Non-planar corrections in ABJM theory from quantum M2 branes

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superconformal theories with $\mathcal{N} > 1$ susy

Integrability: anomalous dims in 4d and 3d conformal theories as exact functions of λ at leading order in large N

$$SU(N)\ \text{SYM}: \quad \lambda = g_{\text{YM}}^2 N, \ \text{ large } N$$

$$F(\lambda, N) = N^2 F_0(\lambda) + F_1(\lambda) + \frac{1}{N^2} F_2(\lambda) + ...$$

beyond large N: $\qquad F_n(\lambda) = ?$

Localization: special supersymmetric observables (free energy on S^d, Wilson loop, some BPS correlators) computed exactly in g_{YM} and N

$$\Delta(\lambda, N) = \Delta_0(\lambda) + \frac{\Delta_1(\lambda)}{N^2} + \frac{\Delta_2(\lambda)}{N^4} + \ldots$$

• Δ_0 controlled by integrability

at weak coupling $\Delta_0 = c_1\lambda + c_2\lambda^2 + \ldots$ while at strong coupling

$$\Delta_0 = \sqrt{\lambda}[a_0 + \frac{a_1}{\sqrt{\lambda}} + \frac{a_2}{(\sqrt{\lambda})^2} + \ldots] \quad \text{or} \quad \Delta_0 = \sqrt[4]{\lambda}[b_0 + \frac{b_1}{\sqrt{\lambda}} + \frac{b_2}{(\sqrt{\lambda})^2} + \ldots]$$

• non-planar correction $\Delta_1(\lambda)$?

• in $\mathcal{N} = 4$ SYM little known even at weak coupling:

non-planar correction to cusp dim $f(\lambda, N)$

$\Delta = S + f(\lambda, N) \log S + \ldots, \quad \mathcal{O} = \mathrm{Tr}\,(\Phi D^S \Phi)$

first appears at 4-loops [Henn, Korchemsky 19]

$$f(\lambda, N) = \frac{1}{(2\pi)^2}\Big[\lambda - \frac{1}{48}\lambda^2 + \frac{11}{11520}\lambda^3 - \big(c_4 + \frac{1}{N^2}d_4\big)\lambda^4 + \mathcal{O}(\lambda^5)\Big]$$

$$c_4 = \frac{73}{20160\times 64} + \frac{1}{8(2\pi)^6}\zeta^2(3)\,, \qquad d_4 = \frac{31}{5040\times 64} + \frac{9}{4}\zeta^2(3)$$

Konishi operator: non-planar correction also appears at 4 loops

$\Delta_1 \sim \zeta(5)\lambda^4 + \ldots$ [Velizhanin 09]

• should be universal weak-coupling scaling

$$\Delta_1\big|_{\lambda\ll 1} = d_4\lambda^4 + ...$$

• what to expect at strong coupling: $\quad \Delta_1\big|_{\lambda\gg 1} \sim \lambda^p$?

string side: requires 1-loop (torus) computation
in GS string in $\mathrm{AdS}_5 \times S^5$: challenging problem

• remarkably, strong-coupling limit of non-planar corrections
can be computed in 3d ABJM theory:
semiclassical M2 brane quantization in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ captures
leading order $\alpha' \sim \frac{1}{\sqrt{\lambda}}$ terms at each order in $g_s^2 \sim \frac{1}{N^2}$

- M2 brane solution: "near-horizon" limit $\mathrm{AdS}_4 \times S^7$

$$ds^2_{11} = \mathrm{L}^2\Big(\tfrac{1}{4}ds^2_{\mathrm{AdS}_4} + ds^2_{S^7}\Big), \quad F_4 \sim N, \quad \Big(\frac{\mathrm{L}}{\ell_P}\Big)^6 = 32\pi^2 N$$

- collective coordinates $\to$ M2 action: [Bergshoeff, Sezgin,Townsend 87]

$$S_{\mathrm{M2}} = T_2 \int d^3\sigma \Big[\sqrt{-\det g_{ab}} + \hat{C}_3\Big]$$

$$g_{ab} = G_{MN}(x)\, \Pi_a^M \Pi_b^N + \ldots\,, \qquad \hat{C}_3 = \tfrac{1}{6}\epsilon^{abc}\, C_{MNK}(x)\, \Pi_a^M \Pi_b^N \Pi_c^K$$

$$\Pi_a^M = \partial_a x^M - i\bar{\theta}\Gamma^M \partial_a \theta\,, \qquad x^M = x^M(\sigma)$$

- $2\kappa^2_{11} = (2\pi)^8\, \ell_P^9\,, \qquad T_2 = \big(\frac{2\pi^2}{\kappa^2_{11}}\big)^{1/3} = \frac{1}{(2\pi)^2 \ell_P^3}$
- relation to 10d picture: $\quad S_{11} \to S_{10} = \frac{1}{2\kappa^2_{10}} \int d^{10}x\, \sqrt{G}\, e^{-2\phi}(R + \ldots)$

$$ds^2_{11} = e^{-\frac{2}{3}\phi}\, ds^2_{10} + e^{\frac{4}{3}\phi}(dx_{11} + e^{-\phi}A)^2, \qquad x_{11} \sim x_{11} + 2\pi R_{11}$$

$$g_{\mathrm{s}} \sim R_{11}\,, \qquad 2\kappa^2_{10} = (2\pi)^7 g_{\mathrm{s}}^2 \alpha'^4$$

• "double dimensional reduction": [Duff, Howe, Inami, Stelle 87]
M2 brane action in 11d background $\rightarrow$
GS superstring action in IIA 10d background

• perturbative string theory – quantum strings
perturbative M-theory – "quantum M2 branes"?

• bosonic membrane action [Dirac 1962]

$$S = -T_2 \int d^3\sigma \sqrt{-\det g}\,, \qquad g_{ab} = \eta_{MN}\partial_a X^M \partial_b X^N$$

can gauge-fix only 3 out of 6 components of 3d metric:
non-linear action in any gauge; non-renormalizable
no mass gap in spectrum of "small" quantum membranes, etc.

Semiclassical expansion of M2 brane path integral

• expand near solution with non-degenerate induced 3d metric: straightforward to quantize M2 brane in static gauge

$$S = S_B + S_F \ , \quad S_B = S_{\rm V} + S_{\rm WZ}, \qquad S_{\rm V} = T_2 \int d^3\sigma \, \sqrt{g}$$

$$S_{\rm WZ} = -i\, T_2 \int d^3\sigma \, \tfrac{1}{3!} \epsilon^{abc} C_{MNK}(X)\, \partial_a X^M \partial_b X^N \partial_c X^K$$

$$S_F = i T_2 \int d^3\sigma \Big[\sqrt{g}\, g^{ab} \partial_a X^M \, \bar\theta\, \Gamma_M \hat{D}_b \theta - \tfrac{1}{2} \epsilon^{abc} \partial_a X^M \partial_b X^N \, \bar\theta\, \Gamma_{MN} \hat{D}_c \theta + ... \Big]$$

$$g_{ab} = \partial_a X^M \partial_b X^N G_{MN}(X), \qquad G_{MN} = E^A_M E^A_N, \qquad \Gamma_M = E^A_M(X) \Gamma_A$$

$$\hat{D}_a = \partial_a X^M \hat{D}_M, \qquad \hat{D}_M = {\rm D}_M - \tfrac{1}{288} (\Gamma^{PNKL}{}_M - 8 \Gamma^{PNK} \delta^L_M) F_{PNKL}$$

