Black holes in scalar-tensor theories

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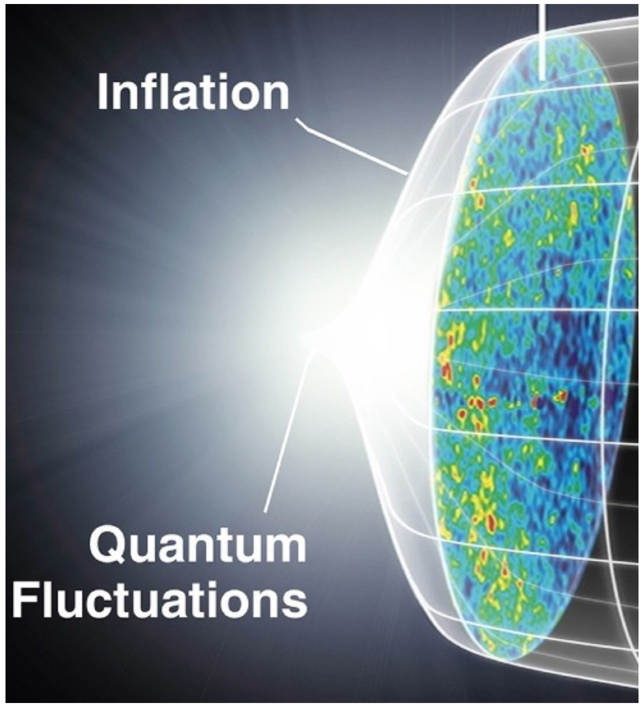
Outline

- Motivation
- From canonical scalar to Horndeski and beyond
- Black holes in general relativity, standard scalar-tensor theories. No-hair theorems
- Black holes in Horndeski theory and beyond, circumventing the no-hair theorem

Motivation

Motivation: inflation

✦ **Acceleration of the Universe at early times**



- Scalar field (inflaton): the field rolls down the potential, quasi de-Sitter solution:

