Models for a Vast Energy Range: Particles Meet Gravity and Cosmology

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Mainly based on

- Salvio (Phys. Rev. D) arXiv:1810.00792, arXiv:1902.09557
- Salvio (Eur. Phys. J. C) arXiv:1907.00983
- Salvio, Veermäe (JCAP) arXiv:1912.13333
- Salvio (Phys. Lett. B) arXiv:2003.10446
- Ghoshal, Salvio (JHEP) arXiv:2007.00005
- Salvio, Scollo arXiv:2104.01334

Introduction: why models for a vast energy range

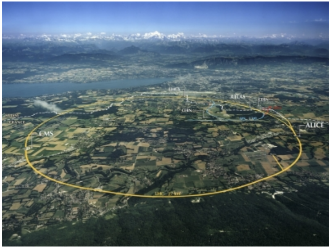
Part 1: simple motivated phenomenological completions of the SM

Part 2: models for an infinite energy range?

Summary

The SM is in agreement (so far) with all experiments performed at the CERN laboratory (LHC)

... located at the border between Switzerland and France.



The SM is a more complex version of QED, where besides e and γ there are other fermions (other leptons and quarks) and other bosons responsible for other forces

In 2012 the last SM particle was discovered at CERN, the Higgs boson, responsible for the masses of the elementary particles
[Englert, Brout (1964)] [Higgs (1964)]

New forces were discovered: the Yukawa interactions between fermions and the Higgs

The Standard Model (SM) and its needed extensions

The SM is very successful but it has certainly to be extended:
e.g. it does not include gravity and does not (completely) account for

- Neutrino Oscillations.
  (Obvious candidates to solve this problem are right-handed neutrinos N_i)
- Dark Matter (DM)
- Baryon Asymmetry of the Universe (BAU)

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$\rightarrow$ Models for a vast energy range

One way to ameliorate this situation:
construct models for the largest possible energy range and address the highest number of SM issues (much more selective than considering these issues separately)

This implies that not only collider bounds, but also observational cosmological/astrophysical bounds should be taken into account

Other SM problems

(besides neutrino oscillations, DM and BAU)

Electroweak vacuum metastability

In order to ensure the absolute stability of the electroweak (EW) vacuum one needs

$$M_t < (171.09 \pm 0.15_{\rm th} \pm 0.25_{\alpha_3} \pm 0.12_{M_h})\,{\rm GeV} = (171.09 \pm 0.31)\,{\rm GeV}$$

[Salvio (2017)], *[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (2013)]*

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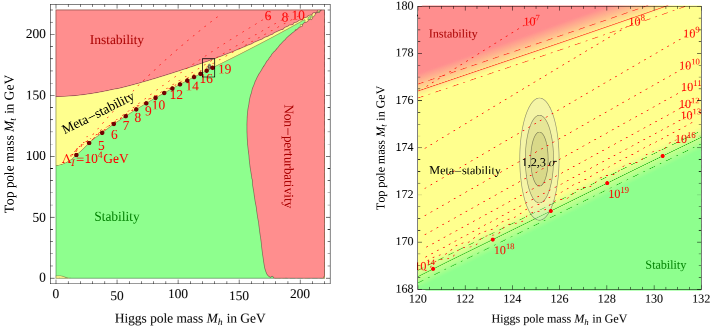
- Phase diagram of the SM:

[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (2013)]



Details on the calculation of the probability of vacuum decay

The probability $\Gamma = d\mathcal{P}/dV\,dt$ per unit time and volume of creating a bubble of true vacuum within a space volume dV and time interval dt is

$$d\mathcal{P} = dt\,dV\;\Lambda_B^4\,e^{-S_B}$$

S_B is the Euclidean action of the bounce of size $R \equiv \Lambda_B^{-1}$: the bounce h is an SO(4) symmetric Euclidean solution

$$h'' + \frac{3}{r}h' = \frac{dV}{dh}, \quad \text{with boundary conditions} \quad h'(0) = 0, \quad h(\infty) = h_{\rm EW}$$

[Coleman (1977); Coleman, Callan (1977)]

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Is metastability a problem during inflation?

Inflation *[Brout, Englert and Gunzig (1978); Guth (1981); Linde (1982); Albrecht and Steinhardt (1982)]*

What it can solve: horizon, flatness, monopole problems

To solve these problems inflation should last enough → lower bounds on

$$N \equiv \ln\left(\frac{a(t_{\rm end})}{a(t_{\rm in})}\right) \equiv \text{number of } e\text{-folds}$$

How it is implemented (slow-roll inflation):

- we assume a scalar field φ (the inflaton)
- at some early time the potential $U(\varphi)$ is large, but quite flat ...
- → the Hubble rate $H \equiv \dot{a}/a$ changes slowly → nearly exponential expansion

The inflaton rolls slowly when ...

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{1}{U}\frac{dU}{d\varphi}\right)^2 \ll 1, \quad \eta \equiv \frac{M_P^2}{U}\frac{d^2U}{d\varphi^2} \ll 1, \quad \text{where } M_P \simeq 2.4\times 10^{18}\text{GeV}$$

... from which we can compute observable inflationary parameters:
the scalar amplitude A_s, its spectral index n_s and the tensor-to-scalar ratio $r = \frac{A_t}{A_s}$

$$A_s = \frac{U/\epsilon}{24\pi^2 M_P^4}, \qquad n_s = 1 - 6\epsilon + 2\eta, \qquad r = 16\epsilon$$

The metastability is a SM problem during inflation

- During inflation the energy were so high that transitions to the true minimum were possible $\rightarrow$ interesting upper bounds on the Hubble rate during inflation *[Joti, Katsis, Loupas, Salvio, Strumia, Tetradis, Urbano (2017)]*
- The condition to have SM Higgs inflation is very similar to the stability bound

$$M_t < (171.43 \pm 0.12_{\rm th} \pm 0.28_{\alpha_3} \pm 0.12_{M_h})\,{\rm GeV} = (171.43 \pm 0.32)\,{\rm GeV}$$

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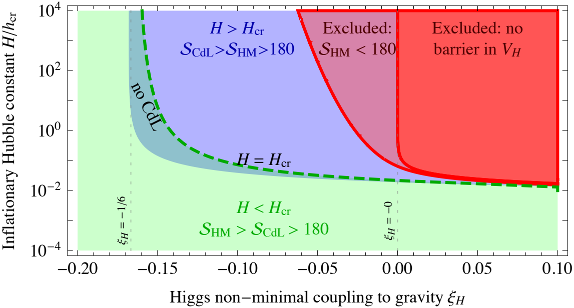
Disclaimer: However, to be sure about the metastability we need more precise measurements and calculations

Upper bounds on the Hubble rate during inflation

The model:

$$\mathscr{L} = \mathscr{L}_{\rm EH} + \mathscr{L}_{\rm SM} + \xi_H |H|^2 R$$

The results:



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  There are dynamical solutions, but not known anthropic solutions

The strong CP problem

One can add a P and CP violating term to the QCD Lagrangian:

$$-\frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu},$$

where

$$G^a_{\mu\nu} \equiv \text{gluon field strength}, \qquad \tilde{G}^a_{\mu\nu} \equiv \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} G^a_{\alpha\beta}$$

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example: $d_{\mathrm{neutron}} \sim |\theta| e \frac{m_\pi^2}{m_{QCD}^3} \sim 10^{-16} |\theta| e \times \mathrm{cm}$

[Baluni (1978); Crewther, Di Vecchia, Veneziano, Witten (1979)]

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This suppression is not enough: experimentally $d_{\text{neutron}} \lesssim 10^{-26} e \times \text{cm}$

$$\rightarrow |\theta| \lesssim 10^{-10}$$

Peccei-Quinn symmetry

Idea by *Peccei and Quinn (1977)*: promote θ to a dynamical variable such that changes in θ are equivalent to redefinitions of the various fields and so have no physical effect.

