$O(D, D)$ and string α'-corrections

Linus Wulff

Masaryk University

Based on 2012.13410 w/ S. Hronek

ITMP Seminar (online) 3 Mar 2021

Dimensional reduction and $O(d, d)$

D-dimensional Einstein gravity reduced on T^d, i.e. assuming metric indep. of y^m, $m = D - d, \ldots, D - 1$, has $GL(d)$ symmetry from internal diffeomorphisms

$$y^m \to M^m{}_n y^n \,, \qquad M \in GL(d)$$

At low energies string theory is described by Einstein gravity + 2-form potential B_{mn} + dilaton ϕ with action

$$S[G, B, \phi] = \int d^D x \sqrt{-G} e^{-2\phi} (R + 4(\nabla\phi)^2 - \frac{1}{12} H^2 + \ldots)$$

where $H = dB$ and $D = 26(10)$ for bosonic(super) string

Its T^d reduction has enhanced $O(d, d)$ symmetry (including all higher derivative α'-corrections in ...) [Meissner, Veneziano; Sen '91]

Double Field Theory (DFT)

This enhanced symmetry arises from T-duality and is far from manifest

It could be made manifest if there was a formulation of the D-dim. theory with $O(D, D)$ symmetry

Since then, reducing on T^d, $O(D, D) \to O(d, d)$

The idea of formulating the string effective action with $O(D, D)$ symmetry if known as DFT [Siegel '93; Hohm, Hull, Zwiebach '09 '10] (see also [Duff; Tseytlin '90])

The name derives from the fact that $O(D, D)$ must act on the coordinates which requires doubling the dimension to $2D$

$$x^m \to X^M = (\tilde{x}_m,\, x^m)$$

Double Field Theory (DFT)

Under an $O(D,D)$ transformation

$$O\eta O^T = \eta\,, \qquad \eta^{MN} = \begin{pmatrix} 0 & \delta^n_m \\ \delta^m_n & 0 \end{pmatrix}$$

X transforms as a vector

$$X^M \to O^M{}_N X^N \qquad O \in O(D,D)$$

However, to describe a D-dim. theory we need to impose an $O(D,D)$ invariant constraint which halves the dimension

This is the section condition (a.k.a. strong constraint)

$$\partial_M \Phi_1 \partial^M \Phi_2 = \partial_m \Phi_1 \tilde{\partial}^m \Phi_2 + \tilde{\partial}^m \Phi_1 \partial_m \Phi_2 = 0 \qquad \forall \Phi_1, \Phi_2$$

The standard solution is to let all fields depend only on x^m.
We will assume this solution throughout this talk.

Double Field Theory (DFT)

A priory such an $O(D,D)$ invariant formulation is not guaranteed to exist.

Nevertheless, at lowest order in α', the string effective action can be written in $O(D,D)$ invariant form in terms of a generalized metric

$$\mathcal{H}^{MN} = \begin{pmatrix} (G - BG^{-1}B)_{mn} & (BG^{-1})_m{}^n \\ -(G^{-1}B)^m{}_n & G^{mn} \end{pmatrix}$$

and generalized dilaton d with

$$e^{-2d} = \sqrt{-G}e^{-2\phi}$$

i.e.

$$S_0[G,B,\phi] = \tilde{S}_0[\mathcal{H},d]$$

α' corrections

Surprising: More symmetry than we would expect

On the other hand: $O(d,d)$ for all d is very constraining. Might expect some trace of it already in D dim.

This DFT action has proven very useful for many purposes: consistent truncations, generalized T-dualities, etc.

But string theory also has α' corrections:

$$S = \int d^D x \sqrt{-G} e^{-2\phi} L \,, \qquad L = L_0 + \alpha' L_1 + \alpha'^2 L_2 + \alpha'^3 L_3 + \dots$$

$L_1 = (\mathrm{Riem})^2 + \dots$ (bos/het)

$L_2 = (\mathrm{Riem})^3 + \dots$ (bos) ($L_2 = CS^2 + \dots$ (het))

$L_3 = \zeta(3)(\mathrm{Riem})^4 + \dots$ (bos/het/type II)

First α' correction

Can we write these in an $O(D, D)$ invariant way?