• 1-loop partition function is well defined (no log UV ∞)

$$Z_{\rm M2} = \int [dX\, d\theta]\, e^{-S[X,\theta]} = \mathcal{Z}_1\, e^{-{\rm T}_2 \bar{S}_{\rm cl}} \Big[1 + \mathcal{O}({\rm T}_2^{-1}) \Big]$$

$$\mathcal{Z}_1 = e^{-\Gamma_1} \ , \qquad \Gamma_1 = \tfrac{1}{2} \sum_k \nu_k \log \det \Delta_k \ , \qquad {\rm T}_2 = {\rm L}^3 T_2 = \tfrac{{\rm L}^3}{(2\pi)^2 \ell_P^3}$$

General points:

• assume $\Sigma^2 \times S^1$ membrane topology to have 10d string limit

• do not sum over M2 topologies: only semiclassical saddles

• string loop expansion is already encoded
in expansion near M2 saddle:
g_s dependence via R_{11} of 11d background in M2 action

• 3d action = 2d action for string modes + tower of massive 2d fields

$$m_n^2 = \frac{n^2}{R_{11}^2} = n^2 \frac{8\pi T}{g_s^2}$$

• integrating out massive modes $\rightarrow$
effective (non-local) 2d theory for string modes depending on g_s
[cf. 2d theory on S^2 with insertions of handle operators]

• explicit example of M2 in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$:
1-loop M2 correction sums leading $T^{-1} \sim \alpha'$ terms at each order in g_s

ABJM theory: [Aharony, Bergman, Jafferis, Maldacena 08]

• one M2-brane: 3d scalar $\mathcal{N} = 8$ multiplet (x^i, θ^i)

theory on N coincident M2 branes $\rightarrow$

3d superconf theory dual to M-theory in $\mathrm{AdS}_4 \times S^7$ [Maldacena 97]

• perturbative description requires extra parameter k:

N M2 branes on $M^{11} = R^{1,2} \times \mathbb{R}^8/\mathbb{Z}_k$

3d theory dual to M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

• $U_k(N) \times U_{-k}(N)$ 3d CS-matter $\mathcal{N} = 6$ superconf. theory

A_m, $\tilde{A}_m$, bi-fundamental 4 scalars Φ^I and 4 fermions ψ_I

$$S = k \int d^3x \left[L_{CS}(A) - L_{CS}(\tilde{A}) + |D\Phi|^2 + V(\Phi) + \bar{\psi} D \psi + \bar{\psi}\psi\Phi^\dagger\Phi \right]$$

$$L_{CS} = \epsilon^{mnp} \mathrm{Tr}\, (A_m \partial_n A_p + \tfrac{2}{3} A_m A_n A_p)\,, \quad V = \mathrm{Tr}\, (\Phi\Phi^\dagger\Phi\Phi^\dagger\Phi\Phi^\dagger) + ...$$

• parameters N and k (analog of $\frac{1}{g^2_{\mathrm{YM}}}$)

• large N, large k, $\lambda \equiv \frac{N}{k}$ = fixed ("string regime")

dual to IIA superstring on $\mathrm{AdS}_4 \times \mathrm{CP}^3$

• large N, fixed k ("M-theory regime")

dual to M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$: $\phi \equiv \phi + \frac{2\pi}{k}$

$$ds^2_{11} = \mathrm{L}^2 \Big(\tfrac{1}{4} ds^2_{\mathrm{AdS}_4} + ds^2_{S^7/\mathbb{Z}_k} \Big), \qquad \mathrm{L} = (2^5 \pi^2 N k)^{1/6} \ell_P$$

$$ds^2_{S^7/\mathbb{Z}_k} = ds^2_{\mathrm{CP}^3} + \frac{1}{k^2} (d\varphi + k\mathrm{A})^2 , \qquad \varphi = k\,\phi \equiv \varphi + 2\pi$$

$$ds^2_{\mathrm{CP}^3} = h_{rs}(w) dw^s d\bar{w}^s, \quad d\mathrm{A} = i h_{rs}(w) dw^r \wedge d\bar{w}^s, \quad h_{rs} = \frac{\delta_{sr} - \frac{w_s \bar{w}_r}{1+|w|^2}}{1+|w|^2}$$

• string regime $k \gg 1$: $\quad ds^2_{10} = L^2 \Big(\tfrac{1}{4} ds^2_{\mathrm{AdS}_4} + ds^2_{\mathrm{CP}^3} \Big), \qquad L = g_{\mathrm{s}}^{1/3}\, \mathrm{L}$

$$g_{\mathrm{s}} = \Big(\frac{\mathrm{L}}{k\,\ell_P} \Big)^{3/2} = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N} , \quad \lambda = \frac{N}{k} , \qquad T = \frac{L^2_{\mathrm{ads}}}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{\sqrt{2}}$$

$$\frac{1}{k^2} = \frac{\lambda^2}{N^2} = \frac{g_{\mathrm{s}}^2}{8\pi\, T}$$

non-planar corrections $\sim 1/k^2$

• ABJM operators are dual to string states in $\mathrm{AdS}_4 \times \mathrm{CP}^3$ and then also to M2 brane states in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

• M-theory expansion $\frac{\mathrm{L}}{\ell_P} \gg 1$:   large N for fixed $k = 1, 2, ...$ or large effective membrane tension

$$\mathrm{T}_2 \equiv \mathrm{L}^3 T_2 = \frac{1}{\pi}\sqrt{Nk} \quad \gg 1$$

• observables like $\Delta(N, k)$ in semiclassical M2 brane expansion

$$F = \mathrm{T}_2 F_0(k) + F_1(k) + (\mathrm{T}_2)^{-1} F_2(k) + ...$$

expansion in large k gives $1/N$, fixed $\lambda = \frac{N}{k}$ expansion

Examples demonstrating that M2 semiclassical expansion
matches ABJM gauge theory results
known from localisation for large N and any k:

- $\frac{1}{2}$ BPS Wilson loop [Giombi, AT 2023]

- Bremsstrahlung function [Beccaria, AT 2024]

$\frac{1}{2}$ BPS circular WL in ABJM

BPS operator $\quad \mathcal{W} = \mathrm{Tr}\, P e^{\int (iA + \Phi^* \Phi + \dots)}$

Localization matrix model (two bi-fund. scalars)

$Z(N,k) = \int d^N x_i \, d^N y_i \; M(x_i, y_j) \; \exp\left[i \frac{k}{4\pi} \sum_{i=1}^N (x_i^2 - y_i^2) \right]$

$M(x_i, y_j) = \prod_{i,j=1}^N \left[\sinh \frac{x_i - x_j}{2} \; \sinh \frac{y_i - y_j}{2} \; (\cosh \frac{x_i - y_j}{2})^{-2} \right]$

$\langle \mathcal{W} \rangle = \langle \exp \sum_i x_i \rangle$ [Drukker, Marino, Putrov 10; Klemm et al 12]

$$\langle \mathcal{W} \rangle = \frac{1}{2\, \sin \frac{2\pi}{k}} \; \frac{\mathrm{Ai}\left[(\frac{\pi^2}{2} k)^{1/3} \left(N - \frac{k}{24} - \frac{7}{3k} \right) \right]}{\mathrm{Ai}\left[(\frac{\pi^2}{2} k)^{1/3} \left(N - \frac{k}{24} - \frac{1}{3k} \right) \right]}$$

