High-Energy Behavior of Scattering Amplitudes in Theories with Purely Virtual Particles

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Outline

Introduction on PVP

Particle Physics and Quantum Gravity

In particle physics

Locality, renormalizability and unitarity (plus symmetries)

$\Downarrow$

Standard Model

Particle Physics and Quantum Gravity

In particle physics

Locality, renormalizability and unitarity (plus symmetries)

$\Downarrow$

Standard Model

In gravity

Einstein gravity

$$S = \int \sqrt{-g} R$$

Unitary ✓ Renormalizable ✗

Stelle gravity

$$S = \int \sqrt{-g} \left[R + R^2 + R_{\mu\nu}^2 \right]$$

Unitary ✗ Renormalizable ✓

Degrees of freedom in Stelle theory

$$S_{\text{QG}}(g) = -\frac{1}{16\pi}\int\sqrt{-g}\left(M_{\text{pl}}^2 R - \frac{\xi}{6}R^2 + \frac{\alpha}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right), \qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi}h_{\mu\nu}.$$

DOF

Massless graviton | Massive scalar ϕ | Massive spin-2 ghost χ

$h_{\mu\nu}$ propagator

$$\frac{i}{p^2}\left[\frac{\Pi^{(2)}_{\mu\nu\rho\sigma}}{(M_{\text{pl}}^2 - \alpha p^2)} - \frac{\Pi^{(0)}_{\mu\nu\rho\sigma}}{2(M_{\text{pl}}^2 - \xi p^2)}\right] + \text{g.f.} = \frac{i}{2M_{\text{pl}}^2}\left[\frac{2\Pi^{(2)}_{\mu\nu\rho\sigma} - \Pi^{(0)}_{\mu\nu\rho\sigma}}{p^2} - \frac{2\Pi^{(2)}_{\mu\nu\rho\sigma}}{p^2 - m_\chi^2} + \frac{\Pi^{(0)}_{\mu\nu\rho\sigma}}{p^2 - m_\phi^2}\right] + \text{g.f.},$$

$$m_\chi^2 = M_{\text{pl}}^2/\alpha, \qquad m_\phi^2 = M_{\text{pl}}^2/\xi.$$

Off-shell χ and ϕ are responsible for renormalizability ✓

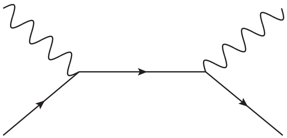
On-shell χ is responsible for the violation of unitarity ✗

Purely Virtual Particles

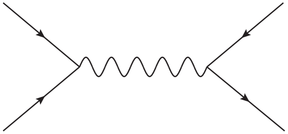
D. Anselmi and MP, JHEP 06 (2017) 066, D. Anselmi JHEP 06 (2017) 086

In quantum field theory a particle can be either "virtual" or "real"(on shell)

Real photon: light



Virtual photon: electromagnetic force



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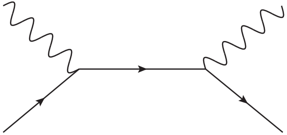

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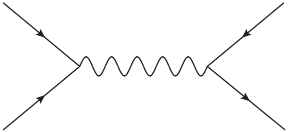
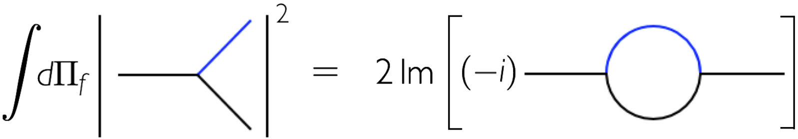


On-shell and virtual parts are related because of the optical theorem and the Feynman prescription



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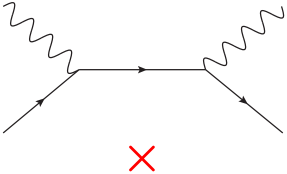

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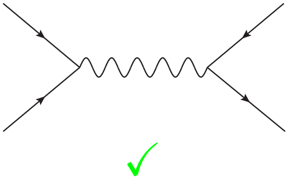

Purely virtual particles (PVP)

- Only mediate interactions
- Contribute to renormalization
- Never be on shell
- Consistent with unitarity.

