Stability of Nonsingular Cosmologies in Galileons with Torsion

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Based on:

- S. Mironov and M. Valencia-Villegas, Phys. Rev. D, vol. 108, no. 2, p. 024057, 2023
- S. Mironov and M. Valencia-Villegas, 2307.06929

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1. Introduction:

Galileons on a spacetime

without Torsion

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- **Motivation**
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1. Introduction: Galileons on a spacetime without Torsion

Horndeski theory:

the most general modification of Einstein's gravity (GR) with a real scalar field, with higher derivatives in the action, but with second order equations of motion [1 - 8]. Rediscovered as **Galileons** [2].

1. Introduction: Galileons on a spacetime without Torsion

On top of GR, $\int \mathrm{d}^4x\,\sqrt{-g}\,R$

consider four general functions $G_2(\phi, X)\,,\;\; G_3(\phi, X)\,,\;\; G_4(\phi, X)\,,\;\; G_5(\phi, X)$

with $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, notation: $G_{4,X} = \partial G_4/\partial X$, (-,+,+,+).

G. Galileons:

$$\mathcal{S} = \int \mathrm{d}^4x\,\sqrt{-g}\,\Big(G_2 - G_3\,\nabla_\mu\nabla^\mu\phi + G_4(\phi, X)\,R + G_{4,X}\left((\nabla_\mu\nabla^\mu\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right)$$

$$+G_5\;\text{ part of the action }\Big)$$

1. Introduction: Galileons on a spacetime without Torsion

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G. Galileons:

$$\mathcal{S} = \int \mathrm{d}^4x \sqrt{-g}\Big(G_2 - G_3\,\nabla_\mu\nabla^\mu\phi + G_4(\phi, X)\,R + G_{4,X}\left((\nabla_\mu\nabla^\mu\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right)$$

$$+G_5\,G^{\mu\nu}\,\nabla_\mu\nabla_\nu\phi - \frac{G_{5X}}{6}\left((\nabla_\mu\nabla^\mu\phi)^3 - 3(\nabla_\mu\nabla^\mu\phi)\,(\nabla_\nu\nabla_\rho\phi)\,\nabla^\nu\nabla^\rho\phi\right.$$

$$\left.+2\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\nu\nabla^\rho\phi)\,\nabla^\mu\nabla_\rho\phi)\right)\Big)$$

1. Introduction: Galileons on a spacetime without Torsion

Motivation for Horndeski theory/ Galileons:

1. Can violate the Null Energy Condition (NEC) in a possibly stable way [9]
   a) What for?

      To avoid the singularity theorems of Penrose and Hawking

      (->Bounce)

2. („Non covariant“) Galileons (Galilean invariance) is an IR modification of gravity inspired by the low energy effective theory of DGP [2].

1. Introduction: Galileons on a spacetime without Torsion

Key aspects of Galileons:

1. There is no Ostrogradsky Ghost

2. Generality: It includes, as special cases, theories ranging from minimally coupled scalars in GR, to k-essence, Brans-Dicke and more general non-minimal couplings.

2. Galileons on a spacetime

with Torsion

2. Galileons on a spacetime

with Torsion

- **Motivation**
- **Resolving ambiguities in the definition of Galileons with torsion**
- **Explicit torsion in the second order formalism. The Action.**
- **Why is this Action interesting?**

2. Galileons on a spacetime with Torsion.

Motivation to introduce Torsion in Galileons:

1. In the torsionless theory, there is already a NO-GO theorem that holds for generic models (there are some special cases):

even if away from the physically relevant phase, there are gradient instabilities in nonsingular models at some time in the evolution [10-17].

-> can a more general spacetime with torsion cure this issue?

Answer so far: partially (there are other issues).

2. Galileons on a spacetime with Torsion.

Motivation to introduce Torsion in Galileons:

General motivations:

2. Torsion has also been studied in relation to nonsingular cosmologies (before Horndeski) [18].
3. Torsion is on the way to introduce spinors [18].
4. Torsion is suggested by demanding local Poincaré invariance [18].

2. Galileons on a spacetime with Torsion:

- **Resolving ambiguities in the definition of Galileons with torsion**

Recall **Torsionless** G4: $\mathcal{S}_4 = \int \mathrm{d}^4x\,\sqrt{-g}\left(G_4(\phi,X)\,R + G_{4,X}\left((\nabla_\mu\nabla^\mu\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right)\right)$

Here, the metric compatible derivative $\nabla_\rho\, g_{\mu\nu} = 0$ on a vector V, is

$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda\,, \qquad \Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right)$$

such that $\Gamma^\rho_{\mu\nu} = \Gamma^\rho_{\nu\mu}$ and $[\nabla_\mu, \nabla_\nu]\,\phi = 0$.

-> There is no ambiguity in $G_{4,X}\;(\nabla_\mu\nabla_\nu\phi)^2$

2. Galileons on a spacetime with Torsion.

- **Resolving ambiguities in the definition of Galileons with torsion**

To go to **Torsionful** G4 take $\nabla \to \tilde{\nabla}$, but then, what is $(\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi)^2$?

Here, the metric compatible derivative $\nabla_\rho \, g_{\mu\nu} = 0$ on a vector is

$$\tilde{\nabla}_\mu V^\nu = \partial_\mu V^\nu + \tilde{\Gamma}^\nu_{\mu\lambda} V^\lambda \,, \qquad \tilde{\Gamma}^\nu_{\mu\lambda} \neq \tilde{\Gamma}^\nu_{\lambda\mu}$$

so, $\left[\tilde{\nabla}_\mu, \tilde{\nabla}_\nu\right] \phi \neq 0$ -> there are two possible contractions with the metric for $G_{4,X}$ $(\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi)^2$, namely

$$G_{4,X} \left(\left(\tilde{\nabla}_\mu \tilde{\nabla}^\mu \phi \right)^2 + c \left(\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi \right) \tilde{\nabla}^\mu \tilde{\nabla}^\nu \phi + s \left(\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi \right) \tilde{\nabla}^\nu \tilde{\nabla}^\mu \phi \right) , \quad c + s = -1$$

2. Galileons on a spacetime with Torsion.

- **Quartic Galileons with Torsion**

The action takes the form

$$\mathcal{S}_{4c} = \int \mathrm{d}^4x\,\sqrt{-g}\left(G_4(\phi,X)\tilde{R} + G_{4,X}\left(\left(\tilde{\nabla}_\mu\tilde{\nabla}^\mu\phi\right)^2 - \left(\tilde{\nabla}_\mu\tilde{\nabla}_\nu\phi\right)\tilde{\nabla}^\nu\tilde{\nabla}^\mu\phi - c\left(\tilde{\nabla}_\mu\tilde{\nabla}_\nu\phi\right)\left[\tilde{\nabla}^\mu,\tilde{\nabla}^\nu\right]\phi\right)\right),$$

c parameterises a family of theories with different dynamics.

2. Galileons on a spacetime <u>with</u> Torsion.

- **Explicit Torsion**

We introduce torsion in the metric (second order) formalism:

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}_{\mu\nu} - \tilde{\Gamma}^{\rho}_{\nu\mu}\,, \qquad K^{\rho}{}_{\mu\nu} = -\frac{1}{2}\left(T_{\nu}{}^{\rho}{}_{\mu} + T_{\mu}{}^{\rho}{}_{\nu} + T^{\rho}{}_{\mu\nu}\right)\,,$$

$$T^{\rho}{}_{\mu\nu} = -T^{\rho}{}_{\nu\mu}$$

$$K_{\mu\nu\sigma} = -K_{\sigma\nu\mu}$$

2. Galileons on a spacetime with Torsion.

- **Explicit Torsion**
- Assume: connection is not an independent field:

$$\tilde{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} - K^{\rho}{}_{\mu\nu}$$

With the torsionful derivative

$$\tilde{\nabla}_{\mu} V^{\nu} = \nabla_{\mu} V^{\nu} - K^{\nu}{}_{\mu\lambda} V^{\lambda}$$

The theory

$$\mathcal{S}_{4c} = \int \mathrm{d}^4 x \sqrt{-g} \left(G_4(\phi, X) \tilde{R} + G_{4,X} \left(\left(\tilde{\nabla}_{\mu} \tilde{\nabla}^{\mu} \phi \right)^2 - \left(\tilde{\nabla}_{\mu} \tilde{\nabla}_{\nu} \phi \right) \tilde{\nabla}^{\nu} \tilde{\nabla}^{\mu} \phi - c \left(\tilde{\nabla}_{\mu} \tilde{\nabla}_{\nu} \phi \right) \left[\tilde{\nabla}^{\mu}, \tilde{\nabla}^{\nu} \right] \phi \right) \right),$$

can be written as follows:

2. Galileons on a spacetime <u>with</u> Torsion.

- **The Action with explicit Torsion**

We consider a set of 3 independent fields: metric, scalar and contortion

$$\mathcal{S}_{4c} = \int \mathrm{d}^4x\,\sqrt{-g}\,\Big(G_4(\phi,X)\tilde{R} + G_{4,X}\Big(\Big(\tilde{\nabla}_\mu\tilde{\nabla}^\mu\phi\Big)^2 - \Big(\tilde{\nabla}_\mu\tilde{\nabla}_\nu\phi\Big)\tilde{\nabla}^\mu\tilde{\nabla}^\nu\phi\Big) + (1-c)G_{4,X}K_{\nu\mu\sigma}\left(K^{\nu\mu\lambda} - K^{\mu\nu\lambda}\right)\tilde{\nabla}_\lambda\phi\tilde{\nabla}^\sigma\phi\Big)\,.$$

with $\tilde{R} = R + K_{\mu\rho\nu}K^{\mu\nu\rho} + K^\mu{}_\mu{}^\nu K_\nu{}^\rho{}_\rho + 2\nabla_\nu K^\mu{}_\mu{}^\nu$

$$\mathcal{S}_{4c} = \int \mathrm{d}^4x\,\sqrt{-g}\,\Big(G_4(R + K_{\mu\rho\nu}K^{\mu\nu\rho} + K^\mu{}_\mu{}^\nu K_\nu{}^\rho{}_\rho + 2\nabla_\nu K^\mu{}_\mu{}^\nu) + G_{4X}(\nabla_\nu\nabla^\nu\phi + K^{\rho\nu}{}_\nu\nabla_\rho\phi)^2 - G_{4X}(K^\rho{}_{\gamma\mu}\nabla_\rho\phi + \nabla_\gamma\nabla_\mu\phi)(K^{\nu\gamma\mu}\nabla_\nu\phi + \nabla^\gamma\nabla^\mu\phi) + (1-c)G_{4,X}K_{\nu\mu\sigma}\left(K^{\nu\mu\lambda} - K^{\mu\nu\lambda}\right)\nabla_\lambda\phi\nabla^\sigma\phi\Big)\,.$$

2. Galileons on a spacetime with Torsion.

Why is this Action interesting?