Reason to be skeptical: DFT does not have an analog of the Riemann tensor [Hohm, Zwiebach '12]

Remarkably Marques and Nunez were able to cast the first α' correction to bos/het string in $O(D, D)$ inv. form [Marques, Nunez '15]

Working in frame-like formulation with gen. vielbein $E_A{}^M$ transforming as

$$E_A{}^M \to \Lambda_A{}^B E_B{}^N O_N{}^M \,, \qquad O \in O(D, D)$$

under $O(D, D)$ and local double Lorentz transf.

$$\Lambda = \begin{pmatrix} \Lambda^{(+)} & 0 \\ 0 & \Lambda^{(-)} \end{pmatrix} \in O(D-1, 1) \times O(D-1, 1)$$

First α' correction

They showed that a certain correction to the double Lorentz transformations

$$E_A{}^M \to (\Lambda_A{}^B + \alpha' \hat{\Lambda}_A{}^B) E_B{}^M$$

induces precisely the known $(\text{Riem})^2$-terms in the action.
Correction depends on 2 parameters

$$\hat{\Lambda}_A{}^B = \hat{\Lambda}_A{}^B(a, b) \qquad \begin{cases} a = b & \Rightarrow \quad \text{bosonic string} \\ b = 0 & \Rightarrow \quad \text{heterotic string} \end{cases}$$

Reproduces the Green-Schwarz correction for het string implied by

$$dH = \alpha' \mathrm{tr}(R \wedge R)$$

Higher α' corrections

Closure requires an infinite series of α' corrections.

Could account also for the α'^2 corrections to bos/het string.

[Baron, Lescano, Marques '18; Baron, Marques '20]

It cannot account for the α'^3 corrections due to the coeff. $\zeta(3)$.

Is it possible to account also for these in DFT?

To address this we setup a systematic way to find $O(D, D)$ invariants, working order by order in fields

Actually, like Marques and Nunez we work in a formulation where $O(D, D)$ is manifest and the problem is to find double Lorentz invariants.

Flux formulation of DFT [Geissbuhler, Marques, Nunez, Penas '13]

We start with the generalized vielbein

$$E_A{}^M = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{(+)} - e^{(+)}B & (e^{(+)})^{-1} \\ -e^{(-)} - e^{(-)}B & (e^{(-)})^{-1} \end{pmatrix}$$

The two ordinary vielbeins $e_m^{(\pm)a}$ ($e^{(\pm)} \cdot e^{(\pm)} = G$) are rotated independently by $\Lambda^{(\pm)}$.

Two constant metrics

$$O(D,D) \text{ metric}: \qquad \eta^{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & -\eta^{ab} \end{pmatrix}$$

$$\text{generalized metric}: \qquad \mathcal{H}^{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & \eta^{ab} \end{pmatrix}$$

Projectors $(P_\pm)^{AB} = \frac{1}{2}(\eta \pm \mathcal{H})^{AB} \rightarrow$ canonical split $A = (\underline{a}, \overline{a})$

Flux formulation of DFT

Just as the ordinary vielbein transforms under both diffeos and local Lorentz, $E_A{}^M$ transforms under gen. diffeos (diff. + B-field transf.) and local double Lorentz transf.

This makes the vielbein awkward to work with. In the Riemannian case we can construct a spin connection $\omega_c{}^{ab}$ from ∂e that transforms only under local Lorentz.

The doubled analog is a gen. diff. scalar constructed from ∂E. It is not hard to show that it is unique and given by

$$F_{ABC} = 3\partial_{[A} E_B{}^M E_{C]M} \qquad (\partial_A \equiv E_A{}^M \partial_M)$$

Taking the gen. dilaton into account there is one more scalar

$$F_A = \partial^B E_B{}^M E_{AM} + 2\partial_A d$$

These gen. fluxes are the analogs of $\omega_c{}^{ab}$ and $\partial_a \phi$.