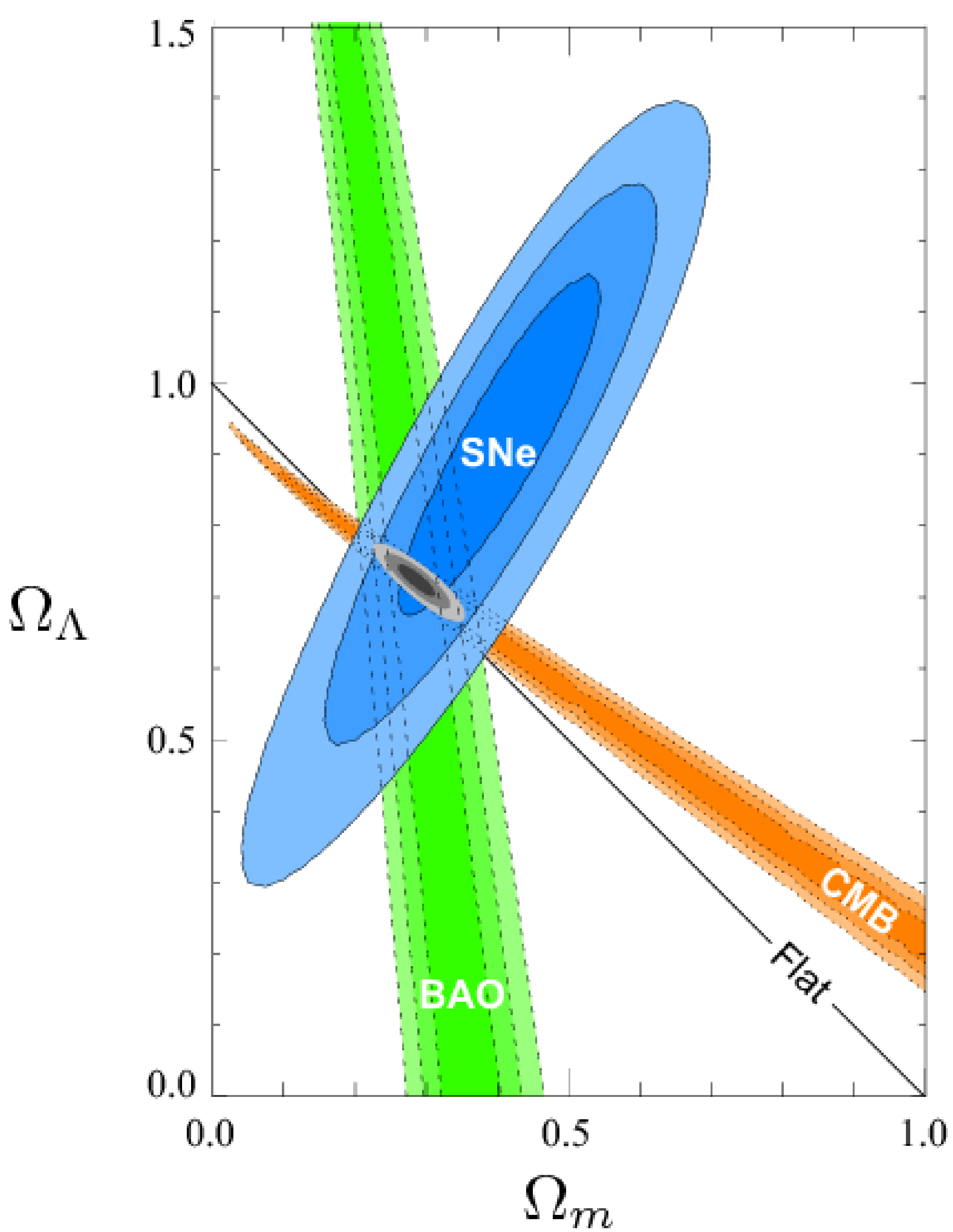
$$a(t) \sim e^{Ht}$$

- Modification of gravity

$$R + \frac{R^2}{6M^2}$$

Motivation: Dark energy

- **Our Universe is accelerating now**



Einstein equations

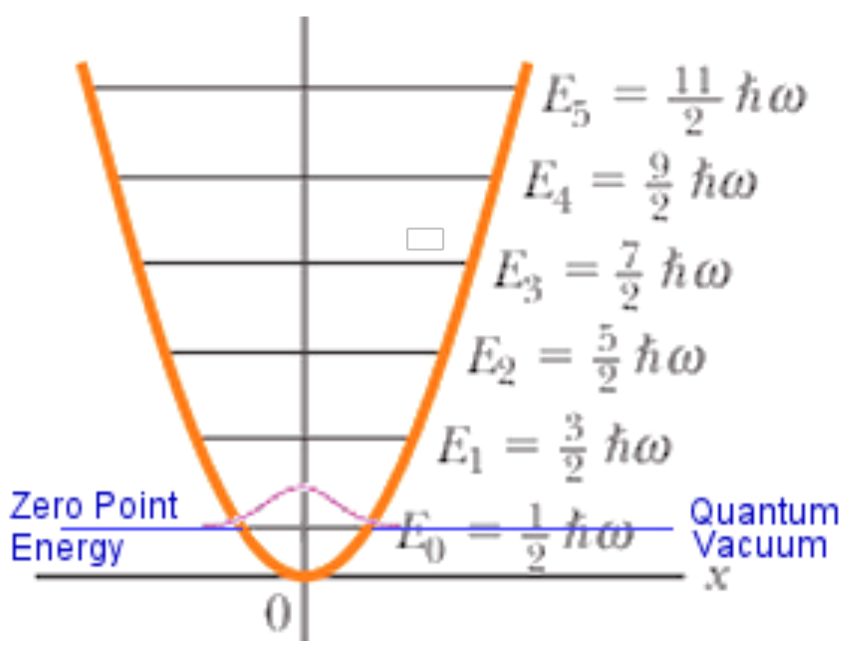
$$G_{\mu\nu} = 8\pi G\, T^{(m)}_{\mu\nu} - \Lambda g_{\mu\nu}$$

Source of acceleration?
add scalar field (quintessence, k-essence)?

Modification of gravity?

Motivation: Cosmological constant problem

- Long standing problem in modern physics: huge discrepancy of the observed value of the cosmological constant and its various theoretical predictions
- The energy density of Dark energy is $10^{-46}\ \mathrm{GeV}^4$. In Planck units it is 10^{-122}.



- Zero-point energy of the quantized fields: $\rho \sim M_{Pl}^4$
- Phase transitions in the early Universe: the electroweak symmetry breaking.

$$|\rho_{EW}| \sim 10^8\ \mathrm{GeV}^4$$

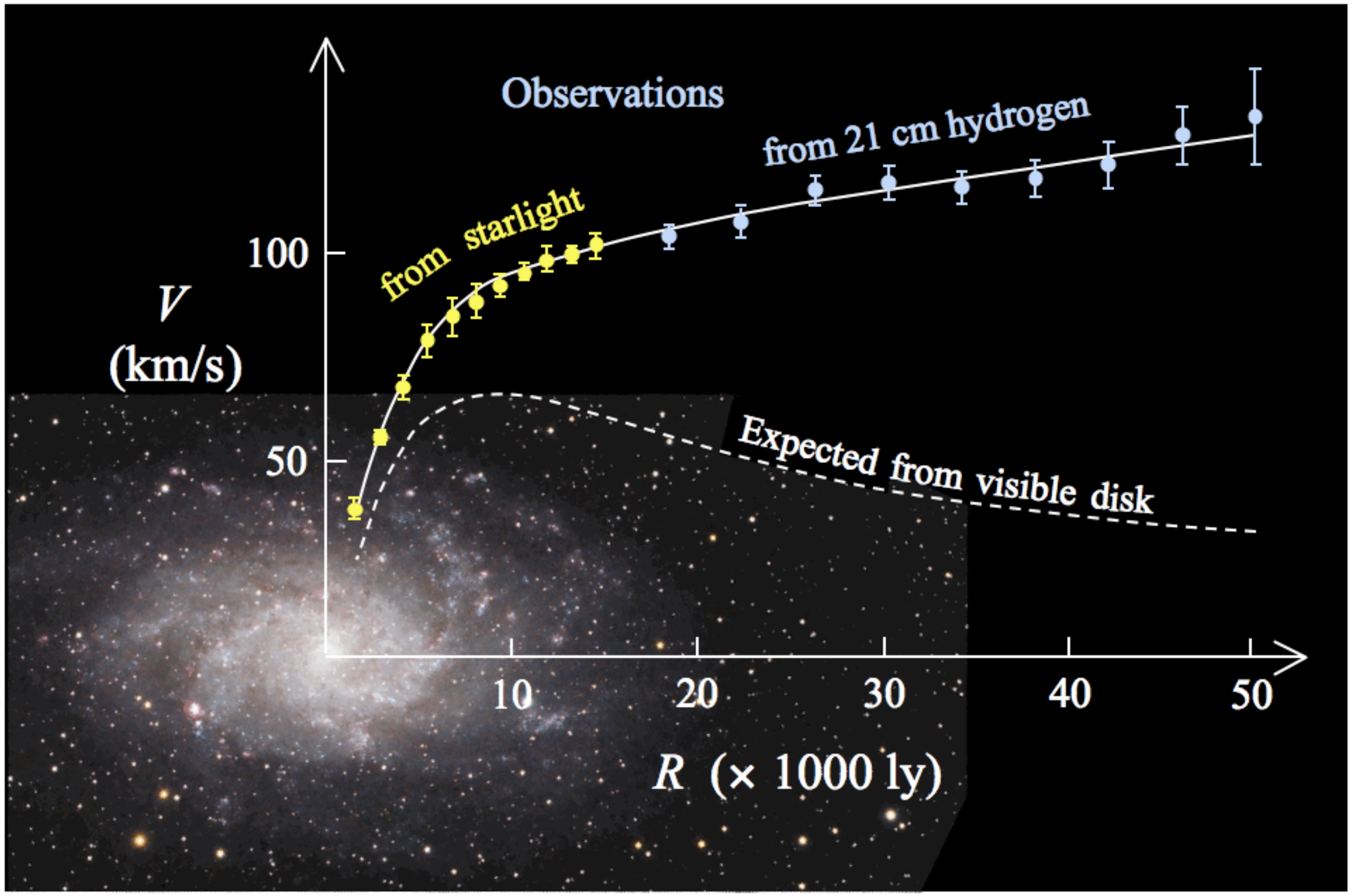
Similarly for QCD phase transition:

$$|\rho_{QCD}| \sim 10^{-2}\ \mathrm{GeV}^4$$

How to cancel these contributions in cosmological constant term?

