_\mu u_\nu + (\sigma_{\mu\lambda} + \omega_{\mu\lambda})\left({\sigma_\nu}^\lambda + {\omega_\nu}^\lambda\right)$
- $g^{(3)}_{\mu\nu} = 8\pi G \ell^2 \left(\ell^2 \varepsilon u_\mu u_\nu + \frac{4}{3}\ell^2 \Delta q_{(\mu} u_{\nu)} + \frac{2}{3}\Delta\tau_{\mu\nu}\right)$, with $\Delta q_\mu = q_\mu - \frac{1}{8\pi G} * c_\mu$ and $\Delta\tau_{\mu\nu} = \tau_{\mu\nu} + \frac{1}{8\pi G k^2} * c_{\mu\nu}$
- Einstein's equations $\to$ $\nabla^{(0)}_\mu T^{\mu\nu} = 0$.

Flat from AdS [arXiv:2309.15182]



The limit $\Lambda = -3k^2 \to 0$ acts as $c \to 0$ limit on the boundary

The boundary geometry becomes a Carrollian spacetime given by

$$\upsilon = \frac{1}{\Omega}\partial_t \,, \quad \mu = -\Omega \mathrm{d}t + b_i \mathrm{d}x^i \quad \text{and} \quad \mathrm{d}\ell^2 = a_{ij}\mathrm{d}x^i \mathrm{d}x^j$$

Carrollian limit on the boundary dynamics

$T^{\mu\nu}$ is expanded in powers of k:

$$\varepsilon = \sum_{n\in\mathbb{Z}} k^{2n}\varepsilon_{(n)}, \quad q^i = \sum_{n\geq 2} \frac{\zeta^i_{(n)}}{k^{2n}} + Q^i + k^2\pi^i + \sum_{n\geq 2} k^{2n}\pi^i_{(n)},$$

$$\tau^{ij} = -\sum_{n\geq 3} \frac{\zeta^{ij}_{(n)}}{k^{2n}} - \frac{\Sigma^{ij}}{k^2} - \Xi^{ij} - k^2 E^{ij} - \sum_{n\geq 2} k^{2n} E^{ij}_{(n)}$$

1. $\mathscr{C}_{ij}$ becomes free and $\sigma_{ij} = 0$ $\quad (k^2\mathscr{C}_{ij} = -2\sigma_{ij})$
2. Finiteness in the flat limit:
   - $\boxed{\varepsilon_{(n)} = 0 \quad \forall n < 0}$, $\boxed{\zeta^i_{(n)} = 0 \quad \forall n \geq 2}$, $\boxed{\zeta^{ij}_{(n)} = 0 \quad \forall n \geq 3}$
   - Q^i, Σ^{ij} and Ξ^{ij} are fixed by the Cotton descendants χ^i, X^{ij} and Ψ^{ij}
3. ε_0 and π_i satisfy Carrollian dynamical equations
4. The others (Chthonian) E_{ij}, $\{E^{ij}_{(n)}, \pi^i_{(n)}, \varepsilon_{(n-1)}\}_{n\geq 2}$ satisfy other flux-balance equations obtained by requiring finiteness in the line element

Ricci-flat spacetime reconstruction

For the infinite set of boundary data $\{a_{ij}, \Omega, b_i, \varepsilon_{(0)}, \pi_i, \mathscr{C}_{ij}, E_{ij}, ...\}$ the bulk spacetime is reconstruced order by order in powers of r as

Ricci-flat spacetime in covariant Newman-Unit gauge

$$
\begin{aligned}
\mathrm{d}s^2_{\text{bulk}} \;=\;& 2\mu \mathrm{d}r + r^2 a_{ij}\mathrm{d}x^i \mathrm{d}x^j - r\left(\theta\mu^2 - 2\varphi_i \mu \mathrm{d}x^i - \mathscr{C}_{ij}\mathrm{d}x^i\mathrm{d}x^j\right) \\
& - \hat{\mathscr{K}}\mu^2 - \left(2 * \hat{\mathscr{D}}_i * \varpi + \hat{\mathscr{D}}_j \mathscr{C}^j{}_i\right)\mu \mathrm{d}x^i \\
& + \left(\left(*\varpi^2 + \frac{\mathscr{C}_{kl}\mathscr{C}^{kl}}{8}\right) a_{ij} + *\varpi \mathscr{C}_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j \\
& + \frac{1}{r}\left(8\pi G \varepsilon_{(0)}\mu^2 - \frac{4}{3}N_i \mu \mathrm{d}x^i - \frac{16\pi G}{3}E_{ij}\mathrm{d}x^i\mathrm{d}x^j\right) + \mathcal{O}\left(\frac{1}{r^2}\right) .
\end{aligned}
$$