Flux formulation of DFT

Gen. diff. invariance requires the action to be constructed from F_A, F_{ABC} and their (∂_A) derivatives

Using the $P_\pm$ projections we have six fields

$$F_{\overline{abc}}\,,\quad F_{\underline{a}\overline{bc}}\,,\quad F_{\overline{a}\underline{bc}}\,,\quad F_{\underline{abc}}\,,\qquad \text{and} \qquad F_{\overline{a}}\,,\quad F_{\underline{a}}$$

$O(D,D)$ and gen. diffeos are manifest but double Lorentz is not. Under these we have

$$\begin{aligned}
\delta F_{\overline{abc}} =& 3\partial_{[\overline{a}}\lambda_{\overline{bc}]} + 3\lambda_{[\overline{a}}{}^{\overline{d}}F_{\overline{bc}]\overline{d}} \\
\delta F_{\underline{a}\overline{bc}} =& \partial_{\underline{a}}\lambda_{\overline{bc}} + \lambda_{\underline{a}}{}^{\underline{d}}F_{\underline{d}\overline{bc}} + 2\lambda_{[\overline{b}}{}^{\overline{d}}F_{|\underline{a}\overline{d}|\overline{c}]} \\
\delta F_{\overline{a}} =& \partial^{\overline{b}}\lambda_{\overline{ba}} + \lambda_{\overline{a}}{}^{\overline{b}}F_{\overline{b}}
\end{aligned}$$

and similarly with projections reversed. Note that

$$\lambda^{(+)} \to \lambda_{\overline{ab}}\,, \qquad \lambda^{(-)} \to \lambda_{\underline{ab}}$$

$F_{\overline{abc}}$ transforms like a 3-form and $F_{\underline{a}\overline{bc}}$ like a connection

No field strengths

To write an invariant action we would normally first construct covariant field strengths, e.g. the Riemann tensor. From those invariant actions are easily constructed.

This approach fails here: F_{ABC} has no indep. covariant field strengths

For example

$$4\partial_{[\overline{a}}F_{\overline{bcd}]} \sim F_{[\overline{ab}}{}^{E}F_{\overline{cd}]E}$$

From $F_{\underline{a}\overline{bc}}$ we can construct something like a Riemann tensor

$$R_{\overline{ab}\underline{cd}} = 2\partial_{[\overline{a}}F_{\overline{b}]\underline{cd}} - F_{\overline{abe}}F^{\overline{e}}{}_{\underline{cd}} + 2F_{[\overline{a}|\underline{c}|}{}^{\underline{e}}F_{\overline{b}]\underline{de}} = -R_{\underline{cd}\overline{ab}}$$

which transforms as

$$\delta R_{\overline{ab}\underline{cd}} = 2\lambda_{[\overline{a}}{}^{\overline{e}}R_{|\overline{e}|\overline{b}]\underline{cd}} + 2\lambda_{[\underline{c}}{}^{\underline{e}}R_{|\overline{ab}\underline{e}|\underline{d}]} - F_{\overline{e}\underline{cd}}\partial^{\overline{e}}\lambda_{\overline{ab}} + F_{\underline{e}\overline{ab}}\partial^{\underline{e}}\lambda_{\underline{cd}}$$

and is non-covariant due to the last 2 terms

Constructing invariants

Note however that $R_{\overline{ab}\underline{cd}}$ is invariant to leading order in fields

$$\delta R_{\overline{ab}\underline{cd}} = \mathcal{O}(F)$$

since $[\partial_{\overline{a}}, \partial_{\overline{b}}] = \mathcal{O}(F)$. It is as close as we can come to a Riemann tensor.

Given the lack of field strengths, how do we construct invariants?

