Quantum states and their back-reacted geometries

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Based on recent work with Yohan Potaux and Deb Sarkar, arXiv:2112.03855

Classical black holes

- Black hole solutions in Einstein theory of gravity $W_{cl} = -\frac{1}{16\pi G}\int R$
- Remarkable stability: Sufficiently large black holes remain to be solutions if added any finite number of local higher curvature terms

Quantum fields

Different states (vacua) for a quantum field:

Hartle-Hawking state: stress-energy tensor is regular at horizon, contains thermal radiation at infinity, describes black hole in thermal equilibrium with Hawking radiation

Boulware state: stress-energy tensor is singular at horizon, vanishing at infinity

Unruh state: stress-energy tensor is regular only at the future horizon, and there is a thermal flux of radiation at future null infinity, describes the process of black hole evaporation

"Quantum Black holes" as solutions to some "quantum" gravitational theory

Some questions we want to answer:

- does the quantum-corrected metric have a horizon?
- if there is a horizon, how does its position and the Hawking temperature change with respect to the classical situation?
- what happens at asymptotic infinity? If there exists a thermal Hawking radiation as in the Hartle-Hawking state, then it curves the spacetime and it is likely that we no longer have Minkowski spacetime.
- how do the answers to the previous questions depend on the choice of the quantum state? What are the back-reacted geometries for the Hartle-Hawking and Boulware states?
- provided the quantum-corrected geometries are horizonless, how close are they to the classical black hole geometry? Can they be considered as the black hole mimickers?

Previous works of York, Sanchez-Lousto, Zaslavsky, Emparan-Kaloper-Fabbri, ..

Semiclassical Gravity: background metric is classical while the matter is quantum

Integrating out the quantum fields one ends up with a modified gravitational action

$$W_{sc}[g] = W_{cl}[g] + W_Q[g]$$

where W_{cl} is classical gravitational action and W_Q is induced by quantum fields

In general W_Q is non-local and very complicated functional of metric

It can be represented as an expansion in curvature. In 4 dimensions

$$W_Q = \int \gamma_i(-\Box_2)\mathcal{R}_1\mathcal{R}_2(i) + \Gamma_i(-\Box_1, -\Box_2, -\Box_3)\mathcal{R}_1\mathcal{R}_2\mathcal{R}_3(i) + O(\mathcal{R}^4)$$

works of Barvinsky, Vilkovisky, Gusev, Zhytnikov (1987-1997)

Rather difficult (although not impossible) to deal with...

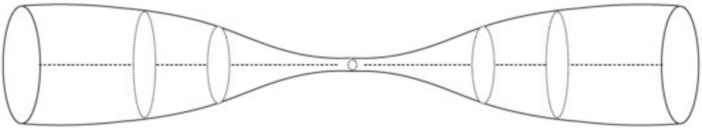
What to expect: Wormholes as black hole mimickers

Replace black hole with a wormhole T. Damour and SS (2007)

$$-g_{tt} \to -g_{tt} + \epsilon^2 , \quad \epsilon^2 \ll 1$$

$$ds^2_{wh} = -(g(r) + \epsilon^2)dt^2 + g(r)^{-1}dr^2 + r^2 d\omega^2_d$$

$$\rho = \int g^{-1}(r)dr , \qquad -\infty < \rho < +\infty$$



Some properties:

- there is no event horizon
- it is replaced by a wormhole throat where $(-g_{tt}) \sim \epsilon^2 \ll 1$
- $t_{throat} \sim \epsilon t_\infty$
- a new time scale:

$$t_H \sim r_+ \ln 1/\epsilon$$

Choosing $\epsilon \sim e^{-S_{BH}}$ one has that t_H is of the order of the Hawking evaporation time

For observation times $t \ll t_H$ no difference with true black holes

Some properties that may be non-generic:

- Z_2-symmetry: $\rho \to -\rho$ (in particular, asymptotically flat on both ends)
- minimum of $(-g_{tt})$ and minimal surface $(\nabla r)^2 = 0$ are in the same place

Earlier 4D analysis (a work with D. Sarkar and C. Berthiere (2017)):

- Local "near-horizon" analysis based on a combination of conformal anomaly and mapping the near-horizon region to $S_1 \times H_3$
- Corresponds to Boulware state
- No-horizon but a narrow throat: $min(-g_{tt})$ and $(\nabla r)^2 = 0$ in the same place
- $min(-g_{tt}) \sim e^{-S_{BH}}$ as anticipated
- Global analysis, integrating equations from the throat to infinity, is desirable but technically complicated and at the moment unavailable