- "M-theory regime": large N at fixed $k > 2$

$$\mathrm{Ai}(x)\Big|_{x \gg 1} \simeq \frac{e^{-\frac{2}{3} x^{3/2}}}{2\sqrt{\pi}\, x^{1/4}} \; \sum_{n=0}^{\infty} \frac{(-\frac{3}{4})^n \, \Gamma(n+\frac{5}{6}) \Gamma(n+\frac{1}{6})}{2\pi \, n! \, x^{3n/2}}$$

$$\langle \mathcal{W} \rangle = \frac{1}{2\sin \frac{2\pi}{k}} \, e^{\pi \sqrt{\frac{2N}{k}}} \left[1 - \frac{\pi \left(k^2 + 32 \right)}{24\sqrt{2}\, k^{3/2}} \frac{1}{\sqrt{N}} + \mathcal{O}(\frac{1}{N}) \right]$$

• "string regime": $N, k \gg 1, \quad \lambda = \frac{N}{k}$ = fixed

$$\langle \mathcal{W} \rangle = \frac{1}{2 \sin \frac{2\pi\lambda}{N}} \, e^{\pi\sqrt{2\lambda}} \Big[1 - \frac{\pi}{24\sqrt{2}} \frac{1}{\sqrt{\lambda}} + \mathcal{O}(\frac{1}{N}) \Big]$$

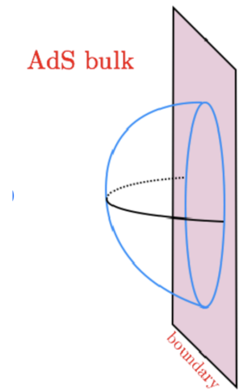
$$= \frac{N}{4\pi\lambda} \, e^{\pi\sqrt{2\lambda}} \big[1 + ... \big]$$

• compare to predictions of dual string theory in $\mathrm{AdS}_5 \times S^5$ and in $\mathrm{AdS}_4 \times \mathrm{CP}^3$

$$\text{SYM}: \quad g_{\mathrm{s}} = \frac{g_{\mathrm{YM}}^2}{4\pi} = \frac{\lambda}{4\pi N}\,, \qquad T = \frac{\sqrt{\lambda}}{2\pi}\,, \qquad \lambda = g_{\mathrm{YM}}^2 N$$

$$\text{ABJM}: \quad g_{\mathrm{s}} = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N}\,, \qquad T = \frac{\sqrt{\lambda}}{\sqrt{2}}\,, \qquad \lambda = \frac{N}{k}$$

$\langle \mathcal{W} \rangle$ = disk part. function expanded near AdS_2 minimal surface



$$\langle \mathcal{W} \rangle = Z_{\rm str} = \frac{1}{g_{\rm s}} Z_1 + \mathcal{O}(g_{\rm s})\,, \qquad\qquad Z_1 = \int [dx] ...\, e^{-T\int d^2\sigma\, L}$$

$$\text{SYM}: \quad \langle \mathcal{W} \rangle = \sqrt{\frac{2}{\pi}} \frac{N}{\lambda^{3/4}}\, e^{\sqrt{\lambda}} + ... \;=\; \frac{1}{2\pi}\, \frac{\sqrt{T}}{g_{\rm s}}\;\; e^{2\pi T} + ...$$

$$\text{ABJM}: \quad \langle \mathcal{W} \rangle = \frac{N}{4\pi\lambda}\, e^{\pi\sqrt{2\lambda}} + ... \;=\; \frac{1}{\sqrt{2\pi}}\, \frac{\sqrt{T}}{g_{\rm s}}\;\; e^{2\pi T} + ...$$

- universal form at strong coupling [Giombi, AT 2020]
- reason: similar dual $\mathrm{AdS}_d \times M^{10-d}$ strings ($d = 4, 5$)

$c_0\sqrt{T}$ from 1-loop string partition function

$$Z_1 \sim \frac{\left[\det(-\nabla^2+\frac{1}{2})\right]^{\frac{2d-2}{2}}\left[\det(-\nabla^2-\frac{1}{2})\right]^{\frac{10-2d}{2}}}{\left[\det(-\nabla^2+2)\right]^{\frac{d-2}{2}}\left[\det(-\nabla^2)\right]^{\frac{10-d}{2}}}$$

$$(Z_1)_\chi \sim (\sqrt{T})^\chi\,, \qquad (Z_1)_{\text{disk}} \sim \sqrt{T}$$

disk with h handles $\chi = 1-2h$: $\; g_s^{-1} \to g_s^\chi, \;\; \sqrt{T} \to (\sqrt{T})^\chi$

• prediction from string theory:

$$\langle \mathcal{W} \rangle = e^{2\pi T} \sum_{h=0}^{\infty} c_h \left(\frac{g_s}{\sqrt{T}}\right)^{2h-1} \left[1+\mathcal{O}(T^{-1})\right]$$

consistent with structure of $1/N$ terms in gauge theory (localization)

SYM case: leading terms exponentiate to $e^{\frac{\pi}{12}\frac{g_s^2}{T}}$ [Gross, Drukker '00]

• hard to directly compute c_h of higher (disk with handles) terms

• string side:

$$\langle \mathcal{W} \rangle = e^{2\pi T} \frac{\sqrt{T}}{g_s} \Big(\big[c_0 + \frac{c_{01}}{T} + ... \big] + \frac{g_s^2}{T} \big[c_1 + \frac{c_{11}}{T} + ... \big] + \big(\frac{g_s^2}{T} \big)^2 \big[c_2 + ... \big] + ... \Big)$$

ABJM: $\frac{g_s^2}{T} \sim \frac{\lambda^2}{N^2} = \frac{1}{k^2}$; subleading corrections $\sim \frac{1}{T} \sim \frac{1}{\sqrt{\lambda}} = \frac{\sqrt{k}}{\sqrt{N}}$

• localization result: leading terms sum into $(\sin \frac{2\pi}{k})^{-1}$:

$$\langle \mathcal{W} \rangle = \frac{1}{2 \sin \frac{2\pi}{k}} e^{\pi \sqrt{\frac{2N}{k}}} \Big[1 + \mathcal{O}(\frac{1}{\sqrt{N}}) \Big] = \frac{1}{2 \sin \big(\sqrt{\frac{\pi}{2}} \frac{g_s}{\sqrt{T}} \big)} e^{2\pi T} \Big[1 + O(T^{-1}) \Big]$$

$$\frac{1}{2 \sin \big(\sqrt{\frac{\pi}{2}} \frac{g_s}{\sqrt{T}} \big)} = \frac{\sqrt{T}}{\sqrt{2\pi} g_s} \Big[1 + \frac{\pi}{12} \frac{g_s^2}{T} + \frac{7\pi^2}{1440} \big(\frac{g_s^2}{T} \big)^2 + ... \Big]$$

• remarkably, can obtain this ABJM analog $\frac{1}{2 \sin(\sqrt{\frac{\pi}{2}} \frac{g_s}{\sqrt{T}})}$

of all-loop SYM factor $e^{\frac{\pi}{12} \frac{g_s^2}{T}}$ as 1-loop M2 brane correction