Unitarity

- Optical theorem

$$\mathrm{S} = 1 + iT, \qquad \mathrm{S}\mathrm{S}^\dagger = 1 \qquad \Leftrightarrow \qquad -i(T - T^\dagger) = TT^\dagger.$$

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$$G + (G)^* + \sum_{\text{cuts}} G_c = 0, \qquad G = iT, \quad G = \text{diagram},$$

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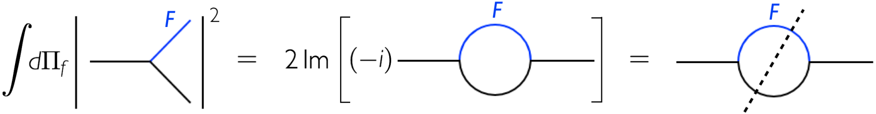
- Goal: consistent projection

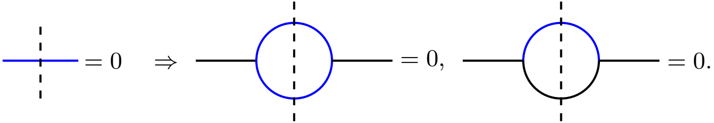


projected away

= 0 by fakeon prescription

Cuts and imaginary parts

PVP cut propagator vanishes. A bubble diagram has only one threshold



More complicated diagrams have multiple thresholds ⇒ Modified functions

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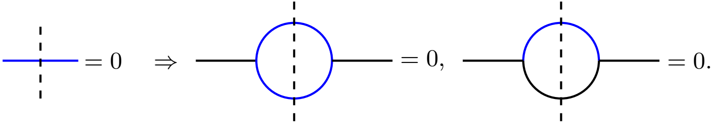

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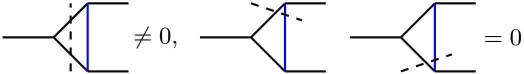

$$\text{Diagram} \to \text{Diagram} - \Delta^{n}_{\text{Diagram}}, \qquad n = \#\text{of PVP inside the loop} \tag{1}$$

For one-loop bubble diagram (this talk): $\Delta^{1,2}_{\text{Bubble}} = \text{Re(Bubble)}$
For triangles and boxes see A. Melis and MP, PRD 108 (2023) 9, 096021.
For general procedure D. Anselmi JHEP 11 (2021) 030.

Threshold decomposition and spectral identities

D. Anselmi, JHEP 11 (2021) 030

$$G_N = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \prod_{a=1}^{N} \frac{1}{(k-p_a)^2 - m_a^2 + i\epsilon_a} = \int \frac{\mathrm{d}^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \left(\prod_{a=1}^{N} \frac{1}{2\omega_a} \right) G_N^s$$

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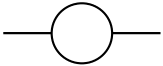
Skeleton diagram

$$G_N^s = \int \frac{\mathrm{d}k^0}{2\pi} \prod_{a=1}^{N} \frac{2\omega_a}{(k^0 - e_a)^2 - \omega_a^2 + i\epsilon_a}, \qquad e_a = p_a^0, \qquad \omega_a = \sqrt{(\mathbf{k} - \mathbf{p}_a)^2 + m_a^2}$$

Spectral identities

$$G^s + (G^s)^* + \sum_{\text{cuts}} G_c^s = 0.$$

which holds threshold by thresholds.