There is an apparent kinetic mixing with Torsion

(In contrast to Einstein-Cartan)

Some questions arise:

1. **Are there more Degrees of Freedom?**
2. **Scalars: A new chance to stable solutions?**

(Recall the No-Go in Torsionless Galileons)

2. Galileons on a spacetime with Torsion.

Why is this Action interesting?

There is an apparent kinetic mixing with Torsion…

- Look closely at the terms

$$G_4\, \nabla K \qquad\qquad G_{4,X}\,(\nabla\phi)\,(\nabla\,\nabla\phi)\,K$$

in the action

$$\begin{aligned}\mathcal{S}_{4c} = \int \mathrm{d}^4x\,\sqrt{-g}\,\Big(& G_4(R + K_{\mu\rho\nu}K^{\mu\nu\rho} + K^{\mu}{}_{\mu}{}^{\nu}K_{\nu}{}^{\rho}{}_{\rho} + 2\nabla_\nu K^{\mu}{}_{\mu}{}^{\nu}) + G_{4X}(\nabla_\nu\nabla^\nu\phi + K^{\rho\nu}{}_{\nu}\nabla_\rho\phi)^2 \\ & - G_{4X}(K^{\rho}{}_{\gamma\mu}\nabla_\rho\phi + \nabla_\gamma\nabla_\mu\phi)(K^{\nu\gamma\mu}\nabla_\nu\phi + \nabla^\gamma\nabla^\mu\phi) \\ & + (1-c)G_{4,X}K_{\nu\mu\sigma}\left(K^{\nu\mu\lambda} - K^{\mu\nu\lambda}\right)\nabla_\lambda\phi\nabla^\sigma\phi\Big)\,.\end{aligned}$$

2. Galileons on a spacetime <u>with</u> Torsion.

Why is this Action interesting?

There is an apparent kinetic mixing with Torsion…

- Look closely at the terms

$$G_4 \nabla K \qquad\qquad G_{4,X}\,(\nabla\phi)\,(\nabla\nabla\phi)\,K$$

(Recall $G_4(\phi, X)$, $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$).

Hence the field equations look like

$$\mathcal{E}_\phi(\tilde{\nabla}^2 K,\, \tilde{\nabla}^2\phi,\, \partial^2 g) = 0\,, \quad \mathcal{E}_{g_{\mu\nu}}(\tilde{\nabla}^2\phi,\, \partial^2 g) = 0\,, \quad \mathcal{E}_{K^\mu{}_{\nu\sigma}}(\tilde{\nabla}^2\phi) = 0\,,$$

e.g.

$$\mathcal{E}_\phi = 2\,G_{4X}\,\partial^\lambda\phi\left(\tilde{\nabla}_\lambda\tilde{\nabla}_\mu K^\nu{}_\nu{}^\mu - \tilde{\nabla}_\nu\tilde{\nabla}^\nu K^\mu{}_{\mu\lambda} + \tilde{\nabla}_\nu\tilde{\nabla}_\mu K^{\nu\mu}{}_\lambda\right) + F(K,\, \tilde{\nabla}K;\, \tilde{\nabla}^2\phi,\, \tilde{\nabla}\phi,\, \tilde{R})$$

3. Torsionful Galileons about the FLRW background

3. Torsionful Galileons about the FLRW background

Consider the following questions at linear order about a spatially flat FLRW background

1. **Are there more Degrees of Freedom?**
2. **Scalars: A new chance to stable solutions?**

(Recall the No-Go in Torsionless Galileons)

3. Torsionful Galileons about the FLRW background

- **Linearization: the perturbed metric**

$$\mathrm{d}s^2 = (\eta_{\mu\nu} + \delta g_{\mu\nu})\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

Spatially flat FLRW background in conformal time

$$\eta_{\mu\nu}\mathrm{d}x^\mu\,\mathrm{d}x^\nu = a^2(\eta)\left(-\mathrm{d}\eta^2 + \delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\right)$$

4 scalars, 2 (2-component) vectors and a (2-component) tensor

perturbation (graviton)

$$\begin{aligned}\delta g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = &\, a^2(\eta)\Big(-2\,\alpha\,\mathrm{d}\eta^2 \\ &+ 2\,(\partial_i B + S_i)\,\mathrm{d}\eta\,\mathrm{d}x^i + (-2\,\psi\,\delta_{ij} \\ &+ 2\,\partial_i\partial_j E + \partial_i F_j + \partial_j F_i + 2\,h_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j\Big)\end{aligned}$$

3. Torsionful Galileons about the FLRW background

- **Linearization:**

The perturbed Horndeski scalar: $\phi = \varphi(\eta) + \Pi$

(In the context of linearized expressions we will also denote with ϕ the background)

The perturbed contortion tensor: $K_{\mu\nu\sigma} = {}^0K_{\mu\nu\sigma} + \delta K_{\mu\nu\sigma}$

- For the background contortion: with $K_{\mu\nu\sigma} = -K_{\sigma\nu\mu}$ on an isotropic and homogeneous spacetime

$$ {}^0K_{0jk} = x(\eta)\delta_{jk} \qquad {}^0K_{ijk} = y(\eta)\epsilon_{ijk} $$

3. Torsionful Galileons about the FLRW background

- **Linearization: structure of the background equations**

- We can solve for *H*, $\ddot{\varphi}(\eta)$ and

$$\mathcal{E}_{0\,K_{ijk}} \;=\; -2\,\epsilon_{ijk}\;G_4\,y/a^6 \;=\; 0 \qquad\qquad y(\eta)\;\equiv\;0$$

$$x(\eta) = -\frac{a^3\,\mathcal{G}_\tau\,(8\,H\,X\,G_{4,X} +\; a\,\dot{\phi}\,(G_3 - 2\,G_{4,\phi}))}{8\,G_4^2}$$

3. Torsionful Galileons about the FLRW background

- **Linearization:**

- For the perturbation of contortion: with $K_{\mu\nu\sigma} = -K_{\sigma\nu\mu}$,

24 independent components

8 scalars,

$$\begin{aligned}
\delta K^{\rm scalar}_{i00} &= \partial_i C^{(1)} \\
\delta K^{\rm scalar}_{ij0} &= \partial_i \partial_j C^{(2)} + \delta_{ij} C^{(3)} + \epsilon_{ijk}\partial_k C^{(4)} \\
\delta K^{\rm scalar}_{i0k} &= \epsilon_{ikj}\partial_j C^{(5)} \\
\delta K^{\rm scalar}_{ijk} &= \left(\delta_{ij}\partial_k - \delta_{kj}\partial_i\right) C^{(6)} + \epsilon_{ikl}\partial_l \partial_j C^{(7)} + \left(\epsilon_{ijl}\partial_l \partial_k - \epsilon_{kjl}\partial_l \partial_i\right) C^{(8)}
\end{aligned}$$

3. Torsionful Galileons about the FLRW background

- **Linearization:**

6 (2-component) vectors

$$\delta K^{\text{vector}}_{i00} = V^{(1)}_i$$
$$\delta K^{\text{vector}}_{ij0} = \partial_i V^{(2)}_j + \partial_j V^{(3)}_i$$
$$\delta K^{\text{vector}}_{i0k} = \partial_i V^{(4)}_k - \partial_k V^{(4)}_i$$
$$\delta K^{\text{vector}}_{ijk} = \delta_{ij} V^{(5)}_k - \delta_{kj} V^{(5)}_i + \partial_j \partial_i V^{(6)}_k - \partial_j \partial_k V^{(6)}_i$$

and 2 (2-component) tensors

$$\delta K^{\text{tensor}}_{ij0} = T^{(1)}_{ij}$$
$$\delta K^{\text{tensor}}_{ijk} = \partial_i T^{(2)}_{jk} - \partial_k T^{(2)}_{ji}$$

3. Torsionful Galileons about the FLRW background

Answer to 1st question at linear order about a spatially flat FLRW background

1. **Are there more Degrees of Freedom?**

Answer: No.

The seeming kinetic terms conspire to cancel out.

Symmetry? Accidental symmetry?