1-loop M2 brane partition function in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

[Giombi, AT 23]

- $\mathrm{AdS}_2 \times S^1$ membrane solution dual to Wilson loop: wrapping AdS_2 of AdS_4 and S^1_φ of $S^7/\mathbb{Z}_k$

$$S_{\mathrm{M2}} = \tfrac{1}{4}\mathrm{T}_2 \mathrm{vol}(\mathrm{AdS}_2)\,\tfrac{2\pi}{k} = -\pi\sqrt{\tfrac{2N}{k}}$$

$e^{-S_{\mathrm{M2}}}$ matches leading factor in $\langle \mathcal{W} \rangle$

- static gauge (σ_0, σ_1) = AdS_2, $\sigma_2 = \varphi$ of radius $R = 1/k$:

κ-gauge: 8+8 3d fluctuations [Sakaguchi, Shin, Yoshida 10]

- Fourier expansion of 3d fields in $\sigma_2 = (0, 2\pi)$:

towers of bosons + fermions on AdS_2: KK masses $\frac{n^2}{R^2} = n^2 k^2$

- B: $2 \perp \mathrm{AdS}_4$ directions: $m^2 = \frac{1}{4}(kn-2)(kn-4)$

  $6\ \mathrm{CP}^3$ directions: $m^2 = \frac{1}{4}kn(kn+2)$

- F: 6+2 towers of 2d spinors: $m = \frac{1}{2}kn \pm 1, \quad m = \frac{1}{2}kn$
- string theory limit $k \to \infty$: $n \neq 0$ modes decouple

  $n = 0$ modes = 2d fluctuations of IIA string in $\mathrm{AdS}_4 \times \mathrm{CP}^3$

1-loop M2 partition function on $\mathrm{AdS}_2 \times S^1$

$$Z_{\rm M2} = Z_1 \, e^{-S_{\rm M2}} \Big[1 + \mathcal{O}\big(\mathrm{T}_2^{-1}\big)\Big] \,, \qquad\qquad S_{M2} = -\tfrac{\pi}{k}\mathrm{T}_2$$

$$Z_1 = \textstyle\prod_{n=-\infty}^{\infty} \mathcal{Z}_n \,, \qquad\qquad \mathcal{Z}_0 = \mathcal{Z}\big(\mathrm{AdS}_4 \times \mathrm{CP}^3 \text{ string on } \mathrm{AdS}_2\big)$$

$$\mathcal{Z}_n = \frac{\Big[\det\big(-\nabla^2 - \frac{1}{2} + (\frac{kn}{2}+1)^2\big)\Big]^{\frac{3}{2}} \Big[\det\big(-\nabla^2 - \frac{1}{2} + (\frac{kn}{2}-1)^2\big)\Big]^{\frac{3}{2}} \det\big(-\nabla^2 - \frac{1}{2} + (\frac{kn}{2})^2\big)}{\det\big(-\nabla^2 + \frac{1}{4}(kn-2)(kn-4)\big) \Big[\det\big(-\nabla^2 + \frac{1}{4}kn(kn+2)\big)\Big]^3}$$

- no 1-loop log UV div in 3d: $\Gamma_1 = -\log Z_1 = -\frac{1}{2} \sum_{n=-\infty}^{\infty} \zeta'_{\rm tot}(0;n)$

$$\Gamma_1 = \sum_{n=1}^{\infty} \log\big(\frac{k^2 n^2}{4} - 1\big) = 2\sum_{n=1}^{\infty} \log\frac{kn}{2} + \sum_{n=1}^{\infty} \log\big(1 - \frac{4}{k^2 n^2}\big)$$

- final expression for $k > 2$

$$Z_1 = e^{-\Gamma_1} = \frac{1}{2\sin\frac{2\pi}{k}}$$

precise agreement with localization result in gauge theory:
non-trivial test of AdS/CFT to all orders in $1/N$

Subleading corrections

• higher loops in M2 semiclassical expansion?

expansion parameter: $(\mathrm{T}_2)^{-1} = \frac{\pi}{\sqrt{2}\,k}\frac{1}{\sqrt{N}}$

$$\langle \mathcal{W} \rangle_{\rm loc} = \frac{1}{2\sin\frac{2\pi}{k}}\, e^{\pi\sqrt{\frac{2N}{k}}}\Big[1 - \frac{\pi(k^2+32)}{24\sqrt{2}\,k^{3/2}}\frac{1}{\sqrt{N}} + \mathcal{O}(\frac{1}{N})\Big]$$

$$= \frac{1}{2\sin\frac{2\pi}{k}}\, e^{\frac{\pi^2}{k}\mathrm{T}_2}\Big[1 - \frac{k^2+32}{24k}\frac{1}{\mathrm{T}_2} + \mathcal{O}(\mathrm{T}_2^{-2})\Big]$$

• compare to string loop expansion (large k limit):

$\frac{k}{T_2} \sim \sqrt{\frac{k}{N}} \sim \frac{1}{\sqrt{\lambda}}$: subleading α' corrections at each order in g_s^2

• $\mathrm{T}_2^{-1} \sim \frac{1}{\sqrt{N}}$ term reproduced by 2-loop M2 contribution?

• suggests conjecture: ∞'s cancel also at 2-loop M2 brane order as at 2 loops in GS string in $\mathrm{AdS}_5 \times S^5$ or $\mathrm{AdS}_4 \times \mathrm{CP}^3$

[Roiban, Tirziu, AT 07; Giombi et al 09; Bianchi et al 14]

• possible reason? hidden symmetry (cf. integrability)?

Non-planar corrections to ABJM Bremsstrahlung function from M2 brane [Beccaria, AT 2024]

$\mathcal{N}=4$ $SU(N)$ SYM: [Correa, Henn, Maldacena, Sever 12]

$$B_{\text{SYM}}(\lambda,N)=\frac{1}{2\pi^2}\lambda\frac{\partial}{\partial\lambda}\log\langle W\rangle_{\text{SYM}}$$

$$\langle W\rangle_{\text{SYM}}=e^{\frac{\lambda}{8N^2}(N-1)}L^1_{N-1}(-\tfrac{\lambda}{4N})=\frac{2N}{\sqrt{\lambda}}I_1(\sqrt{\lambda})\Big(1+\frac{\lambda^{3/2}}{96N^2}\Big[\frac{I_2(\sqrt{\lambda})}{I_1(\sqrt{\lambda})}-\frac{12}{\sqrt{\lambda}}\Big]+...\Big)$$

$$B_{\text{SYM}}(\lambda,N)=\mathrm{B}_{\text{SYM}}(\lambda)+\frac{1}{128\pi^2}\frac{\lambda^{3/2}}{N^2}+...$$

$$\mathrm{B}_{\text{SYM}}(\lambda)=\frac{\sqrt{\lambda}}{4\pi^2}\frac{I_2(\sqrt{\lambda})}{I_1(\sqrt{\lambda})}=\frac{1}{4\pi^2}\sqrt{\lambda}-\frac{3}{8\pi^2}+....$$

same from small cusp or w-wound circle [Lewkowycz, Maldacena 13]

$$B(\lambda,N)=\frac{1}{4\pi^2}\frac{\partial}{\partial \mathrm{w}}\log\langle W\rangle\Big|_{\mathrm{w}=1}$$