Motivation: Dark matter

- Galaxy rotation curves



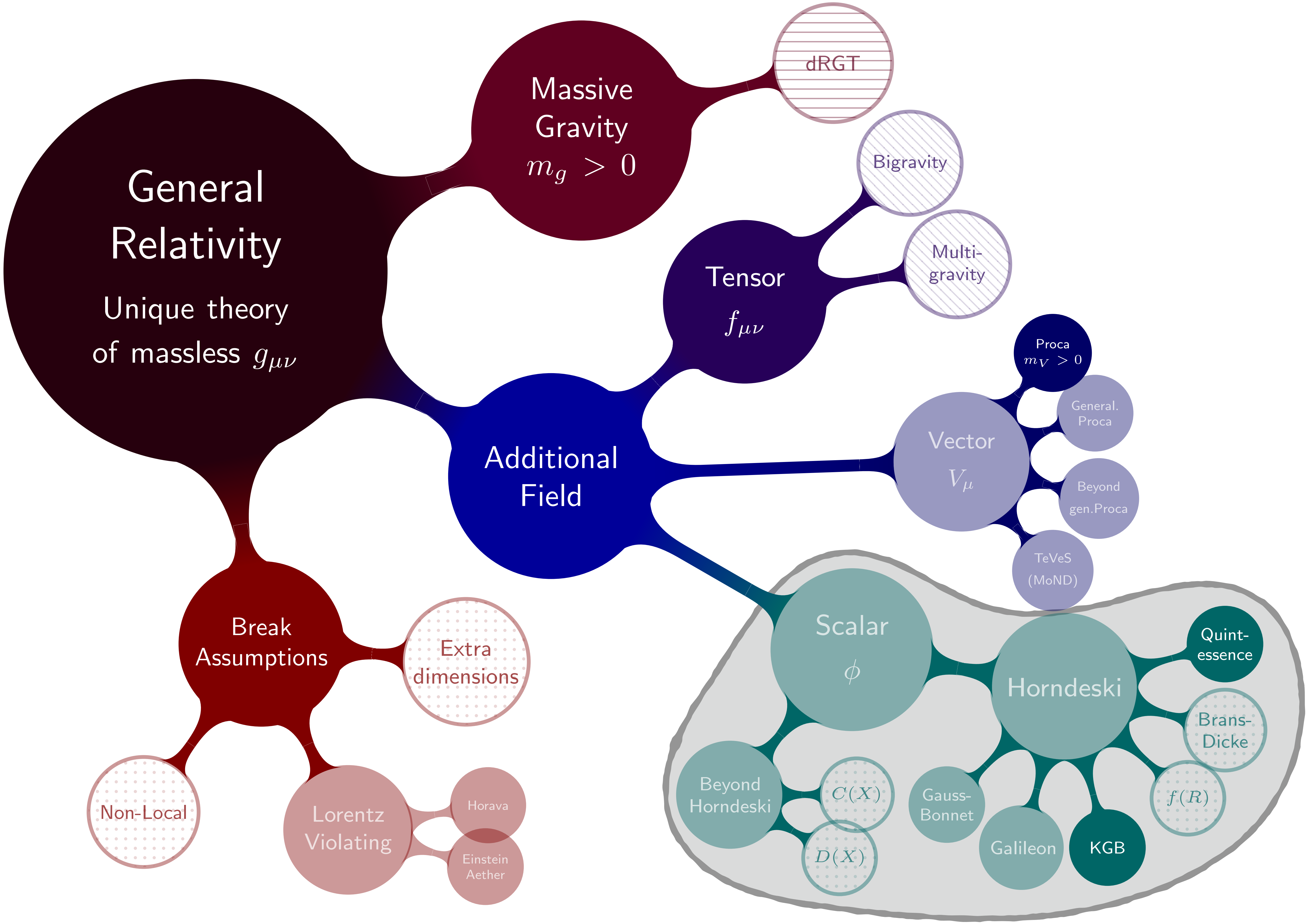
Extra matter content: scalar particles, clouds of scalar field or MOND?

Theoretical motivation for gravity modifications

- Theoretical curiosity
- Establishing benchmarks to compare with GR
- Make gravity renormalisable

Historical detour

- At the advent of general relativity: observational evidence pointed towards shortcomings of Newton's gravitational theory.
- In particular, the advance of the perihelion of Mercury, which deviated from Kepler's well-established laws describing planetary motion.
- The existence of a planet in an even closer orbit to the sun, Vulcan, was hypothesized to explain the advance of Mercury in terms of newton's gravity.
- The presence of an unknown substance, ether, was put forward, mediating and slightly modifying the prediction of Kepler's laws to account for observational data.
- In fact, it was only after General Relativity theory was put forward, this slight difference was accounted for as, rather, a fundamental modification of gravity theory from Newton's theory to General Relativity.



[from Ezquiaga,Zumalacárregui'18]

Modification of gravity

Why scalar tensor models?

- ✦ Simple (the simplest?)
- ✦ Many theories related to scalar-tensor theories in specific regimes:
  - Kaluza-Klein reduction of higher-dimensional theories (i.e. DGP)
  - Massive (bi) gravity
  - Vector-tensor theories
  - f(R)

$$R + \frac{R^2}{6M^2} \quad \rightarrow \quad R - \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Starobinsky inflation

Modification of gravity

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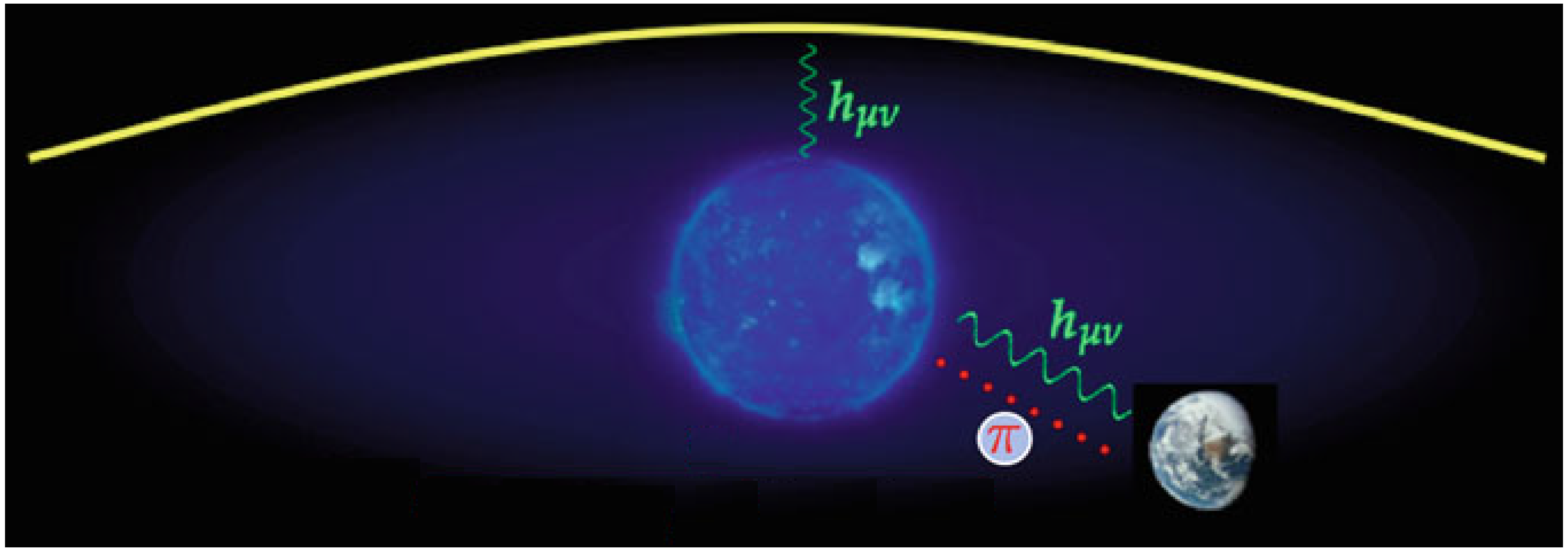


From canonical scalar to Horndeski and beyond

Scalar theories

Canonical scalar field

$$S = \int d^4x \, \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 \right)$$

Canonical scalar field with non-linear potential

$$S = \int d^4x \, \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

Majority of inflationary models and quintessence.