2. **Scalar: A new chance to stable solutions?**

3. Torsionful Galileons about the FLRW background

- **Quadratic Action:** $\mathcal{S}_{4c} = \mathcal{S}^{Tensor} + \mathcal{S}_c^{Scalar}$

$$\mathcal{S}^{Tensor} = \frac{1}{2}\int \mathrm{d}\eta\, \mathrm{d}^3x \ \left(v_1\, (\dot{h}_{ij})^2 + v_2\, (\partial_k h_{ij})^2 + v_3\, (T^{(1)}_{ij})^2 + v_4\, (\partial_k T^{(2)}_{ij})^2 + v_5\, h_{ij}\, T^{(1)}_{ij} + v_6\, \dot{h}_{ij}\, T^{(1)}_{ij} + v_7 (h_{ij})^2\right)$$

$$T^{(1)}_{ij} = \frac{2\, a^2\, X\, G_{4,X}}{G_4 + \, 2\, X\, G_{4,X}} \dot{h}_{ij} - 2\, x\, h_{ij} \qquad\qquad T^{(2)}_{ij} \equiv 0$$

$$\mathcal{S}_\tau = \int \, \mathrm{d}\eta\, \mathrm{d}^3x\, a^4 \, \left[\tfrac{1}{2\, a^2}\left(\mathcal{G}_\tau \left(\dot{h}_{ij}\right)^2 - \mathcal{F}_\tau (\partial_k\, h_{ij})^2\right)\right]$$

$$\mathcal{G}_\tau = 2 \frac{G_4^2}{G_4 + \, 2\, X\, G_{4,X}}\,, \qquad c_g^2 = \mathcal{F}_\tau / \mathcal{G}_\tau \leq 1 \qquad X = \frac{\dot{\phi}^2}{2\, a^2}$$

$$\mathcal{F}_\tau = 2\, G_4\,,$$

3. Torsionful Galileons about the FLRW background

- **Quadratic Action:** $\mathcal{S}_{4c} = \mathcal{S}^{Tensor} + \mathcal{S}_c^{Scalar}$

$\mathcal{S}_c^{Scalar} =$

3. Torsionful Galileons about the FLRW background

- **Quadratic Action:** $\mathcal{S}_{4c} = \mathcal{S}^{Tensor} + \mathcal{S}_c^{Scalar}$

$$
\begin{aligned}
\mathcal{S}_c^{Scalar} = \frac{1}{2} \int \mathrm{d}\eta\, \mathrm{d}^3x \Big(c\, \big(f_7\, \partial_i B\, \partial_i C^{(1)} + f_{51}\, (\partial_i B)^2 + f_{52}\, (\partial_i C^{(1)})^2\big) \\
+ \Big(f_1\, \alpha\, \Pi + f_2\, C^{(3)}\, \psi + f_3\, \alpha\, \psi + f_4\, \Pi\, \psi + f_5\, C^{(3)}\, \Pi + f_6\, C^{(3)}\, \alpha + f_8\, \partial_i C^{(3)}\, \partial_i E \\
+ f_9\, \partial_i B\, \partial_i C^{(3)} + f_{10}\, \partial_i C^{(2)}\, \partial_i C^{(3)} + f_{11}\, \partial_i C^{(4)}\, \partial_i C^{(5)} + f_{12}\, \partial_i B\, \partial_i C^{(6)} + f_{13}\, \partial_i C^{(1)}\, \partial_i C^{(6)} \\
+ f_{14}\, \partial_i B\, \partial_i \Pi + f_{15}\, \partial_i E\, \partial_i \Pi + f_{16}\, \partial_i C^{(1)}\, \partial_i \Pi + f_{17}\, \partial_i C^{(2)}\, \partial_i \Pi + f_{18}\, \partial_i C^{(3)}\, \partial_i \Pi + f_{19}\, \partial_i C^{(6)}\, \partial_i \Pi \\
+ f_{20}\, \partial_i B\, \partial_i \alpha + f_{21}\, \partial_i E\, \partial_i \alpha + f_{22}\, \partial_i C^{(2)}\, \partial_i \alpha + f_{23}\, \partial_i C^{(6)}\, \partial_i \alpha + f_{24}\, \partial_i \alpha\, \partial_i \Pi + f_{25}\, \partial_i E\, \partial_i \psi \\
+ f_{26}\, \partial_i B\, \partial_i \psi + f_{27}\, \partial_i C^{(2)}\, \partial_i \psi + f_{28}\, \partial_i \Pi\, \partial_i \psi + f_{29}\, \partial_i \alpha\, \partial_i \psi + f_{30}\, \partial_i \partial_j C^{(7)}\, \partial_i \partial_j C^{(8)} + f_{31}\, \psi\, \dot{\Pi} \\
+ f_{32}\, \alpha\, \dot{\Pi} + f_{33}\, C^{(3)}\, \dot{\Pi} + f_{34}\, \alpha\, \dot{\psi} + f_{35}\, C^{(3)}\, \dot{\psi} + f_{36}\, \dot{\Pi}\, \dot{\psi} + f_{37}\, \partial_i C^{(3)}\, \partial_i \dot{E} + f_{38}\, \partial_i \alpha\, \partial_i \dot{E} \\
+ f_{39}\, \partial_i B\, \partial_i \dot{\Pi} + f_{40}\, \partial_i E\, \partial_i \dot{\Pi} + f_{41}\, \partial_i C^{(2)}\, \partial_i \dot{\Pi} + f_{42}\, \partial_i C^{(6)}\, \partial_i \dot{\Pi} + f_{43}\, \partial_i \dot{E}\, \partial_i \dot{\Pi} + f_{44}\, \partial_i B\, \partial_i \dot{\psi} \\
+ f_{45}\, \partial_i C^{(2)}\, \partial_i \dot{\psi} + f_{46}\, \partial_i \dot{E}\, \partial_i \dot{\psi} + f_{47}\, (C^{(3)})^2 + f_{48}\, \alpha^2 + f_{49}\, \psi^2 + f_{50}\, \Pi^2 + f_{53}\, (\partial_i C^{(4)})^2 \\
+ f_{54}\, (\partial_i C^{(6)})^2 + f_{55}\, (\partial_i \Pi)^2 + f_{56}\, (\partial_i \psi)^2 + f_{57}\, (\partial_i \partial_j C^{(8)})^2 + f_{58}\, \dot{\Pi}^2 + f_{59}\, \dot{\psi}^2 \Big) \Big)\,,
\end{aligned}
$$

3. Torsionful Galileons about the FLRW background

- **Final Quadratic Action:**

$$S_{4c} = \frac{1}{2}\int \mathrm{d}\eta\,\mathrm{d}^3x\,a^4\,\left[\frac{1}{a^2}\left(\mathcal{G}_\tau\left(\dot{h}_{ij}\right)^2 - \mathcal{F}_\tau(\partial_k\,h_{ij})^2\right) + \frac{1}{a^2}\left(\dot{\psi}\left(\mathcal{G}_{\mathcal{S}\mathrm{I}} - c\,\frac{1}{a^2}\,\mathcal{G}_{\mathcal{S}\mathrm{II}}\,\partial_i\partial_i\right)\dot{\psi} - \mathcal{F}_{\mathcal{S}}(\partial_i\psi)^2\right)\right]$$

The no-ghost, stability and subluminality conditions

$$\mathcal{G}_\tau > 0\,, \mathcal{F}_\tau > 0\,, \mathcal{F}_{\mathcal{S}} > 0\,, \mathcal{G}_{\mathcal{S}} > 0 \qquad c\,\mathcal{G}_{\mathcal{S}II} = \frac{8\,c\,{G_{4,X}}^3\,{G_4}^3}{(G_4 + c\,X\,G_{4,X})\,{(G_{4,X}\,G_{4,\phi} - G_4\,G_{4,\phi\,X})}^2} > 0$$

- One tensor perturbation
- No dynamical vector perturbation
- One scalar perturbation
- Theory with c=0 is special

3. Torsionful Galileons about the FLRW background

Table 1. Classification of the scalar according to the parameter *c* of the theory.

	$c < 0$	$c = 0$	$0 < c \leq 2$	$c > 2$
Scalar mode	Non wave-like dispersion relation. *Not a ghost* (in high momentum) if the graviton is healthy*.	*Wave-like* dispersion relation.	Non wave-like dispersion relation. *A ghost* (in high momentum) if the graviton is healthy*.	Non wave-like dispersion relation.
Graviton	Is massless. The no ghost, stability and subluminality conditions ($\mathcal{G}_\tau > 0$, $\mathcal{F}_\tau > 0$, $\frac{\mathcal{F}_\tau}{\mathcal{G}_\tau} < 1$) are satisfied if $G_4 > -2\,X\,G_{4,X} > 0$.			
Vector sector	Non dynamical.			

4. Stability:

the NO-GO in the Torsionful theory (c=0)

4. Stability

Answer to 2nd question at linear order about a spatially flat FLRW background

1. **Are there more Degrees of Freedom?**

Answer: No.

