Primordial gravitational waves from spontaneous breaking of discrete symmetries

Sabir Ramazanov (ITMP MSU, Moscow)

Beware: lots of hep-ph and astro-ph!

Based on 2406.17053, 2410.21971, 2307.04582

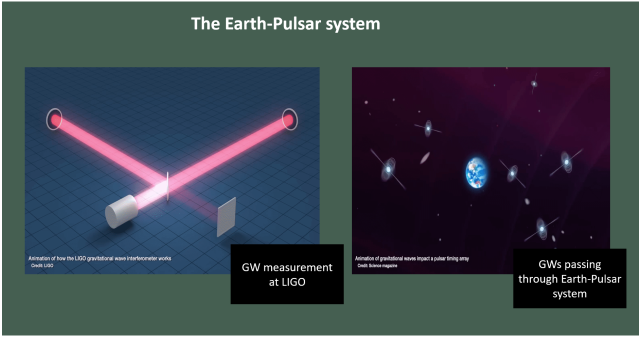
with E. Babichev, I. Dankovsky, D. Gorbunov,

R. Samanta, A. Vikman

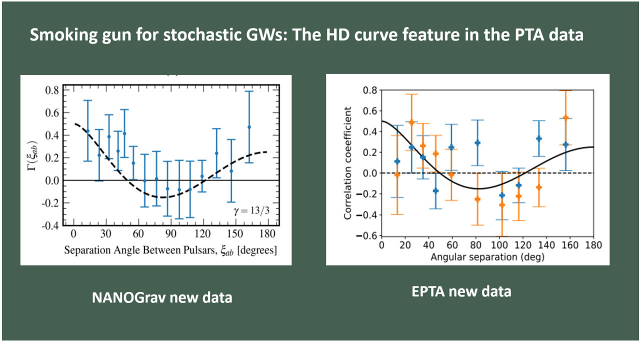
Strong evidence of stochastic GW background has been reported: NANOGrav, EPTA+InPTA, CnPTA, PPTA

$$\Omega_{gw}(f) \simeq 5.8 \cdot 10^{-8} \cdot \left(\frac{f}{30 \text{ nHz}} \right)^n \qquad n = 1.8 \pm 0.6$$

$$\Omega_{gw}(f) \equiv \frac{d\rho_{gw}}{\rho_{tot}\, d \ln f}$$



Hellings-Downs curve'83



$$\langle \frac{\delta\nu_a}{\nu_a}\frac{\delta\nu_b}{\nu_b}\rangle \propto \frac{1}{2} - \frac{1}{4}\left(\frac{1-\cos\xi_{ab}}{2}\right) + \frac{3}{2}\left(\frac{1-\cos\xi_{ab}}{2}\right)\ln\left(\frac{1-\cos\xi_{ab}}{2}\right)$$

Supermassive black hole binaries (SMBHB) mergers are often quoted as the most common source of the background found, but...

- GW driven SMBHBs predict $n = 2/3$ (or $\gamma = 5 - n = 13/3$) versus the NANOGrav $n = 1.8 \pm 0.6$, excluded at more than 2σ CL
- final pc problem
- SMBHs are difficult to produce, $M \sim 10^{10}\ M_{\text{sun}}$

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NANOGrav: Afzal et al'23

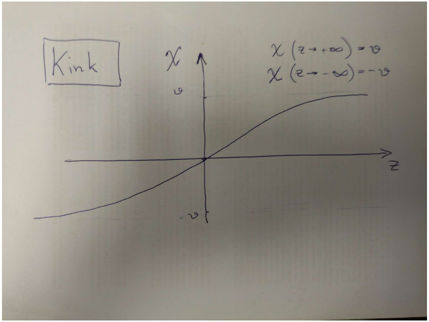
"*...we investigate potential cosmological interpretations of this signal, specifically cosmic inflation, scalar-induced GWs, first-order phase transitions, cosmic strings, and domain walls. We find that, with the exception of stable cosmic strings of field theory origin, all these models can reproduce the observed signal.*"

NB Cosmological =primordial=(in this context) operating at radiation domination or earlier epoch

Domain walls arise in models with spontaneous breaking of discrete symmetries, e.g., Z_2 Zel'dovich, Kobzarev, and Okun'74