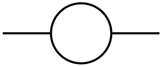


$$B^s = -\frac{i}{e_1 - e_2 - \omega_1 - \omega_2 + i\epsilon} - \frac{i}{e_2 - e_1 - \omega_1 - \omega_2 + i\epsilon}.$$

Using $\frac{i}{x+i\epsilon} = \mathcal{P}\frac{i}{x} + \pi\delta(x)$ everywhere

$$\boxed{B^s = -i\mathcal{P}_2 - \Delta^{12} - \Delta^{21}}$$

$$\mathcal{P}^{ab} = \mathcal{P}\frac{1}{e_a - e_b - \omega_a - \omega_b}, \qquad \mathcal{P}_2 = \mathcal{P}^{ab} + \mathcal{P}^{ba}, \qquad \Delta^{ab} = \pi\delta(e_a - e_b - \omega_a - \omega_b)$$



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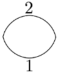
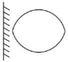
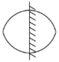
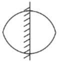
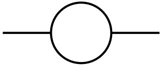
Diag. / Terms				
$\mathcal{P}_2$	$-i$	i	0	0
Δ^{12}	-1	-1	2	0
Δ^{21}	-1	-1	0	2



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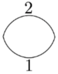
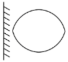
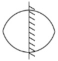
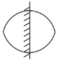
Diag. / Terms				
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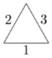
If we want to include PVP, we kill all the Δ's that contain at least one PVP frequency.

Diag. / Terms								
$\mathcal{P}_3$	$-i$	i	0	0	0	0	0	0
$\Delta^{12}\mathcal{Q}^{13}$	-1	-1	2	0	0	0	0	0
$\Delta^{23}\mathcal{Q}^{21}$	-1	-1	0	2	0	0	0	0
$\Delta^{31}\mathcal{Q}^{32}$	-1	-1	0	0	2	0	0	0
$\Delta^{21}\mathcal{Q}^{23}$	-1	-1	0	0	0	2	0	0
$\Delta^{32}\mathcal{Q}^{31}$	-1	-1	0	0	0	0	2	0
$\Delta^{13}\mathcal{Q}^{12}$	-1	-1	0	0	0	0	0	2
$\Delta^{12}\Delta^{13}$	i	$-i$	$2i$	0	0	0	0	$-2i$
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$$\mathcal{P}_3 = \mathcal{P}^{12}\mathcal{P}^{13} + \mathrm{cycl} + (e \to -e), \qquad \mathcal{Q}^{ab} = \mathcal{P}^{ab} - \mathcal{P}\frac{1}{e_a - e_b - \omega_a + \omega_b}.$$

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$$\mathcal{P}_{\mathrm{ABC}} = \mathcal{P}^{12}\mathcal{P}^{13} + \mathrm{cycl} + (e \to -e), \qquad \mathcal{Q}^{ab} = \mathcal{P}^{ab} - \mathcal{P}\frac{1}{e_a - e_b - \omega_a + \omega_b}.$$

Scattering in quantum gravity

Graviton scattering in quantum gravity

In Einstein gravity (unitary, nonrenormalizable)

$$S_{\mathrm{H}}(g) = -\frac{1}{2\kappa^2}\int \sqrt{-g}R, \qquad \kappa^2 = 8\pi G, \qquad g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu} \tag{2}$$

Tree level amplitudes

$$\mathcal{A}^{\mathrm{EG}}_{hh\to hh} \sim \kappa^2 s, \qquad s = \text{ c.o.m. energy squared} \tag{3}$$

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Tree level amplitudes

P. Donà, S. Giaccari, L. Modesto, L. Rachwal, Y. Zhu JHEP 08 (2015) 038.

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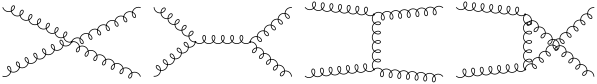

Equivalent formulation of Stelle gravity

D. Anselmi and MP, JHEP 11 (2018) 021.

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Equivalent action

auxiliary fields ϕ, $\chi_{\mu\nu}$ + Weyl transformation + field redefinitions.

$$\Downarrow$$

$$S(g,\phi,\chi) = -\frac{1}{2\kappa^2}\int\sqrt{-g}\left[\zeta R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + S_{\chi\chi}(g,\chi) + S_{\rm int}(g,\chi,\phi) \tag{7}$$

$$V(\phi) = \frac{m_\phi^2}{\kappa^2}(1-e^{\tilde{\kappa}\phi})^2, \qquad S_{\chi\chi} = -S_{\text{Pauli-Fierz}}$$