A simple approach is the following: [Utiyama '56]

Suppose we have a $U(1)$ gauge field with transformation

$$A_\mu \to A_\mu + \partial_\mu \lambda$$

but we don't know anything about connections and field strengths

Constructing invariants

We take a general Lagrangian

$$L(A_\mu, \partial_\mu A_\nu, \partial_\mu \partial_\nu A_\rho, \ldots)$$

The requirement that L be invariant under $A_\mu \to A_\mu + \partial_\mu \lambda$ becomes

$$\sum_{n=1}^{N} \frac{\partial L}{\partial(\partial_{\mu_1 \cdots \mu_{n-1}} A_{\mu_n})} \partial_{\mu_1 \cdots \mu_n} \lambda = 0$$

Since λ is an arbitrary function all its derivatives are independent and each term in the sum must vanish

$$\frac{\partial L}{\partial(\partial_{\mu_1 \cdots \mu_{n-1}} A_{\mu_n})} \partial_{\mu_1 \cdots \mu_n} \lambda = 0 \qquad n = 1, 2, \cdots$$

or

$$\frac{\partial L}{\partial(A_\mu)} = 0\,, \qquad \frac{\partial L}{\partial(\partial_{(\mu} A_{\nu)})} = 0\,, \qquad \frac{\partial L}{\partial(\partial_{(\mu\nu} A_{\rho)})} = 0\,, \qquad \cdots$$

Which implies that L depends only on $F_{\mu\nu}$ and its derivatives

$O(D, D)$ invariants: Leading order

The same approach can be used for Yang-Mills theory and gravity

We will use the same idea to construct $O(D, D)$ invariants

To simplify the problem further we work order by order in fields

In our case the leading order transformations are

$$\delta F_{\overline{abc}} \sim 3\partial_{[\overline{a}}\lambda_{\overline{bc}]} \,, \qquad \delta F_{\underline{a}\overline{bc}} \sim \partial_{\underline{a}}\lambda_{\overline{bc}} \,, \qquad \delta F_{\overline{a}} \sim \partial^{\overline{b}}\lambda_{\overline{ba}}$$

and similarly with projections reversed

We take

$$S = \int dX e^{-2d} \left(L_n(F, \partial F, \ldots) + \ldots \right)$$

with L_n of order n in the fields.

$O(D, D)$ invariants: Leading order

Double Lorentz inv. at leading order says

$$\delta L_n = \mathcal{O}(F^n)$$

and analyzing this condition one finds that L_n must be constructed from

(a) $R_{\overline{ab}\underline{cd}}$ $(\sim 2\partial_{[\overline{a}} F_{\overline{b}]\underline{cd}})$

(b) $\partial^{\overline{a}} F_{\overline{a}}$

(c) $\partial_{\underline{a}} F_{\overline{b}} + \partial^{\overline{c}} F_{\underline{a}\overline{bc}}$

(d) $F_{\overline{abc}} \partial^{\overline{a}} \Phi_1 \partial^{\overline{b}} \Phi_2 \partial^{\overline{c}} \Phi_3 + 3 F_{\underline{a}\overline{bc}} \partial^{[\underline{a}} \Phi_1 \partial^{\overline{b}} \Phi_2 \partial^{\overline{c}]} \Phi_3$

and their derivatives (expressions with projections reversed are not independent).

$O(D,D)$ invariants: Subleading order

First we will construct the usual 2-derivative action

There is only one scalar of dimension 2 in the list so we take

$$L = 4\partial^{\bar{a}}F_{\bar{a}} + L_2$$

Requiring this to the invariant (up to total derivatives) to the next order in fields fixes

$$L = 4\partial^{\bar{a}}F_{\bar{a}} - 2F_{\bar{a}}F^{\bar{a}} + F_{\underline{a}\overline{bc}}F^{\underline{a}\overline{bc}} + \frac{1}{3}F_{\overline{abc}}F^{\overline{abc}}$$

which coincides with the DFT action in the flux formulation.