2. **Scalar: A new chance to stable solutions?**

Answer: Partially for *c*=0

Now the NO-GO on the Torsionful theory holds on different assumptions

4. Stability

Details for theory with c=0

$$\mathcal{S} = \int \mathrm{d}^4x\,\sqrt{-g}\,\Big(G_2 - G_3\,\tilde{\nabla}_\mu\tilde{\nabla}^\mu\phi + G_4\,\tilde{R}$$
$$+G_{4,X}\Big(\big(\tilde{\nabla}_\mu\tilde{\nabla}^\mu\phi\big)^2 - \big(\tilde{\nabla}_\mu\tilde{\nabla}_\nu\phi\big)\,\tilde{\nabla}^\nu\tilde{\nabla}^\mu\phi\Big)\Big)$$

Scalar sector very similar to torsionless Galileons, **Except** $\boxed{\mathcal{G}_\tau \neq T}$

$$\boxed{\begin{aligned}\mathcal{S}_s = \int \mathrm{d}\eta\,\mathrm{d}^3x\,a^4\,\Big(&-3\,\frac{\mathcal{G}_\tau}{a^2}\,\dot{\psi}^2 \\ &+\frac{\mathcal{F}_\tau}{a^2}\,(\partial_i\psi)^2 + 6\,\frac{\Theta}{a}\,\alpha\,\dot{\psi} + 2\,\frac{T}{a^2}\,\partial_i\alpha\,\partial_i\psi \\ &+2\frac{\partial_i\partial_i B}{a^2}\,\Big(a\,\Theta\,\alpha - \mathcal{G}_\tau\,\dot{\psi}\Big) + \Sigma\,\alpha^2\Big)\end{aligned}}$$

$$\mathcal{G}_\tau = 2\,\frac{G_4^2}{G_4 + 2\,X\,G_{4,X}}\,,\quad \mathcal{F}_\tau = 2\,G_4\,,\quad T = \mathcal{F}_\tau\,(c_g^2 - 2)$$

$$\Theta = \frac{4\,\mathcal{G}_\tau^2\,\theta}{a\,\mathcal{F}_\tau^4}\,,\quad \Sigma = \frac{2\,\mathcal{G}_\tau^3\,\sigma}{a\,\mathcal{F}_\tau^6}\,,$$

4. Stability

How do we get $\boxed{\mathcal{G}_\tau \neq T}$ in the torsionful Galileons?

Answer: a nontrivial torsion scalar coupled to the dynamical scalar mode

$$C^{(3)} = -\frac{2\,a^2\,X\,G_{4,X}}{G_4 + 2\,X\,G_{4,X}}\dot{\psi} + 2\,x\,\psi - \frac{a^3\,(2\,G_4\,H + \Theta) + a^2\,\dot{\phi}\,G_{4,\phi}}{2\,G_4}\alpha$$

4. Stability

Using the equation for B, $\alpha = \frac{1}{a}\frac{\mathcal{G}_\tau}{\Theta}\dot{\psi}$ the action reads,

$$\mathcal{S}_s = \int \mathrm{d}\eta\,\mathrm{d}^3x\,a^4\left(\frac{1}{a^2}\,\mathcal{G}_\mathcal{S}\,\dot{\psi}^2 - \frac{1}{a^2}\,\mathcal{F}_\mathcal{S}\,(\partial_i\psi)^2\right)$$

with $\mathcal{G}_\mathcal{S} = 3\,\mathcal{G}_\tau + \frac{\mathcal{G}_\tau^2\,\Sigma}{\Theta^2}\,,\;\; \mathcal{F}_\mathcal{S} = \frac{1}{a^2}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{a\,\mathcal{G}_\tau\,T}{\Theta}\right) - \mathcal{F}_\tau$

Follow a similar reasoning as in (Rubakov, 2016) in relation to wormholes, or as initially proved for a subclass of generalized Galileons in (Libanov, Mironov and Rubakov, 2016) and then extended to the full Horndeski action in (Kobayashi, 2016)…

4. Stability: the NO-GO in the Torsionful theory (c=0)

No-Go for nonsingular, all-time stable and sub/ luminal solutions

For (up to quartic) Galileons on a spacetime with torsion the following assumptions for a first order perturbative expansion about FLRW are mutually inconsistent:

I) Nonsingular cosmology: namely, there is a lower bound on the scale factor $a(\eta) > b_1 > 0.$

II) The graviton and the scalar mode are not ghosts and they suffer no gradient instabilities: $\mathcal{G}_\mathcal{T} > 0\,, \mathcal{F}_\mathcal{T} > 0\,, \mathcal{F}_\mathcal{S} > 0\,, \mathcal{G}_\mathcal{S} > 0.$

III) …

4. Stability: the NO-GO in the Torsionful theory (c=0)

No-Go for nonsingular, all-time stable and sub/ luminal solutions

…

III) The graviton is always sub/ luminal: $(c_g)^2 \leq 1$

IV) There is a lower bound $\mathcal{F}_\tau(\eta) > b_2 > 0$ *as* $\eta \to \pm\infty$ *(no „Strong gravity" at linear order (Ageeva, Petrov and Rubakov, 2021)).*

V) Θ *Vanishes at most a finite amount of times (To cover generic theories not defined by the equation* $\Theta \equiv 0$ *(Mironov and Shtennikova, 2023))*

4. Stability:

The argument in Galileons $$\mathcal{F}_\mathcal{S} = \frac{1}{a^2}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{a\,\mathcal{G}_\tau\,T}{\Theta}\right) - \mathcal{F}_\tau$$

With Torsion **/** without Torsion

$$T = \mathcal{F}_\tau\,(c_g^2 - 2) \qquad\qquad T = \mathcal{G}_\tau$$

With Torsion: (I)-(III) imply

$$\boxed{N =: \frac{a\,\mathcal{G}_\tau\,\mathcal{F}_\tau\,(c_g^2 - 2)}{\Theta} \neq 0\,,}$$

Because Θ is a regular function of *H and* ϕ

4. Stability:

$$\mathcal{F}_{\mathcal{S}} = \tfrac{1}{a^2} \frac{\mathrm{d}}{\mathrm{d}\eta} \left(\tfrac{a\, \mathcal{G}_\tau\, T}{\Theta} \right) - \mathcal{F}_\tau$$

Now integrate $\mathcal{F}_{\mathcal{S}} > 0$

$$\boxed{\begin{gathered} \Delta N = N_f - N_i > I(\eta_i,\, \eta_f)\,, \\ I(\eta_i,\, \eta_f) = \int_{\eta_i}^{\eta_f} \mathrm{d}\eta\, a^2\, \mathcal{F}_\tau\,, \end{gathered}}$$

with N_f and N_i the values of N at some (conformal) times η_f and η_i respectively

4. Stability

A) $a^2\,\mathcal{F}_\tau \;>\; 0$

$$I(\eta_i) = I(\eta_i,\, \eta_f)\big|_{\eta_f}$$ positive, growing with η_i

$$I(\eta_f) = I(\eta_i,\, \eta_f)\big|_{\eta_i}$$ positive, growing with η_f

B) $\Delta N > 0$

C) $I(\eta_i)$ not convergent as $\eta_i \rightarrow -\infty$

$I(\eta_f)$ not convergent as $\eta_f \rightarrow \infty$

4. Stability

Now, take

$$-\infty < N_i < 0$$

Since $N \neq 0$, follows

$$N_f(\eta_f) < 0$$

and

$$|N_i| > \Delta N = |N_i| - |N_f|$$

so, there exist η_C such that

$$\boxed{\text{if } \eta_f > \eta_c, I(\eta_f) > |N_i| > \Delta N}$$

4. Stability

Similarly, take

$$\infty > N_f > 0$$

Since $N \neq 0$, follows

$$N_i(\eta_i) > 0$$

and

$$N_f > \Delta N = N_f - N_i$$

so, there exist η_C such that

$$\boxed{\text{if } \eta_i < \eta_c,\, I(\eta_i) > N_f > \Delta N}$$

4. Stability

Namely,

$$\Delta N \not> I(\eta_i, \eta_f)$$

And

$$\boxed{\mathcal{F}_{\mathcal{S}} \not> 0}$$

No-Go for nonsingular, all-time stable and sub/ luminal solutions

4. Stability: example

A model with an all-time stable non singular cosmology with a short period of superluminality of the graviton

4. Stability: example

By-pass the no-go? short-lived superluminality

$$c_g \geq \sqrt{2}\, c$$

Let

- τ_s : width of the superluminal phase
- τ_b : width of the bounce phase
- η_s : center of superluminal phase
- η_b : center of bounce phase

4. Stability: reconstruct S

Out of G2, G3, G4 reconstruct a Lagrangian for the fixed solutions

(Inverse method, see also [19])

$$a = (\tau_b^2 + \eta^2)^{\frac{1}{4}}\,, \;\; H = \frac{\dot{a}}{a^2} = \frac{\eta}{2\,(\tau_b^2+\eta^2)^{\frac{5}{4}}}\,, \quad \phi = \eta\,,$$

Thus, in linearized expressions

$$X = \frac{\dot{\phi}^2}{2\,a^2}$$

$$X = 1/(2\,(\tau_b^2 + \eta^2)^{\frac{1}{2}})$$

4. Stability: example

Demand GR asymptotics as $\eta \to \pm\infty$

$$G_2(\phi, X) \to \frac{1}{2\,a^2}\,\dot{\xi}^2\,, \quad G_4(\phi, X) \to \frac{1}{2}\,, \quad M_{pl}^2/8\pi = 1$$

$$G_3(\phi, X) \to 0\,, \qquad x(\eta) \to 0\,.$$

ξ is some invertible function of the Horndeski scalar.