Vertices in $S_{\rm int}$

✓ $\quad h\chi\chi, \quad h\phi\phi, \quad ,hh\chi\chi, \quad hh\phi\phi, \quad \chi\phi\phi, \quad \chi\chi\phi\phi \quad$ etc... (8)

✗ $\quad hh\chi, \quad hh\phi, \quad$ etc.. (9)

Unitarity Bounds and Perturbativity

Unitarity bounds and perturbativity

Usual derivation of Unitarity bounds for 2-to-2 scattering of scalars

$$\sigma(s) = \frac{1}{32\pi s}\int_{-1}^{1} \mathrm{d}v |\mathcal{M}(s,v)|^2, \qquad v \equiv \cos\theta. \tag{10}$$

$$S = 1 + iT, \qquad S^\dagger S = 1, \qquad -i\left(T - T^\dagger\right) = \frac{1}{2}T^\dagger T \tag{11}$$

$$\Downarrow$$

$$\mathrm{Im}\mathcal{M}(s,1) = \sqrt{\kappa(s,m_1^2,m_2^2)}\sum_X \sigma_X(s) \geq \sqrt{\kappa(s,m_1^2,m_2^2)}\sigma(s), \tag{12}$$

$$\kappa(x,y,z) = x^2 + y^2 + z^2 - xy - xz - yz \tag{13}$$

Partial wave expansion in Legendre polynomials

$$\mathcal{M}(s,v) = 16\pi \sum_{j=0}^{\infty}(2j+1)A_j(s)P_j(v), \qquad \sigma(s) = \frac{16\pi}{s}\sum_{j=0}^{\infty}(2j+1)|A_j(s)|^2 \tag{14}$$

Unitarity bounds and perturbativity

$$\sum_{j=0}^{\infty}(2j+1)\mathrm{Im}A_j(s) \geq \frac{\sqrt{\kappa(s,m_1^2,m_2^2)}}{s}\sum_{j=0}^{\infty}(2j+1)|A_j(s)|^2. \tag{15}$$

For elastic scattering only we have the equality, which leads to

$$|\mathcal{A}_j| \leq 1, \qquad 0 \leq \mathrm{Im}\mathcal{A}_j \leq 1, \qquad |\mathrm{Re}\mathcal{A}_j| \leq \frac{1}{2}, \qquad \mathcal{A}_j(s) = \frac{\sqrt{\kappa(s,m_1^2,m_2^2)}}{s}A_j(s) \tag{16}$$

So far Unitarity $\Rightarrow$ (15). Therefore, if (15) is violated $\Rightarrow$ Unitarity is violated.

Unitarity bounds and perturbativity

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So far Unitarity $\Rightarrow$ (15). Therefore, if (15) is violated $\Rightarrow$ Unitarity is violated.

- Typical argument:

$$\mathcal{M} = \mathcal{M}^{\mathrm{tree}} + \mathcal{M}^{\mathrm{loops}}, \quad \mathcal{M}^{\mathrm{tree}} = 16\pi A_0^{\mathrm{tree}} + \ldots, \quad A_0^{\mathrm{tree}} \sim s\text{-channel diagrams} \qquad (17)$$

✗ if $\mathcal{A}_0^{\mathrm{tree}}$ violates (16) at some scale, then the theory violates unitarity at some scale.

Unitarity **and** perturbativity ($|\mathcal{M}^{\mathrm{loop}}| < |\mathcal{M}^{\mathrm{tree}}|$) $\Rightarrow$ (16) for $\mathcal{A}_0^{\mathrm{tree}}$

✓ if $\mathcal{A}_0^{\mathrm{tree}}$ violates (16) at some scale, then **either** unitarity or perturbativity is violated at some scale

Diagrammatic optical theorem

Unitarity can be checked by means of the cutting equations

$$2\mathrm{Im}\,(-iG) = -\sum_{\mathrm{c}} G_{\mathrm{c}} \tag{18}$$

which hold for any diagram G for any local QFT.