• ABJM: $\quad \mathrm{B}_{\text{ABJM}}(\lambda)$ for $\frac{1}{2}$ or $\frac{1}{6}$ BPS WL

from $\frac{\partial}{\partial\alpha}\log\langle W\rangle\big|_{\alpha=0}$ over latitude angle [Bianchi et al 14, 18]

$$\mathrm{B}_{\text{ABJM}}(\lambda) = \frac{1}{2\pi}\sqrt{\frac{\lambda}{2}} - \frac{1}{4\pi^2} - \frac{1}{96\pi}\frac{1}{\sqrt{2\lambda}} + \dots$$

• more direct way: $B_{\text{ABJM}} \sim \frac{\partial}{\partial \mathrm{w}}\log\langle W\rangle\big|_{\mathrm{w}=1}$ assuming that w-wrapped WL same as WL in w-fundamental rep

• $B_{\text{ABJM}}(\lambda, N)$ from localization [Armanini, Griguolo, Guerrini 24]

localization result for w-fundamental $\frac{1}{2}$ BPS WL [Klemm et al 12]

$$\langle W\rangle_{\text{ABJM}} = \frac{1}{2\sin\frac{2\pi\mathrm{w}}{k}}\,\frac{\mathrm{Ai}\Big[(\frac{\pi^2}{2}k)^{1/3}\big(N-\frac{k}{24}-\frac{1}{3k}-\frac{2\mathrm{w}}{k}\big)\Big]}{\mathrm{Ai}\Big[(\frac{\pi^2}{2}k)^{1/3}\big(N-\frac{k}{24}-\frac{1}{3k}\big)\Big]}$$

• large N, fixed $k>2$ expansion:

$$\langle W\rangle_{\text{ABJM}} = \frac{1}{2\sin\frac{2\pi\mathrm{w}}{k}}\, e^{\pi\mathrm{w}\sqrt{\frac{2N}{k}}}\Big[1 - \frac{\pi\mathrm{w}\big(k^2+24\mathrm{w}+8\big)}{24\sqrt{2}\,k^{3/2}}\frac{1}{\sqrt{N}} + \mathcal{O}\big(\tfrac{1}{N}\big)\Big]$$

$$B_{\text{ABJM}}(k,N) = \frac{1}{4\pi^2}\frac{\partial}{\partial\mathrm{w}}\log\langle W\rangle\Big|_{\mathrm{w}=1} = \frac{1}{2\pi}\sqrt{\frac{N}{2k}} - \frac{1}{2\pi k}\cot\frac{2\pi}{k} + \mathcal{O}\big(\tfrac{1}{\sqrt{N}}\big)$$

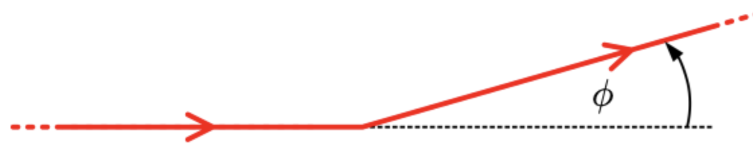
$$B_{\text{ABJM}}(k,N) = \Big[\frac{1}{2\pi}\sqrt{\frac{N}{2k}} - \frac{1}{4\pi^2} + ...\Big] \quad + \frac{1}{3k^2} + \frac{4\pi^2}{45k^4} + \frac{32\pi^4}{945k^6} + ...$$

$$= \mathrm{B}_{\text{ABJM}}(\lambda) \qquad + \frac{\lambda^2}{3N^2} + \frac{4\pi^2\lambda^4}{45N^4} + \frac{32\pi^4\lambda^6}{945N^6} + ...$$

non-planar term $-\frac{1}{2\pi k}\cot\frac{2\pi}{k} = -\frac{\lambda}{2\pi N}\cot\frac{2\pi\lambda}{N}$

can be found from 1-loop M2 brane correction in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

corresponding to line with small spatial cusp [Beccaria, AT 2024]



$$\log Z_{\text{cusp}} \;\sim\; \Gamma_{\text{cusp}} = -\phi^2\, B(\lambda, N) + \ldots$$

$$B^{1-\text{loop}}_{\text{M2}} = -\tfrac{1}{\pi^2} \sum_{n=-\infty}^{\infty} E_n = -\tfrac{1}{4\pi^2} + \tfrac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{k^2 n^2 - 2}$$

$$= -\tfrac{1}{2\pi k} \cot \tfrac{2\pi}{k} \;, \qquad\qquad k > 2$$

$$B^{1-\text{loop}}_{\text{M2}} = \tfrac{1}{4\pi^2} \;, \qquad k = 1, 2$$

String in $AdS_4 \times \mathrm{CP}^3$ representing cusped Wilson line

[Drukker, Giombi, Ricci, Trancanelli 2007] [Correa, Aguilera, Silva 2014]

$$ds^2_{\mathrm{AdS}_3\times S^1} = \tfrac{1}{4}L^2(-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\theta^2) + \tfrac{1}{4}L^2 d\psi^2$$

$$\rho = \rho(\theta)\,, \quad \psi = \psi(\theta)\,, \quad \theta(\sigma) \in \left[\tfrac{\alpha}{2}, \pi - \tfrac{\alpha}{2}\right], \quad \psi(\sigma) \in \left[-\tfrac{\beta}{2}, \tfrac{\beta}{2}\right], \quad \phi \equiv \alpha$$

$$ds^2 = \tfrac{1}{4}L^2 \frac{1-\varepsilon^2}{\mathrm{cn}^2\sigma}(-d\tau^2 + d\sigma^2)\,, \qquad R^{(2)} = -2\big(1 + \tfrac{\varepsilon^2}{1-\varepsilon^2}\mathrm{cn}^4\sigma\big)$$

$$\theta(\sigma) = \tfrac{\pi}{2} + \tfrac{p^2}{b\sqrt{p^2+b^4}}\left[\sigma - \Pi\left(\tfrac{b^4}{p^2+b^4}, \mathrm{am}(\sigma + \mathrm{K})|\varepsilon^2\right) + \Pi\left(\tfrac{b^4}{p^2+b^4}|\varepsilon^2\right)\right]$$

$$\psi(\sigma) = \tfrac{bq}{\sqrt{p^2+b^4}}\,\sigma$$

small angle limit: $p \gg 1$ with fixed q, $\alpha = \pi\varepsilon + ...$

$$b = p + \tfrac{1-q^2}{2p} + \ldots, \quad \varepsilon^2 = \tfrac{1-q^2}{p^2} + \ldots, \quad \alpha = \tfrac{\pi}{p} + \ldots, \quad \beta = \tfrac{\pi q}{p} + \ldots .$$

$\beta = 0$, zero cusp angle α or $\varepsilon = 0$: $\mathrm{cn}\,\sigma \to \cos\sigma$, get AdS_2 metric

$$ds^2 = \tfrac{1}{4}L^2 \frac{1}{\cos^2\sigma}(-d\tau^2 + d\sigma^2)$$

M2 in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ representing cusped Wilson line

use 1st order small-angle perturbation theory near straight line (AdS_2)

