String theory in AdS3, boundary currents, and the Schwarzian

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Motivational puzzle: how to explain thermodynamics of near-extremal BHs?

- Extremal BH $M \to Q$, $T \to 0$ (Reissner-Nordstrom soln of Einstein-Maxwell theory)

- Large entropy $S_{\rm BH} = \frac{1}{4G_N} A_H = \pi Q^2$

$$Z_{\rm BH} = e^{S_{\rm BH}}$$

but no symmetry! (cf. usual quantum systems).

- Breakdown in statistical thermal description.

"Uncontrollable thermodynamic fluctuations"

[Preskill, Schwarz, Shapere, Trivedi, Wilczek '91]

The emergence of the (nearly) gapless Schwarzian mode resolves this puzzle

- Nearly gapless mode in the near-horizon AdS2 region of nearly extremal BHs.

- Large quantum fluctuations at low temperatures.

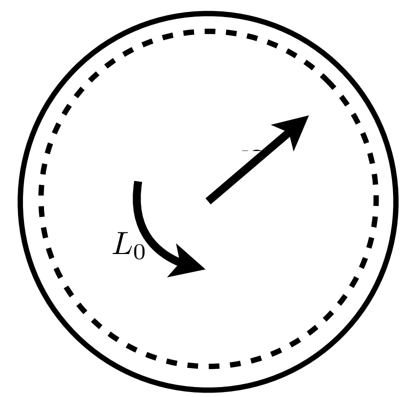
$$Z_{\rm BH} = T^{3/2} \, e^{S_{\rm BH}} \qquad\qquad T \to 0$$

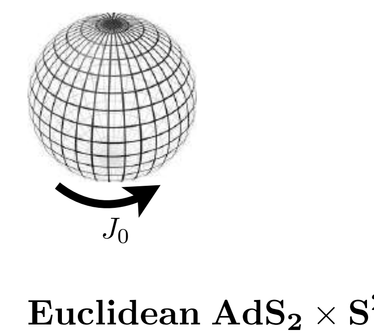
- Conclusion: BH is like a conventional QM system, with only a few low-lying excitations.

[Sachdev '15; Almheiri, Kang '16; Maldacena, Stanford, Yang '16; Moitra, Trivedi, Vishal '18, Maldacena, Turiaci, Yang '19; Ghosh, Maxfield,Turiaci '19; Iliesiu,Turiaci, 20; Iliesiu, Kruthoff,Turiaci, Verlinde, 20; Heydeman, Iliesiu, Turiaci, Zhao, '20;…]

Schwarzian modes come from subtle boundary excitations

- Zero modes in AdS2 coming from "pure gauge transformations" which don't vanish at boundary. [Camporesi, Higuchi '95]

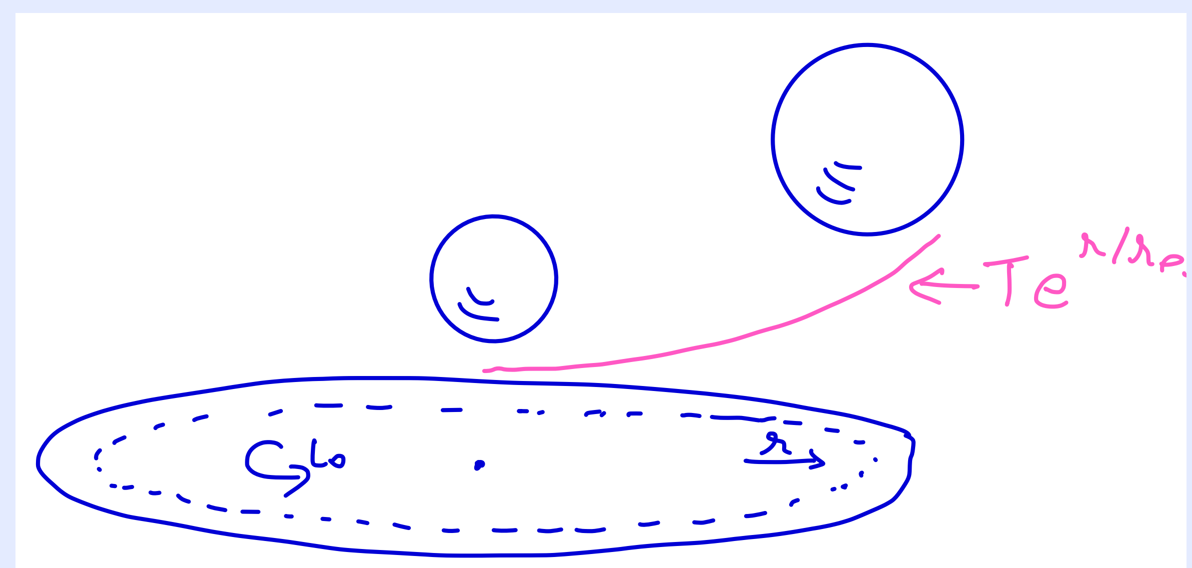




- Regulate zero-modes by turning on a small temperature.