- Absence of ghosts $\Rightarrow$ (18) is the diagrammatic version of the unitarity equation.
- The cutting equations (18) relies only on the diagramamtic expansion.
- No assumptions on the behavior of G.

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- No assumptions on the behavior of G.

$$\Downarrow$$

If $\mathcal{A}_0^{\mathrm{tree}}$ violates $|\mathcal{A}_0^{\mathrm{tree}}| \leq 1$ etc... at some scale, then perturbativity is violated at some scale.

$O(N)$ models

$O(N)$ models and large-N diagrammatics

$$\mathcal{L}(\varphi) = \frac{1}{2}\partial_\mu \varphi^a \partial^\mu \varphi^a - \frac{1}{2} m^2 \varphi^a \varphi^a - \frac{g}{8}(\varphi^a \varphi^a)^2, \qquad a = 1, \dots N \tag{19}$$

Identify the orders in N

$$-\frac{g}{8}(\varphi^a \varphi^a)^2 \to \frac{1}{2g}\Omega^2 - \frac{1}{2}\Omega \varphi^a \varphi^a, \qquad \tilde{g} = gN \ \ \text{('t Hooft coupling)} \tag{20}$$

Study the limit $N \to \infty$ by keeping $\tilde{g}$ fixed.

- Each Ω internal line gives $1/N$
- Each closed φ loop gives N

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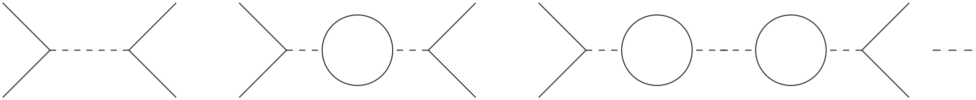
Consider the scattering of $\varphi^a\varphi^a \to \varphi^b\varphi^b$ with $a \neq b$



All the bubble insertions are order $1/N$ and can be resummed

Higher-derivative $O(N)$ models

$$\mathcal{L}(\varphi) = \frac{1}{2}\partial_\mu \varphi^a F_n\left(\frac{-\Box}{M^2}\right)\partial^\mu \varphi^a - \frac{1}{2}m^2 \varphi^a F_n\left(\frac{-\Box}{M^2}\right)\varphi^a - \frac{1}{8}\varphi^2 G_r\left(\frac{-\Box}{M^2}\right)\varphi^2, \qquad \varphi^2 \equiv \varphi^a\varphi^a, \quad (21)$$

$$F_n(z) = 1 + \sum_{i=1}^{n} f_i z^i, \qquad G_r(z) = \sum_{i=0}^{r} \lambda_i z^i, \quad (22)$$

- Auxiliary fields and 't Hooft couplings

$$-\frac{1}{8}\varphi^2 G_r\left(\frac{-\Box}{M^2}\right)\varphi^2 \to \frac{1}{2}\Omega G_r^{-1}\left(\frac{-\Box}{M^2}\right)\Omega - \frac{1}{2}\Omega\varphi^2, \qquad \tilde{\lambda}_i = \lambda_i N \quad (23)$$

- Propagators

$$iD_\Omega(p^2) = iG_r(p^2), \qquad iD_\varphi^{ab}(p) = \frac{i\delta^{ab}}{(p^2 - m^2 + i\epsilon)F_n(p^2/M^2)} \equiv i\delta^{ab} D_{\mathrm{HD}}(p^2) \quad (24)$$

Bubble diagrams

$$D_{\mathrm{HD}}(p^2) = \frac{a_0}{p^2 - m^2 + i\epsilon} + \sum_{i=1}^{n} \frac{a_i}{p^2 - M_i^2 + i\epsilon}, \qquad \text{with} \qquad \sum_{i=0}^{n} a_i = 0 \tag{25}$$