- bosonic modes

$$\Delta = \partial_\tau^2 - \partial_\sigma^2 + \tfrac{1-\varepsilon^2}{\mathrm{cn}^2\sigma}\, m^2$$

$$m_0^2 = 3 + \tfrac{1}{2} R^{(2)} - \tfrac{1}{2}|2 + R^{(2)}| = 2 - 2\cos^4\sigma\,\varepsilon^2 + \dots$$

$$m_n^2 = \tfrac{1}{4}(kn-2)(kn-4) - \cos^4\sigma\,\varepsilon^2 + \dots, \qquad n = \pm 1, \pm 2, \dots$$

$$E\int d\tau = -\log \mathcal{Z}_1 = \tfrac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{8}\big(\log\det\Delta_{B_{p,n}} - \log\det\Delta_{F_{p,n}}\big)$$

$$E = \tfrac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{8}\sum_{\ell}\Big[\omega_{B_{p,\ell}}(n) - \omega_{F_{p,\ell}}(n)\Big]$$

$$\Big(-\partial_\sigma^2 + \tfrac{1-\varepsilon^2}{\mathrm{cn}^2\sigma} m^2\Big) X_\ell = \omega_\ell^2 X_\ell$$

$$\Big[-i\,\gamma^1\Big(\partial_\sigma + \frac{\mathrm{sn}\,\sigma\,\mathrm{dn}\,\sigma}{2\,\mathrm{cn}\,\sigma}\Big) + \frac{\sqrt{1-\varepsilon^2}}{\mathrm{cn}\,\sigma}\, m\Big]\Psi_\ell = \gamma^0\,\omega_\ell\Psi_\ell$$

• for scalars with the Dirichlet boundary conditions in AdS_2:

$$X_{h,\ell}(\sigma) = \frac{\sqrt{\ell!\,\Gamma(\ell+2h)\,(\ell+h)}}{2^{h-\frac{1}{2}}\,\Gamma(\ell+h+\frac{1}{2})}\cos^h\sigma\,P_\ell^{(h-\frac{1}{2},h-\frac{1}{2})}(\sin\sigma)$$

$$\omega_\ell = \ell + h\,, \qquad h = \tfrac{1}{2}\big(1+\sqrt{1+4m^2}\big)\,, \qquad \ell = 0,1,2,\ldots$$

• expansion in small ε

$$\omega_\ell^{\mathrm{CP}^3} = (\ell+h_n)(1-\tfrac{1}{4}\varepsilon^2) - \tfrac{h_n(h_n-1)}{4(\ell+h_n)}\varepsilon^2 + \ldots .$$

$$\omega_\ell^{\mathrm{AdS}_4} = (\ell+h_n)(1-\tfrac{1}{4}\varepsilon^2) - \Big[\tfrac{h_n(h_n-1)+1}{4(h_n+\ell)} + \tfrac{h_n(h_n-1)}{4(h_n+\ell)(h_n+\ell+1)(h_n+\ell-1)}\Big]\varepsilon^2 + \ldots$$

$$\omega_\ell^{\mathrm{F}} = (\ell+h_n)\big(1-\tfrac{1}{4}\varepsilon^2\big) + \tfrac{1}{4}\big(\tfrac{1}{2}-h_n\big)\Big[\tfrac{h_n-\frac{1}{2}}{\ell+h_n+\frac{1}{2}} + \tfrac{h_n-\frac{1}{2}}{(\ell+h_n-\frac{1}{2})(\ell+h_n+\frac{1}{2})}\Big]\varepsilon^2 + \ldots$$

• string mode $n=0$ contribution:

$$E_0 = \sum_{\ell=0}^{\infty} \frac{1}{4(\ell+1)(\ell+2)}\, \varepsilon^2 + \mathcal{O}(\varepsilon^4) = \frac{1}{4}\varepsilon^2 + \mathcal{O}(\varepsilon^4)$$

• M2 mode contribution:

$$\sum_{n\neq 0} E_n = -2\sum_{n=1}^{\infty} \frac{1}{k^2n^2-4}\, \varepsilon^2 + \cdots = \left(-\frac{1}{4} + \frac{\pi}{2k}\cot\frac{2\pi}{k}\right) \varepsilon^2 + \ldots .$$

$$E = \sum_{n=-\infty}^{\infty} E_n = \frac{\pi}{2k}\cot\frac{2\pi}{k}\, \varepsilon^2 + \mathcal{O}(\varepsilon^4)\ , \qquad \varepsilon = \frac{\alpha}{\pi}$$

• in general [Drukker, Forini 2011; Griguolo et al 2012]

$$\Gamma_{\rm cusp}(\lambda, N; \alpha, \beta) = -(\alpha^2 - \beta^2)\, B(k,N) + \ldots$$

β = angle in internal space; $\alpha = \pm\beta$ is BPS limit

• similar computation for β^2 term gives same result for $B(k,N)$

Non-planar corrections to ABJM operator dimensions

[Giombi, Kurlyand, AT 2024; Beccaria, Kurlyand, AT 2025]

• use semiclassical M2 brane quantization
to get predictions for $1/N$ strong coupling corrections to non-BPS ABJM observables not determined by localization or integrability

• string regime: non-planar corrections to dim's of operators with large spins can be found from string loop corrections to energies of semiclassical strings in $\mathrm{AdS}_4 \times \mathrm{CP}^3$

• M-theory regime: semiclassical quantization of corresponding spinning M2 branes with topology $(R_t \times S^1) \times S^1$:
dependence on finite $k = \frac{\lambda}{N}$ from
dependence of M2 action on 11d background with $R_{11} = 1/k$

Null cusp anomalous dimension

$$\Delta = S + f(\lambda, N)\log S + ..., \qquad \mathcal{O} = \mathrm{Tr}\,(\Phi D^S \Phi)$$

$f(\lambda, N)$ at strong coupling beyond planar limit?

• membrane analog of fast rotating folded string in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

$$ds^2 = \mathrm{L}^2\Big(\tfrac{1}{4}\big[-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\,(d\psi^2 + \cos^2\psi\, d\phi^2)\big] + \tfrac{1}{k^2}d\varphi^2\Big)$$

$$C_3 = -\tfrac{3}{8}R^3\cosh\rho\,\sinh^2\rho\,\sin\psi\, dt\wedge d\rho\wedge d\phi$$

$$t = \kappa\sigma_0, \qquad \rho = \kappa\sigma_1, \qquad \phi = \kappa\sigma_0, \qquad \psi = 0, \qquad \varphi = \sigma_2 \in (0, 2\pi)$$

$$S_{\mathrm{M2}} = T_2\int d^3\sigma\sqrt{-\det g} + T_2\int C_3 \;+\; \text{fermionic terms}$$

• classical energy as in string case: $\kappa = \frac{1}{\pi}\log S$

$$E - S = 2T\log S, \qquad T = \tfrac{2\pi}{k}T_2\mathrm{L}^3 = \sqrt{\tfrac{\lambda}{2}}\,, \qquad \mathrm{L}^6 = 32\pi^2 Nk\ell_P^6$$

• 1-loop correction: static gauge $t = \kappa\sigma_0,\ \ \rho = \kappa\sigma_1,\ \ \varphi = \sigma_2$

8+8 fluctuations in modes in σ_2: $X \sim \sum_n e^{in\sigma_2}X_n, \quad n = 0, \pm 1, ...$