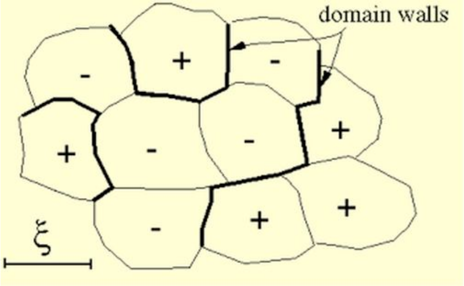
$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{\lambda \cdot (\chi^2 - v^2)^2}{4}$$

Static localized 1+1 solution   Kink $\chi(z) = v \cdot \tanh\left(\sqrt{\frac{\lambda}{2}} \cdot v \cdot z\right)$



Domain walls are embeddings of kinks into $1+3$

Domain walls separate regions, where $\chi = \pm v$



Domain wall tension: $\sigma_{wall} = \int_{-\infty}^{+\infty} dz' \, T_{00}(z') = \frac{2\sqrt{2\lambda} v^3}{3}$

$\delta_{wall} \sim \sqrt{\frac{2}{\lambda}} \frac{1}{v} \sim \frac{1}{m_\chi}$

http://www.ctc.cam.ac.uk/

$$\boxed{v = \text{const}} \quad \text{vs} \quad \boxed{v(t) \propto T(t) \propto \frac{1}{a(t)}}$$

Standard domain walls vs Melting domain walls

$\boxed{v = \text{const}}$ vs $\boxed{v(t) \propto T(t) \propto \frac{1}{a(t)}}$

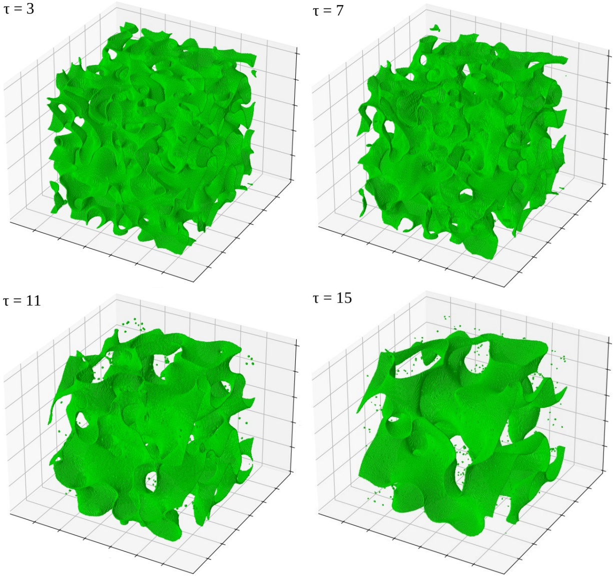
Standard domain walls vs Melting domain walls

Goals: study evolution of both standard and melting domain walls and their gravitational waves at radiation domination with

CosmoLattice $N = 512, 1024, 2048$

Figueroa, Florio, Torrenti, Valkenburg'20 '21

Evolution of standard domain walls



Scaling: O (1) domain wall in the horizon volume $\sim H^{-3}$.
with a curvature radius $\sim H^{-1}$ Ryden, Press, Spergel'89

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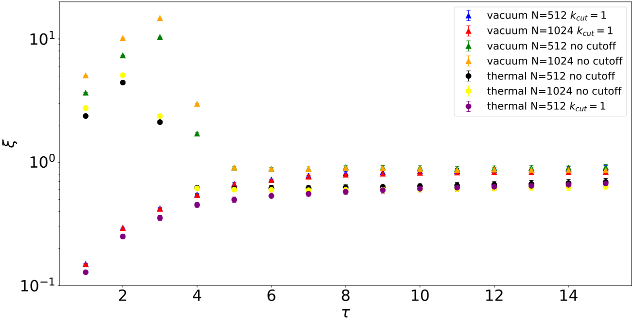
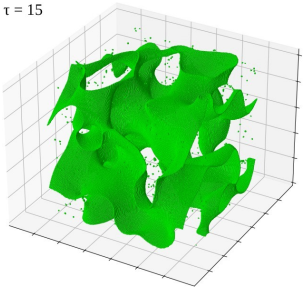
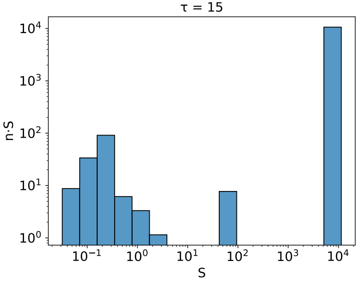
$$\xi = \frac{St}{a(t)V} = \text{const} \qquad \rho_{wall} \sim \frac{\sigma_{wall} H^{-2}}{H^{-3}} \sim \sigma_{wall} H \qquad \delta_{wall} \sim 0.05 H^{-1}_{scaling}$$







$$r \equiv \frac{S(closed\ \ walls)}{S(long\ \ wall)} \simeq 0.01$$

How does the system enter scaling?