This is implement through a global chiral $U(1)$ (the Peccei-Quinn symmetry, $U(1)_{\mathrm{PQ}}$): some colored fermions are charged under $U(1)_{\mathrm{PQ}}$

$\Longrightarrow$ because of chiral anomaly a field redefinition leads to

$$\theta \to \theta + \Delta\theta$$

Field redefinitions cannot affect physics so any value of θ is equivalent to

$$\theta = 0$$

(the P and CP conserving value)

Peccei-Quinn symmetry and axions

In the presence of fermion masses

$\rightarrow$ the condensing field, which gives mass to fermions, is charged under $U(1)_{\rm PQ}$

Since any colored fermion is (or seems to be) massive
$\rightarrow U(1)_{\rm PQ}$ is spontaneously broken

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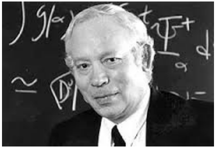
This leads to a pseudo Goldstone boson called *the axion*
[Weinberg (1978); Wilczek (1978)]



Example: axions, right-handed (sterile) neutrinos

The $a\nu$MSM *[Salvio (2015, 2018)]*

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \mathscr{L}_N + \mathscr{L}_{\rm axion} + \text{gravity part}$$

Right-handed neutrino sector:

$$\mathscr{L}_N = i\overline{N}_i \partial\!\!\!/ N_i + \left(\frac{1}{2} N_i M_{ij} N_j + Y_{ij} L_i H N_j + \text{h.c.}\right)$$

Axion sector (KSVZ):

$$\mathscr{L}_{\rm axion} = i \sum_{j=1}^{2} \overline{q}_j D\!\!\!\!/\, q_j + |\partial_\mu A|^2 - (y\, q_2 A q_1 + h.c.)$$
$$-\lambda_A (|A|^2 - f_a^2/2)^2 - \lambda_{HA}(|H|^2 - v^2)(|A|^2 - f_a^2/2)$$

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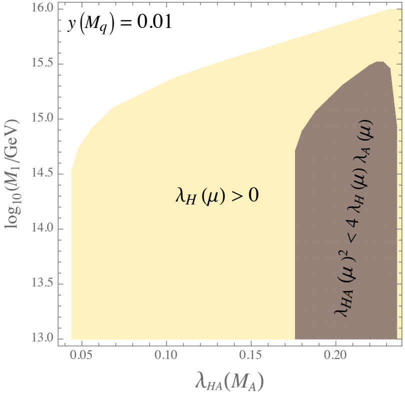
In some region of the parameter space the model account for

- **Neutrino oscillations** (through right-handed neutrinos which symmetrize the SM field content)
- **DM** (through the axion and the lightest sterile neutrino)
- **Baryogenesis** (through leptogenesis triggered by the right-handed neutrinos)

The $a\nu$MSM and absolute stability



- In the plot we set
  - the SM and low-energy neutrino parameters around the central values
  - the lightest neutrino mass $m_1 = 0$, $M_2 = 10^{14}$GeV
  - $f_a = 10^{11}$GeV and $\lambda_A(M_A) = 0.05$

The $a\nu$MSM and inflation

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This model was further studied by several scientists, e.g.
[Ballesteros, Redondo, Ringwald, Tamarit (2016)]
who proposed a variant where the Majorana masses M_i are generated by $\langle A \rangle$

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Inflation can be triggered by the Higgs and/or by $|A|$
[Salvio (2015, 2018)], *[Ballesteros, Redondo, Ringwald, Tamarit (2016)]*

By sitting at the frontier between stability and metastability (criticality) one can avoid further new physics or strong coupling at subplanckian energies in the case of Higgs inflation *[Salvio (2017, 2018)]*

The $a\nu$MSM and criticality

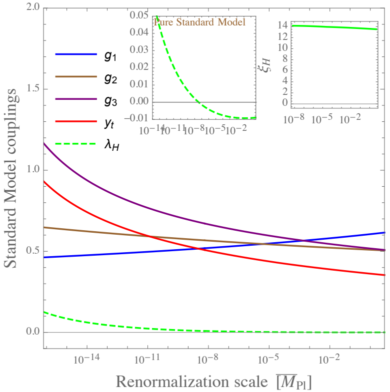

Figure: *Representative RG evolution of the relevant SM parameters close to criticality (λ_H is nearly zero at the Planck scale).*

The $a\nu$MSM and inflation: results

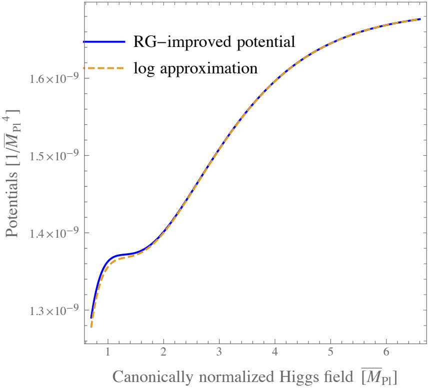

Figure: *RG-improved potential and its log-approximation close to criticality.*

Inflationary observables: $n_s \approx 0.96$, $r \sim 0.01$, $A_s \approx 2.1 \times 10^{-9}$ in agreement with the most recent Planck results (2018)

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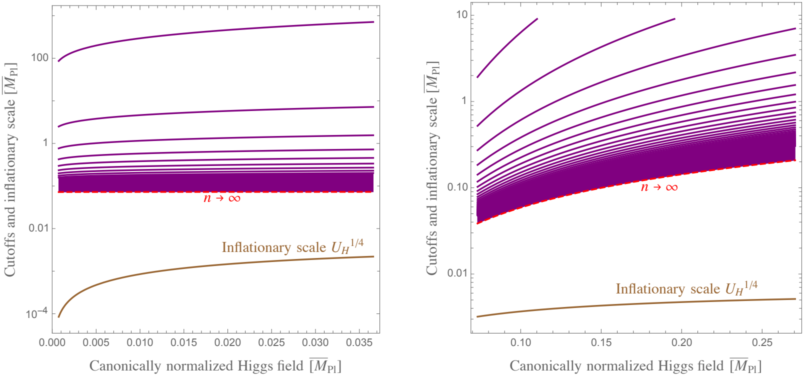


Figure: *The cutoff of the theory obtained by reading the coefficients of the dimension-n operators $\delta h'^n$ (for $n > 4$ and varying n) is compared to the inflationary scale.*

The $a\nu$MSM and dark matter (DM)

Work in collaboration with Simone Scollo

There are three possible sources of DM in the aνMSM

- axion
- lightest sterile neutrino
- Primordial black holes?

Axion dark matter

The axion is a good dark matter candidate

Axions are produced non-thermally through

- Misalignement mechanism *[Preskill, Wise, Wilczek (1983); Abbott, Sikivie (1983); Dine, Fischler (1983); Turner (1986)]*
  A recent calculation gives *[Ballesteros, Redondo, Ringwald, Tamarit (2016)]*

  $$\Omega_a h^2 = (0.12 \pm 0.02) \left(\frac{f_a}{1.92 \times 10^{11} \text{GeV}} \right)^{1.165}$$

  If $\Omega_a = \Omega_{\rm DM}$ this fixes f_a

  Higgs inflation features a high reheating temperature, $T_{\rm RH} \gtrsim 10^{13}$ GeV, thanks to the sizable couplings between the Higgs and other SM particles *[Bezrukov, Gorbunov, Shaposhnikov (2008)]*, *[Bellido, Figueroa, Rubio (2009)]*.
  Thus the PQ phase transition occurs after inflation in this case

- String decay *[Davis (1986); Harari, Sikivie (1987); Davis, Shellard (1989); Battye, Shellard (1997); etc]*
  It was estimated in the KSVZ model by *[Ballesteros, Redondo, Ringwald, Tamarit (2016)]*

Sterile-neutrino DM

The lightest sterile neutrino N_1 with mass m_s can contribute a fraction Ω_s of $\Omega_{\rm DM}$

It can be produced through a mixing θ with the active neutrinos. θ can receive a contribution from the mixing $\theta_{\alpha 1}$ of N_1 with the active neutrino of any flavour $\alpha \in \{e, \mu, \tau\}$:

$$\theta^2 = \sum_{\alpha=e,\mu,\tau} |\theta_{\alpha 1}|^2$$

- Non resonantly *[Dodelson, Widrow (1994)]*

  For a standard quark-hadron crossover transition, $T_{\rm QCD} \approx 170\ {\rm MeV}$, one obtains *[Abazajian (2005)]*

  $$m_s \approx 3.4\ {\rm keV} \left(\frac{\sin^2(2\theta)}{10^{-8}}\right)^{-0.615} \left(\frac{\Omega_{\rm s}}{0.26}\right)^{0.5}$$