In this talk: 2d dilaton gravity

I will focus on global structure of the back-reacted solutions in the semiclassical gravity depending on the choice of the quantum vacuum

Two dimensions: things are simplified

For a 2d conformal field theory the quantum action is know to be the Polyakov action

$$W_Q[g] = \frac{c}{96\pi} \int R \frac{1}{\Box} R$$

This makes the two-dimensional gravitational models quite attractive as toy models to address the quantum black holes and the information problem

Callan, Giddings, Harvey and Strominger (1992) and many publications since then

2D dilaton gravity

In two dimensions one needs to introduce an extra field (dilaton) to define dynamical gravity

String inspired 2d dilaton gravity

$$I_0 = \frac{1}{2\pi}\int_M d^2x\sqrt{-g}e^{-2\phi}(R + 4(\nabla\phi)^2 + 4\lambda^2)$$

This action is distinguished by the fact that in this theory the one-loop beta function vanishes, Russo-Tseytlin (1992).

Witten (1991), Mandal, Sengupta, Wadia (1991)

Trace of gravitational equations implies that $R = -2\Box\phi$

Trace free gravitational equations $\nabla_\mu\nabla_\nu\phi = \frac{1}{2}g_{\mu\nu}\Box\phi$

imply Killing vector $\xi_\mu = \epsilon_\mu^{\ \nu}\partial_\nu\phi$

General solution is static

$$ds^2 = -g(x)dt^2 + \frac{1}{g(x)}dx^2\,, \quad g(\phi) = 1 - ae^{2\phi}\,, \quad \phi = -\lambda x$$

It describes a 2d black hole with Hawking temperature $T_H = \frac{\lambda}{2\pi}$, mass $M = \frac{a\lambda}{\pi}$ and entropy $S_{cls} = 2e^{-2\phi_h} = 2a$. Curvature singularity is at $\phi = +\infty$.

Quantum CFT on a 2d black hole background

Local form of Polyakov action

$$I_1 = -\frac{\kappa}{2\pi}\int d^2x(\frac{1}{2}(\nabla\psi)^2 + \psi R)\,, \quad \kappa = \frac{N}{24}$$

Equation for ψ: $\Box\psi = R$

Equations of motion in dilaton gravity: $R = -2\Box\phi$

Relation to dilaton: $\psi = -2\phi + w$, $\Box w = 0$

For static metric $R = -g''(x)$ so that $w'(x) = \frac{C}{g(x)}$, where C is constant

As we will see in a moment w (or constant C) contains information about the choice of quantum state

Renormalized stress-energy tensor on the (fixed) dilaton black hole background

Energy density: $T^0_0 = \frac{\kappa}{\pi}[3\lambda^2 g(x) - 2\lambda^2 + \frac{C(C+4\lambda)}{4g(x)}]$

At infinity ($x \to \infty$): $T^0_0 = \frac{\kappa}{4\pi}(C + 2\lambda)^2$

Hartle-Hawking state (regular at horizon): $C = 0$ or $C = -4\lambda$

At infinity ($x \to \infty$) $T^0_{0\,HH} = \frac{\kappa\lambda^2}{\pi} = \frac{\pi}{6}NT^2$ (thermal Hawking radiation)

At horizon $x \to x_h$: $T^0_{0\,HH} = -\frac{2\kappa\lambda^2}{\pi}$

Boulware state: $C = -2\lambda$

$T^0_{0\,B} = 0$ at infinity (no radiation) and is singular at horizon ($g(x_h) = 0$)

In general: a *family of states* parametrised by C

$$T^0_0(C_1) - T^0_0(C_2) = \frac{\kappa}{4\pi g(x)}(C_1 - C_2)(C_1 + C_2 + 4\lambda)$$

Equivalence between C_1 and $C_2 = -(C_1 + 4\lambda)$

Boulware state is symmetric under this map

2d RST model

To address the issue of back-reaction we consider RST model

$$I = I_0 + I_1 + I_2\,, \quad I_2 = -\frac{\kappa}{2\pi}\int d^2x\sqrt{-g}\,\phi R$$

Russo, Susskind, Thorlacius (1992)