- Generic bubble diagram

$$B_{ij}(p^2) \equiv \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(p+q)^2 - M_i^2 + i\epsilon} \frac{1}{q^2 - M_j^2 + i\epsilon}, \tag{26}$$

$$B_{ij}(p^2, \tau) = \tau \mathrm{Re} B_{ij}(p^2) + i \mathrm{Im} B_{ij}(p^2), \qquad (\tau = 0 \text{ for PVP}) \tag{27}$$

- Ω self energy is $N\Sigma(p^2, \tau)$

$$\Sigma(p^2, \tau) = \int \frac{\mathrm{d}^D q}{(2\pi)^D} D_{\mathrm{HD}}(p+q) D_{\mathrm{HD}}(q) = \sum_{i,j=0}^{n} a_i a_j B_{ij}(p^2, \tau) \tag{28}$$

High-energy expansion

$$\mathrm{Re}B_{ij}(s,\tau) = -\frac{\tau}{16\pi}\left(1 - \frac{M_i^2 + M_j^2}{s}\right) + \mathcal{O}(1/s^2) \equiv \mathrm{Re}B_{ij}^{(1)}(s,\tau) + \mathcal{O}(1/s^2) \tag{29}$$

- For k particles with $\tau = 1$ and $n+1-k$ with $\tau \neq 1$

$$\mathrm{Re}\Sigma(s,\tau) = (1-\tau)\sum_{i,j=0}^{k} a_i a_j B_{ij}^{(1)}(s,1) + \mathcal{O}(1/s^2), \qquad \mathrm{Im}\Sigma(s,\tau) = \mathcal{O}(1/s^2) \tag{30}$$

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- For $\tau = 1$ (standard particles and ghosts)

$$\Sigma(s,1) = \mathcal{O}(1/s^2) \tag{31}$$

- For $\tau = 0$ (k standard particles and $n+1-k$ PVP)

$$\mathrm{Re}\Sigma(s,0) = \mathcal{O}(1/s^0), \qquad \mathrm{Im}\Sigma(s,0) = \mathcal{O}(1/s^2) \tag{32}$$

This difference is crucial in the resummed Ω propagator

$$iD(s,\tau) = \frac{1}{N}\frac{iG_r(s,\tilde{\lambda}_i)}{1 - iG_r(s,\tilde{\lambda}_i)\Sigma(s,\tau)} \tag{33}$$

Explicit example

$$F_1(z) = 1 - z, \qquad G_1(z) = \lambda_0 - \lambda_1 z \tag{34}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi_a)\left(1 + \frac{\Box}{M^2}\right)(\partial^\mu \varphi_a) - \frac{m^2}{2}\varphi_a \left(1 + \frac{\Box}{M^2}\right)\varphi_a - \frac{1}{8}\varphi^2\left(\lambda_0 + \frac{\lambda_1 \Box}{M^2}\right)\varphi^2 \tag{35}$$

Propagators

$$iD_\varphi^{ab}(p^2) = -\frac{iM^2\delta^{ab}}{(p^2 - m^2 + i\epsilon)(p^2 - M^2 + i\epsilon)}, \qquad \text{with } a_0 = -a_1 = \frac{M^2}{M^2 - m^2} \tag{36}$$

$$iD_\Omega(p^2) = \lambda_0 - \frac{\lambda_1 p^2}{M^2} \tag{37}$$

Properties

- Superrenormalizable
- For $\tau = 1$ there are N standard particles of mass m and N ghosts of mass M (violates opt th).
- For $\tau = 0$ there are N standard particles of mass m and N PVP of mass M (unitary).
- Setting $F_1(z) = 1$ give a nonrenormalizable theory.

Why not doing directly Stelle gravity instead of toy models?