[L. Iliesiu, S.M., G.J.Turiaci '22]

- Schwarzian modes: large diffeomorphisms which change the shape of the boundary.



The partition function of the Schwarzian vanishes at low temperatures

- Dual CFT1 has H=0 $\Longrightarrow$ invariance under local time-reparameterizations. [Maldacena, Stanford, Yang ’16]

- *Not* a symmetry of AdS2 background $\Longrightarrow$ spontaneously broken — except for SL(2) isometries.

- Ultra-locality of measure $\Longrightarrow$ $Z_{\text{Scwh}} = T^{3/2}$

[Stanford, Witten ’17]

…(except for supersymmetric BHs)

[L. Iliesiu, S.M., G.J.Turiaci ’22]

- In supersymmetric case, additional fermionic and R-symmetry currents

$$Z_{\text{Super-Scwh}} = T^0$$

Heydeman, Iliesiu, Turiaci, Zhao, ’20

- The partition function of the BH is exponentially large, in agreement with microscopic string theory calculations

$$Z_{\text{BH}} = e^{S_{\text{BH}}} + \dots$$

[L. Iliesiu, S.M., G.J.Turiaci ’22]

Main question today: Can we isolate the Schwarzian mode in string theory?

- BTZ is a good candidate — controllable stringy BH.
- Nevertheless, non-trivial problem, even in supergravity. (Need to lift the near-horizon modes to the boundary.)

- Trick/indirect method: BTZ = thermal AdS3 (Euclidean) (cf Cardy formula). [Ghosh, Maxfield, Turiaci, '19]
- Boundary gravitons (Brown-Henneaux) in AdS3 gravity.

$\Longleftrightarrow$ Virasoro module in dual CFT2 $L_{-n}|\ \rangle\,,\quad n = 2, 3, \ldots$

Problem maps to boundary currents in AdS3, = Virasoro module in dual CFT2

- Modular transf. + "Twist gap"

$$\Longrightarrow \quad Z_{\rm BTZ} = \chi_{\mathbb{1}}(-1/\tau)\chi_{\mathbb{1}}(1/\bar\tau) + \ldots$$

Virasoro character

$$\chi_{\mathbb{1}}(\tau) = \frac{(1-q)q^{-\frac{c-1}{24}}}{\eta(\tau)},$$

$$q = e^{2\pi i \tau} = e^{-\beta}$$

- Near-extremal BH $\quad \beta_L \to \infty\,, \qquad \beta_R \to 0\,,$

$$\chi_{\mathbb{1}}\left(\frac{2\pi i}{\beta_L}\right) \sim 2\pi \left(\frac{2\pi}{\beta_L}\right)^{3/2} \exp\left[\frac{\beta_L}{24} + \frac{c}{24}\frac{(2\pi)^2}{\beta_L}\right] \qquad \chi_{\mathbb{1}}\left(\frac{2\pi i}{\beta_R}\right) \sim \exp\left[\frac{c}{24}\frac{(2\pi)^2}{\beta_R}\right]$$

$$Z_{\rm BTZ} = T^{3/2} \; e^{S_{\rm BH}}$$

[Ghosh, Maxfield, Turiaci, '19]

Do these boundary currents exist in the spectrum of string theory in AdS3?

- Bosonic AdS3 and superstrings with NS-NS fluxes have known RNS description using SL(2) WZW model.

  [Maldacena, Ooguri '00] [Maldacena, Ooguri, Son '00]

- The low-lying spectrum of string states are local fields of (super)gravity, with global $\mathfrak{psu}(1,1|2) \oplus \overline{\mathfrak{psu}}(1,1|2)$ symmetry.

- Currents, with dimension (1,0) in spacetime and infinite number of symmetry generators less obvious.