Now we can consider higher derivative invariants

$O(D,D)$ invariants: Subleading order

At leading order they must be built from

(a) $R_{\overline{ab}\underline{cd}}$ $(\sim 2\partial_{[\overline{a}}F_{\overline{b}]\underline{cd}})$

(b) $\partial^{\overline{a}}F_{\overline{a}}$

(c) $\partial_{\underline{a}}F_{\overline{b}} + \partial^{\overline{c}}F_{\underline{a}\overline{bc}}$

(d) $F_{\overline{abc}}\partial^{\overline{a}}\Phi_1\partial^{\overline{b}}\Phi_2\partial^{\overline{c}}\Phi_3 + 3F_{\underline{a}\overline{bc}}\partial^{[\underline{a}}\Phi_1\partial^{\overline{b}}\Phi_2\partial^{\overline{c}]}\Phi_3$

But (b) and (c) are proportional to the lowest order equations of motion, i.e. removable by field redefinition

Furthermore (d) arises only at dim 10 (α'^4) or higher (take $\Phi_i \sim R_{\overline{ab}\underline{cd}}$)

Up to order α'^3 we only need to consider $R_{\overline{ab}\underline{cd}}$ and its derivatives:

$$R^n\,,\quad n=2,3,4\,,\qquad \partial^2R^2\,,\quad \partial^4R^2\,,\quad \partial^2R^3$$

$O(D, D)$ invariants: Subleading order

The terms with derivatives can again be removed by field redefinitions

We therefore take

$$L = (R^n) + L_{n+1} + \dots\,, \qquad n = 2, 3, 4$$

and require $\delta L = \mathcal{O}(F^{n+1})$+total derivatives

We will only look at a small subset of the conditions namely the terms in δL involving $\partial_{\overline{c}}\lambda_{\overline{ab}}$ and $\partial_{\underline{c}}\lambda_{\overline{ab}}$

Define the derivatives of L

$$G^{\overline{ab}\underline{cd}} = \frac{\partial(R^n)}{\partial R_{\overline{ab}\underline{cd}}}\,, \; G^{\overline{abc}} = \frac{\partial L_{n+1}}{\partial F_{\overline{abc}}}\,, \; G^{\overline{ab}\underline{c}} = \frac{\partial L_{n+1}}{\partial F_{\underline{c}\overline{ab}}}\,, \; G^{\overline{a}} = \frac{\partial L_{n+1}}{\partial F_{\overline{a}}}$$

$O(D, D)$ invariants: Subleading order

The two conditions become

$$-G^{\overline{ab}\underline{de}} F^{\overline{c}}{}_{\underline{de}} + \eta^{\overline{c}[\overline{a}} G^{\overline{b}]} + 3G^{\overline{abc}} + H^{\overline{abc}} = \dots , \quad G^{\overline{ab}\underline{c}} + H^{\overline{ab}\underline{c}} = \dots$$

where

$$H^{\overline{abc}} = [\Phi_1 \partial^{\overline{c}} \Phi_2]^{\overline{ab}} , \qquad H^{\overline{ab}\underline{c}} = [\Phi_1 \partial^{\underline{c}} \Phi_2]^{\overline{ab}}$$

arise from freedom in using the section cond. for contracted derivatives

On the RHS

$$\dots = \text{total derivatives} + F_A\text{-terms} + \partial^{\overline{a}} F_{\overline{a}BC}\text{-terms}$$

which we neglect. In particular this includes the terms coming from modifying the double Lorentz transformations.

R^2 invariants

We take

$$L = R_{\bar{a}\bar{b}\underline{cd}} R^{\bar{a}\bar{b}\underline{cd}} + L_3 + \mathcal{O}(F^4)$$

Our two conditions read

$$-2R^{\bar{a}\bar{b}\underline{de}} F^{\bar{c}}{}_{\underline{de}} + \eta^{\bar{c}[\bar{a}} G^{\bar{b}]} + 3G^{\bar{a}\bar{b}\bar{c}} + H^{\bar{a}\bar{b}\bar{c}} = \ldots , \quad G^{\bar{a}\bar{b}\underline{c}} + H^{\bar{a}\bar{b}\underline{c}} = \ldots$$