- Resonantly *[Shi, Fuller (1998)]*: similar to the Dodelson-Widrow mechanism but there is a resonant enhancement due to a primordial lepton asymmetry

Sterile-neutrino DM

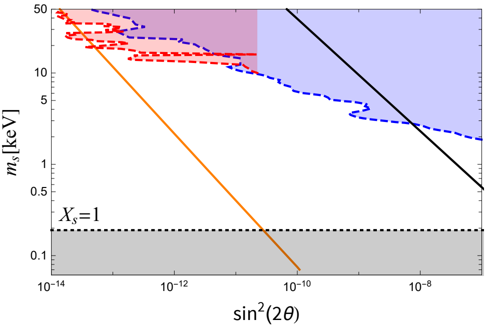
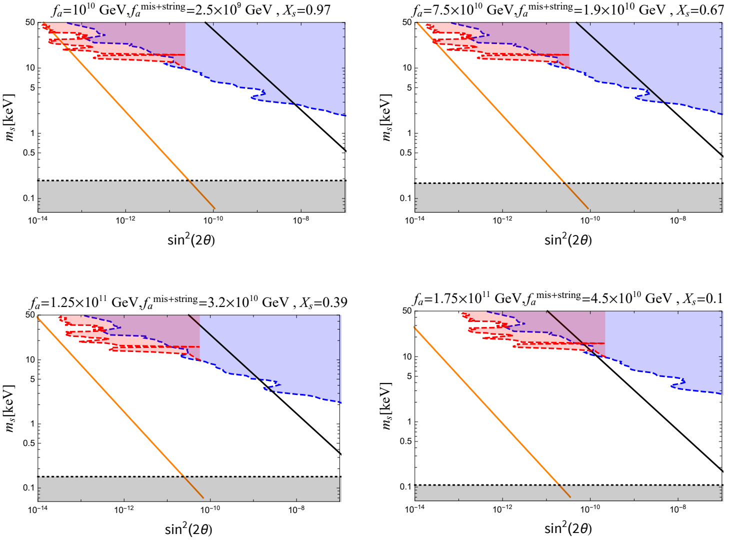


Figure:
The black line is the non-resonant sterile-neutrino production.
The region between the black and orange line is the resonant sterile-neutrino production.
The upper constraints are given by X-rays searches and the bound in dashed black is a phase-space bound related to Pauli's exclusion principle

The allowed regions have $m_s \sim$ keV and a very small θ

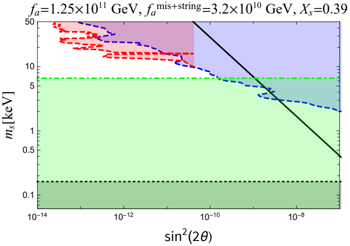
Axion-sterile-neutrino DM

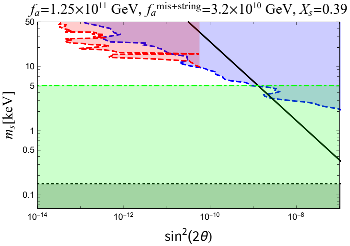


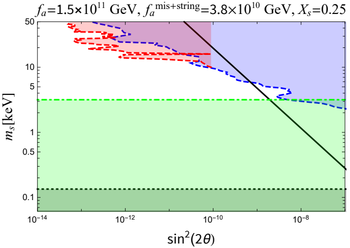
If you include some estimate (subject however to large uncertainties) of structure formation bounds one finds a small region of parameter space allowed for resonantly produced sterile neutrino DM.

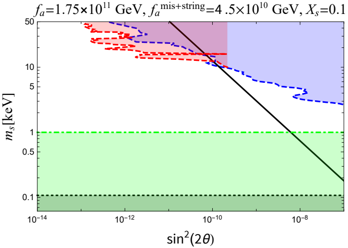
Axion-sterile-neutrino DM

Adding the structure formation bounds in the non-resonant case (green dot-dashed lines) *[Palazzo, Cumberbatch, Slosar, Silk (2007)]*









An allowed region appears only for $X_s \lesssim 0.3$

Primordial black holes?

- Primordial black holes may be generated if the curvature power spectrum has a peak of order $\sim 10^{-2}$ *[Hertzberg, Yamada (2017)]*.
- This is about 7 orders of magnitude larger than at ~ 60 e-folds before the end of inflation (the $A_s \sim 10^{-9}$ measured by Planck).

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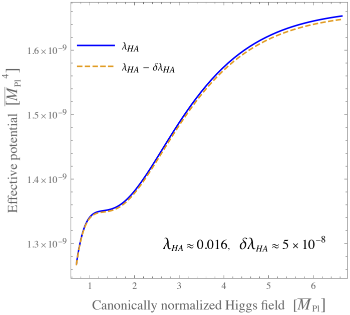

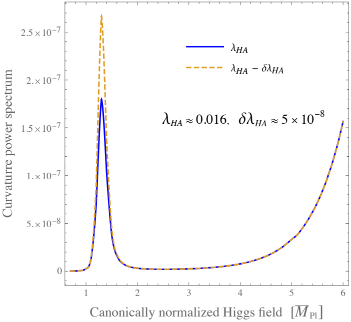

We approach criticality by lowering λ_{HA} but the number of e-folds become too large before reaching the required height

Introduction: why models for a vast energy range

Part 1: simple motivated phenomenological completions of the SM

Part 2: models for an infinite energy range?

Summary

Why $E \to \infty$ and how?

Requiring BSM models to hold from the Hubble up to the Planck scale removed some of the ambiguity in model building. Can we reduce further this ambiguity by requiring the theory to hold in an infinite energy range?

Apparently no: we cannot perform experiments at infinite energy
However, that requirement is very strong and can predict some effect at low energies (not all models can be UV completed)

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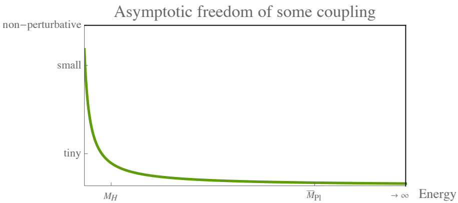
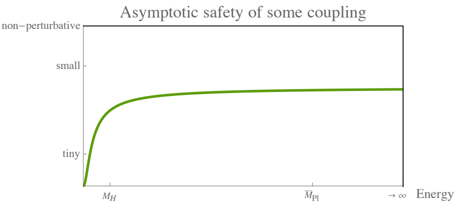
However, that requirement is very strong and can predict some effect at low energies (not all models can be UV completed)

We are confident that a field theory holds up to infinite energy if

- All couplings flow to zero at infinite energy (total asymptotic freedom): similarly to what happens in QCD, but extended to *all* fundamental interactions
- Or all couplings flow to an interacting fixed point: total asymptotic safety (TAS)
- Or, *more generally*, some couplings flow to zero and others to an interacting (typically non perturbative) fixed point, henceforth total asymptotic freedom/safety (TAFS)





Is the TAFS programme compatible with gravity?

(Einstein) gravitational interactions increase with energy

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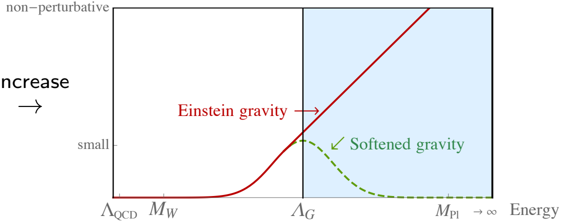
Idea (*softened gravity*):

consider theories where the increase of the gravitational coupling $\rightarrow$ *stops at some* $\Lambda_G \ll M_{\rm Pl}$.