- Vertex operators written down using dimension 0 field in AdS3 (*not* identity). [Giveon, Kutasov, Seiberg '98]

Goal: extract currents from one-loop partition function of string theory in AdS3

[Ferko, Murthy, Rangamani ’24]

- Revisit string 1-loop amplitude. We find:
  - Virasoro character in bosonic string one-loop k>3
  - Super-Virasoro character in superstring one-loop

- Further,
  - Clarify mismatch: AdS/CFT vs naive supergravity about gauge fields and currents in superstring
  - Comment about thermal string theory vs AdS/CFT.
  - Remaining puzzle for K3 (time permitting).

Thermal AdS3 is a solid torus

- Non-perturbative gravitational theory on AdS3 = boundary CFT2 (= spacetime CFT2)

- Euclidean AdS3 $ds^2 = k\,(\cosh^2\rho\, dt_{\rm E}^2 + d\rho^2 + \sinh^2\rho\, d\varphi^2)$

  Thermal identification $(t_{\rm E}, \varphi) \sim (t_{\rm E} + \beta, \varphi + i\,\beta\,\mu)$

$$\tau = \frac{\beta\mu + i\,\beta}{2\pi}\,, \qquad q = e^{2\pi i\,\tau}$$

Modular parameter of boundary torus

- Saddles of spacetime theory: thermal AdS3, BTZ BH, …

  Perturbation expansion around each saddle

$$\mathcal{Z}_{\rm CFT}(\tau,\bar\tau)\Big|_{\rm thermal\ AdS_3} = \exp\Big(S_{\rm tree} + \mathcal{Z}_{\rm ws}(\tau,\bar\tau) + \ldots\Big)$$

Bosonic string theory on AdS3 = maps from worldsheet to solid torus

- One-loop partition function of strings [Maldacena, Ooguri, Son '00]

$$\mathcal{Z}_{\rm ws}(\tau,\bar\tau) \;=\; \int_{\mathcal{F}} \frac{d^2\mathfrak{t}}{\mathfrak{t}_2}\, \mathcal{Z}_{\mathfrak{sl}(2)_k}(\mathfrak{t},\bar{\mathfrak{t}};\tau,\bar\tau)\, \mathcal{Z}_X(\mathfrak{t},\bar{\mathfrak{t}})\, |\eta(\mathfrak{t})|^4$$

$$\mathfrak{t} \;=\; \mathfrak{t}_1 + i\,\mathfrak{t}_2 \qquad \mathfrak{z} \;=\; e^{2\pi i\,\mathfrak{t}}$$

b-c ghosts

Modular parameter of worldsheet torus

SL(2) WZW model

$$\mathcal{Z}_{\mathfrak{sl}(2)_k}(\mathfrak{t},\bar{\mathfrak{t}};\tau,\bar\tau) \;=\; \frac{\beta\sqrt{k-2}}{2\pi\sqrt{\mathfrak{t}_2}} \sum_{n,m\in\mathbb{Z}} \frac{\exp\left(-\frac{k\,\beta^2}{4\pi\,\mathfrak{t}_2}\,|m-n\,\mathfrak{t}|^2 + \frac{2\pi}{\mathfrak{t}_2}\,(\mathrm{Im}(\mathfrak{u}_{n,m}))^2\right)}{|\vartheta_1(\mathfrak{u}_{n,m},\mathfrak{t})|^2}$$

$$\mathfrak{u}_{n,m} \;=\; \tau\,(n\,\mathfrak{t}-m)$$

Internal CFT

$$\mathcal{Z}_X(\mathfrak{t},\bar{\mathfrak{t}}) \;=\; \sum_{h_{\rm int},\,\bar h_{\rm int}} D(h_{\rm int},\bar h_{\rm int})\,\mathfrak{z}^{h_{\rm int}}\,\bar{\mathfrak{z}}^{\bar h_{\rm int}}$$

Single-string spectrum

- One-loop spectrum has a well-known spacetime (Hamiltonian) interpretation

$$\mathcal{Z}_{\rm ws}(\tau,\bar\tau) \;=\; -\beta \sum_{m=1}^{\infty} f(m\tau, m\bar\tau)$$

$$f(\tau,\bar\tau) \;=\; \frac{1}{\beta} \sum_{\mathcal{H}_{\text{single-string}}} e^{-\beta E - i\beta\mu\ell}$$