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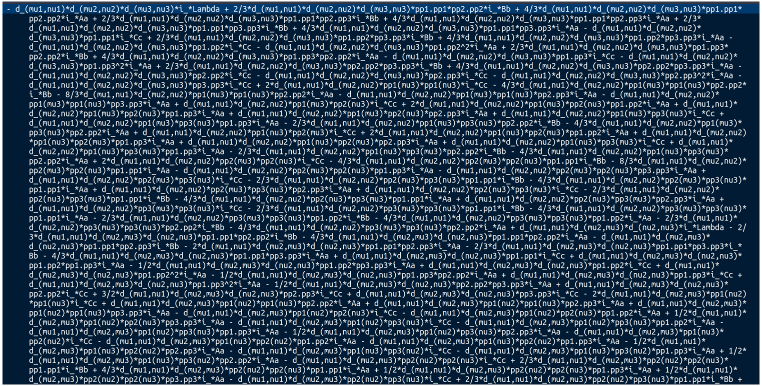
- d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*i_*Lambda + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp2*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*
    pp2.pp2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp3*i_*Aa + 2/3*
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*
    d_(mu3,nu3)*pp1.pp1*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*pp3.pp3*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2^2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*
    pp2.pp2*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*
    d_(mu3,nu3)*pp1.pp3^2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*pp3.pp3*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp3*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp3^2*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp3.pp3*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*i_*Cc - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp2*
    i_*Bb - 8/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*
    pp1(mu3)*pp1(nu3)*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp1.pp2*i_*Aa + d_(mu1,nu1)*
    d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp1.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*i_*Cc +
    d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*pp1.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*
    pp3(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*pp1.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)
    *pp1(nu3)*pp2(mu3)*pp1.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*i_*Cc + d_(mu1,nu1)*
    d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*pp1.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*
    pp2.pp2*i_*Aa + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*i_*Cc - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp1.pp1*i_*Bb - 8/3*d_(mu1,nu1)*d_(mu2,nu2)*
    pp2(mu3)*pp2(nu3)*pp1.pp1*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp1.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp3.pp3*i_*Aa +
    d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*
    pp1.pp1*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*
    pp2(nu3)*pp3(mu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*pp1.pp1*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*pp2.pp3*i_*Aa +
    d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*
    pp1.pp1*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp2*i_*Aa - 2/3*d_(mu1,nu1)*
    d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*i_*Lambda - 2/
    3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*
    d_(nu2,nu3)*pp1.pp1*pp2.pp3*i_*Bb - 2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp3.pp3*i_*
    Bb - 4/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*
    pp1.pp2*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2*i_*Cc + d_(mu1,nu1)*
    d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2^2*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3*i_*Cc +
    d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3^2*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp2.pp2*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*
    pp2.pp2*i_*Cc + 3/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp2.pp3*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp3.pp3*i_*Cc - 2*d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)
    *pp1(nu3)*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp1(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp1(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*
    pp1(nu2)*pp1(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*pp1.pp2*i_*Aa + 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp1.pp2*i_*Aa -
    d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*
    pp2(nu2)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp1.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*pp1.pp3*i_*Aa + 1/2*
    d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*
    pp1.pp1*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp1.pp1*i_*Aa + 1/2*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp1.pp3*i_*Aa + 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp3(nu3)*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp3(nu3)*pp1.pp1*i_*Bb +
```

First 40 lines of 3-leg vertex in Stelle gravity. Total is 790 lines, txt file 120 kB

Why not doing directly Stelle gravity instead of toy models?