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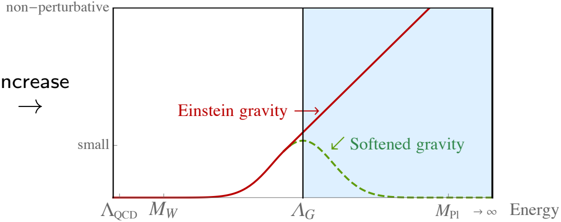

Examples of softened gravity can be string theory, non-local quantum gravity or quadratic-in-curvature gravity

TAFS phenomenology

In order to eliminate the Landau poles or run into a non-perturbative regime so far we needed to avoid U(1) gauge factors (obvious in the TAF case)

$\rightarrow$ explanation of the electric charge quantization

(we will discuss the more general TAFS case later)

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We find 2 options: $\text{SU}(4)_{\text{PS}} \times \text{SU}(2)_L \times \text{SU}(2)_R$, $\text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R$

that, unlike SU(5) and SO(10), are not severely constrained by proton decay

$$\rightarrow M_{\text{NP}} \sim 10 \text{ TeV}$$

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- TAFS models predict new physics

$$W_R\,, \quad Z'\,, \quad H'\,, \quad \text{etc}$$

[Giudice, Isidori, Salvio, Strumia (2014); Pelaggi, Strumia, Vignali (2015)]

TAF Pati-Salam

	Fields	spin	generations	$SU(2)_L$	$SU(2)_R$	$SU(4)_{PS}$
skeleton model	$\psi_L = \begin{pmatrix} \nu_L & e_L \\ u_L & d_L \end{pmatrix}$	1/2	3	$\bar{2}$	1	4
	$\psi_R = \begin{pmatrix} \nu_R & u_R \\ e_R & d_R \end{pmatrix}$	1/2	3	1	2	$\bar{4}$
	ϕ_R	0	1	1	2	$\bar{4}$
	$\phi = \begin{pmatrix} H_U^0 & H_D^+ \\ H_U^- & H_D^0 \end{pmatrix}$	0	1	2	$\bar{2}$	1
extra fields	ψ	1/2	$N_\psi \leq 3$	2	$\bar{2}$	1
	Q_L	1/2	2	1	1	10
	Q_R	1/2	2	1	1	$\overline{10}$
	Σ	0	1	1	1	15

TAF Trinification

Another TAF model can be built by extending the minimal trinification

Field	spin	generations	$SU(3)_L$	$SU(3)_R$	$SU(3)_c$	Δb_L	Δb_R	Δb_c
$Q_R = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ d_R^1 & d_R^2 & d_R^3 \\ d_R'^1 & d_R'^2 & d_R'^3 \end{pmatrix}$	$1/2$	3	1	3	$\bar{3}$	0	1	1
$Q_L = \begin{pmatrix} u_L^1 & d_L^1 & \bar{d}_R'^1 \\ u_L^2 & d_L^2 & \bar{d}_R'^1 \\ u_L^3 & d_L^3 & \bar{d}_R'^3 \end{pmatrix}$	$1/2$	3	$\bar{3}$	1	3	1	0	1
$L = \begin{pmatrix} \bar{\nu}_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ e_R & \nu_R & \nu' \end{pmatrix}$	$1/2$	3	3	$\bar{3}$	1	1	1	0
H	0	3	3	$\bar{3}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0

Note that right-handed neutrinos are included

A TAF axion sector

Field content: two extra quarks q and $\bar{q}$ that are doublets of a new SU(2) (called SU(2)$_a$) plus one complex scalar A such that we can write

$$\mathscr{L}_y = -y\bar{q}Aq + \text{H.c.}$$

and a potential

$$V_A = -m^2 \text{Tr}(A^\dagger A) + \lambda_1 \text{Tr}^2(A^\dagger A) + \lambda_2 |\text{Tr}(AA)|^2$$

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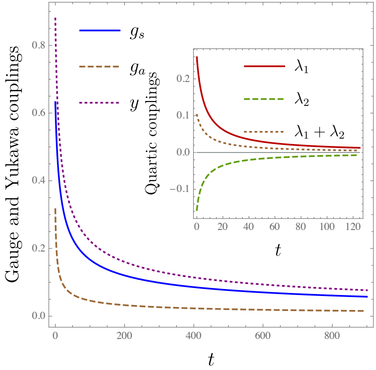


$$t \equiv \ln(\mu^2/\mu_0^2)/(4\pi)^2$$

Values of $(\tilde{\lambda}_1, \tilde{\lambda}_2) \equiv (t\lambda_1, t\lambda_2)$

Δ	unstable vacuum	stable vacuum
28/3	(0.183, −3.23)	(1.68, −0.951)
26/3	(0.149, −1.05)	(0.575, −0.343)
8	(0.145, −0.598)	(0.349, −0.231)

λ_1, λ_2 and y are predicted at low energies because only isolated values can be TAF

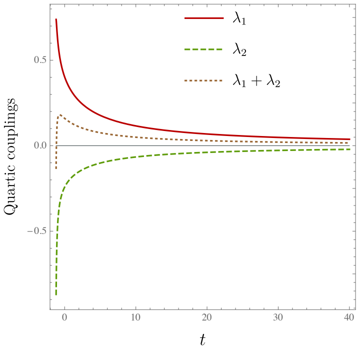
A TAF axion sector with f_a generated quantum mechanically

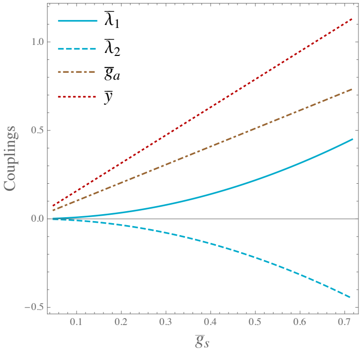
An even more predictive model can be obtained by generating f_a quantum mechanically *[Coleman, Weinberg (1973)]*

$$V_A = \lambda_1 \mathrm{Tr}^2(A^\dagger A) + \lambda_2 |\mathrm{Tr}(AA)|^2$$

f_a corresponds to a scale at which $\boxed{\lambda_1 + \lambda_2 = 0}$ (the flat direction $A = A^\dagger$ appears)

This condition can be obtained by choosing approprately g_a
(which is then also predicted)

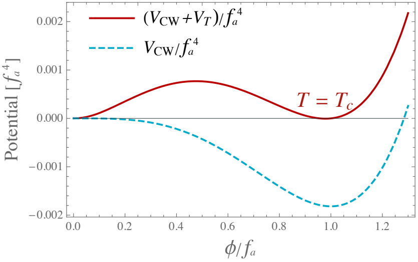




A strong first-order PQ phase transition and gravitational waves

- In collaboration with Anish Ghoshal (JHEP)

Unlike the $a\nu$MSM the TAF axion model gives a strong first-order phase transition



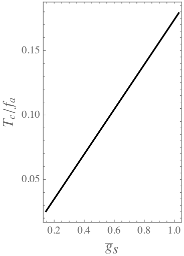


Figure: Left plot: *The effective potential setting the QCD gauge coupling at the PQ scale to $\bar{g}_s \approx 0.91$ and adding a constant such that it vanishes at $\phi = 0$.* **Right plot:** *the critical temperature T_c divided by f_a as a function of $\bar{g}_s$.*

The bubbles created are diluted by the expansion of the universe and they cannot collide until T reaches the nucleation temperature T_n, i.e. $\Gamma \sim H_I^4$.
The inverse duration of the phase transition

$$\beta_n \equiv \left[\frac{1}{\Gamma}\frac{d\Gamma}{dt}\right]_{T_n}$$

is very small in CW symmetry breaking (as almost scale invariant) and the phase transition lasts a long time (supercooling)

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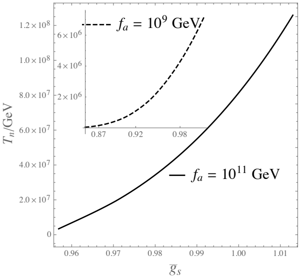
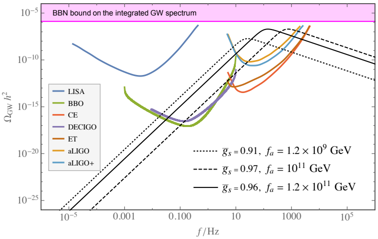
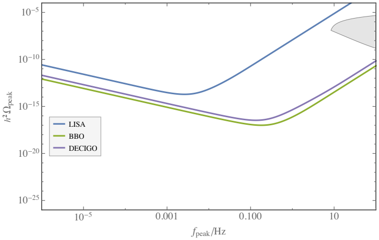