- Single-string spectrum

$$f(\tau,\bar\tau) \;=\; \frac{4}{i\beta\,(k-2)} \int\limits_{-\infty}^{\infty} d\zeta\,\zeta \int_{-\frac{1}{2}}^{\frac{1}{2}} d\mathfrak{t}_1 \int_0^{\infty} d\mathfrak{t}_2\, e^{4\pi\left(a_k - \frac{\zeta^2}{k-2}\right)\mathfrak{t}_2 + 2i\beta\zeta}\, \mathcal{P}_{\rm bos}(\mathfrak{z},\bar{\mathfrak{z}};q,\bar q)$$

$$\mathcal{P}_{\rm bos}(\mathfrak{z},\bar{\mathfrak{z}};q,\bar q) \;=\; \frac{Z_X(\mathfrak{z},\bar{\mathfrak{z}})}{\left|q^{\frac{1}{2}} - q^{-\frac{1}{2}}\right|^2} \left| \prod_{n=1}^{\infty} \frac{1-\mathfrak{z}^n}{(1-q\,\mathfrak{z}^n)\,(1-q^{-1}\,\mathfrak{z}^n)} \right|^2 .$$

The appearance of Virasoro modules

- Single-string states can be separated into discrete states and a continuum

$$f(\tau,\overline{\tau}) \;=\; f_{\text{tachyon}}(\tau,\overline{\tau}) + f_{\text{disc}}(\tau,\overline{\tau}) + f_{\text{cont}}(\tau,\overline{\tau})$$

- (There are also the spectral flow sectors associated with long strings in AdS3 — not important today)

- Discrete states have a *universal* low-lying spectrum

$$f_{\text{disc}}(\tau,\overline{\tau}) \;=\; 1 + \underbrace{\frac{q^2}{1-q} + \frac{\overline{q}^2}{1-\overline{q}} + \frac{(q\,\overline{q})^2}{|1-q|^2}}_{N=1} + \underbrace{\frac{1+q^4}{1-q}\,\frac{1+\overline{q}^4}{1-\overline{q}}\,(q\overline{q})^{-\frac{3}{2}+\sqrt{(k-2)+\frac{1}{4}}}}_{N=2} + \cdots\,.$$

Virasoro module!

TTbar deformation

Mass gap

[Ferko, Murthy, Rangamani ’24]

Superstrings on AdS3 x S3 x T4/K3

- Euclidean AdS3 x S3 + NSNS flux

$$ds^2 = k\Big(\cosh^2\rho\, dt_{\rm E}^2 + d\rho^2 + \sinh^2\rho\, d\varphi^2 + d\theta^2 + \cos^2\theta\, d\phi_1^2 + \sin^2\theta\, d\phi_2^2\Big)$$

$$B = k\Big(\sinh^2\rho\, dt_{\rm E}\wedge d\varphi + \sin^2\theta\, d\phi_1 \wedge d\phi_2\Big)$$

- Solvable RNS worldsheet string theory

$$SL(2)_k \times SU(2)_k \times T^4/K3$$

- Thermal identification with twist around the S3

$$(t_{\rm E},\varphi,\phi_1,\phi_2) \sim (t_{\rm E},\varphi+2\pi,\phi_1,\phi_2) \sim (t_{\rm E},\varphi,\phi_1+2\pi,\phi_2) \sim (t_{\rm E},\varphi,\phi_1,\phi_2+2\pi)$$

$$(t_{\rm E},\varphi,\phi_1,\phi_2) \sim (t_{\rm E}+\beta,\varphi+i\,\beta\,\mu,\phi_1+i\,\beta\,\nu_1,\phi_2+i\,\beta\,\nu_2)\,.$$

The building blocks of the individual CFTs partition functions are known

- Need to twist the SL(2)

$$\mathcal{Z}_{\mathfrak{sl}(2)}(\mathfrak{t},\bar{\mathfrak{t}};\tau,\bar{\tau}) \;=\; \frac{\beta\sqrt{k}}{2\pi\sqrt{\mathfrak{t}_2}} \sum_{n,m\in\mathbb{Z}} \frac{\exp\left(-\frac{k\,\beta^2}{4\pi\,\mathfrak{t}_2}\left|m-n\,\mathfrak{t}\right|^2+\frac{2\pi}{\mathfrak{t}_2}\left(\mathrm{Im}(\mathfrak{u}_{n,m})\right)^2\right)}{\left|\vartheta_1(\mathfrak{u}_{n,m},\mathfrak{t})\right|^2}\,,$$