What about non-linear kinetic term? Makes sense because

- Hydrodynamics
- Gauss-Bonnet gravity
- Euler–Heisenberg Lagrangian
- Fluctuations of a brane in an extra dimension
- …

Nonlinear scalar theories

$$X = \partial_\mu \phi \partial^\mu \phi$$

definition of X

standard massless scalar

$$S = \int d^4x \; X$$

$$\partial_\mu \varphi \partial^\mu \varphi \to \partial_\mu \varphi \partial^\mu \varphi + (\partial_\mu \varphi \partial^\mu \varphi)^2 + \ldots$$

$$X \to X + X^2 + \ldots$$

Nonlinear kinetic term

$$S = \int d^4x \; K(X)$$

K-essence, k-inflation in cosmology

[Armendariz-Picon et al'99; Chiba et al'00]

$$\mathcal{G}^{\mu\nu}(\partial\varphi)\, \nabla_\mu \nabla_\nu \varphi = 0$$

Quasi-linear second order PDE

Even more non-linear scalar?

Monge-Ampère equation

Monge'1784, Ampère'1820

$$A(u_{xx}u_{yy} - u_{xy}^2) + Bu_{xx} + Cu_{xy} + Du_{yy} + E = 0$$

- to find a surface with a prescribed Gaussian curvature
- optimizing transportation costs

$$u_{xx}u_{yy} - u_{xy}^2 = 0$$ first galileon in history

Not in the class of quasi-linear equations

Generalizing scalar field theory

Guiding principles?

- **locality**
- **Lorentz invariance**
- **no pathological propagating modes** **(hard to see *a priori*)**

↓

OK, but at least
no Ostrogradsky ghost
(an extra d.o.f. generically appearing
in a higher order EoMs)

Ostrogradsky ghost

Ostrogradsky'1850

$$S = \int L(q, \dot{q}, \ddot{q}) dt \quad \rightarrow \quad \frac{dL}{dq} - \frac{d}{dt}\frac{dL}{d\dot{q}} + \frac{d^2}{dt^2}\frac{dL}{d\ddot{q}} = 0$$

Extra degree of freedom

Canonical variables: $q_2 = \dot{q} \quad \rightarrow \quad \pi = \frac{dL}{d\dot{q}} - \frac{d}{dt}\frac{dL}{d\ddot{q}} \qquad \pi_2 = \frac{dL}{d\ddot{q}}$

Hamiltonian: $H = \pi q_2 - \pi_2 \ddot{q}(q, q_2, \pi_2) - L$

- Hamiltonian is unbounded from below.
- New propagating degree of freedom appear: ghost.
- An assumption of the theorem is non-degeneracy

Similar in field theory

$$S = \int d^4x \left[\frac{1}{2}\left(\Box\phi\right)^2\right] \quad \Rightarrow \quad S = \int d^4x \left[\frac{1}{2}(\nabla_\mu \eta \nabla^\mu \eta) - \frac{1}{2}(\nabla_\mu \xi \nabla^\mu \xi) + \frac{1}{4}(\eta - \xi)^2\right]$$

One d.o.f. is a ghost

Non-quasi-linear theory?

$$\phi_{tt}\phi_{xx} - \phi_{tx}^2 = 0$$

Monge'1784, Ampère'1820

Can it be obtained from the variational principle?

$$\mathcal{L} = \partial\phi\,\partial\,\phi\,\partial^2\phi$$

$$\cdots$$

$$\mathcal{L} = \partial_\mu\phi\,\partial^\mu\phi\,\Box\phi$$

$$S = \int d^4x\,\mathcal{L} \quad \rightarrow \quad (\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) = 0$$

Miraculously the 3d order derivative terms cancel out when varying the action

Horndeski theory

Horndeski'1974

Construct the theory:

- 4D
- only one metric and one scalar field
- local
- diffeomorphism invariance
- EOMs are of the second order

$$S=\int d^4x\,F\left[g,\partial g,\partial^2 g,\partial^3 g,...\varphi,\partial\varphi,\partial^2\varphi,\partial^3\varphi,...\right] \;\Rightarrow\; E[g,\partial g,\partial^2 g,\varphi,\partial\varphi,\partial^2\varphi]=0$$

$$G_2(X,\phi),\;G_3(X,\phi),\;G_4(X,\phi),\;G_5(X,\phi)$$

$$\mathcal{L}_2=G_2\left(X,\phi\right)$$

$$\mathcal{L}_3=G_3\left(X,\phi\right)\Box\phi$$

$$\mathcal{L}_4=G_4(X,\phi)\,R+G_{4,X}(X,\phi)\left[\left(\Box\phi\right)^2-\left(\nabla\nabla\phi\right)^2\right]$$

$$\mathcal{L}_5=G_{5,X}\left(X,\phi\right)\left[\left(\Box\phi\right)^3-3\Box\phi\left(\nabla\nabla\phi\right)^2+2\left(\nabla\nabla\phi\right)^3\right]-6G_5\left(X,\phi\right)\,G_{\mu\nu}\nabla^\mu\nabla^\nu\phi$$

Historical detour (2)

Monge'1784, Ampère'1820

Monge-Ampère equation

$$A(u_{xx}u_{yy} - u_{xy}^2) + Bu_{xx} + Cu_{xy} + Du_{yy} + E = 0$$

Fairlie,Govaerts'1991; Fairlie,Govaerts,Morozov'1991

The procedure:

$$\mathcal{L}_n = F_n(\partial\varphi)W_{n-1}, \quad W_0 = 1$$

$$W_n = \mathcal{E}\mathcal{L}_n$$

$$\mathcal{L}_1 = (\partial\varphi)^2 \to W_1 = \Box\varphi \to$$

$$\mathcal{L}_2 = (\partial\varphi)^2\Box\varphi \to \mathcal{E}\mathcal{L}_2 = (\Box\varphi)^2 - (\nabla\nabla\varphi)^2$$

Dvali et al'00; Luty et al'03

DGP: brane model of gravity

$$\mathcal{L}_{DGP} = -\frac{M_P^2}{4}h^{\mu\nu}\left(\mathcal{E}h\right)_{\mu\nu} - 3(\partial\pi)^2 - \frac{r_c^2}{M_P}(\partial\pi)^2\Box\pi + \frac{1}{2}h^{\mu\nu}T_{\mu\nu} + \frac{1}{M_P}\pi T$$

Nicolis et al'09

Galileons as generalization of DGP scalar

Going beyond Horndeski?