Summary

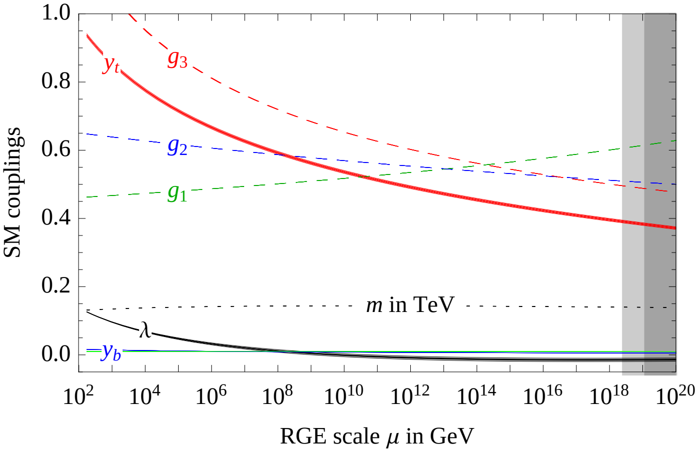
- SM extensions valid in a vast energy range allow us to reduce the arbitrariness we have in model building
- **Bottom-up road.** It was proposed a model (aνMSM) that combine the idea of axions and right-handed neutrinos and accounts for *all* the observational evidence for new physics and solve the strong-CP problem as well as the metastability issue of the SM. In particular we have discussed:
  1. How critical Higgs inflation can be implemented in a viable way
  2. Multicomponet axion-sterile-neutrino DM (work in collaboration with Simone Scollo)
- **Top-down road.** Theories with UV fixed points (TAFS) are even more predictive and their phenomenology should be investigated in detail.
  1. We have mentioned some examples that provide viable SM extensions
  2. We have discussed a TAF axion sector and shown how this predict physics testable with gravitational wave detectors (work in collaboration with Anish Ghoshal)

Thank you very much for your attention!

Extra slides

The consistency seems ok (up to the Planck mass $M_{\rm Pl}$)

Some couplings diverge as a function of the energy μ (Landau poles), but above $M_{\rm Pl}$



Solutions of the renormalization group equations (RGEs) of the most relevant SM parameters

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Qualitative origin of the stability bound

$$V_{\text{eff}} = V + V_1 + V_2 + \dots$$

$$V(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2, \quad V_1(h) = \frac{1}{(4\pi)^2}\sum_i c_i m_i(h)^4 \left(\ln \frac{m_i(h)^2}{\mu^2} + d_i\right), \quad \dots$$

where $h^2 \equiv 2|H|^2$ and c_i and d_i are ~ 1 constants

By substituting bare parameters $\rightarrow$ renormalized ones

$$\Longrightarrow \frac{\partial V_{\text{eff}}}{\partial \mu} = 0$$ and one is free to choose μ to improve perturbation theory

- Since at large fields, $h \gg v$, we have $m_i(h)^2 \propto h^2$, we choose $\mu^2 = h^2$, then

$$V_{\text{eff}}(h) = \frac{\lambda(h)}{4}\left(h^2 - v(h)^2\right)^2 + \dots = -\frac{m(h)^2}{2}h^2 + \frac{\lambda(h)}{4}h^4 + \dots$$

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So for $h \gg v$

$$V_{\text{eff}}(h) \approx \frac{\lambda(h)}{4}h^4$$

- M_h contributes positively to $\lambda \rightarrow$ lower bound on M_h
- y_t contributes negatively to the running of $\lambda \rightarrow$ upper bound on M_t

Procedure to extract the stability bound

Steps of the procedure

- $V_{\rm eff}$, including relevant parameters
- RGEs of the relevant couplings
- Values of the relevant parameters (also called *threshold corrections* or *matching conditions*) at the EW scale (e.g. at M_t) ...

Finally impose that $V_{\rm eff}$ at the EW vacuum is the absolute minimum!

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State-of-the-art loop calculation

- Two loop $V_{\rm eff}$ including the leading couplings $= \{\lambda, y_t, g_3, g_2, g_1\}$
- Three loop RGEs for $\{\lambda, y_t, g_3, g_2, g_1\}$ and one loop RGE for $\{y_b, y_\tau\}$ ...
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Previous calculations

[Cabibbo, Maiani, Parisi, Petronzio (1979); Casas, Espinosa, Quiros (1994, 1996); Bezrukov, Kalmykov, Kniehl, Shaposhnikov (2012); Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori, Strumia (2012)]

Input values of the SM observables

(used to fix relevant parameters: λ, y_t, g_1, g_2)

$$M_h = (125.09 \pm 0.21_{\rm stat.} \pm 0.11_{\rm syst.})\,\text{GeV}.$$

[ATLAS and CMS Collaborations (2015)]

M_W	$=$	80.384 ± 0.014 GeV	Mass of the W boson [1]
M_Z	$=$	91.1876 ± 0.0021 GeV	Mass of the Z boson [2]
M_h	$=$	125.15 ± 0.24 GeV	(source quoted above)
M_t	$=$	$173.34 \pm 0.76 \pm 0.3$ GeV	Mass of the top quark [3]
$V \equiv (\sqrt{2}G_\mu)^{-1/2}$	$=$	246.21971 ± 0.00006 GeV	Fermi constant [4]
$\alpha_3(M_Z)$	$=$	0.1184 ± 0.0007	SU(3)$_c$ coupling (5 flavors) [5]

[1] TeVatron average: FERMILAB-TM-2532-E. LEP average: CERN-PH-EP/2006-042

[2] 2012 Particle Data Group average, pdg.lbl.gov

[3] ATLAS, CDF, CMS, D0 Collaborations, arXiv:1403.4427. Plus an uncertainty $\mathcal{O}(\Lambda_{\rm QCD})$ because of non-perturbative effects [Alekhin, Djouadi, Moch (2013)]

[4] MuLan Collaboration, arXiv:1211.0960

[5] S. Bethke, arXiv:1210.0325

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Step 1: effective potential

RG-improved tree level potential (V)

Classical potential with couplings replaced by the running ones

One loop (V_1)

$V_{\rm eff}$ depends mainly on the top, W, Z, h and Goldstone squared masses in the classical background h: in the Landau gauge ... they are

$$t \equiv \frac{y_t^2 h^2}{2}, \quad w \equiv \frac{g_2^2 h^2}{4}, \quad z \equiv \frac{(g_2^2 + 3g_1^2/5)h^2}{4}, \quad m_h^2 \equiv 3\lambda h^2 - m^2, \quad g \equiv \lambda h^2 - m^2$$

$\to (4\pi)^2 V_1$ is (in a suitable renormalization scheme, called $\overline{\rm MS}$)

$$\frac{3w^2}{2}\left(\ln\frac{w}{\mu^2} - \frac{5}{6}\right) + \frac{3z^2}{4}\left(\ln\frac{z}{\mu^2} - \frac{5}{6}\right) - 3t^2\left(\ln\frac{t}{\mu^2} - \frac{3}{2}\right) + \frac{m_h^4}{4}\left(\ln\frac{m_h^2}{\mu^2} - \frac{3}{2}\right) + \frac{3g^2}{4}\left(\ln\frac{g}{\mu^2} - \frac{3}{2}\right)$$

In order to keep the logarithms in the effective potential small we choose

$$\mu = h$$

Indeed, t, w, z, m_h^2 and g are $\propto h^2$ for $h \gg m$

Two loop (V_2)

It is very complicated, but always depend on t, w, z, m_h^2, g plus g_i

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Step 2: running couplings

For a generic parameter p we write the RGE as

$$\frac{dp}{d\ln\mu^2} = \frac{\beta_p^{(1)}}{(4\pi)^2} + \frac{\beta_p^{(2)}}{(4\pi)^4} + \dots$$

They were computed before in the literature up to three loops

(very long and not very illuminating expressions at three loops)