- Also twist the SU(2)

$$\rho_{n,m} \;=\; \rho\,(n\,\mathfrak{t}-m)$$

$$\mathcal{Z}^{(n,m)}_{\mathfrak{su}(2)}(\mathfrak{t},\bar{\mathfrak{t}};\rho,\bar{\rho}) \;=\; e^{\frac{\pi\,k\,\mathrm{Im}(\rho)^2}{\mathfrak{t}_2}\,|n\,\mathfrak{t}-m|^2-\frac{\pi}{\mathfrak{t}_2}\,(k-2)\,(\mathrm{Im}(\rho_{n,m}))^2} \sum_{\ell=0}^{\frac{k}{2}-1} \left|\frac{\Theta^{(k)}_{2\ell+1}(\rho_{n,m},\mathfrak{t})-\Theta^{(k)}_{-2\ell-1}(\rho_{n,m},\mathfrak{t})}{\vartheta_1\,(\rho_{n,m},\mathfrak{t})}\right|^2$$

$$\Theta^{(k)}_{l}(\rho,\mathfrak{t}) \;=\; \sum_{n\in\mathbb{Z}+\frac{l}{2k}} \mathfrak{z}^{k\,n^2}\,r^{n\,k}\,,\qquad \mathfrak{z}=e^{2\pi i t}\,,\qquad r=e^{2\pi i\rho}$$

[Ferko, Murthy, Rangamani ’24]

RNS fermions handled in light-cone gauge or (better) through super-ghosts

$$\mathcal{Z}_{\rm IIB} = \frac{\beta\sqrt{k}}{2\pi}\int_{\mathcal{F}}\frac{d^2\mathfrak{t}}{\mathfrak{t}_2^{\frac{3}{2}}}\left(\sum_{\Gamma^{4,4}}\mathfrak{z}^{p_L^2}\,\bar{\mathfrak{z}}^{p_R^2}\right)\sum_{n,m}\frac{\exp\left(-\frac{k}{\mathfrak{t}_2}\left(\frac{\beta^2}{4\pi}+\pi\,{\rm Im}(\rho)^2\right)|m-n\,\mathfrak{t}|^2+\frac{2\pi\,k\,{\rm Im}(\rho_{n,m})^2}{\mathfrak{t}_2}\right)}{|\eta(\mathfrak{t})|^{12}\,|\vartheta_1(\mathfrak{u}_{n,m},\mathfrak{t})|^2}$$
$$\times\sum_{\ell=0}^{\frac{k}{2}-1}\left|\frac{\Theta^{(k)}_{2\ell+1}(\rho_{n,m},\mathfrak{t})-\Theta^{(k)}_{-2\ell-1}(\rho_{n,m},\mathfrak{t})}{\vartheta_1\left(\rho_{n,m},\mathfrak{t}\right)}\right|^2\times\begin{cases}\left|{\rm Z}^{(n,m)}_{\rm PF}\right|^2, & \text{periodic fermions,}\\ \left|{\rm Z}^{(n,m)}_{\rm AF}\right|^2 & \text{antiperiodic fermions.}\end{cases}$$

- The twisted fermions, after spin-structure sum, Riemann identities, assemble into spacetime fermions

$$\begin{aligned}{\rm Z}^{(n,m)}_{\rm PF}(\mathfrak{t};\tau,\rho) &= \vartheta_1^2(\mathfrak{v}^+_{n,m},\mathfrak{t})\,\vartheta_1^2(\mathfrak{v}^-_{n,m},\mathfrak{t})\,,\\ {\rm Z}^{(n,m)}_{\rm AF}(\mathfrak{t};\tau,\rho) &= \vartheta_1^2(\mathfrak{v}^+_{n,m},\mathfrak{t})\,\vartheta_1^2(\mathfrak{v}^-_{n,m},\mathfrak{t})+(-1)^n\,\vartheta_2^2(\mathfrak{v}^+_{n,m},\mathfrak{t})\,\vartheta_2^2(\mathfrak{v}^-_{n,m},\mathfrak{t})\\ &\quad+(-1)^m\,\vartheta_4^2(\mathfrak{v}^+_{n,m},\mathfrak{t})\,\vartheta_4^2(\mathfrak{v}^-_{n,m},\mathfrak{t})-(-1)^{n+m}\vartheta_3^2(\mathfrak{v}^+_{n,m},\mathfrak{t})\,\vartheta_3^2(\mathfrak{v}^-_{n,m},\mathfrak{t})\,,\end{aligned}$$