Naive expectations: formation of closed walls $\Longrightarrow$ particle production *a la* the case of cosmic strings Seems that it does not work!

The energy loss mechanism appears to be direct radiation from a long wall

Garagounis and Hindmarsh'02

Theoretical analysis of domain wall evolution:
work in progress with Batyr Gafarov and Mauricio Valencia-Villegas

Domain wall problem

$$\rho_{wall} \sim \sigma_{wall} H \sim \sigma_{wall} \cdot \frac{T^2}{M_{Pl}} \quad \text{vs} \quad \rho_{rad} \sim T^4$$

$$\frac{\rho_{wall}}{\rho_{rad}} \propto \frac{1}{T^2(t)} \propto a^2(t) \Longrightarrow$$

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Domain walls are too energetic
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Possible solution: explicitly break Z_2-symmetry

E.g., $V_{bias}(\chi) = \epsilon v \chi^3$

For properly chosen $\epsilon \Longrightarrow$ destruction happens at radiation stage

Implementation of the bias term is a work in progress.

Domain walls emit gravitational waves

- By construction, domain walls are spatially inhomogeneous.

$$\left(\frac{\partial^2}{\partial \tau^2} + \frac{2a'}{a} \frac{\partial}{\partial \tau} - \frac{\partial^2}{\partial x^2} \right) h_{ij}^{TT} = 16\pi G_N T_{ij}^{TT}$$

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- Domain walls are very energetic.

  Most energetic gravitational waves are emitted, when the domain wall network is being destroyed.

$$\rho_{gw} = \frac{1}{32\pi G_N a^2} \cdot \langle \frac{\partial h_{ij}^{TT}}{\partial\tau}\frac{\partial h_{ij}^{TT}}{\partial\tau}\rangle \qquad \rho_{gw} \sim \frac{\sigma_{wall}^2}{M_{Pl}^2} \Longrightarrow \frac{\rho_{gw}}{\rho_{rad}} \propto a^4$$

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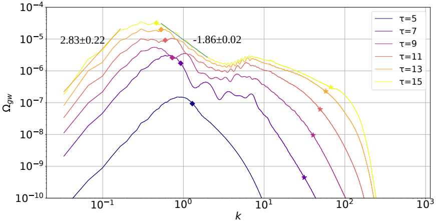
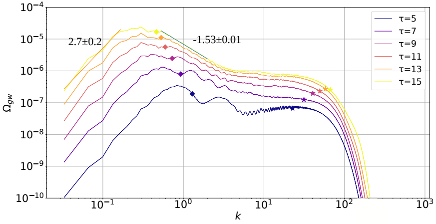
- Emission occurs at a characteristic frequency $\sim H^{-1}$ (scaling!!)

  100 MeV $\lesssim T \lesssim 10^{10}$ GeV $\Longrightarrow$

  frequency is in a wide range covering PTAs and Einstein Telescope





Earlier numerical simulations: Hiramatsu, Kawasaki, Saikawa'13
Kitajima et al'23
Ferreira et al'23

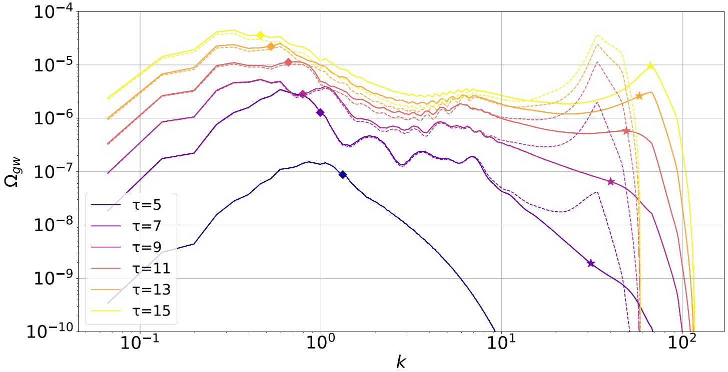
Good agreement in the IR frequency part:

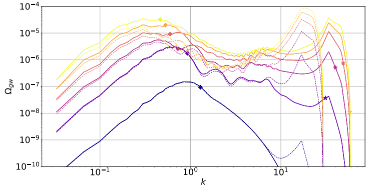
$\Omega_{gw}(k) \propto k^3$ for $k\tau_f \ll 1$ by causality!