- No more than 2 derivatives in EOMs to avoid the Ostrogradsky ghost
- When the equations of motion are of higher oder, in general it means a new degree of freedom which is a ghost
- Break assumption of the Ostrogradsky theorem => a possibility to have higher order EOMs

Extension of Horndeski: + 2 extra functions

EOMS contain three derivatives

Degenerate Higher-Order Scalar-Tensor (DHOST) theories

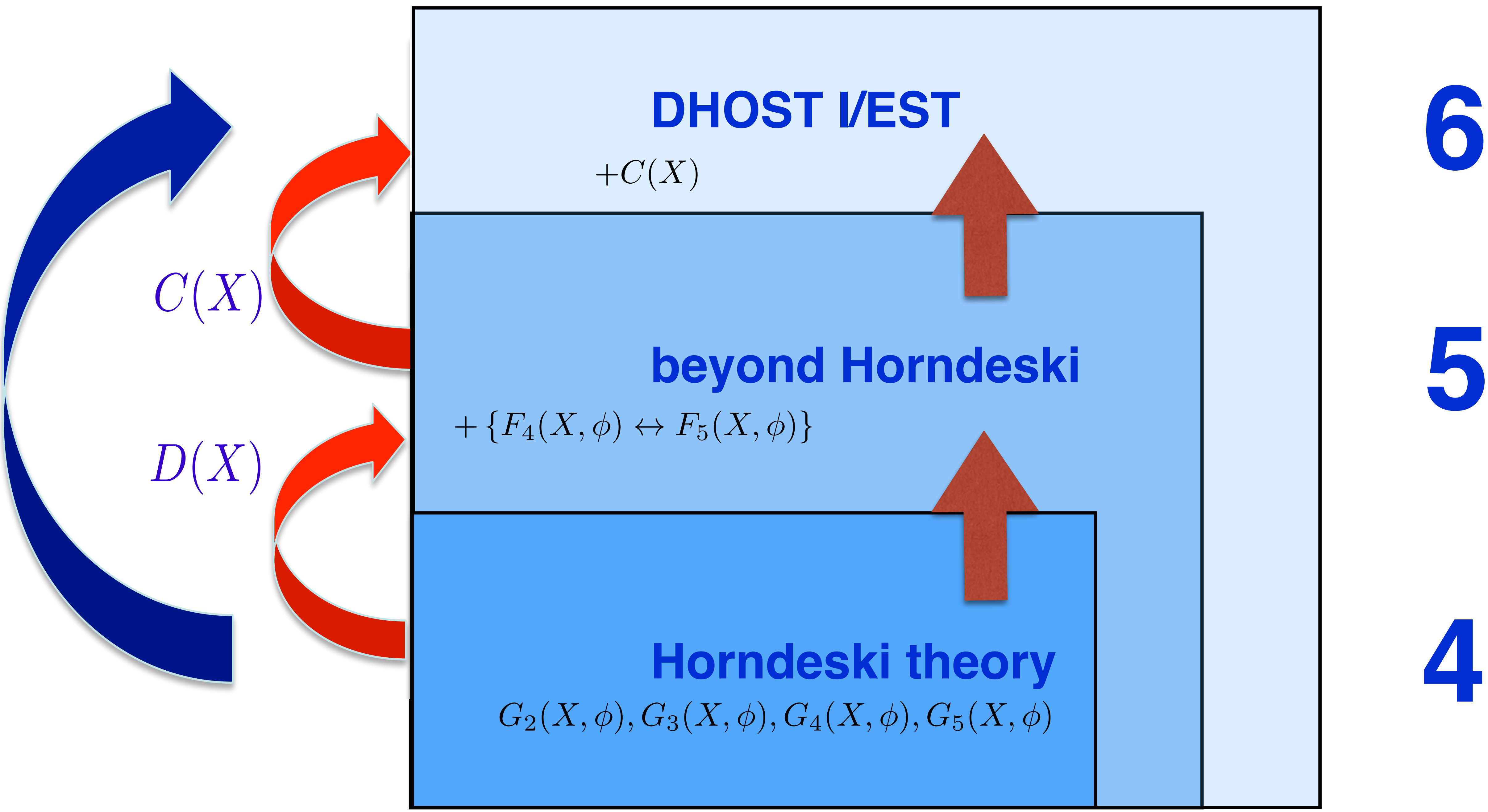
or

Extended scalar-tensor (EST) theories

Zumalacárregui&García-Bellido'14
Gleyzes et al'15
Deffayet et al'15
Langlois and Noui'15
Crisostomi et al'16
Motohashi et al'16

Horndeski, beyond and DHOST

$$g_{\mu\nu} \quad \longrightarrow \quad \tilde{g}_{\mu\nu} = C(X,\phi)g_{\mu\nu} + D(X,\phi)\partial_\mu\phi\partial_\nu\phi$$