$\kappa = (N-24)/24$

A combination of dilaton and $g^{\mu\nu}T_{\mu\nu} = 0$ ($T_{\mu\nu} = T^{(0)}_{\mu\nu} + T^{(1)}_{\mu\nu} + T^{(2)}_{\mu\nu}$) leads to

$$(R + 2\Box\phi)(\kappa - 2e^{-2\phi}) = 0$$

One solution is de Sitter spacetime with constant dilaton,

$$R = -2\lambda^2\,, \quad 2\phi = -\ln\frac{\kappa}{2} = \mathrm{const}$$

The other is with non-constant dilaton with same relations as in classical case

$$R = -2\Box\phi\,, \quad \psi = -2\phi + w\,, \quad \Box w = 0$$

This simplifies integration of field equations

Quantum-corrected black hole (HH state)

Hartle-Hawking state: $w = 0$, $\psi = -2\phi$ trace-free gravitational equations reduce to

$$\nabla_\mu \nabla_\nu F(\phi) = \frac{1}{2} g_{\mu\nu} \Box F(\phi)\,, \quad F(\phi) = \phi - \frac{\kappa}{4} e^{2\phi}$$

This implies (as in classical case) Killing vector $\xi_\mu = \epsilon_\mu^{\ \nu} \partial_\nu F(\phi)$.
General solution is

$$ds^2 = -g(x)dt^2 + \frac{1}{g(x)} dx^2\,, \qquad -\lambda x = F(\phi)$$

$$g(\phi) = g_{HH}(\phi) = 1 + \kappa\phi e^{2\phi} - a e^{2\phi} \qquad\qquad SS(1995)$$

Curved metric at asymptotic infinity ($\phi \to -\infty$) $g_0(\phi) = 1 + \kappa\phi e^{2\phi}$ is due to the presence of thermal radiation (Minkowski spacetime is not a solution)

Curvature singularity is now at finite value of $\phi = \phi_{cr}$: $F'(\phi_{cr}) = 1 - \frac{\kappa}{2} e^{2\phi_{cr}} = 0$

For $a > a_{cr} = \frac{\kappa}{2}(1 - \ln \frac{\kappa}{2})$ there exists a horizon at $\phi_h < \phi_{cr}$ with same Hawking temperature $T_H = \frac{\lambda}{2\pi}$ as in classical case

There exists another asymptotically flat solution ("beyond singularity") for $\phi > \phi_{cr}$ with a horizon at $\phi'_h > \phi_{cr}$

Solutions with naked singularity for $a < a_{cr}$

Solution for a general quantum state

A general $C-$state: $\psi = -2\phi + w$, $\quad w'(x) = \frac{C}{g(x)}$

The corresponding static asymptotically flat solution is

$$ds^2 = -g(x)dt^2 + \frac{1}{g(x)}dx^2\,, \quad \frac{dx}{d\phi} = h(\phi)$$

$$g(x) = e^{2\phi} Z(\phi)\,, \quad h(\phi) = \frac{1}{2\lambda} e^{2\phi} Z'(\phi)$$

$$Z + A(C)\ln Z = e^{-2\phi} g_{HH}(\phi)$$

$$g_{HH}(\phi) = 1 + \kappa\phi e^{2\phi} - a e^{2\phi}$$

A generic quantum state is characterized by two real numbers (A, κ)

Here $A(C) = \frac{\kappa}{16\lambda^3} C(C + 4\lambda)$. Boulware state: $C = -2\lambda$ and hence $A = -\frac{\kappa}{4\lambda} < 0$

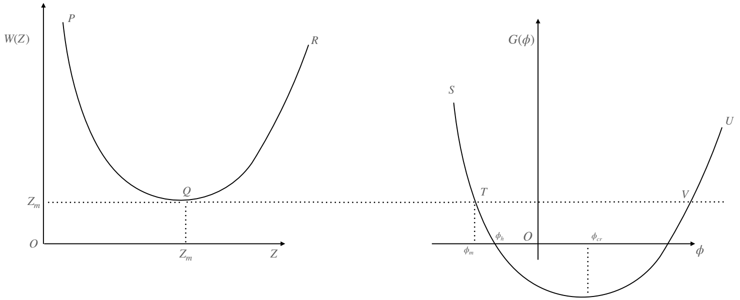
When $A(C) = 0$ the 2d space-time is the quantum-corrected black hole corresponding to the Hartle-Hawking state

Back-reacted geometry for Boulware state in more detail

$$W(Z) = G(\phi)$$

$$W(Z) = Z - Z_m \ln \frac{Z}{Z_m}\,, \quad G(\phi) = e^{-2\phi} + \kappa\phi - a\,, \quad Z_m = \kappa/2$$

First case to consider is $a > a_{cr} = -Z_m \ln Z_m$ (positive mass)



In this case $W(Z_m) > G(\phi_{cr})$: Admissible branches R-Q-P (S-T-S)