Symmetrizing the 1st in $(\bar{b}\bar{c})$ $G^{\bar{a}\bar{b}\bar{c}}$ drops out $\rightarrow$ Constraint on form of R^2-terms

It turns out to be satisfied in the present case and we find

$$\begin{aligned} G^{\bar{a}\bar{b}\bar{c}} &= R^{[\bar{a}\bar{b}}{}_{\underline{de}} F^{\bar{c}]\underline{de}} + \ldots & H^{\bar{a}\bar{b}\bar{c}} &= -2\partial^{\bar{c}} F^{[\bar{a}}{}_{\underline{de}} F^{\bar{b}]\underline{de}} + \ldots \\ G^{\bar{a}\bar{b}\underline{c}} &= 2\partial^{\underline{c}} F^{[\bar{a}}{}_{\underline{de}} F^{\bar{b}]\underline{de}} + \ldots & H^{\bar{a}\bar{b}\underline{c}} &= -2\partial^{\underline{c}} F^{[\bar{a}}{}_{\underline{de}} F^{\bar{b}]\underline{de}} + \ldots \end{aligned}$$

Note: $G^{\bar{a}\bar{b}\bar{c}}$ dep. on $F^{\bar{c}\underline{de}}$ but $G^{\bar{a}\bar{b}\underline{c}}$ is indep. of $F^{\bar{a}\bar{b}\bar{c}}$

R^2 and R^3 invariants

Seems to be in conflict with the integrability condition

$$\frac{\partial G^{\overline{abc}}}{\partial F_{\overline{d}\underline{ef}}} = \frac{\partial G^{\underline{ef}\overline{d}}}{\partial F_{\overline{abc}}} \qquad \Rightarrow \qquad \eta^{\overline{d}[\overline{c}} R^{\overline{a}\overline{b}]}{}_{\underline{ef}} \sim \ldots$$

This is satisfied here however, since $R_{\overline{ab}\underline{cd}} \sim 2\partial_{[\overline{a}} F_{\overline{b}]\underline{cd}}$

Indeed, going on to include the terms we neglected we reproduce the result of Marques and Nunez including the required correction to the double Lorentz transformations which they took as input. Demonstrates its uniqueness.

Repeating the calculation for $L = R^{\overline{a}}{}_{\overline{b}\underline{cd}} R^{\overline{bc}\underline{ef}} R_{\overline{ca}\underline{f}}{}^{\underline{d}} + \ldots$ it fails already in the first step

$\Rightarrow$ No $O(D,D)$ invariant R^3 terms exist ((Riem)^3 terms arise by completion of R^2 however)

R^4 invariants

There are now 8 possible structures for the leading terms

$$L = c_1 I_1 + \dots c_8 I_8 + L_5 + \mathcal{O}(F^6)$$

with

$$\begin{aligned}
I_1 &= R_{\overline{ab}\underline{ef}} R^{\overline{ab}\underline{ef}} R_{\overline{cd}\underline{gh}} R^{\overline{cd}\underline{gh}} , & I_5 &= R_{\overline{ab}\underline{ef}} R_{\overline{cd}}{}^{\underline{ef}} R^{\overline{bc}}{}_{\underline{gh}} R^{\overline{da}\underline{gh}} , \\
I_2 &= R_{\overline{ab}\underline{ef}} R^{\overline{cd}\underline{ef}} R^{\overline{ab}\underline{gh}} R_{\overline{cd}\underline{gh}} , & I_6 &= R_{\overline{ab}\underline{ef}} R^{\overline{cd}\underline{fg}} R^{\overline{ab}}{}_{\underline{gh}} R_{\overline{cd}}{}^{\underline{he}} , \\
I_3 &= R_{\overline{ab}\underline{ef}} R^{\overline{ab}\underline{fg}} R_{\overline{cd}\underline{gh}} R^{\overline{cd}\underline{he}} , & I_7 &= R_{\overline{ab}\underline{ef}} R^{\overline{bc}\underline{fg}} R_{\overline{cd}\underline{gh}} R^{\overline{da}\underline{he}} , \\
I_4 &= R_{\overline{ab}\underline{ef}} R^{\overline{bc}\underline{ef}} R_{\overline{cd}\underline{gh}} R^{\overline{da}\underline{gh}} , & I_8 &= R_{\overline{ab}\underline{ef}} R^{\overline{bc}\underline{fg}} R_{\overline{cd}}{}^{\underline{he}} R^{\overline{da}}{}_{\underline{gh}} .
\end{aligned}$$