- B: 6 CP^3 fluctuations: $m^2 = \frac{1}{4}kn(kn+2)$

  2 from $\psi, \tilde{\phi}$ mixing: $m_\pm^2 = 3 + \frac{1}{4}k^2n^2 \pm \sqrt{1+\frac{9}{4}k^2n^2}$
- F: 6+2 with $m = \frac{1}{2}kn \pm 1$ and $m = \frac{1}{2}kn$

$$\Gamma = -\log Z = \sum_{r=1}^{\infty}(T_2)^{-r+1}\Gamma_r(k) \;\sim \kappa^2 \int d\sigma_0 \quad \to \quad E \sim \log S$$

$$\Gamma_1 = \frac{1}{2}\frac{1}{(2\pi)^2} V \int d^2p \left[X_0(p^2) + 2\sum_{n=1}^{\infty} X_n(p^2)\right]$$

$$\begin{aligned}
X_n(p^2) = {}& \log\left[p^2 + 3 + \tfrac{1}{4}k^2n^2 + \sqrt{1+\tfrac{9}{4}k^2n^2}\right] \\
& + \log\left[p^2 + 3 + \tfrac{1}{4}k^2n^2 - \sqrt{1+\tfrac{9}{4}k^2n^2}\right] \\
& +3\log\left[p^2 + \tfrac{1}{4}(kn)^2 + \tfrac{1}{2}kn\right] + 3\log\left[p^2 + \tfrac{1}{4}(kn)^2 - \tfrac{1}{2}kn\right] \\
& -3\log\left[p^2 + (1+\tfrac{1}{2}kn)^2\right] - 3\log\left[p^2 + (1-\tfrac{1}{2}kn)^2\right] - 2\log\left[p^2 + (\tfrac{1}{2}kn)^2\right]
\end{aligned}$$

• string part ($n=0$) [McLoughlin, Roiban; Alday, Arutyunov, Bykov 08]

$$\frac{1}{4\pi}\int_0^\infty dp^2\, X_0(p^2) = \frac{1}{4\pi}\int_0^\infty dp^2\Big[\log(p^2+4)+\log(p^2+2)$$
$$+4\log p^2 - 6\log(p^2+1)\Big] = -\frac{5}{2\pi}\log 2$$

• $f = \left[\sqrt{2\lambda} - \frac{5}{2\pi}\log 2 + \frac{c_2}{\sqrt{\lambda}} + \frac{c_3}{(\sqrt{\lambda})^2} + \ldots\right] + \mathcal{O}(\frac{1}{k^2})$
$$\equiv \mathrm{f}_0(\lambda) + \mathrm{f}_1(k) + \ldots$$

c_n from string σ-model loops; planar $\mathrm{f}_0(\lambda)$ fixed by integrability (same as $AdS_5\times S^5$ one up to $\sqrt{\lambda}\to h(\lambda)$ [Gromov, Vieira 08])

• M2 brane mode contributions: $\quad Y_n \equiv \frac{1}{2\pi}\int_0^\infty dp^2\, X_n(p^2)$

$$Y_n = -\frac{1}{8\pi}\Big[-3((kn)^2+8)\log((kn)^2-4) + 18kn\log\frac{kn-2}{kn+2}$$
$$+(kn)^2\log(kn)^2 + ((kn)^2+12)\log((kn)^4-12(kn)^2+128)$$
$$+2\sqrt{9(kn)^2+4}\,\log\frac{(kn)^2+12+2\sqrt{9(kn)^2+4}}{(kn)^2+12-2\sqrt{9(kn)^2+4}}\Big] = \sum_{m=1}^\infty \frac{d_m}{(kn)^{2m}}$$

• $\mathrm{f}_1(k) = \sum_{n=1}^{\infty} Y_n = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{d_m}{(kn)^{2m}}$

finite with $\sim \zeta(2m)$ coeffs in expansion in $1/k^{2m}$

$$\mathrm{f}_1(k) = \sum_{n=1}^{\infty} Y_n = \frac{2\pi}{3k^2} + \frac{2\pi^3}{45k^4} + ... = \frac{2\pi\lambda^2}{3N^2} + \frac{2\pi^3\lambda^4}{45N^4} + ...$$

• non-planar correction at weak coupling $\qquad \mathrm{f}\big|_{\lambda\ll 1} \sim \frac{\lambda^4}{N^2} + ...$

prediction at strong coupling $\qquad \mathrm{f}\big|_{\lambda\gg 1} \sim \frac{\lambda^2}{N^2} + ...$

• higher loop M2 corrections to $f(\lambda, N) = f(\mathrm{T}_2, k)$

$f = \frac{4\pi}{k}\mathrm{T}_2 + \mathrm{f}(k) + \frac{q_2(k)}{\mathrm{T}_2} + \frac{q_3(k)}{(\mathrm{T}_2)^2} + ..., \qquad \mathrm{T}_2 = L^3 T_2 = \frac{1}{\pi}\sqrt{kN}$

should give subleading at strong coupling corrections

at each order in $1/N^2$ or in $\frac{g_s^2}{T}$:

$$f(\lambda, N) = \sqrt{2\lambda}\big[1 - \frac{5\log 2}{2\pi\sqrt{2\lambda}} + ...\big]$$
$$+ \frac{2\pi\lambda^2}{3N^2}\Big[1 + \frac{a_1}{\sqrt{\lambda}} + ...\Big] + \frac{2\pi^3\lambda^4}{45N^4}\Big[1 + \frac{b_1}{\sqrt{\lambda}} + ...\Big] + ...$$

Non-planar corrections to cusp anomaly in SYM ?

$\mathcal{N}=4$ SYM dual to $AdS_5\times S^5$ superstring:
no direct analog of M2 brane picture as in ABJM
but localization results for WL's suggest close analogy

$$\text{ABJM}:\qquad \langle W\rangle=\frac{\sqrt{T}}{\sqrt{2\pi}\,g_s}\,e^{2\pi T}\Big[1+\frac{\pi}{12}\frac{g_s^2}{T}+\frac{7\pi^2}{1440}\Big(\frac{g_s^2}{T}\Big)^2+\ldots\Big]$$

$$T=\sqrt{\frac{\lambda}{2}},\qquad g_s=\frac{\sqrt{\pi}}{N}(2\lambda)^{5/4},\qquad \lambda=\frac{N}{k},\qquad \frac{g_s^2}{T}\sim\frac{\lambda^2}{N^2}$$

$$\text{SYM}:\qquad \langle W\rangle=\frac{\sqrt{T}}{2\pi\,g_s}\,e^{2\pi T}\Big[1+\frac{\pi}{12}\frac{g_s^2}{T}+\frac{\pi^2}{288}\Big(\frac{g_s^2}{T}\Big)^2+\ldots\Big]$$

$$T=\frac{\sqrt{\lambda}}{2\pi},\qquad g_s=\frac{g_{YM}^2}{4\pi},\qquad \lambda=g_{YM}^2N,\qquad \frac{g_s^2}{T}\sim\frac{\lambda^{3/2}}{N^2}$$

same universal form when expressed in terms of (T,g_s),
and similar coefficients

• surprisingly, same $\frac{g_s^2}{T}$ term found in ABJM cusp anomaly

$$f_{\rm ABJM}(T,g_s) = \frac{1}{\pi}\Big[2\pi T - \tfrac{5}{2}\log 2 + \mathcal{O}(T^{-1}) \ + \frac{\pi}{12}\frac{g_s^2}{T} + \frac{\pi^2}{1440}\big(\frac{g_s^2}{T}\big)^2 + ...\Big]$$

$$= \frac{1}{\pi}\Big[\pi\sqrt{2\lambda} - \tfrac{5}{2}\log 2 + \mathcal{O}(\tfrac{1}{\sqrt{\lambda}}) + \frac{2\pi^2}{3}\frac{\lambda^2}{N^2} + \frac{2\pi^3}{45}\frac{\lambda^4}{N^4} + ...\Big]$$