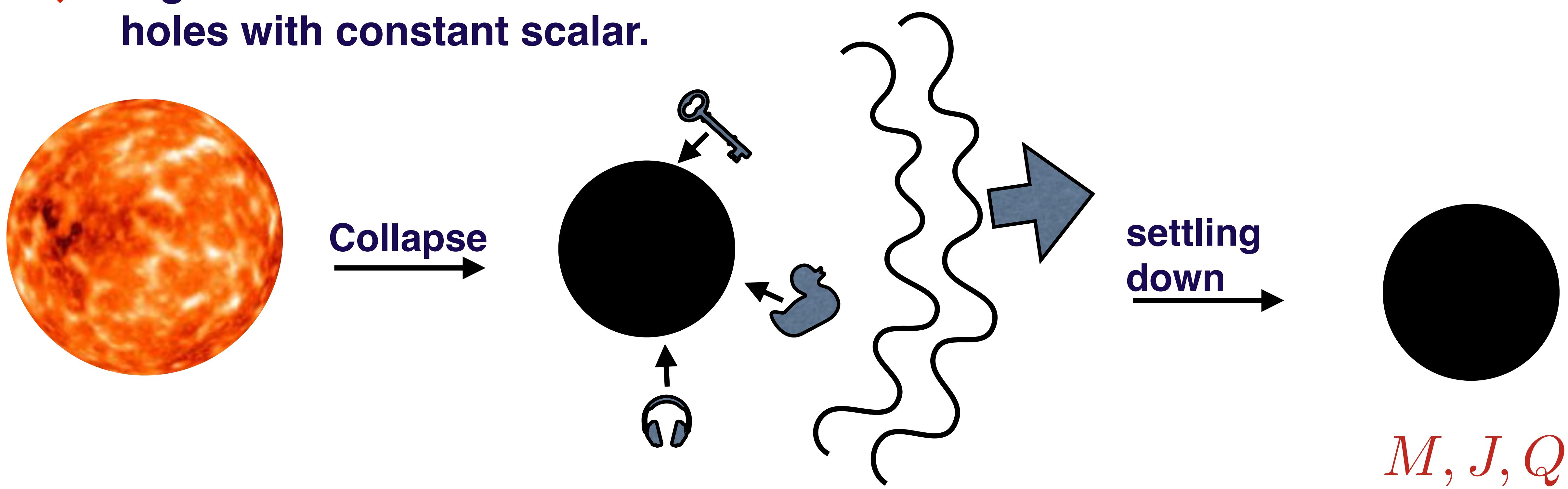


Black holes

Black holes are bald (?)

- Gravitational collapse...
- Black holes eat or expel surrounding matter
- Their stationary phase is characterised by a limited number of charges
- No details about collapse
- Black holes are bald

- **No hair theorems/arguments dictate that adding degrees of freedom lead to trivial (General Relativity) or singular solutions.**
- **E.g. in the standard scalar-tensor theories BH solutions are GR black holes with constant scalar.**



Example of hairy black hole

BBMB solution

Bocharova et al'70, Bekenstein'74

Conformally coupled scalar field:

$$S[g_{\mu\nu},\phi] = \int \sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{12}R\phi^2\right)d^4x$$

Static spherically symmetric (nontrivial) solution:

$$ds^2 = -\left(1-\frac{m}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1-\frac{m}{r}\right)^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Secondary scalar hair:

$$\phi = \sqrt{\frac{3}{4\pi G}}\frac{m}{r-m}$$

NB. The geometry is of that of extremal RN.
The scalar field is unbounded at r=m

Shift-symmetric Horndeski

$$\mathcal{L}_2 = G_2\,(X, \cancel{\phi})$$
$$\mathcal{L}_3 = G_3\,(X, \cancel{\phi})\,\Box\phi$$
$$\mathcal{L}_4 = G_4(X, \cancel{\phi})\,R + G_{4,X}(X, \cancel{\phi})\left[(\Box\phi)^2 - (\nabla\nabla\phi)^2\right]$$
$$\mathcal{L}_5 = G_{5,X}\,(X, \cancel{\phi})\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla\nabla\phi)^2 + 2\,(\nabla\nabla\phi)^3\right] - 6G_5\,(X, \cancel{\phi})\,G_{\mu\nu}\nabla^\mu\nabla^\nu\phi$$

Arbitrary $G_2(X),\ G_2(X),\ G_4(X),\ G_5(X)$

Conserved current because of shift-symmetry: $J^\mu = \dfrac{\delta S}{\delta(\partial_\mu\phi)}$

No hair for galileon

Hui,Nicolis'12

Shift-symmetric galileon, with arbitrary $G_2(X),\ G_2(X),\ G_4(X),\ G_5(X)$

Assume that:

(i) spacetime and scalar field is static spherically symmetric,

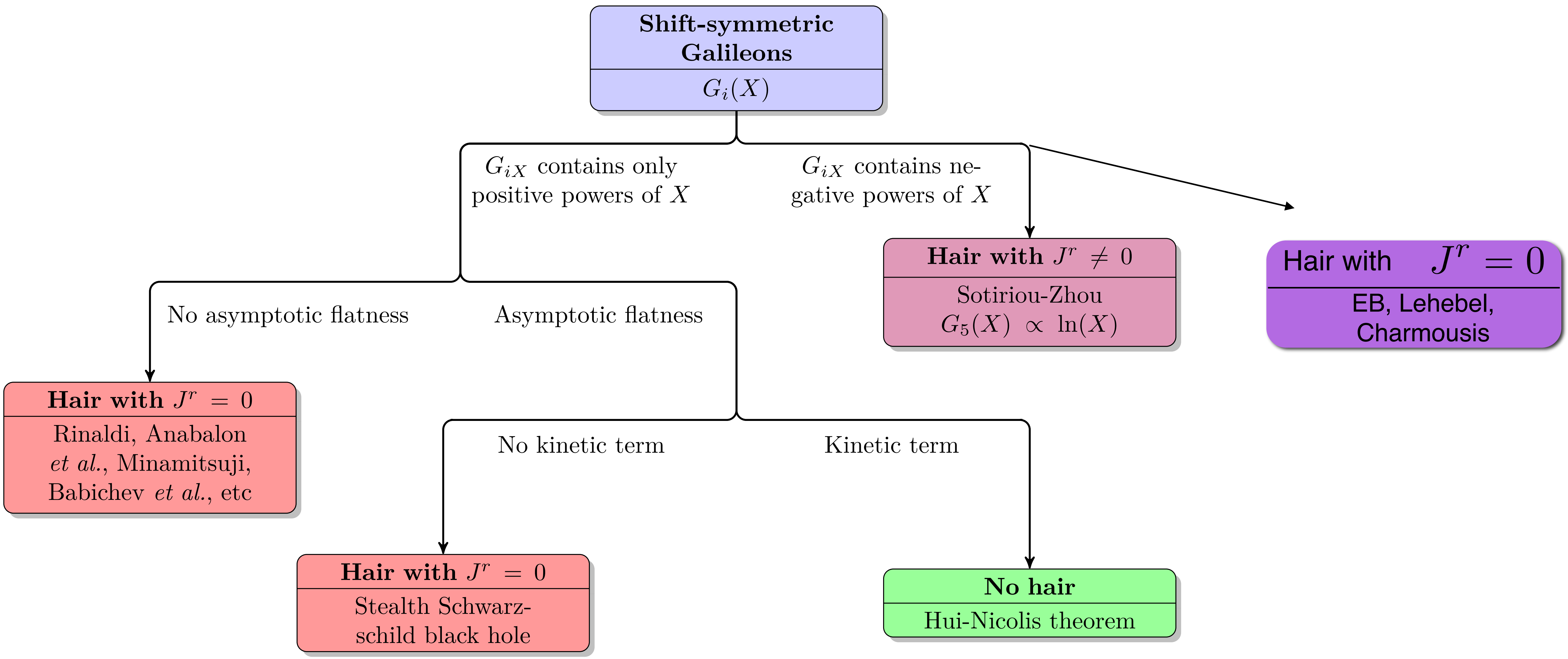
$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \qquad \phi = \phi(r)$$

(ii) spacetime is asymptotically flat, and $\phi' \to 0 \text{ as } r \to \infty$
and the norm of the current J^2 is finite (at the horizon)

(iii) there is a canonical kinetic term in the action and G_i are such that their derivatives $dG(X)_i/dX$ contain only positive or zero powers of X

A no-hair theorem then follows: the metric is Schwarzschild and the scalar field is constant

Avoiding no-hair theorem



Constructing hairs

EB, Charmousis'13

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{A(r)} + \rho(r)^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2).$$

$$\phi = qt + \psi(r)$$

Time-dependent scalar !