And we assume $\eta_s < \eta_b = 0$

4. Stability: example

The following Ansatz for the model has enough structure

$$
\begin{aligned}
G_2(\phi, X) &= g_{20}(\phi) + g_{21}(\phi)\, X + g_{22}(\phi)\, X^2 \,, \\
G_3(\phi, X) &= g_{30}(\phi) + g_{31}(\phi)\, X \,, \\
G_4(\phi, X) &= \frac{1}{2} + g_{40}(\phi) + g_{41}(\phi)\, X \,.
\end{aligned}
$$

4. Stability: example

Indeed, g_{20}, g_{21}, g_{22}, g_{30}, g_{31}, g_{40} and g_{41} can be solved algebraically from the following 7 equations

$$\mathcal{F}_\tau(g_{40},\, g_{41}) = 1\,,$$

$$T(g_{40},\, g_{41}) = -1 - \tfrac{5}{4}\mathrm{Sech}\left(\tfrac{\eta-\eta_s}{\tau_s}\right) + 3\,\mathrm{Sech}\left(\tfrac{\eta-\eta_s}{\tau_s}\right)^2$$

$$G_3(g_{30},\, g_{31}) = \mathrm{Sech}\left(\tfrac{\eta}{\tau_b}\right)$$

$$\boxed{\mathcal{F}_\mathcal{S} > 0}$$

$$\Theta(g_{30},\, g_{31}) = -H_s$$

$$\mathcal{G}_\mathcal{S} = \mathcal{F}_\mathcal{S}$$

$$H_s = \frac{\eta - \eta_s\, S}{2\left(\tau_b^2\,(1-S) + \tau_s^2\, S + (\eta - \eta_s\, S)^2\right)^{\frac{5}{4}}}$$

$$\mathcal{E}_{g_{00}} = 0$$

$$\mathcal{E}_{g_{33}} = 0$$

$$S = \mathrm{Sech}\left(\frac{\tau_s}{\tau_b}\,\frac{(\eta-\eta_s)}{\eta_s}\right)$$

$$\Theta \overset{\eta\to\pm\infty}{\longrightarrow} -H$$

4. Stability: example

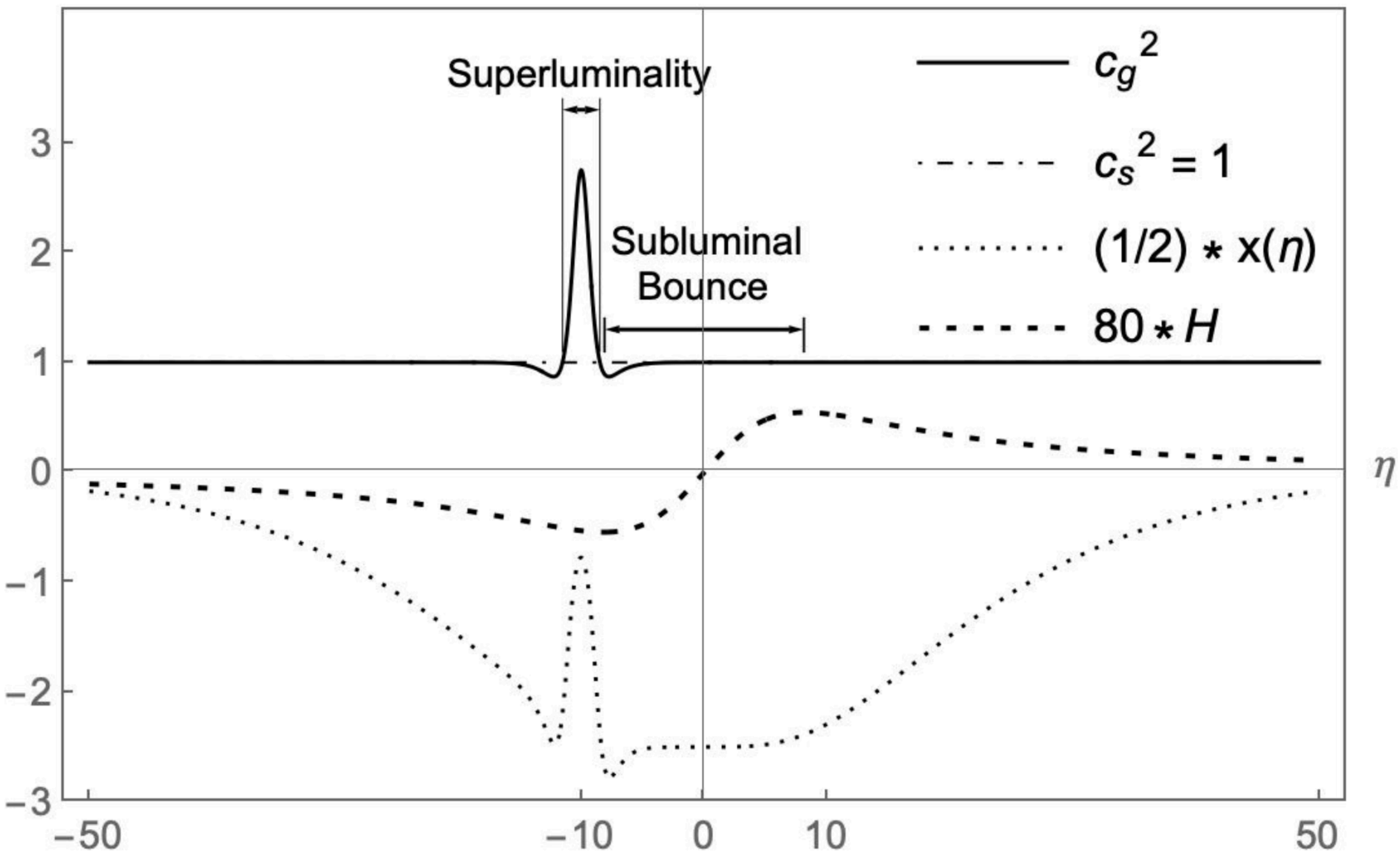


FIG. 1: Hubble parameter for a bounce at $\eta_b = 0$ with $\tau_b = 10$. Speed of sound for the scalar mode c_s^2. Speed of the graviton c_g^2 with *short* superluminality phase ($\tau_s << \tau_b$) happening at $\eta_s = -10$ before the bounce (For convenience displaying the graphs we choose here $\tau_b = 10\,\tau_s$). The graviton quickly becomes subluminal around η_s and approaches luminality from below in the past, and during the bounce phase and future. Torsion background $x(\eta)$ exponentially vanishing in the asymptotic past and future.

$$\mathcal{G}_\tau > 0\,, \mathcal{F}_\tau > 0\,, \mathcal{F}_\mathcal{S} > 0\,, \mathcal{G}_\mathcal{S} > 0$$

4. Stability: example

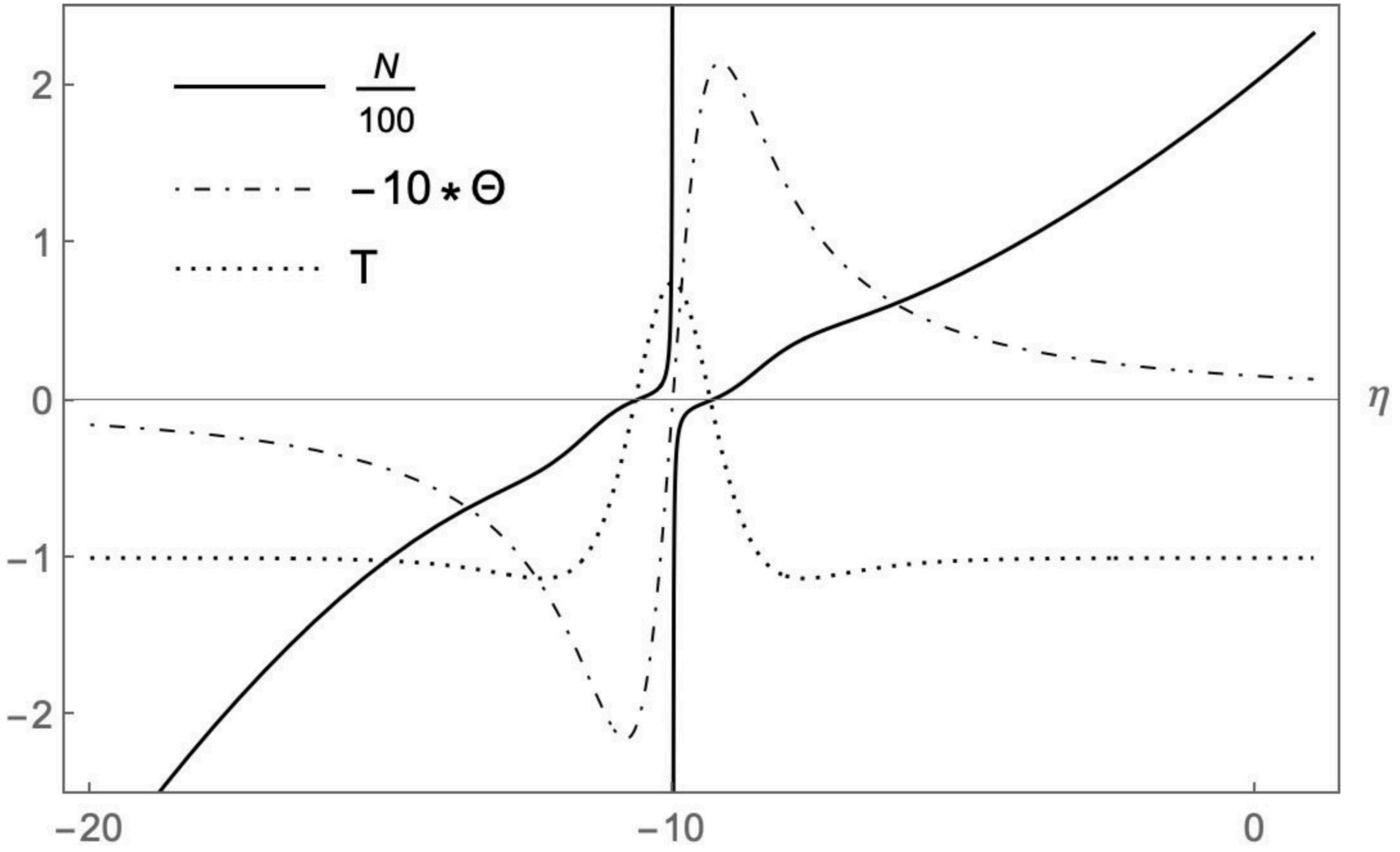


FIG. 2: By-passing the no-go theorem: this choice for T (16) does not satisfy the all-time negativity condition, which critically means that the graviton is superluminal during a brief stage of evolution, around $\eta_s = -10$ as shown in Figure 1, and that the function N in equation (8) vanishes. Hence, the no-go theorem does not hold and we can build all-time stable solutions.
$(\tau_b = 10,\ \tau_s = 1,\ \eta_b = 0,\ \eta_s = -10)$