[Ferko, Murthy, Rangamani ’24]

Atick-Witten twist vs CFT2 twist

- On one hand, the twists of the fermions are decided by demanding worldsheet modular invariance

$$(\mathcal{Z}^{\pm}_{\mathrm{AF}})^{(n,m)} \;=\; \mathrm{Tr}_{\mathrm{NS}}\left(\left[\frac{1-e^{i\pi F}}{2}\frac{1+e^{i\pi\, n}}{2}+\frac{1+e^{i\pi F}}{2}\,e^{i\pi\, m}\,\frac{1-e^{i\pi\, n}}{2}\right]e^{2\pi i\,\mathfrak{t}\,L_0}\right) - \mathrm{Tr}_{\mathrm{R}}\left(\left[\frac{1\pm e^{i\pi F}}{2}\frac{1-e^{i\pi\, n}}{2}+\frac{1+e^{i\pi F}}{2}\,e^{i\pi\, m}\,\frac{1-e^{i\pi\, n}}{2}\right]e^{2\pi i\,\mathfrak{t}\,L_0}\right)$$

- On the other hand, boundary SCFT2 R-symmetry ⇨

$$Z_{\mathrm{CFT}}(\tau,\overline{\tau},\rho,\overline{\rho}) \;=\; \mathrm{Tr}\,\exp\Big(2\pi i\,\tau\,L_0^{\mathrm{CFT}} - 2\pi i\,\overline{\tau}\,\overline{L}_0^{\mathrm{CFT}} + 2\pi i\,\rho\,J_{\mathrm{CFT}} - 2\pi i\,\overline{\rho}\,\overline{J}_{\mathrm{CFT}}\Big)$$

- Nicely enough, two notions agree for the partition function

$$\mathrm{Z}_{\mathrm{PF}}(\mathfrak{t};\tau,\rho+1) \;=\; \mathrm{Z}_{\mathrm{AF}}(\mathfrak{t};\tau,\rho)\,, \qquad \mathrm{Z}_{\mathrm{PF}}(\mathfrak{t};\tau,\rho+2) \;=\; \mathrm{Z}_{\mathrm{PF}}(\mathfrak{t};\tau,\rho)$$

[Ferko, Murthy, Rangamani ’24]

Useful to organize spectrum in terms of spacetime symmetries

- Spacetime symmetry $\mathfrak{psu}(1,1|2) \oplus \overline{\mathfrak{psu}}(1,1|2)$

reduced $\mathfrak{psu}(1,1|2)$ *character*

$$\chi_{\rm g}(\ell) = \chi_{\rm g}(\ell; r, q) = q^{\ell}\,\chi_{\ell}(r) - 2\,q^{\ell+\frac{1}{2}}\,\chi_{\ell-\frac{1}{2}}(r) + q^{\ell+1}\,\chi_{\ell-1}(r)\,,$$

su(2) characters

reduced $\mathfrak{psu}(1,1|2) \oplus \overline{\mathfrak{psu}(1,1|2)}$ *characters*

$$\boldsymbol{\xi}^{(1)}(j) = \chi_{\rm g}(j)\,\overline{\chi}_{\rm g}(j)\,, \quad \boldsymbol{\xi}^{(\frac{3}{2})}(j) = \chi_{\rm g}(j+\frac{1}{2})\,\overline{\chi}_{\rm g}(j)\,, \quad \boldsymbol{\xi}^{(2)}(j) = \chi_{\rm g}(j+1)\,\overline{\chi}_{\rm g}(j)\,.$$

- Supergravity modes [Deger, Kaya, Sezgin, Sundell '98]

$$s = 2: \qquad \sum_{j\geq 0} \boldsymbol{\xi}^{(2)}(j) + \overline{\boldsymbol{\xi}}^{(2)}(j)\,,$$

$$s = \frac{3}{2}: \qquad 2\,\boldsymbol{\xi}^{(\frac{3}{2})}(0) + 2\,\overline{\boldsymbol{\xi}}^{(\frac{3}{2})}(0) + \sum_{j\geq\frac{1}{2}} \mathbf{4}_5\,\boldsymbol{\xi}^{(\frac{3}{2})}(j) + \mathbf{4}_5\,\overline{\boldsymbol{\xi}}^{(\frac{3}{2})}(j)\,,$$

$$s = 1: \qquad \sum_{j\geq 1/2} \mathbf{5}_5\,\boldsymbol{\xi}^{(1)}(j) + \boldsymbol{\xi}^{(1)}(j+\tfrac{1}{2})\,.$$