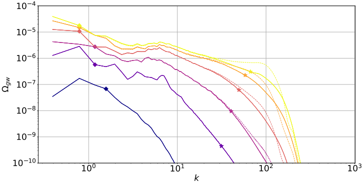
Some potentially relevant discrepancies in the UV part:

$n \approx -1.5$ vs $n = -1$ and unusual power excess followed by the exponential falloff

Compare results obtained with 1024^3 and 2048^3 lattices.







Fitting to PTA data

$$\Omega_{gw,peak} h_0^2 \approx 1.0(0.6)\times 10^{-10} \cdot \left(\frac{100\ \text{MeV}}{T_{dec}}\right)^4 \cdot \frac{\sigma_{wall}^2}{(100\ \text{TeV})^6} \cdot \left(\frac{10}{g_*(T_{dec})}\right)^{4/3} ,$$

$$f_{peak} \approx 0.7 H_{dec} \cdot \frac{a_{dec}}{a_0} \approx 7.5\ \text{nHz} \left(\frac{T_{dec}}{100\ \text{MeV}}\right) \cdot \left(\frac{g_*(T_{dec})}{10}\right)^{1/6} ,$$

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$$\Omega_{gw}(f) = \Omega_{peak} \cdot \left(\frac{f}{f_{peak}}\right)^3 \qquad f \ll f_{peak} \qquad \text{Causality}$$

$n = 1.8 \pm 0.6$ 68% CL NANOGrav 15 yr

Not bad, but probably one could do better.

Melting domain walls

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{\lambda(\chi^2 - v^2(T))^2}{4} \qquad v \propto T \propto \frac{1}{a}$$

$$\sigma_{wall} = \frac{2\sqrt{2\lambda} v^3}{3} \propto T^3$$

$$\rho_{wall} \simeq \sigma_{wall} H \propto T^5 \qquad \frac{\rho_{wall}}{\rho_{rad}} \propto T(t) \propto \frac{1}{a(t)}$$

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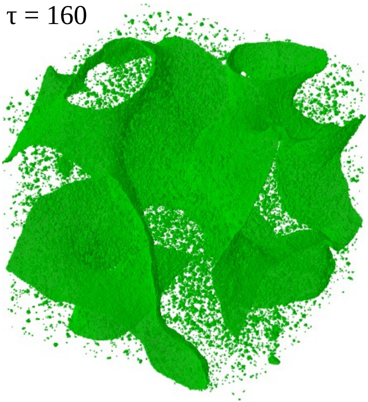
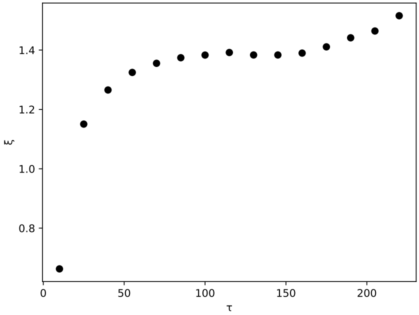
Energy density of domain walls redshifts faster than radiation $\Longrightarrow$ no domain wall problem

Vilenkin'81

$$\xi = \frac{St}{a(t)V} \approx \text{const}$$

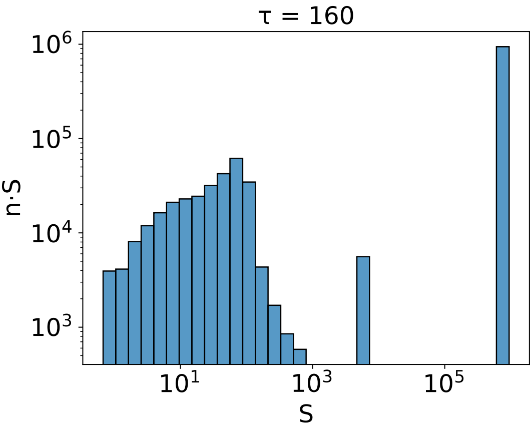
$$\delta_{wall} \sim 0.05 H^{-1}_{scaling}$$





$$r = \frac{S(\text{closed walls})}{S(\text{long wall})} \simeq 0.3 \Longrightarrow$$

It sounds plausible that melting domain walls enter scaling by formation of collapsing closed walls



Melting domain walls: the strongest GW signal comes from the earliest times. Opposite to standard walls!