4. Stability: example

The solutions for the Lagrangian functions take the following form

as $\eta \to \pm\infty$

$$g_{40} = -g_{41}\, X = \frac{5}{8}\, e^{\mp \frac{(\eta - \eta_s)}{\tau_s}}$$

$$g_{30} = -g_{31}\, X = \frac{3}{2}\frac{\eta_s}{\eta^2} e^{\mp \frac{\tau_s}{\tau_b}\frac{(\eta-\eta_s)}{|\eta_s|}}$$

$$x = \mp\eta\, e^{\mp \frac{\eta}{\tau_b}}$$

$$g_{20} = -\frac{\tau_b^2}{2}\ (\pm\eta)^{-5}\ ,\ \ g_{21}\, X = \frac{3}{4}\ (\pm\eta)^{-3}$$

$$g_{22}\, X^2 \ = \ \mp\frac{3}{4}(\pm\eta)^{-3}\frac{\tau_s}{\tau_b} e^{\mp \frac{\tau_s}{\tau_b}\frac{(\eta-\eta_s)}{|\eta_s|}}$$

4. Stability: example

Plot of the analytical solutions for g_{20}, g_{21}, g_{22}, g_{30}, g_{31}, g_{40} and g_{41}

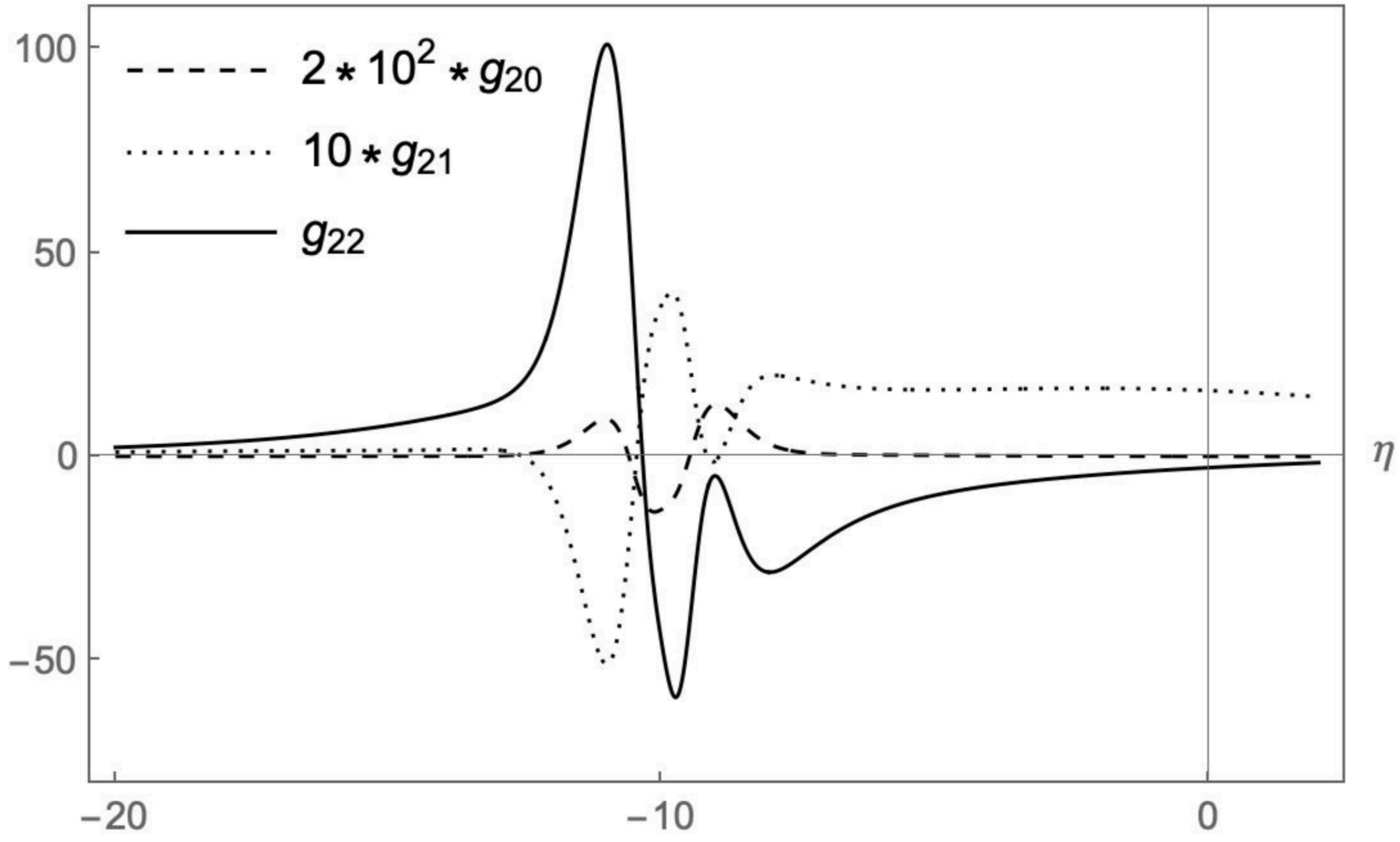
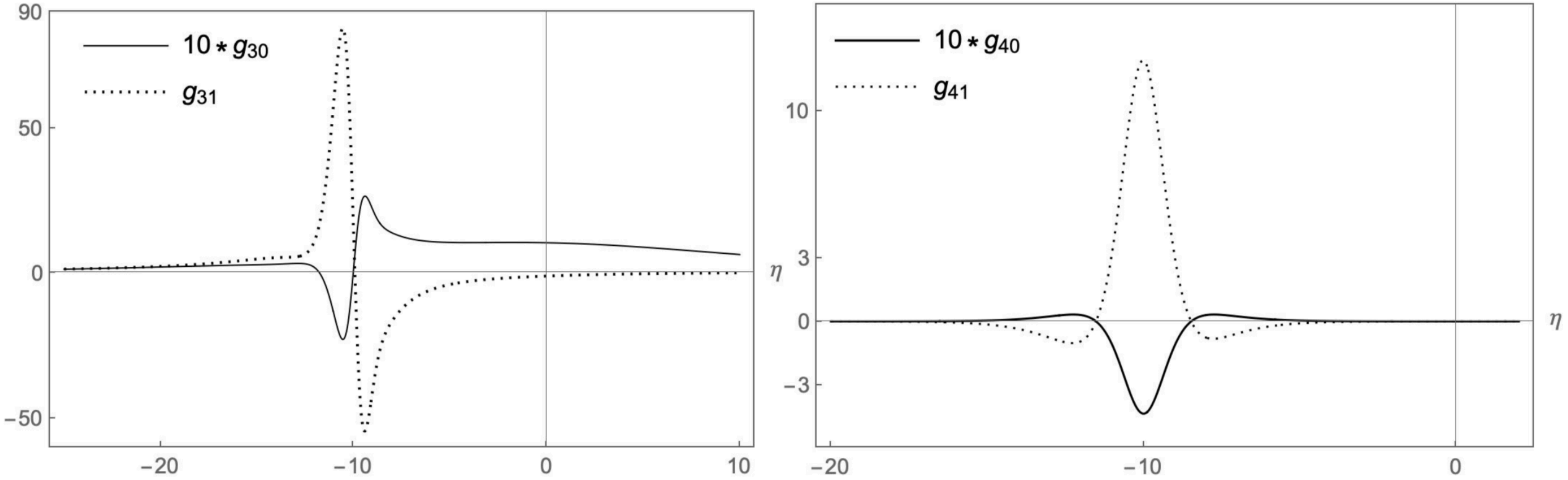


FIG. 3: Everywhere regular Lagrangian functions g_{20}, g_{21} and g_{22}.

4. Stability: example

Plot of the analytical solutions for g_{20}, g_{21}, g_{22}, g_{30}, g_{31}, g_{40} and g_{41}



(a) g_{30} and g_{31}. (b) g_{40} and g_{41}.

FIG. 4: Everywhere regular Lagrangian functions.

4. Stability: example

Asymptotic Lagrangian

The leading expressions in the Ansatz

$$G_2(\phi, X) = g_{20}(\phi) + g_{21}(\phi)\,X + g_{22}(\phi)\,X^2\,,$$
$$G_3(\phi, X) = g_{30}(\phi) + g_{31}(\phi)\,X\,,$$
$$G_4(\phi, X) = \frac{1}{2} + g_{40}(\phi) + g_{41}(\phi)\,X\,.$$

as $\eta \to \pm\infty$

Are $G_4 = \frac{1}{2}$, $g_{20} = -\frac{\tau_b^2}{2}\,(\pm\eta)^{-5}$, $g_{21}\,X = \frac{3}{4}\,(\pm\eta)^{-3}$

Now, with the leading solutions $a = \eta^{\frac{1}{2}}$, $H = \frac{1}{2}\eta^{-\frac{3}{2}}$, $\phi = \eta$, $X = 1/(2\,\eta)$

The corresponding action to S in the asymptotic past and future is

$$\boxed{S^{\infty} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,(R - \partial_\mu\xi\,\partial^\mu\xi)} \qquad \xi = \sqrt{\tfrac{3}{2}}\ln(\phi)$$

Indeed, the leading solutions satisfy $\ddot{\xi} + 2\,a\,H\,\dot{\xi} = 0\,,\ \dot{\xi}^2 - 6\,a^2\,H^2 = 0$

Conclusions

- Classification of the scalar according to the parameter *c* of the theory.