Single-string spectrum: supergravity modes

- Single-string spectrum:

$$
\begin{aligned}
f_{\rm PF}(\tau,\rho,\overline{\tau},\overline{\rho})\Big|_{\rm sugra} &= 1 + \frac{\boldsymbol{\xi}^{(2)}(0) - 2\,\boldsymbol{\xi}^{(\frac{3}{2})}(0)}{1-q} + \frac{\overline{\boldsymbol{\xi}}^{(2)}(0) - 2\,\overline{\boldsymbol{\xi}}^{(\frac{3}{2})}(0)}{1-\overline{q}} \\
&+ \frac{\boldsymbol{\xi}^{(1)}(1) - 2\,\boldsymbol{\xi}^{(\frac{3}{2})}\left(\frac{1}{2}\right) - 2\,\overline{\boldsymbol{\xi}}^{(\frac{3}{2})}\left(\frac{1}{2}\right) + 4\,\boldsymbol{\xi}^{(1)}\left(\frac{1}{2}\right)}{|1-q|^2} \\
&+ \sum_{\frac{1}{2}\le j\le \frac{k}{2}-1} \frac{\boldsymbol{\xi}^{(2)}(j) + \overline{\boldsymbol{\xi}}^{(2)}(j) + \boldsymbol{\xi}^{(1)}(j) + 4\,\boldsymbol{\xi}^{(1)}\left(j+\frac{1}{2}\right) + \boldsymbol{\xi}^{(1)}(j+1)}{|1-q|^2} \\
&- 2 \sum_{\frac{1}{2}\le j\le \frac{k}{2}-1} \frac{\boldsymbol{\xi}^{(\frac{3}{2})}(j) + \overline{\boldsymbol{\xi}}^{(\frac{3}{2})}(j) + \boldsymbol{\xi}^{(\frac{3}{2})}\left(j+\frac{1}{2}\right) + \overline{\boldsymbol{\xi}}^{(\frac{3}{2})}\left(j+\frac{1}{2}\right)}{|1-q|^2}\,.
\end{aligned}
$$

Supergravity modes

[Ferko, Murthy, Rangamani ’24]

Single-string spectrum: currents

$$\frac{\boldsymbol{\xi}^{(2)}(0) - 2\,\boldsymbol{\xi}^{(\frac{3}{2})}(0)}{1-q} = \frac{q\,\chi_1(r) - 2\,q^{\frac{3}{2}}\,\chi_{\frac{1}{2}}(r) + q^2}{1-q} - 2\,\frac{q^{\frac{1}{2}}\,\chi_{\frac{1}{2}}(r) - 2\,q}{1-q}$$

su(2) R currents

supercurrent (gravitini)

boundary gravitons

4 spin-half fermions

4 U(1) currents

N=(4,4) superconformal algebra

Symmetries of T4 (promoted to current algebra)

[Ferko, Murthy, Rangamani ’24]

How does this agree with the expectations for currents?

- Currents of (4,4) algebra expected from boundary: Brown-Henneaux analysis.

- The 4+4 U(1) currents (+superpartners) correspond to gauge fields from NSNS excitations in the T4 directions.

- In the boundary theory, this corresponds to overall (center of mass) T4 in $\mathrm{Sym}^N(T^4)$

How does this agree with the expectations for currents?

- Naively, RR gauge fields could also give boundary currents (another 4+4).

- Harmonic analysis suggests corresponding gauge fields in spectrum.

- However, our analysis shows that it is not consistent with modular invariant string 1-loop amplitude.

- Conclusion agrees with recent analysis of boundary theory, extra currents decoupled sector of theory.