The only consistent solution for this ansatz is when $J^r = 0$

$$-qJ^r = \mathcal{E}_{tr} f$$

The norm of the current:

$$J^\mu J_\mu = -A(J^t)^2 + (J^r)^2/A,$$

The physical requirement of no-hair theorem is automatically satisfied by virtue of EOMs.

Explicit example

EB, Charmousis'13

$$\mathcal{L}^{\Lambda\mathrm{CGJ}} = R - \eta(\partial\phi)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda.$$

follows from general galileon with $G_4 = 1 + \beta X$ and $G_2 = -2\Lambda + 2\eta X.$

The general solution is given by the solution of the algebraic equation:

$$(q\beta)^2\left(\kappa + \frac{r^2}{2\beta}\right)^2 - \left(2\kappa + (1-2\beta\Lambda)\frac{r^2}{2\beta}\right)k(r) + C_0 k^{3/2}(r) = 0,$$

$$h(r) = -\frac{\mu}{r} + \frac{1}{\beta r}\int \frac{k(r)}{\kappa + \frac{r^2}{2\beta}}dr, \qquad f = \frac{(\kappa + \frac{r^2}{2\beta})^2\beta h}{k(r)},$$

$$\psi' = \pm\frac{\sqrt{r}}{h(\kappa + \frac{r^2}{2\beta})}\left(q^2(\kappa + \frac{r^2}{2\beta})h' - \frac{1+2\beta\Lambda}{4\beta^2}(h^2r^2)'\right)^{1/2}.$$

Explicit solutions

Asymptotically flat (no standard kinetic term)

$$f = h = 1 - \frac{\mu}{r} \qquad \psi' = \pm q\sqrt{\mu r}/(r-\mu).$$

Asymptotically static universe:

$$h = 1 - \frac{\mu}{r}, \quad f = \left(1 - \frac{\mu}{r}\right)\left(1 + \frac{\eta r^2}{\beta}\right) \qquad \psi' = \pm\frac{q}{h}\sqrt{\frac{\mu}{r(1+\frac{\eta}{\beta}r^2)}}$$

Asymptotically dS/AdS:

$$f = h = 1 - \frac{\mu}{r} - \frac{\Lambda_{\text{eff}}}{3}r^2, \qquad \psi' = \pm\frac{q}{h}\sqrt{1-h}, \qquad \Lambda_{\text{eff}} = -\frac{1}{2\beta}$$

For above solutions $X = \text{const}$

Extension to other theories

The solutions are almost identical for the theory: Kobayashi, Tanahashi'14

$$\mathcal{L} = G_2(X) + G_4(X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$

Beyond Horndeski: EB,Charmousis,Langlois,Saito'15

$$\mathcal{L}^{\rm bH} = R + F_{\rm J}(X)G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi,$$

Cubic Galileon: EB,Charmousis,Lehebel,Moskalets'16

$$\mathcal{L} = \zeta\,(R - 2\,\Lambda) - \eta\,(\partial\phi)^2 + \gamma\,\Box\phi\,(\partial\phi)^2$$

Gauss-Bonnet term

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \lambda\phi\hat{G}\right]$$

Gauss-Bonnet invariant: $\hat{G} = R_{\mu\nu\sigma\alpha}R^{\mu\nu\sigma\alpha} - 4R_{\mu\nu}R^{\mu\nu} + R^2$

Horndeski theory with $G_5 \propto \ln|X| \Rightarrow$ assumption (iii) is violated.

EoM for the scalar: $\Box\phi = -\lambda\hat{G}$

Source for the scalar: it cannot be trivial in BH background

Campbell et al'92
Kanti et al'96
Sotiriou and Zhou'13

J^2 diverges at the horizon => violation of the condition (ii) as well — EB, Charmousis, Lehebel'16

Stealth Kerr solution

- A stealth Kerr solution [Charmousis+'19], where the metric is Kerr and the scalar field such that

$$g = g_{\text{Kerr}}$$

$$X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = X_0 = \text{const.}$$

$$\phi = q\left[t + \int \frac{\sqrt{2Mr(a^2+r^2)}}{\Delta}\mathrm{d}r\right]$$

- The scalar ϕ is the Hamilton-Jacobi potential of the Kerr space-time
- The metric g_{Kerr} is regular everywhere apart from the ring singularity and
- The scalar field is regular at $r > 0$.

Disforming the Kerr metric

- Starting from the Kerr solution, we perform the transformation:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu}^{\text{(Kerr)}} - \frac{D}{q^2}\,\partial_\mu\phi\,\partial_\nu\phi$$

$$\phi = q\left[t + \int \frac{\sqrt{2Mr(a^2+r^2)}}{\Delta}\mathrm{d}r\right]$$

where D and q are constants.

- The line element is now

$$\mathrm{d}\tilde{s}^2 = -\left(1 - \frac{2\tilde{M}r}{\rho^2}\right)\mathrm{d}t^2 - 2D\frac{\sqrt{2\tilde{M}r(a^2+r^2)}}{\Delta}\mathrm{d}t\mathrm{d}r + \frac{\rho^2\Delta - 2\tilde{M}(1+D)rD(a^2+r^2)}{\Delta^2}\mathrm{d}r^2$$
$$- \frac{4\sqrt{1+D}\tilde{M}ar\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\varphi + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta\right]\mathrm{d}\varphi^2 + \rho^2\mathrm{d}\theta^2$$

with $\tilde{M} = M/(1+D)$ and the rescaling $t \to \sqrt{1+D}t$

- The scalar again defines a geodesic direction, since

$$\tilde{X} = \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \frac{X}{1+D}$$

Properties of disformed Kerr: non-circularity and asymptotics

- If $a = 0$, there exists a diffeomorphism $\mathrm{d}t \to \mathrm{d}T + f(r)\mathrm{d}r$ that brings the metric to the Schwarzschild solution with rescaled mass [EB & Esposito-Farese'17]