Asymptotic behaviour:

$\phi \to -\infty$ and $Z \to \infty$. This is where we expect the solution to approach the classical black hole solution

$$Z(\phi) = e^{-2\phi} - (a - a_{cr}) + \dots ,$$
$$h(\phi) = -1/\lambda + \dots ,$$
$$g(\phi) = 1 - (a - a_{cr}) e^{2\phi} + \dots .$$

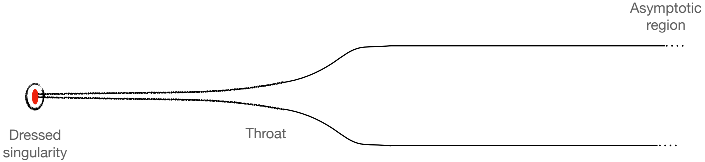
Notice the absence of the term $\kappa \phi e^{2\phi}$ (since no radiation at infinity, no its back-reaction)

Other properties:

No regular horizon: $g(\phi)$ does not vanish for any finite ϕ ($Z \neq 0$)

- Null singularity at $\phi = -\infty$ ($Z = 0$)
- Minimal value of dilaton, $(\nabla\phi)^2 = 0$, at $Z = Z_m$ ($\phi = \phi_m$): a throat ("minimal surface") of a wormhole?
- No Z_2 symmetry
- $(-g_{tt})$ monotonically decreases everywhere, its value in the throat

$$g(\phi_m) = \frac{\kappa}{2} e^{2\phi_m} \simeq \frac{\kappa}{2a} = \frac{\kappa}{S_{BH}(a)}$$



Other cases:

Flat space (linear dilaton vacuum, $\phi = -\lambda x$, $g_B(x) = 1$) is now exact solution, it corresponds to $a = a_{cr}$
Note: More generally, Minkowski spacetime is a solution for a quantum state $(A, \kappa = -2A)$

For $a < a_{cr}$ the solution has a naked singularity at $\phi = \phi_{cr}$

More solutions: "twisted" solutions, solutions with $Z < 0$ etc. All of them have either naked or null singularity

What is the right quantum state for a non-physical field (such as ghost)?

In QFT ghosts participate in Feynman diagrams but do not appear in asymptotic states

In gravitational context ghosts may contribute to conformal anomaly (negative contribution to κ)

However, we do not want to see a thermal gas of ghost particles at infinity

So that the most suitable choice for the quantum state of a ghost is the Boulware state

Earlier discussions of ghosts, negative κ and respective Hawking radiation: Strominger (1992), Bilal-Callan (1992)

Hybrid quantum state

Physical fields ($\kappa_1 > 0$) and ghosts ($\kappa_2 < 0$) so that total $\kappa = \kappa_1 + \kappa_2$

Respectively, $A_1 = 0$ (Hartle-Hawking state) and $A_2 = -\kappa_2/2$ (Boulware state)

so that total $A = A_1 + A_2 = -\kappa_2/2 > 0$

Master equation in this case:

$$W(Z) = G(\phi)$$

$$W(Z) = Z + Z_m \ln Z/Z_m \,, \quad G(\phi) = e^{-2\phi} + \kappa\phi - a\,,$$

$$Z_m = (-\kappa_2)/2 > 0$$

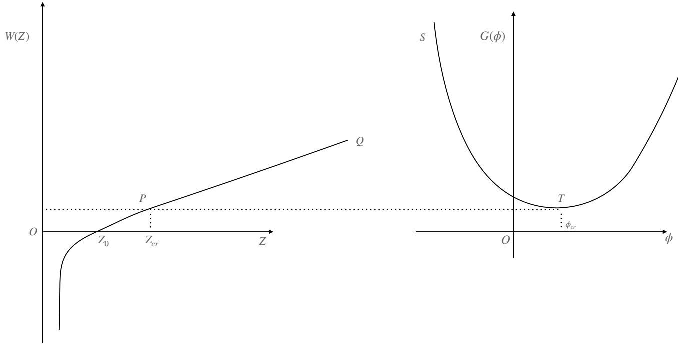
W(Z) is monotonic function in this case taking values in interval $(-\infty, +\infty)$

Asymptotically ($\phi \to -\infty$): $\quad g(\phi) = 1 + \kappa_1 \phi e^{2\phi} + \dots$

so that only physical fields contribute (thermal radiation of physical particles)

Shape of $G(\phi)$ depends on sign of $\kappa = \kappa_1 + \kappa_2$

<u>Positive κ</u>



Naked singularity at $\phi = \phi_{cr}$

$\underline{\kappa = \kappa_1 + \kappa_2 = 0}$

In this case $G(\phi) = e^{-2\phi} - a$ is monotonic but bounded $G(\phi) > -a$

Respectively $Z > Z_\infty > 0, \quad W(Z_\infty) = -a$

A naked (time-like) singularity at $\phi = +\infty$, $Z = Z_\infty$.