The symmetrized 1st condition

$$-G^{\bar{a}(\bar{b}}{}_{\underline{de}} F^{\bar{c})\underline{de}} + \frac{1}{2}\eta^{\bar{a}(\bar{b}} G^{\bar{c})} - \frac{1}{2}\eta^{\bar{c}\bar{b}} G^{\bar{a}} = \dots$$

now has a unique solution

R^4 invariants

It takes the form

$$L_4 = \bar{t}_8 \underline{t}_8 R^4$$

where

$$\bar{t}_{\bar{a}_1 \cdots \bar{a}_8} M_1^{\bar{a}_1 \bar{a}_2} M_2^{\bar{a}_3 \bar{a}_4} M_3^{\bar{a}_5 \bar{a}_6} M_4^{\bar{a}_7 \bar{a}_8} = 8\mathrm{tr}(M_1 M_2 M_3 M_4) - 2\mathrm{tr}(M_1 M_2)\mathrm{tr}(M_3 M_4) + \mathrm{cylc}(2,3,4)$$

and similarly with underlined indices.

Remarkably this is precisely the right structure to match the α'^3 string correction [Gross, Witten '86;...]

$$t_8 t_8 R^4 + \tfrac{1}{8}\varepsilon_{10}\varepsilon_{10} R^4$$

at leading order (2nd term is a total derivative at leading order)

R^4 invariants

From the condition

$$-G^{\overline{ab}\underline{de}}F^{\overline{c}}{}_{\underline{de}} + \eta^{\overline{c}[\overline{a}}G^{\overline{b}]} + 3G^{\overline{abc}} + H^{\overline{abc}} = \ldots$$

we now extract $G^{\overline{abc}} \sim \underline{t}^{\underline{d}_1\cdots\underline{d}_8}F^{[\overline{a}}{}_{\underline{d}_1\underline{d}_2}K^{\overline{bc}]}{}_{\underline{d}_3\cdots\underline{d}_8}$ with

$$K^{\overline{ab}}{}_{\underline{d}_1\cdots\underline{d}_6} \sim -\tfrac{3}{2}\partial^{\overline{a}}F^{\overline{c}}{}_{\underline{d}_1\underline{d}_2}\partial^{\overline{b}}F^{\overline{d}}{}_{\underline{d}_3\underline{d}_4}\partial_{\overline{c}}F_{\overline{d}\underline{d}_5\underline{d}_6}$$

But the integrability conditions again require

$$\frac{\partial G^{\overline{abc}}}{\partial F_{\overline{d}\underline{ef}}} \sim t_8 K$$

to be a total derivative. However, it is not hard to show that this is not the case. Therefore R^4 cannot be completed with $\mathcal{O}(F^5)$ terms to an $O(D,D)$ invariant.

Conclusions

- We have argued that DFT in flux formulation admits a Riem^2 correction but no Riem^3 or Riem^4
- This suggests that DFT cannot account for the $\zeta(3)$Riem^4 correction in string theory at α'^3
- While an $O(D, D)$ invariant appears not to exist, an $O(d, d)$ inv. form on T^d must exist. Indeed, this was verified recently for $d = 9$ [Codina, Hohm, Marques '20]

- Can the obstruction we find be given a geometric interpretation?
- Even if $O(D, D)$ is broken, can we still exploit it to constrain α' corrections?