- We extended the no-go argument of (Rubakov, 2016) (Libanov, Mironov and Rubakov, 2016) (Kobayashi, 2016) to up to quartic Galileons (c=0) on a spacetime with torsion (Horndeski-Cartan)

    - *in generic models it is not possible to obtain a nonsingular FLRW cosmology that is always free of gradient instabilities against the scalar perturbation and an eternally sub/ luminal graviton.*

Conclusions

- A spacetime with torsion can support all-time linearly stable nonsingular solutions in Galileons if there exists at an arbitrary time a superluminal phase for the graviton and by at least an amount

$$c_g \geq \sqrt{2\,c}$$

- This unphysical phase can formally happen as a deep UV inconsistency (arbitrarily short) and unrelated to the physically relevant length scales that are pertinent to these models, such as time and much longer width of a bounce.

- At least in what concerns the stability and speed of solutions, this shows that Horndeski-Cartan theory is fundamentally different to Horndeski on a torsionless geometry, in contrast to e.g. the equivalence of Einstein-Cartan.

Open questions:

- Accidental symmetry [20 - 23]?

- Lorentz invariant UV completions for models with all-time stable nonsingular cosmologies (Adams *et.al.* , 2006), (Dubovsky *et.al.* , 2006)?

- G5 changes the picture?

References

[1] G. W. Horndeski, “Second-order scalar-tensor field equations in a four-dimensional space,” International Journal of Theoretical Physics, vol. 10, no. 6, pp. 363–384, 1974.

[2] A. Nicolis, R. Rattazzi, and E. Trincherini, “Galileon as a local modification of gravity,” Physical Review D, vol. 79, no. 6, p. 064036, 2009.

[3] C. Deffayet, G. Esposito-Farese, and A. Vikman, “Covariant galileon,” Physical Review D, vol. 79, no. 8, p. 084003, 2009.

[4] C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, “Imperfect Dark Energy from Kinetic Gravity Braiding,” JCAP, vol. 10, p. 026, 2010.

[5] A. Padilla and V. Sivanesan, “Covariant multi-galileons and their generalisation,” JHEP, vol. 04, p. 032, 2013.

[6] D. B. Fairlie, J. Govaerts, and A. Morozov, “Universal field equations with covariant solutions,” Nucl. Phys. B, vol. 373, pp. 214–232, 1992.

[7] T. Kobayashi, “Horndeski theory and beyond: a review,” Reports on Progress in Physics, vol. 82, no. 8, p. 086901, 2019.

[8] S. Arai, K. Aoki, Y. Chinone, R. Kimura, T. Kobayashi, H. Miyatake, D. Yamauchi, S. Yokoyama, K. Akitsu, T. Hiramatsu et al., “Cosmological gravity probes: connecting recent theoretical developments to forthcoming observations,” arXiv preprint arXiv:2212.09094, 2022.

References

[9] V. A. Rubakov, “The null energy condition and its violation,” Physics-Uspekhi, vol. 57, no. 2, p. 128, 2014.

[10] M. Libanov, S. Mironov, and V. Rubakov, “Generalized Galileons: instabilities of bouncing and Genesis cosmolo- gies and modified Genesis,” JCAP, vol. 08, p. 037, 2016.

[11] T. Kobayashi, “Generic instabilities of nonsingular cos- mologies in Horndeski theory: A no-go theorem,” Phys. Rev. D, vol. 94, no. 4, p. 043511, 2016.

[12] R. Kolevatov and S. Mironov, “Cosmological bounces and Lorentzian wormholes in Galileon theories with an extra scalar field,” Phys. Rev. D, vol. 94, no. 12, p. 123516, 2016.

[13] S. Mironov, “Mathematical Formulation of the No-Go Theorem in Horndeski Theory,” Universe, vol. 5, no. 2, p. 52, 2019.

[14] S. Akama and T. Kobayashi, “ Generalized multi-Galileons, covariantized new terms, and the no-go theo- rem for nonsingular cosmologies,” Phys. Rev. D, vol. 95, no. 6, p. 064011, 2017.

[15] P. Creminelli, D. Pirtskhalava, L. Santoni, and E. Trincherini, “Stability of Geodesically Complete Cos- mologies,” JCAP, vol. 11, p. 047, 2016.

[16] S. Mironov and A. Shtennikova, “Stable cosmological so- lutions in Horndeski theory,” JCAP, vol. 06, p. 037, 2023.

References

[17] Y. Ageeva, P. Petrov, and V. Rubakov, “Nonsingular cosmological models with strong gravity in the past,” Phys. Rev. D, vol. 104, no. 6, p. 063530, 2021.

[18] F. W. Hehl, P. Von Der Heyde, G. D. Kerlick and J. M. Nester, “General Relativity with Spin and Torsion: Foundations and Prospects,” Rev. Mod. Phys. 48 (1976), 393-416.

[19] S. Mironov, V. Rubakov, and V. Volkova, “Bounce be- yond Horndeski with GR asymptotics and γ-crossing,” JCAP, vol. 10, p. 050, 2018.

[20] A. Golovnev and T. Koivisto, Cosmological perturbations in modified teleparallel gravity models, J. Cosmol. Astropart. Phys. 11 (2018) 012.

[21] A. Golovnev and M.-J. Guzman, Nontrivial Minkowski backgrounds in f(T) gravity, Phys. Rev. D 103, 044009 (2021).

[22] J. Beltrán Jime´nez and K.F. Dialektopoulos, Non-linear obstructions for consistent new general relativity, J. Cosmol. Astropart. Phys. 01 (2020) 018.

[23] J. Beltrán Jime´nez, L. Heisenberg, T.S. Koivisto, and S. Pekar, Cosmology in f(Q) geometry, Phys. Rev. D 101, 103507 (2020)

Additional Material

Non wave like dispersion relation

$$\mathcal{L}_{toy} = \left(1 - c\, p^2\right) \dot{x}_1^2 - p^2\, x_1^2\,,$$

equivalent

$$\mathcal{L}_{toy'} = \dot{x}_1^2 - p^2\, x_1^2 + c\, p^2\ \left(2\, x_2\, \dot{x}_1 + x_2^2\right) \qquad\qquad (x_2 = -\dot{x}_1)$$

Key terms in action $\dot{\psi}^2,\ p^2\psi^2,\ p^2(C^{(6)})^2$

$$p^2\, f_{23}\, C^{(6)}\alpha \qquad\qquad \alpha = \dot{\psi} + \ldots$$

$$C^{(6)} = -c\, \frac{1}{f_{13}}\ \left(2\, f_{52}\, C^{(1)}\ +\ f_7\, B\right)$$

Additional Material

Classification of the scalar for nonzero *c*

In high momentum

$$c\,\mathcal{G}_{\mathcal{S}II} = \frac{8\,c\,{G_{4,X}}^3\,{G_4}^3}{(\,G_4 + c\,X\,G_{4,X})\,(G_{4,X}\,G_{4,\phi} - G_4\,G_{4,\phi\,X})^2} > 0$$

$$\mathcal{F}_{\mathcal{S}} > 0$$

From tensor sector $\mathcal{G}_\tau > 0\,,\;\mathcal{F}_\tau > 0\,,$ and sub/ luminality $G_4 > -2\,X\,G_{4,X} > 0$

$$X = \frac{1}{2\,a^2}\dot{\varphi}^2 \;>\; 0 \qquad\qquad G_4 > 0\,,\;\; G_{4,X} < 0\,,$$

then

$$c\,\mathcal{G}_{\mathcal{S}II} > 0 \qquad\qquad \frac{c}{(\,G_4 + c\,X\,G_{4,X})} < 0$$

Rewrite

$$G_4 > -2\,X\,G_{4,X} > 0 \qquad\qquad (\,G_4 + c\,X\,G_{4,X}) > (c-2)\,X\,G_{4,X}$$

Additional Material

Background equations $\mathcal{E}_\varphi = 0\,,\ \mathcal{E}_{g_{00}} = 0\,,\ \mathcal{E}_{g_{ij}} = 0\,,\ \mathcal{E}_{K_{ij0}} = 0\,,$

$$\mathcal{E}_{g_{00}} = (\,x +\ a\,\dot{a})\left(\frac{3\,G_4(\,x -\ a\,\dot{a})}{a^8} - \frac{3\,G_{4,\phi}\,\dot{\varphi}}{a^6} + \frac{6\,G_{4,X}\,(2\,x + a\,\dot{a})\dot{\varphi}^2}{a^{10}} - \frac{3\,G_{4,\phi X}\,\dot{\varphi}^3}{a^8} + \frac{3\,G_{4,XX}(\,x +\ a\,\dot{a})\,\dot{\varphi}^4}{a^{12}}\right)\,,$$

$$\mathcal{E}_{g_{ij}} = \delta_{ij}\,\left(\frac{G_4\Big(-\,x^2 +\ a^4(\frac{\dot{a}^2}{a^2} + 2(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}))\Big)}{a^8} + \frac{G_{4,\phi\phi}\,\dot{\varphi}^2}{a^4} + \frac{2\,G_{4,XX}(\,x +\ a\,\dot{a})\,\dot{\varphi}^3(-\,\ddot{\varphi} + \frac{\dot{a}\,\dot{\varphi}}{a})}{a^{10}}\right.$$
$$+ \frac{G_{4,\phi X}\,\dot{\varphi}^2(-2\,x\,\dot{\varphi} +\ a^2(\,\ddot{\varphi} - \frac{3\,\dot{a}\,\dot{\varphi}}{a}))}{a^8} + \frac{G_{4,\phi}(-\,x\,\dot{\varphi} +\ a^2(\,\ddot{\varphi} + \frac{\dot{a}\,\dot{\varphi}}{a}))}{a^6}$$
$$\left. + \frac{G_{4,X}\,\dot{\varphi}\Big(-\,x^2\,\dot{\varphi} +\ a^4(-\frac{2\,\ddot{\varphi}\,\dot{a}}{a} + \frac{\dot{a}^2\,\dot{\varphi}}{a^2} - 2(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2})\,\dot{\varphi}) +\ a^2(-4\,x\,\ddot{\varphi} + \frac{6\,x\,\dot{a}\,\dot{\varphi}}{a} - 2\,\dot{\varphi}\,\dot{x})\Big)}{a^{10}}\right)\,,$$