In the middle, for $\phi = \phi_{min}$, metric component $(-g_{tt})$ takes a minimal value

$$min(-g_{tt}) \sim e^{-2a/\kappa_1} = e^{-S_{BH}/\kappa_1}$$

That is extremely small for large S_{BH}.

This is a throat. The spacetime geometry outside the throat mimics a black hole in the limit of large entropy S_{BH} or small κ_1.

Negative $\kappa = \kappa_1 + \kappa_2$

Master equation:

$$W(Z) = G(\phi)$$

$$W(Z) = Z + Z_m \ln Z/Z_m\,, \quad G(\phi) = e^{-2\phi} - |\kappa|\phi - a\,, \quad Z_m = |\kappa_2|/2\,, \quad |\kappa| = |\kappa_2| - \kappa_1 > 0$$

Both functions $W(Z)$ and $G(\phi)$ are monotonic and change in interval $(-\infty, +\infty)$

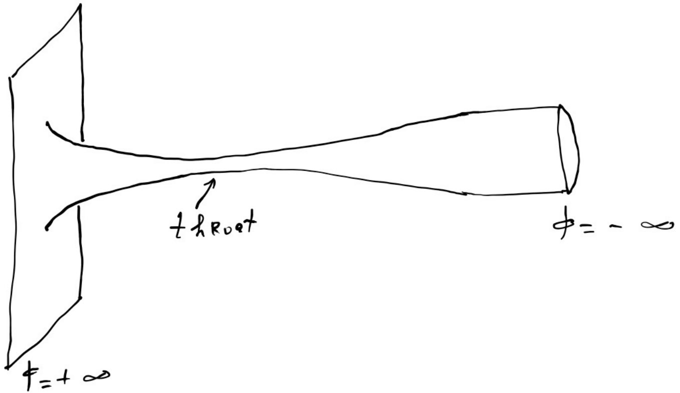
No curvature singularity, spacetime is asymptotically flat on both ends, $\phi = -\infty$ and $\phi = +\infty$

Metric function has a minimum (a throat) in the middle

$$min(-g_{tt}) \sim e^{-S_{BH}/|\kappa_2|}$$

Note: a similar back-reacted geometry for any state (A, κ), $A > 0$, $\kappa < 0$.

It is the most interesting solution: everywhere regular spacetime with a narrow throat that mimics the classical black hole



A curious solution: horizon free spacetime and still an outside observer sees the thermal Hawking radiation of the physical particles at the classical Hawking temperature

Lessons for four dimensions

- If all fields are in HH state then the semiclassical spacetime has a regular horizon and the spacetime is a deformation of the classical black hole.
  Open question in 4d: what is the appropriate modification at infinity due to the thermal Hawking radiation? Is the metric still static?

- If at least one of fields (physical or ghosts) is in Boulware state then the semiclassical geometry is horizon free; instead of horizon a narrow throat is formed; outside the throat the spacetime is a mimicker of a black hole

- On the other side of the throat many things may happen (depending on the parameters in the theory): a null singularity, a time-like singularity or a new asymptotically flat region

What are the possible implications for the information paradox?

- Need to consider a dynamical picture of semiclassical analog of black hole formation
- Details have to be elaborated. However in the optimistic scenario when horizon is replaced by a throat and spacetime has a new asymptotically flat region no reasons to think that information may be fundamentally lost.

- *The Hartle-Hawking state appears to be the only state whose back-reacted geometry has a regular horizon*
- *Back-reacted geometry of the Boulware state in semiclassical gravity generically has a throat where* $(-g_{tt})$ *is small but non-zero*
- *Minimal non-zero value of* $-g_{tt}$ *is bounded by the classical BH entropy:* $1/S_{BH}$ *or* $e^{-S_{BH}}$ *as in four dimensions*
- *The hybrid quantum state with negative total* κ *is closest we can get to a completely regular horizon free spacetime that is a perfect mimicker to a classical black hole. Only physical fields contribute to Hawking radiation at infinity in this case.*

An important urgent problem

Repeat this (global) analysis in four dimensions

Thank you for your attention!