Here $\pi^i = *\psi^i - N^i$ and $8\pi G\varepsilon_{(0)} = 2M + \frac{1}{4}\mathscr{C}^{ij}\hat{\mathscr{N}}_{ij}$.

Flux-balance equations

The remaining equations to be satisfied are

$R_{tt} = R_{ti} = 0 \to \lim_{k\to 0} \nabla_\mu T^{\mu\nu} = 0$

$$\frac{1}{\Omega}\hat{\mathscr{D}}_t \varepsilon_{(0)} + \hat{\mathscr{D}}_i Q^i = F(\hat{\mathscr{N}}^{ij}, \mathscr{C}^{ij})$$

$$\frac{1}{\Omega}\hat{\mathscr{D}}_t \pi^i + \frac{1}{2}\hat{\mathscr{D}}^i \varepsilon_{(0)} - \hat{\mathscr{D}}_j \Xi^{ij} + 2 * \varpi * Q^i = F^i(\hat{\mathscr{N}}^{ij}, \mathscr{C}^{ij})$$

The above corresponds to the flux balance equations for the Bondi mass aspect M and angular momentum aspects N^i which are mapped as Carrollian fluid equations with external force $\{F(\hat{\mathscr{N}}, \mathscr{C}), F^i(\hat{\mathscr{N}}, \mathscr{C})\}$

Ressumable case

Resumation can be performed by tuning the boundary structure so that the bulk line element becomes exact for a subclass of solutions.

Resummation: Algebraically special Petrov type

- Solutions that admit at least one multiple principal null direction (caracterized by the properties of the Weyl tensor).
- Examples: Type D, II, III, N. Most of the known black hole solutions are type D.

Conditions on the boundary

1. $\mathscr{C}_{ij}(t, \mathbf{x}) = 0$
2. All Chthonian degrees of freedom are discarded
3. $\pi^i = *\psi^i$ (sub-leading Carroll-Cotton current)

Ressumable case

Under these conditions we get the exact line

$$\mathrm{d}s^2_{\text{res.}} = \mu\left[2\mathrm{d}r + 2\left(r\varphi_j - *\hat{\mathscr{D}}_j *\varpi\right)\mathrm{d}x^j - \left(r\theta + \hat{\mathscr{K}}\right)\mu\right] + \rho^2\mathrm{d}\ell^2 + \frac{\mu^2}{\rho^2}\left[8\pi G\varepsilon r + *\varpi c_{(0)}\right]$$

with $\rho^2 = r^2 + *\varpi^2$ and principal null direction $\mathsf{k} = \partial_r$.

The solution space is finite $\{\varepsilon, \Omega, b_i, a_{ij}\}$ and gives account for all possible algebraically special Ricci-flat solutions.

Conclusions and future directions

Some conclusions

- We have constructed general Carroll dynamical equations valid for curved and time-dependent Carrollian geometries.
- We also saw that in the presence of Carroll isometries, conservation of Noether charges is not guaranteed
- We found that a reconstruction of Ricci-flat spacetimes in terms of Carrollian boundary data exists and it can be reached from a vanishing Λ limit of the AdS instance in the appropriate gauge.

Future directions

- Carrollian reconstruction of Ricci-flat spacetimes in higher dimensions?
- Holographic Carrollian stress tensor from gravity action (null Brown-York stress tensor)
- Where does gravitational radiation is encoded in the Carrollian approach for flat holography? [G. Arenas-Henriquez, F. Diaz and D. Rivera-Betancour (to appear)]
- Computation of charges from boundary Carrollian dynamics. Comparison with bulk approaches [H.Godazgar, M.Godazgar and C.N. Pope (18-21)]. Where are the Newman-Penrose charges in this formalism?

Thanks!