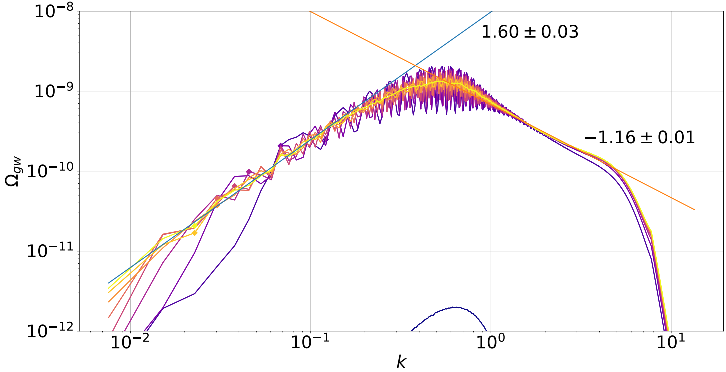
The earliest relevant time: the onset of scaling $\tau_{scaling}$

$$k_{peak} \sim \frac{2\pi}{\tau_{scaling}}$$

Later time emission at $\tau > \tau_{scaling}$ feeds into lower frequencies.

$\rho_{wall} \propto \frac{1}{a^5}$ +scaling+statistical homegeneity+isotropy

$$\Longrightarrow \Omega_{gw} \propto k^2 \qquad \frac{2\pi}{\tau_f} \ll k \ll k_{peak}$$



Very good agreement with the NANOGrav 15 yr $n = 1.8 \pm 0.6$

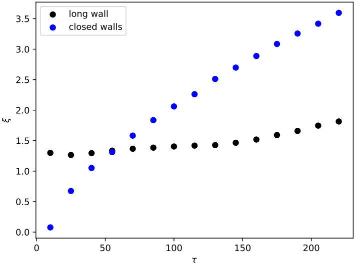
No violation of causality: causal tail $n = 3$ is shfited towards very small frequencies

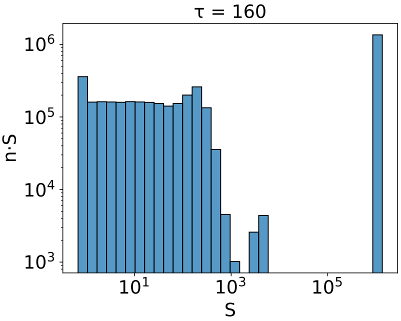
The theoretical prediction $n = 2$ assumes an infinite duration of melting DWs/ finite duration of the source $\Longrightarrow$ small tilt

Scaling is violated for large initial scalar fluctuations $\delta\chi_i \gtrsim 0.1 v_i$

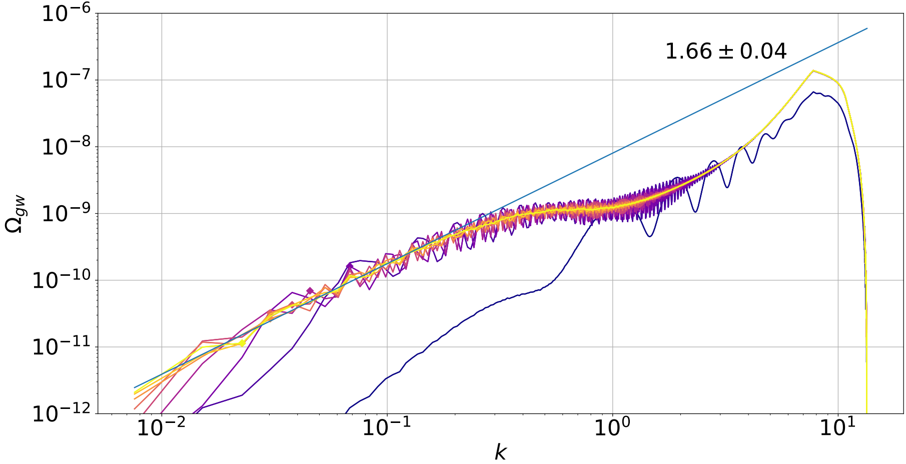
This violation is mainly due to abundant production of closed walls

It is likely to be of non-physical origin $\Longrightarrow$ small scalar fluctuations of non-topological origin are misinterpreted as closed walls





Effect of scaling violation on GW spectrum: peak $\to$ inflection point, but remarkably the IR part of the spectrum is almost unaffected



$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{\lambda \cdot \chi^4}{4} + \frac{g^2 \chi^2 \phi^\dagger \phi}{2}$$