[Aharony, Urbach ’24]

Consequences on near-extremal BTZ BHs

- We can read off the thermodynamics of near-extremal BTZ BHs

$$\mathcal{Z}_{\rm CFT}(\tau,\overline{\tau})\Big|_{\rm BTZ} = \mathcal{Z}_{\rm CFT}\left(-\frac{1}{\tau},-\frac{1}{\overline{\tau}}\right)\Big|_{\rm thermal\ AdS_3}$$

Read off from string spectrum

- We want large BH, but small temperature

$$T = \frac{1}{\pi\, i}\,\frac{1}{\overline{\tau}-\tau}\,, \qquad \Omega = \pi\,(\tau+\overline{\tau})\,.$$

$$\tau \to i\,\infty\,, \qquad \overline{\tau}\to -i\,0$$

Consequences on near-extremal BTZ BHs

$$\mathcal{Z}_{\rm CFT}(\tau,\overline{\tau})\Big|_{\rm BTZ} = e^{S_{\rm tree}-\frac{2\pi i}{24}\left(\frac{1}{\tau}-\frac{1}{\overline{\tau}}\right)+\cdots} \times \left|\frac{\left(1-e^{-\frac{2\pi i}{\tau}}\right)}{\eta(-\frac{1}{\tau})}\right|^2 \times \prod_{n_1,n_2=0}^{\infty} \frac{1}{1-e^{-2\pi i\left(\frac{2+n_1}{\tau}-\frac{2+n_2}{\overline{\tau}}\right)}} \times \cdots .$$

$$\underset{\overline{\tau}\to -i\,0}{\sim} \quad e^{\frac{2\pi i}{24}(c-1)\left(\frac{1}{\tau}-\frac{1}{\overline{\tau}}\right)} \times \frac{\left(1-e^{-\frac{2\pi i}{\tau}}\right)}{(-i\,\tau)^{\frac{1}{2}}\;\eta(\tau)}$$

$$\underset{\tau\to i\,\infty}{\sim} \quad (-i\,\tau)^{-\frac{3}{2}}\;\exp\left(-\frac{2\pi i\,(c-1)}{24\,\overline{\tau}} - \frac{2\pi i\,\tau}{24}\right)$$

- In the canonical ensemble, [cf. Ghosh, Maxfield, Turiaci, '19]

$$\widetilde{\mathcal{Z}}_{\rm CFT}(T,J)\Big|_{\rm BTZ} \sim T^{\frac{3}{2}}\,J^{-\frac{3}{4}}\,e^{2\pi\sqrt{\frac{c}{6}J}}$$

- Our analysis shows that these results hold at finite k = non-zero α' [Ferko, Murthy, Rangamani '24]

Consequences on near-extremal BTZ BHs (supersymmetric theory)

- Anti-periodic fermions. Zero modes from SL(2), SU(2), T4.

$$\mathcal{Z}^{\mathrm{NS}}_{\mathrm{CFT}}(\tau,\rho,\overline{\tau},\overline{\rho})\Big|_{\mathrm{BTZ}} \sim e^{S_{\mathrm{tree}}}\,\frac{T^5}{T^5_{\mathrm{gap}}}\sum_{m\in\mathbb{Z}}\frac{\left(m+\frac{\rho}{2}\right)\,e^{\frac{T}{T_{\mathrm{gap}}}\left(1-\left(m+\frac{\rho}{2}\right)^2\right)}}{\sin(\pi\rho)}$$

- Canonical ensemble $T^{3/2}$

- Periodic fermions. Bosons + fermions

$$\mathcal{Z}^{R}_{\mathrm{CFT}}(\tau,\rho,\overline{\tau},\overline{\rho})\Big|_{\mathrm{BTZ}} \sim e^{S_{\mathrm{tree}}}\,\frac{T}{T_{\mathrm{gap}}}\,\frac{\cos^4(\frac{\pi}{2}\,\rho)}{\sin(\pi\rho)}\sum_{m\in\mathbb{Z}}\frac{\left(m+\frac{\rho}{2}\right)\,e^{\frac{T}{T_{\mathrm{gap}}}\left(1-4\left(m+\frac{\rho}{2}\right)^2\right)}}{\left(1-4\,\left(m+\frac{\rho}{2}\right)^2\right)^2}$$

- After absorbing T4 fermions, index $\sim\; T^0$ as $T\to 0$

[Ferko, Murthy, Rangamani ’24]

Further directions

- String theory on AdS3 x S3 x K3.
  Dual heterotic AdS3 x S3 x T4.

  [work in progress,
  Murthy, Rangamani]

- String theory on AdS3 x S3 x S3 x S1

- Generalize results to RR flux. k=1?

- AdS5? Analyze Metsaev-Tseytlin string with this viewpoint? (Note only finite symmetry charges.)

- Harder: Schwarzian in higher-dimensional theory?

Спасибо!