- In general case, there are still two Killing vectors $\xi_{(t)} = \partial_t$ and $\xi_{(\varphi)} = \partial_\varphi$, however

$$\xi_{(t)} \wedge \xi_{(\varphi)} \wedge \mathrm{d}\xi_{(t)} = -D\frac{4a^2\tilde{M}r\sqrt{2\tilde{M}r(a^2+r^2)}\cos\theta\sin^3\theta}{\rho^4}\mathrm{d}t \wedge \mathrm{d}r \wedge \mathrm{d}\theta \wedge \mathrm{d}\varphi \neq 0$$

  This means we cannot write the metric in a form that is invariant under $(t,\varphi) \to (-t,-\varphi)$

- Asymptotically Kerr with small corrections:

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}s^2_{\mathsf{Kerr}} + \frac{D}{1+D}\left[\mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}}{r^3}\right)\mathrm{d}T^2 + \mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}^{3/2}}{r^{7/2}}\right)\alpha_i\mathrm{d}T\mathrm{d}x^i + \mathcal{O}\left(\frac{\tilde{a}^2}{r^2}\right)\beta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]$$

- The (assymptotically) observable mass and angular momentum are $\tilde{M} = M/(1+D)$ and $\tilde{a} = a\sqrt{1+D}$ instead of M and a.

Properties of disformed Kerr: Important surfaces

- *Ergosphere (static limit)*: static timelike observers can no longer exist, the Killing vector $l^\mu = (1,0,0,0)$ becomes null. I.e. $\tilde{g}_{tt} = 0$, or

$$\tilde{g}_{tt} = 0 \quad \Rightarrow \quad r_E = \tilde{M} + \sqrt{\tilde{M}^2 - a^2\cos^2\theta}$$

- *Stationary limit.* Observers constant (r,θ), with a 4-velocity $u = \partial_t + \omega\partial_\varphi$. These observers cease to exist at the surface $\tilde{g}_{tt}\tilde{g}_{\varphi\varphi} - \tilde{g}_{t\varphi}^2 = 0$, i.e.

$$P(r,\theta) \equiv r^2 + a^2 - 2\tilde{M}r + \frac{2\tilde{M}Da^2 r\sin^2\theta}{\rho^2(r,\theta)} = 0$$

The surface is *timelike* and thus cannot be a horizon.

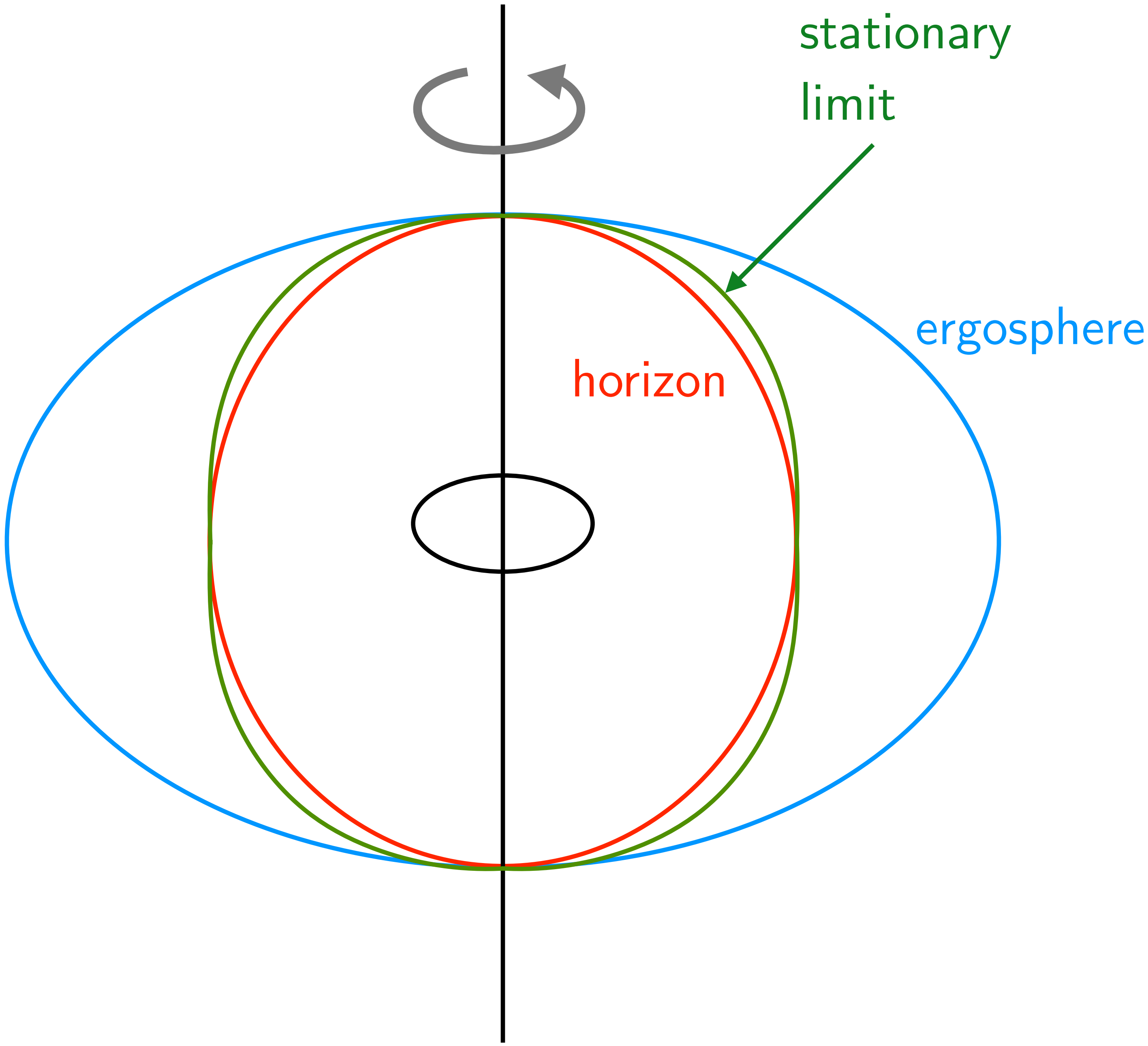
- *Horizon*: a null hypersurface of the form $r = R(\theta)$. The normal has components

$$n_\mu = (0, 1, -R'(\theta), 0)$$

The condition $n^2 = 0$ yields

$$R'(\theta)^2 + P(R,\theta) = R'(\theta)^2 + R^2 + a^2 - 2\tilde{M}R + \frac{2\tilde{M}Da^2 R\sin^2\theta}{\rho^2(R,\theta)} = 0$$

Surfaces



Conclusions

- No-go theorem black holes in Horndeski theory
- However one requires many assumptions
- A number of ways to construct hairy black holes
- Also rotating hairy black holes
- Interesting aspects to study: effects of non-circularity, consequences of strange properties of the horizon, thermodynamics etc.
- Signatures of modified gravity and constraints.