2104.13722

χ is cold

ϕ is in thermal equilibrium with plasma

$$0 < g^2 \ll 1$$

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$$\langle \phi^\dagger \phi \rangle_T = \frac{\mathcal{N} T^2}{12} \Longrightarrow V_{eff} = \frac{\lambda \cdot \chi^4}{4} - \frac{\mathcal{N} g^2 T^2 \chi^2}{24}$$

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$T \propto \frac{1}{a(t)} \Longrightarrow Z_2$-symmetry breaking at early times

$$v^2 = \frac{\mathcal{N} g^2 T^2}{12\lambda}$$

Fitting to NANOGrav

$$f_{peak} \simeq \frac{15\ \text{nHz}\sqrt{\mathcal{N}}}{g_*^{1/3}(T_{sc})} \cdot \left(\frac{g}{10^{-18}}\right)$$

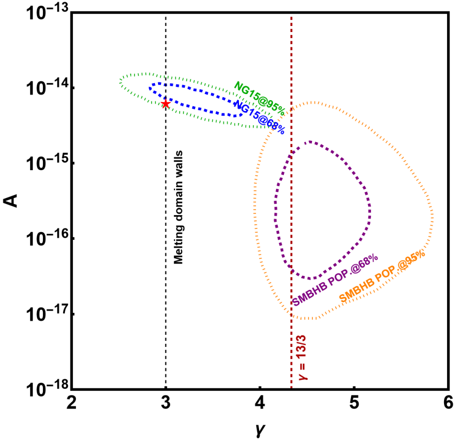
$$\Omega_{gw,peak} h_0^2 \simeq \frac{5 \cdot 10^{-11} \mathcal{N}^4}{g_*^{7/3}(T_{sc}) \cdot \beta^2}$$

Vanilla region: $\beta \equiv \frac{\lambda}{g^4} \simeq 1$ $\mathcal{N} \gg 1$

The field χ should be extremely weakly coupled!

Not an unfamiliar situation in physics, cf. axions, but we deal with a different group of underlying symmetries.

$$g^2 \simeq 10^{-36} \quad \lambda \simeq 10^{-72} \quad \boxed{\mathcal{N} = 24}$$

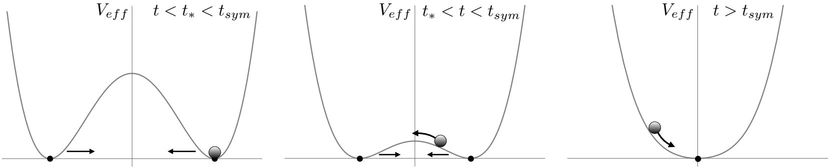


$$A = \sqrt{\frac{3\Omega_{gw,peak} H_0^2}{2\pi^2 f_{peak}^2}}$$

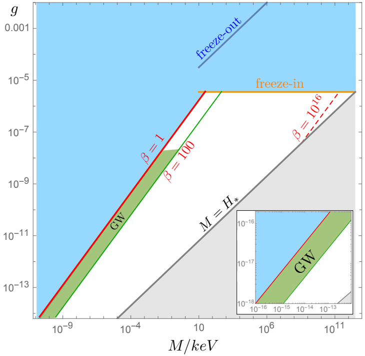
A bit of dark matter

Slightly break conformal invariance $\Longrightarrow$ dark matter
2104.13722, 2112.12608,

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{M^2 \cdot \chi^2}{2} - \frac{\lambda \cdot \chi^4}{4} + \frac{g^2 \chi^2 \phi^\dagger \phi}{2} \, .$$



Abundance constraint: $M \simeq 3\times 10^{-13}\text{eV}\cdot\frac{\beta^{3/5}}{\sqrt{\mathcal{N}}}\cdot\left(\frac{g}{10^{-18}}\right)^{7/5}$



$M \simeq 10^{-12} - 10^{-13}$ eV $\Longrightarrow$ superradiance Zel'dovich

- Evolution of standard domain walls has been revisited with a publicly available CosmoLattice: low number of closed walls+important features of GW spectrum in UV.
- Melting domain walls avoid the problem of overclosing the Universe+ the spectral index of GWs is in excellent agreement with PTA data.
- The field constituting melting domain walls is extremely weakly coupled in the PTA range. However, the model is not limited to PTA, also LISA, TianQin, Einstein Telescope...
- The same field can be also a dark matter candidate with possible implications for Kerr black holes.

Thanks for your attention!!!