One loop RGEs for λ, y_t^2, g_i^2 and m^2

$$\begin{aligned}
\beta_\lambda^{(1)} &= \lambda\left(12\lambda + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}\right) - 3y_t^4 + \frac{9g_2^4}{16} + \frac{27g_1^4}{400} + \frac{9g_2^2g_1^2}{40},\\
\beta_{y_t^2}^{(1)} &= y_t^2\left(\frac{9y_t^2}{2} - 8g_3^2 - \frac{9g_2^2}{4} - \frac{17g_1^2}{20}\right),\\
\beta_{g_1^2}^{(1)} &= \frac{41}{10}g_1^4, \quad \beta_{g_2^2}^{(1)} = -\frac{19}{6}g_2^4, \quad \beta_{g_3^2}^{(1)} = -7g_3^4,\\
\beta_{m^2}^{(1)} &= m^2\left(6\lambda + 3y_t^2 - \frac{9g_2^2}{4} - \frac{9g_1^2}{20}\right)
\end{aligned}$$

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Step 3: threshold corrections

$$\lambda(M_t) = 0.12604 + 0.00206\left(\frac{M_h}{\text{GeV}} - 125.15\right) - 0.00004\left(\frac{M_t}{\text{GeV}} - 173.34\right) \pm 0.00030_{\text{th}}$$

$$\frac{m(M_t)}{\text{GeV}} = 131.55 + 0.94\left(\frac{M_h}{\text{GeV}} - 125.15\right) + 0.17\left(\frac{M_t}{\text{GeV}} - 173.34\right) \pm 0.15_{\text{th}}$$

$$y_t(M_t) = 0.93690 + 0.00556\left(\frac{M_t}{\text{GeV}} - 173.34\right) - 0.00042\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.00050_{\text{th}}$$

$$g_2(M_t) = 0.64779 + 0.00004\left(\frac{M_t}{\text{GeV}} - 173.34\right) + 0.00011\,\frac{M_W - 80.384\,\text{GeV}}{0.014\,\text{GeV}}$$

$$g_Y(M_t) = 0.35830 + 0.00011\left(\frac{M_t}{\text{GeV}} - 173.34\right) - 0.00020\,\frac{M_W - 80.384\,\text{GeV}}{0.014\,\text{GeV}}$$

$$g_3(M_t) = 1.1666 + 0.00314\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} - 0.00046\left(\frac{M_t}{\text{GeV}} - 173.34\right)$$

The theoretical uncertainties on these quantities are much lower than those used in previous determinations of the stability bound

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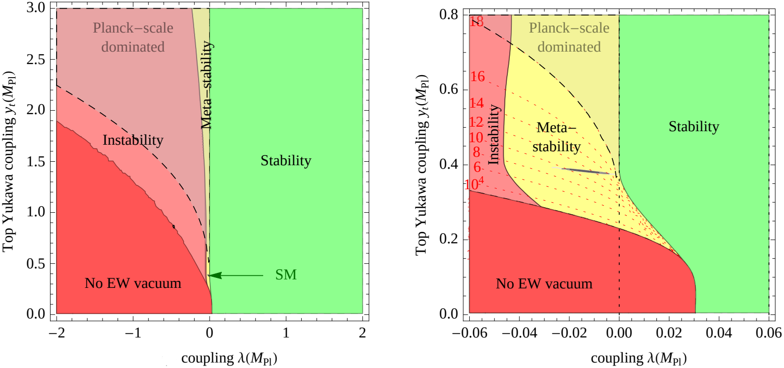
The SM phase diagram in terms of Planck scale couplings

$y_t(M_{\rm Pl})$ versus $\lambda(M_{\rm Pl})$

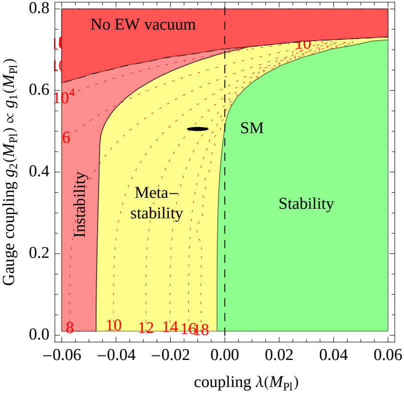


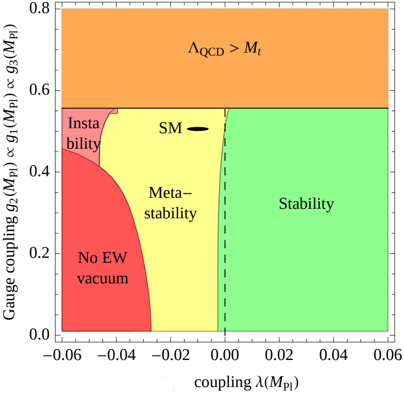
"Planck-scale dominated" corresponds to $\Lambda_I > 10^{18}$ GeV

"No EW vacuum" corresponds to a situation in which λ is negative at the EW scale

The SM phase diagram in terms of Planck scale couplings

Gauge coupling g_2 at $M_{\rm Pl}$ versus $\lambda(M_{\rm Pl})$

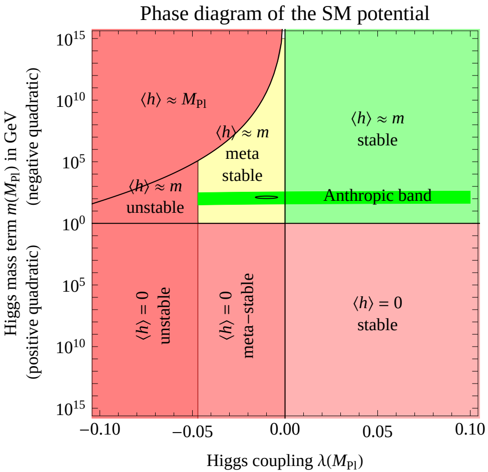




Left: $g_1(M_{\rm Pl})/g_2(M_{\rm Pl}) = 1.22, y_t(M_{\rm Pl})$ *and* $g_3(M_{\rm Pl})$ *are kept to the SM value*

Right: *a common rescaling factor is applied to* g_1, g_2, g_3. $y_t(M_{\rm Pl})$ *is kept to the SM value*

The SM phase diagram in terms of potential parameters



If $\lambda(M_{\rm Pl}) < 0$ there is an upper bound on m requiring $\langle h \rangle \neq 0$ at the EW scale.

This bound is, however, much weaker than the anthropic bound of
[Agrawal, Barr, Donoghue, Seckel (1997); Schellekens (2014)]

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Tunneling probability

The probability of creating a bubble of the absolute minimum in $dV\,dt$ was found by *[Kobzarev, Okun, Voloshin (1975); Coleman (1977); Callan, Coleman (1977)]*

$$d\wp = dt\,dV\;\Lambda_B^4\,e^{-S(\Lambda_B)}$$

$S(\Lambda_B) \equiv$ the action of the bounce of size $R = \Lambda_B^{-1}$, given by $S(\Lambda_B) = \dfrac{8\pi^2}{3|\lambda(\Lambda_B)|}$

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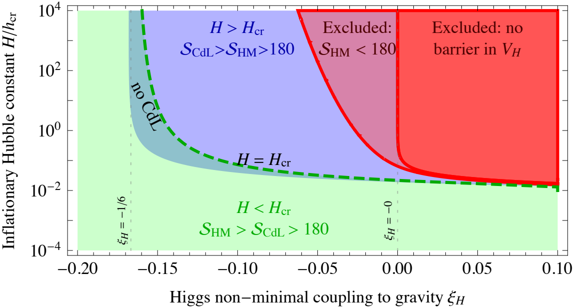
Upper bounds on the Hubble rate during inflation

The model:

$$\mathscr{L} = \mathscr{L}_{\rm EH} + \mathscr{L}_{\rm SM} + \xi_H |H|^2 R$$

The results:



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h inflation: definition

In the h inflation model the role of the inflaton is played by h

The model: *[Bezrukov, Shaposhnikov (2008)]*

$$\mathscr{L} = \mathscr{L}_{\rm EH} + \mathscr{L}_{\rm SM} + \xi |H|^2 R$$

h inflation: classical analysis

The part of S that depends on $g_{\mu\nu}$ and H *only* $\longrightarrow$

$$S_{gH} = \int d^4x\sqrt{-g}\left[\left(\frac{M_P^2}{2} + \xi|H|^2\right)R + |D_\mu H|^2 - V(H)\right]$$

The non-minimal coupling can be eliminated through a *conformal* transformation ...