$$\mathcal{E}_{K_{ij0}} = \delta_{ij}\,\left(\frac{2\,G_4\,x}{a^6} - \frac{G_{4,\phi}\,\dot{\varphi}}{a^4} + \frac{2\,G_{4,X}(\,x +\ a\,\dot{a})\,\dot{\varphi}^2}{a^8}\right)\,,$$

$$\mathcal{E}_\varphi = -\frac{2\,a^2}{\dot{\varphi}^2}\,\left(\dot{\mathcal{E}}_{g_{00}} + (5\,\mathcal{E}_{g_{00}} + 3\,\mathcal{E}_{g_{ii}})\,\frac{\dot{a}}{a}\right) + \frac{6\,x}{\dot{\varphi}^2}\left(\dot{\mathcal{E}}_{K_{ii0}} + 4\,\mathcal{E}_{K_{ii0}}\frac{\dot{a}}{a}\right)\,,$$

Additional Material

Gauge transformations

$$\alpha \to \alpha - \dot{\xi}^0 - \frac{\dot{a}}{a}\xi^0 \qquad B \to B + \xi^0 - \dot{\xi} \qquad \psi \to \psi + \frac{\dot{a}}{a}\xi^0 \qquad E \to E - \xi$$

$$S_i \to S_i - \dot{\xi}_i \qquad F_i \to F_i - \xi_i \qquad h_{ij} \to h_{ij}\,,$$

$$\Pi \to \Pi - \xi^0 \dot{\varphi}\,,$$

$$T_{ij}^{(1)} \to T_{ij}^{(1)} \qquad T_{ij}^{(2)} \to T_{ij}^{(2)} \qquad V_i^{(5)} \to V_i^{(5)} \qquad V_i^{(6)} \to V_i^{(6)}$$

$$V_i^{(1)} \to V_i^{(1)} + \dot{\xi}_i\, x \qquad V_i^{(2)} \to V_i^{(2)} + \xi_i\, x - \dot{\omega}_i\, y \qquad V_i^{(3)} \to V_i^{(3)} + \xi_i\, x + \dot{\omega}_i\, y \qquad V_i^{(4)} \to V_i^{(4)} + \dot{\omega}_i\, y$$

$$C^{(1)} \to C^{(1)} + \dot{\xi}\, x \qquad C^{(2)} \to C^{(2)} + 2\xi\, x \qquad C^{(3)} \to C^{(3)} + \xi^0\, \dot{x} + \dot{\xi}^0\, x \qquad C^{(4)} \to C^{(4)} - \dot{\xi}\, y$$

$$C^{(5)} \to C^{(5)} + \dot{\xi}\, y \qquad C^{(6)} \to C^{(6)} + \xi^0\, x \qquad C^{(7)} \to C^{(7)} + \xi\, y - \frac{1}{|\vec{p}|^2}\dot{y}\xi^0 \qquad C^{(8)} \to C^{(8)} - \xi\, y + \frac{1}{|\vec{p}|^2}\dot{y}\xi^0\,,$$

$$\xi_k = \epsilon_{ijk}\partial_i\omega_j \qquad \partial_i\omega_i = 0$$

Additional Material

Coefficients in scalar sector (c=0)

$$\theta = -2\,a\,G_4\,H(\,G_4{}^2+4(2\,G_{4,X}{}^2-\,G_{4,XX}\,G_4)\,X^2)-(\,G_{4,\phi}(\,G_4{}^2+4\,G_{4,X}{}^2\,X^2-2\,G_4\,X(\,G_{4,X}+2\,G_{4,XX}\,X))$$
$$+\,G_4\,X(-(\,G_{3,X}-2\,G_{4,\phi X})(\,G_4+2\,G_{4,X}\,X)+\,G_3(3\,G_{4,X}+2\,G_{4,XX}\,X)))\,\dot{\phi}\,,$$

$$\sigma = a\,\big(-24\,H^2(\,G_4{}^4+16\,G_{4,X}{}^4\,X^4+8\,G_{4,X}{}^2\,G_4\,X^3(\,G_{4,X}-2\,G_{4,XX}\,X)+2\,G_4{}^2\,X^2(15\,G_{4,X}{}^2+8\,G_{4,XX}{}^2\,X^2$$
$$+4\,G_{4,X}\,X(\,G_{4,XX}-\,G_{4,XXX}\,X))-\,G_4{}^3\,X(\,G_{4,X}+4\,X(4\,G_{4,XX}+\,G_{4,XXX}\,X)))$$
$$+\,X(-3\,G_3{}^2(\,G_4{}^2+2\,X^2(5\,G_{4,X}{}^2+8\,G_{4,XX}{}^2\,X^2+4\,G_{4,X}\,X(\,G_{4,XX}-\,G_{4,XXX}\,X))-\,G_4\,X(11\,G_{4,X}+4\,X(5\,G_{4,XX}$$
$$+\,G_{4,XXX}\,X)))+6\,G_3(-\,X(\,G_4+2\,G_{4,X}\,X)(2(\,G_{3,XX}-2\,G_{4,\phi XX})\,X(\,G_4+2\,G_{4,X}\,X)$$
$$+\,G_{3,X}(5\,G_4-2\,X(\,G_{4,X}+4\,G_{4,XX}\,X))+2\,G_{4,\phi X}(-5\,G_4+2\,X(\,G_{4,X}+4\,G_{4,XX}\,X)))$$
$$+2\,G_{4,\phi}(\,G_4{}^2+2\,X^2(5\,G_{4,X}{}^2+8\,G_{4,XX}{}^2\,X^2+4\,G_{4,X}\,X(\,G_{4,XX}-\,G_{4,XXX}\,X))$$
$$-\,G_4\,X(11\,G_{4,X}+4\,X(5\,G_{4,XX}+\,G_{4,XXX}\,X))))+4((\,G_4+2\,G_{4,X}\,X)^2(\,G_{2,X}(\,G_4+2\,G_{4,X}\,X)$$
$$-2\,G_{3,\phi}(\,G_4+2\,G_{4,X}\,X)+\,X(-3(\,G_{3,X}-2\,G_{4,\phi X})^2\,X+2(\,G_{2,XX}-\,G_{3,\phi X})(\,G_4+2\,G_{4,X}\,X)))$$
$$+3\,G_{4,\phi}\,X(\,G_4+2\,G_{4,X}\,X)(2(\,G_{3,XX}-2\,G_{4,\phi XX})\,X(\,G_4+2\,G_{4,X}\,X)+\,G_{3,X}(5\,G_4-2\,X(\,G_{4,X}+4\,G_{4,XX}\,X))$$
$$+2\,G_{4,\phi X}(-5\,G_4+2\,X(\,G_{4,X}+4\,G_{4,XX}\,X)))-3\,G_{4,\phi}{}^2(\,G_4{}^2+2\,X^2(5\,G_{4,X}{}^2+8\,G_{4,XX}{}^2\,X^2$$
$$+4\,G_{4,X}\,X(\,G_{4,XX}-\,G_{4,XXX}\,X))-\,G_4\,X(11\,G_{4,X}+4\,X(5\,G_{4,XX}+\,G_{4,XXX}\,X))))))$$
$$+24\,H\,\big(\,X((\,G_4+2\,G_{4,X}\,X)((2\,G_{3,X}-5\,G_{4,\phi X})\,G_4{}^2+\,G_4(-(\,G_{3,X}+2\,G_{4,\phi X})\,G_{4,X}+(\,G_{3,XX}-2\,G_{4,\phi XX})\,G_4)\,X$$
$$+2((\,G_{3,X}-4\,G_{4,\phi X})\,G_{4,X}{}^2+((\,G_{3,XX}-2\,G_{4,\phi XX})\,G_{4,X}-2(\,G_{3,X}-2\,G_{4,\phi X})\,G_{4,XX})\,G_4)\,X^2)$$
$$+\,G_3(-6\,G_{4,X}\,G_4{}^2+3\,G_4(\,G_{4,X}{}^2-3\,G_{4,XX}\,G_4)\,X-2(3\,G_{4,X}{}^3-2\,G_{4,X}\,G_{4,XX}\,G_4+\,G_{4,XXX}\,G_4{}^2)\,X^2$$
$$-4(\,G_{4,X}{}^2\,G_{4,XX}-2\,G_{4,XX}{}^2\,G_4+\,G_{4,X}\,G_{4,XXX}\,G_4)\,X^3))$$
$$+\,G_{4,\phi}(-\,G_4{}^3+4\,G_{4,X}{}^2\,X^3(\,G_{4,X}+2\,G_{4,XX}\,X)-2\,G_4\,X^2(9\,G_{4,X}{}^2+8\,G_{4,XX}{}^2\,X^2+4\,G_{4,X}\,X(\,G_{4,XX}-\,G_{4,XXX}\,X))$$
$$+2\,G_4{}^2\,X(3\,G_{4,X}+\,X(9\,G_{4,XX}+2\,G_{4,XXX}\,X))))\big)\;\dot{\phi}\,.$$