• conjecture: coincidence of g_s^2 string 1-loop terms in ABJM and SYM WL's extends to non-planar part of cusp anomaly:

$$f_{\rm SYM}(T,g_s) = \frac{1}{\pi}\Big[2\pi T - 3\log 2 + \mathcal{O}(T^{-1}) + \frac{\pi}{12}\frac{g_s^2}{T} + \gamma_1\pi^2\big(\frac{g_s^2}{T}\big)^2 + ...\Big]$$

$$= \frac{1}{\pi}\Big[\sqrt{\lambda} \ - 3\log 2 + \mathcal{O}(\tfrac{1}{\sqrt{\lambda}}) + \frac{1}{12}\frac{\lambda^{3/2}}{N^2} + \frac{\gamma_1}{36}\frac{\lambda^3}{N^4} + ...\Big] \ .$$

prediction: $1/N^2$ correction to $f_{\rm SYM}$ that scales as λ^4 at $\lambda \ll 1$ should scale as $\lambda^{3/2}$ at $\lambda \gg 1$

Corrections to dimensions of short ABJM operators

"slow" spinning strings in $\mathrm{AdS}_4 \times \mathrm{CP}^3$

$1 \ll J \ll \sqrt{\lambda}$ and $E \sim \sqrt{\sqrt{\lambda} J} + ...$

dual to "short" gauge theory operators with $\Delta\big|_{\lambda \gg 1} \sim \lambda^{1/4}$

• planar limit may be determined from integrability

• non-planar corrections: semiclassical M2 branes in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

Example: M2 brane with two spins J_1, J_2 in $S^7/\mathbb{Z}_k$

generalizing spinning string in CP^3

dual to ABJM operator $\mathcal{O} = \mathrm{Tr}\left[(\Phi^1 \Phi_2^\dagger)^{J_1} \Phi^3 \Phi_4^\dagger)^{J_2}\right]$

"long" (or "fast") string with $J_= J_2$ [Frolov, AT 03; Bandres, Lipstein 09]

$$E_0 = \sqrt{4J^2 + \bar{\lambda}}\,, \qquad \bar{\lambda} \equiv 2\pi^2 \lambda, \qquad T = \sqrt{\frac{\lambda}{2}} = \frac{\sqrt{\bar{\lambda}}}{2\pi}$$

"short" (or "slow") string: $E_0 = \sqrt{2\sqrt{\bar{\lambda}} J}$ (as in flat space)

• 1-loop string correction in $AdS_4 \times CP^3$
get prediction for planar part of dimension $\Delta_0(\lambda, J) = E_{\rm str}$

$$E_{\rm str} = 2\bar{\lambda}^{1/4} J^{1/2} + \tfrac{1}{2} + \tfrac{1}{2}\bar{\lambda}^{-1/4} J^{1/2} - \tfrac{9}{4}\zeta(3)\bar{\lambda}^{-3/4} J^{3/2} + ...$$

(yet to be reproduced from spectral curve approach)
• including rest of M2 brane 1-loop correction:

$$E_{\rm M2} = 2\bar{\lambda}^{1/4} J^{1/2} + \tfrac{1}{2} + \tfrac{1}{2}\bar{\lambda}^{-1/4} J^{1/2} - \tfrac{9}{4}\zeta(3)\bar{\lambda}^{-3/4} J^{3/2} + ...$$

$$+ \frac{1}{k^2}\Big[\zeta(2)\big(- 4\bar{\lambda}^{3/4} J^{-3/2} + 8\bar{\lambda}^{1/4} J^{-1/2}\big) + ...\Big] + \mathcal{O}(\frac{1}{k^4})$$

$\frac{1}{k^2} = \frac{\lambda^2}{N^2} = \frac{g_s^2}{4\sqrt{\bar{\lambda}}}$:

large-tension limit of 1-loop (torus) string correction in $AdS_4 \times CP^3$
• prediction for $\lambda \gg 1$ limit of leading non-planar
correction to Δ of dual "short" operator: $\lambda^{11/4}/N^2$
(cf. λ^2/N^2 in cusp anomaly)

Similar results in (S, J) sector:

• fast (S, J) folded string or M2 brane $\rightarrow$ generalized cusp anomaly

$$\mathcal{S} \gg \mathcal{J} \gg 1, \quad x \equiv \frac{\log \mathcal{S}}{\pi \mathcal{J}} = \text{fixed}, \qquad \mathcal{S} = \frac{S}{\sqrt{\bar{\lambda}}}, \quad \mathcal{J} = \frac{J}{\sqrt{\bar{\lambda}}}$$

$$E = S + f(\lambda, N, x) \log S + \ldots, \qquad f = f_0(\lambda, x) + q_0(k, x) + \ldots$$

$$f_0(\lambda, x) = \frac{\sqrt{\bar{\lambda}}}{\pi} \frac{\sqrt{1 + x^2}}{x} + \mathrm{f}_0(\lambda, x)$$

$$q_0(x, k) = -\frac{5 \log 2}{2\pi} + \frac{\pi}{x} \Big(\tfrac{2}{3} \sqrt{1 + x^2} + \frac{1}{\sqrt{1 + x^2}} \Big) \frac{1}{k^2}$$

$$+ \frac{\pi^3}{x^3} \Big[\tfrac{2}{45} (\sqrt{1 + x^2})^3 + \tfrac{19}{45} \sqrt{1 + x^2} - \tfrac{1}{60} \frac{1}{\sqrt{1 + x^2}} \Big] \frac{1}{k^4} + \ldots .$$

• circular (S, J) string/membrane: close analogy with $J_1 = J_2$ case

Concluding remarks

• evidence that quantum M2 brane theory is well defined
in semiclassical expansion

• susy observables like BPS WL (and instanton part of S^3 free energy):
quantising M2 brane opens way to precision tests
of $\mathrm{AdS}_4/\mathrm{CFT}_3$ beyond planar limit

• beyond susy observables: dimensions of ABJM operators
dual to spinning strings/membranes:
predictions for non-planar corrections at strong coupling
(leading α' terms in string higher genus contributions)

• structure of localization result for WL suggests that higher loop M2 brane corrections should also be well defined
hidden symmetry of M2 theory that constrains loop corrections?
[cf. integrability in GS string or $T\bar{T}$ in 2d theories]

• M2 world-volume theory with $\Sigma^2 \times S^1$ topology:
effective 2d theory with massive KK towers ($m^2 \sim k^2 \sim g_s^{-2}$):
effective string action with non-local terms
related to small handle resummation?

• is there a way to do similar computations
of non-planar corrections at strong coupling
in $\mathcal{N} = 4$ SYM dual to $AdS_5 \times S^5$ string theory?