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} \equiv \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{2\xi|H|^2}{M_P^2}$$

In the unitary gauge, where the only scalar field is the radial mode $\phi \equiv \sqrt{2|H|^2}$

$$S_{gH} = \int d^4x\sqrt{-\hat{g}}\left[\frac{M_P^2}{2}\hat{R} + K\frac{(\partial\phi)^2}{2} - \frac{V}{\Omega^4}\right]$$

where $K \equiv (\Omega^2 + 6\xi^2\phi^2/M_P^2)/\Omega^4$ and we set the gauge fields to zero.

The ϕ kinetic term can be made canonical through $\phi = \phi(\chi)$ defined by

$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2\phi^2/M_P^2}{\Omega^4}}$$

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$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2\phi^2/M_P^2}{\Omega^4}}$$

This is what we want in order to have slow-roll ...

Thus, χ feels a potential $$U \equiv \frac{V}{\Omega^4} = \frac{\lambda(\phi(\chi)^2 - v^2)^2}{4(1 + \xi\phi(\chi)^2/M_P^2)^2} \underset{\phi > M_P/\sqrt{\xi}}{\simeq} \frac{\lambda}{4\xi^2}M_P^4$$

h inflation: classical analysis

All parameters can be fixed through experiments and observations ...

ξ can be fixed requiring the WMAP normalization *[WMAP Collaboration (2013)]*

$$\frac{U(\phi = \phi_{WMAP})}{\epsilon(\phi = \phi_{WMAP})} \simeq (0.02746 M_P)^4$$

ϕ_{WMAP} is fixed by requiring $\quad N = \int_{\phi_{\rm end}}^{\phi_{WMAP}} \frac{U}{M_P^2} \left(\frac{dU}{d\phi}\right)^{-1} \left(\frac{d\chi}{d\phi}\right)^2 d\phi \simeq 59$

[Bezrukov, Gorbunov, Shaposhnikov (2009); Garcia-Bellido, Figueroa, Rubio (2009)]

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This leads to $\xi \simeq 4.7 \times 10^4 \sqrt{\lambda}$ and indicates that xi has to be large ...

h inflation: quantum analysis

Two regimes *[Bezrukov, Shaposhnikov, (2009)]*:

- small fields: $\phi \ll M_P/\xi$ (the SM is recovered)
- large fields: $\phi \gg M_P/\xi$ (chiral EW action with VEV set to $\phi/\Omega \simeq M_P/\sqrt{\xi}$) $\rightarrow$ decoupling of ϕ in the inflationary regime

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State-of-the-art calculation of the bound on M_h to have inflation:

- Two loop effective potential U_{eff} in the inflationary regime including the effect of ξ and the leading SM couplings $= \{\lambda, y_t, g_3, g_2, g_1\}$
- Three loop SM RGE from the EW scale up to M_P/ξ for $\{\lambda, y_t, g_3, g_2, g_1\}$ ...
- Two loop RGE for the same SM couplings and one loop RGE for ξ in the chiral EW theory
- Two loop threshold corrections at the top mass, for these SM couplings

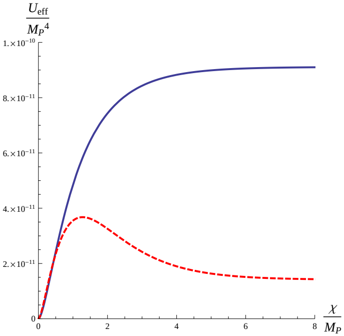
Previous calculations: *[Bezrukov, Magnin, Shaposhnikov (2009); Bezrukov, Shaposhnikov (2009); Allison (2013)]*

Bound on M_h to have h inflation

Derivation

1. We fix ξ as in the classical case, but with U replaced by $U_{\rm eff}$.
   ... this already gives $\xi_{\rm inf} \equiv \xi(M_P/\sqrt{\xi_t})$, where conventionally $\xi_t = \xi(M_t)$

2. If M_h is too small (or M_t is too large) we go from the blue behavior to the red one! When the slope is negative the Higgs cannot roll towards the EW vacuum



We set the th. errors to zero and the input parameters to the central values, except M_t:

- *Solid line:* $M_t = 171.43$*GeV* *(ξ fixed as described above)*
- *Dashed line:* $M_t = 171.437$*GeV* *($\xi_t = 300$)*

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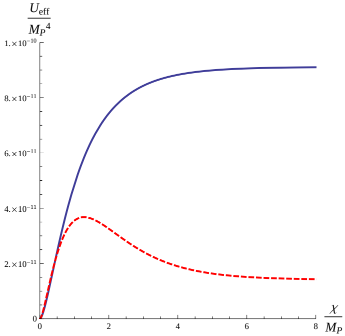

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Result (bound to have h inflation):

$$M_h > 129.4\,\text{GeV} + 2.0(M_t - 173.34\,\text{GeV}) - 0.5\,\text{GeV}\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.3_{\rm th}\,\text{GeV}$$

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More details on right-handed neutrinos

$$Y = \frac{U_\nu^* D_{\sqrt{m}} \mathcal{R} D_{\sqrt{M}}}{v}$$

where

$$\begin{aligned} D_{\sqrt{m}} &\equiv \text{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3}), \\ D_{\sqrt{M}} &\equiv \text{diag}(\sqrt{M_1}, \sqrt{M_2}, \sqrt{M_3}) \end{aligned}$$

and U_ν is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix:
it can be decomposed as $U_\nu = V_\nu P_{12}$, where ($s_{ij} \equiv \sin(\theta_{ij})$, $c_{ij} \equiv \cos(\theta_{ij})$)

$$V_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

$$P_{12} = \begin{pmatrix} e^{i\beta_1} & 0 & 0 \\ 0 & e^{i\beta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mathcal{R}$ is a generic complex orthogonal matrix. One can show that the simpler and realistic case of two right-handed neutrinos below M_{Pl} can be recovered by setting $m_1 = 0$ and

$$\mathcal{R} = \begin{pmatrix} 0 & 0 & 1 \\ \cos z & -\sin z & 0 \\ \xi \sin z & \xi \cos z & 0 \end{pmatrix}$$

where z is a complex parameter and $\xi = \pm 1$.
(In the plot ξ is irrelevant and we set $z = 0$)

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An example, renormalizable quantum gravity

Lagrangian:

$$\mathscr{L} = -\frac{\bar{M}_{\rm Pl}^2}{2} R + \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm NP}$$

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The theory is renormalizable *[Stelle (1977)]*

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One can absorb the loop divergences and compute δM_h from loops:

$$\delta M_h^2 \sim \frac{\bar{M}_{\rm Pl}^2 f_2^4}{(4\pi)^2}$$

[Salvio, Strumia (2014)]

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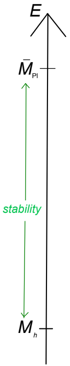

One can absorb the loop divergences and compute δM_h from loops:

$$\delta M_h^2 \sim \frac{\bar{M}_{\rm Pl}^2 f_2^4}{(4\pi)^2}, \quad \text{naturalness} \rightarrow f_2 \sim \sqrt{\frac{4\pi M_h}{\bar{M}_{\rm Pl}}} \sim 10^{-8}$$

[Salvio, Strumia (2014)]

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Renormalizability and spectrum of quadratic Gravity

Quadratic Gravity is renormalizable

However, looking at the spectrum:

(i) massless graviton

(ii) scalar z with mass $M_0^2 \sim \frac{1}{2} f_0^2 \bar{M}_{\rm Pl}^2$

(iii) massive spin-2 ghost with mass $M_2^2 = \frac{1}{2} f_2^2 \bar{M}_{\rm Pl}^2$

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(iii) is the manifestation of the Ostrogradsky theorem: Lagrangians that depend non-degenerately on the second derivatives have Hamiltonians unbounded from below

However, one can obtain a ***quantum*** Hamiltonian bounded from below by introducing an indefinite metric on the Hilbert space, which is anyhow required by renormalizability

One addresses the remaining problem of having a probabilistic interpretation by introducing positively defined metrics (implied by the frequency approach to quantum probabilities)

Predictions of Starobinsky inflation

$$n_s \approx 1 - \frac{2}{N} \overset{N \approx 60}{\approx} 0.967, \qquad r \approx \frac{12}{N^2} \overset{N \approx 60}{\approx} 0.003$$

The scalar amplitude $A_s = f_0^2 N^2 / 48\pi^2$ is reproduced for $f_0 \approx 1.8 \times 10^{-5}$

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A Pati-Salam model without Landau poles

	Fields	spin	generations	$SU(2)_L$	$SU(2)_R$	$SU(4)_{PS}$
skeleton model	$\psi_L = \begin{pmatrix} \nu_L & e_L \\ u_L & d_L \end{pmatrix}$	1/2	3	$\bar{2}$	1	4
	$\psi_R = \begin{pmatrix} \nu_R & u_R \\ e_R & d_R \end{pmatrix}$	1/2	3	1	2	$\bar{4}$
	ϕ_R	0	1	1	2	$\bar{4}$
	$\phi = \begin{pmatrix} H_U^0 & H_D^+ \\ H_U^- & H_D^0 \end{pmatrix}$	0	1	2	$\bar{2}$	1
extra fields	ψ	1/2	$N_\psi \leq 3$	2	$\bar{2}$	1
	Q_L	1/2	2	1	1	10
	Q_R	1/2	2	1	1	$\overline{10}$
	Σ	0	1	1	1	15

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Peccei-Quinn phase transition: supercooling

Decay rate per unit volume Γ of the false vacuum $\phi = 0$ into the true vacuum $\phi \neq 0$

Nucleation temperature T_n

Peccei-Quinn phase transition: supercooling

Decay rate per unit volume Γ of the false vacuum $\phi = 0$ into the true vacuum $\phi \neq 0$

For $T < T_c$ we have [Coleman (1977); Callan, Coleman (1980); Linde (1981); Linde (1983)]

$$\Gamma \approx \max\left(T^4\left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3/T},\ \frac{1}{R_4^4}\left(\frac{S_4}{2\pi}\right)^2 e^{-S_4}\right)$$

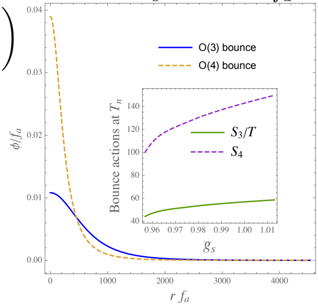
S_d is the action

$$S_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}\int_0^\infty dr\, r^{d-1}\left(\frac{1}{2}\phi'^2 + V_{\rm eff}(\phi,T)\right)$$

evaluated at the O(d) bounce:

$$\phi'' + \frac{d-1}{r}\phi' = \frac{dV_{\rm eff}}{d\phi}, \quad \phi'(0) = 0, \quad \lim_{r\to\infty}\phi(r) = 0$$

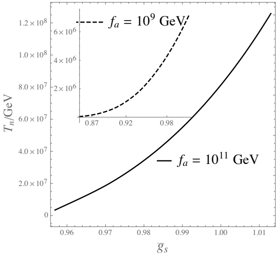
$\bar{g}_s = 0.91$ and
$T \approx 1\times 10^{-2}\, T_c \approx 2\times 10^{-3} f_a$



Nucleation temperature T_n

ϕ is trapped ($\phi = 0$) until $T \ll T_c$; the universe features a phase of strong supercooling and the universe inflates with Hubble rate $H_I = \sqrt{\bar{\beta}} f_a^2/(4\sqrt{3}\bar{M}_{\rm Pl})$.
T_n corresponds to $\Gamma/H_I^4 \sim 1$ or

$$\frac{S_3}{T_n} - \frac{3}{2}\ln\left(\frac{S_3/T_n}{2\pi}\right) = 4\ln\left(\frac{T_n}{H_I}\right)$$



Peccei-Quinn phase transition: reheating and duration

Reheating

Duration of the phase transition

Peccei-Quinn phase transition: reheating and duration

Reheating

At the end of supercooling the universe should be reheated. This occurs thanks to the unavoidable coupling between the axion and the SM sectors due to gluons, leading to

$$\Gamma_{\phi\to gg} \sim \frac{\bar{y}^2 \bar{g}_s^4 M_\phi^3}{(4\pi)^5 M_Q^2} \qquad \to \qquad T_{\rm RH} = \left(\min\left(\frac{45\Gamma_{\phi\to gg}^2 \bar{M}_{\rm Pl}^2}{4\pi^3 g_*}, \frac{15\bar{\beta} f_a^4}{8\pi^2 g_*}\right)\right)^{1/4}$$

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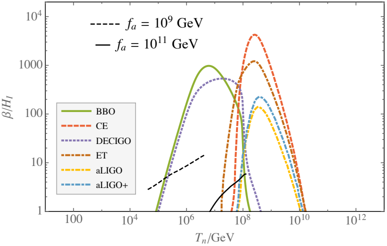
Duration of the phase transition

The inverse of the duration of the phase transition is defined by

$$\beta \equiv \left[\frac{1}{\Gamma}\frac{d\Gamma}{dt}\right]_{T_n}$$

This quantity, for fast reheating, can be computed with the formula

$$\frac{\beta}{H_I} = \left[T\frac{d}{dT}(S_3/T) - 4\right]_{T=T_n}$$



Monopole dilution

Strong supercooling and the corresponding inflation dilute the density $n(T)$ of monopoles due to SU(2)$_a \to$ U(1)$_a$

In a strong first-order phase transition monopoles may be created by bubble collisions and well-known estimates *[Preskill (1984)]* lead to

$$\frac{n(T_n)}{T_n^3} \gtrsim p \left(\frac{T_n}{C M_P}\right)^3, \tag{1}$$

where

- p is the probability that the scalar field configuration is topologically non trivial,
- $C = 0.6/\sqrt{g_*(T_n)}$

Even for $p \approx 1$, setting $g_*(T_n)$ of order 10^2 (a realistic setup given the existing TAF SM sectors) the theoretical bound in (1) is amply compatible with the bound coming from the fact that the mass density of monopoles must not exceed the limit on the total mass density imposed by the observed Hubble constant and deceleration parameter. Indeed, the latter bound is around $n(T_0)/T_0^3 \lesssim 10$ eV$/M_m$, where T_0 is today's temperature, M_m is the monopole mass and $M_m \sim 4\pi f_a/\bar{g}_a$, $n(T_0)/T_0^3 \lesssim n(T_n)/T_n^3$ and the window 10^8 GeV $\lesssim f_a \lesssim 10^{12}$ GeV have been used.

Gravitational waves

Because of supercooling and inflation the main source of GWs are vacuum bubble collisions *[Caprini et al (2015)]*

$$h^2\Omega_{\rm GW}(f) \approx 1.29\times 10^{-6}\left(\frac{H(T_{\rm RH})}{\beta}\right)^2\left(\frac{100}{g_*(T_{\rm RH})}\right)^{1/3}\frac{3.8(f/f_{\rm peak})^{2.8}}{1+2.8(f/f_{\rm peak})^{3.8}},$$

where

$$f_{\rm peak}\approx 3.79\times 10^2\frac{\beta}{H(T_{\rm RH})}\frac{T_{\rm RH}}{10^{10}{\rm GeV}}\left(\frac{g_*(T_{\rm RH})}{100}\right)^{1/6}\ {\rm Hz}$$

$\Omega_{\rm GW}$ is subject to a big-bang nucleaosynthesis (BBN) bound, which depends on the effective number of neutrinos $N_{\rm eff}$

$$h^2\bar{\Omega}_{\rm GW}\equiv\int_{f_{\rm BBN}}^{f_{\rm UV}}\frac{df}{f}h^2\Omega_{\rm GW}(f) < 1.3\times 10^{-6}\,\frac{N_{\rm eff}-3.046}{0.234}$$

where $f_{\rm BBN}\sim 10^{-11}$Hz and $f_{\rm UV}$ is some UV cutoff, which in our case can be conservatively taken to be Λ_G

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