```
- d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*i_*Lambda + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp2*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*
    pp2.pp2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp3*i_*Aa + 2/3*
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*
    d_(mu3,nu3)*pp1.pp1*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*pp3.pp3*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2^2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*
    pp2.pp2*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*
    d_(mu3,nu3)*pp1.pp3^2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*pp3.pp3*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp3*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp3^2*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp3.pp3*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*i_*Cc - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp2*
    i_*Bb - 8/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*
    pp1(mu3)*pp1(nu3)*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp1.pp2*i_*Aa + d_(mu1,nu1)*
    d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp1.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*i_*Cc +
    d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*pp1.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*
    pp3(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*pp1.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)
    *pp1(nu3)*pp2(mu3)*pp1.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*i_*Cc + d_(mu1,nu1)*
    d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*pp1.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*
    pp2.pp2*i_*Aa + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*i_*Cc - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp1.pp1*i_*Bb - 8/3*d_(mu1,nu1)*d_(mu2,nu2)*
    pp2(mu3)*pp2(nu3)*pp1.pp1*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp1.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp3.pp3*i_*Aa +
    d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*
    pp1.pp1*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*
    pp2(nu3)*pp3(mu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*pp1.pp1*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*pp2.pp3*i_*Aa +
    d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*
    pp1.pp1*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp2*i_*Aa - 2/3*d_(mu1,nu1)*
    d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*i_*Lambda - 2/
    3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*
    d_(nu2,nu3)*pp1.pp1*pp2.pp3*i_*Bb - 2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp3.pp3*i_*
    Bb - 4/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*
    pp1.pp2*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2*i_*Cc + d_(mu1,nu1)*
    d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2^2*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3*i_*Cc +
    d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3^2*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp2.pp2*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*
    pp2.pp2*i_*Cc + 3/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp2.pp3*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp3.pp3*i_*Cc - 2*d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)
    *pp1(nu3)*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp1(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp1(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*
    pp1(nu2)*pp1(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*pp1.pp2*i_*Aa + 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp1.pp2*i_*Aa -
    d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*
    pp2(nu2)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp1.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*pp1.pp3*i_*Aa + 1/2*
    d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*
    pp1.pp1*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp1.pp1*i_*Aa + 1/2*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp1.pp3*i_*Aa + 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp3(nu3)*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp3(nu3)*pp1.pp1*i_*Bb +
```

First 40 lines of 3-leg vertex in Stelle gravity. Total is 790 lines, txt file 120 kB

4-leg vertex txt file is 4 MB

Why not doing directly Stelle gravity instead of toy models?

```
- d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*i_*Lambda + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp2*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*
    pp2.pp2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp3*i_*Aa + 2/3*
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*
    d_(mu3,nu3)*pp1.pp1*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*pp3.pp3*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2^2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*
    pp2.pp2*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*
    d_(mu3,nu3)*pp1.pp3^2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*pp3.pp3*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp3*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp3^2*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp3.pp3*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*i_*Cc - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp2*
    i_*Bb - 8/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*
    pp1(mu3)*pp1(nu3)*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp1.pp2*i_*Aa + d_(mu1,nu1)*
    d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp1.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*i_*Cc +
    d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*pp1.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*
    pp3(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*pp1.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)
    *pp1(nu3)*pp2(mu3)*pp1.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*i_*Cc + d_(mu1,nu1)*
    d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*pp1.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*
    pp2.pp2*i_*Aa + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*i_*Cc - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp1.pp1*i_*Bb - 8/3*d_(mu1,nu1)*d_(mu2,nu2)*
    pp2(mu3)*pp2(nu3)*pp1.pp1*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp1.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp3.pp3*i_*Aa +
    d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*
    pp1.pp1*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*
    pp2(nu3)*pp3(mu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*pp1.pp1*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*pp2.pp3*i_*Aa +
    d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*
    pp1.pp1*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp2*i_*Aa - 2/3*d_(mu1,nu1)*
    d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*i_*Lambda - 2/
    3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*
    d_(nu2,nu3)*pp1.pp1*pp2.pp3*i_*Bb - 2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp3.pp3*i_*
    Bb - 4/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*
    pp1.pp2*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2*i_*Cc + d_(mu1,nu1)*
    d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2^2*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3*i_*Cc +
    d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3^2*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp2.pp2*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*
    pp2.pp2*i_*Cc + 3/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp2.pp3*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp3.pp3*i_*Cc - 2*d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)
    *pp1(nu3)*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp1(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp1(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*
    pp1(nu2)*pp1(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*pp1.pp2*i_*Aa + 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp1.pp2*i_*Aa -
    d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*
    pp2(nu2)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp1.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*pp1.pp3*i_*Aa + 1/2*
    d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*
    pp1.pp1*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp1.pp1*i_*Aa + 1/2*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp1.pp3*i_*Aa + 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp3(nu3)*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp3(nu3)*pp1.pp1*i_*Bb +
```

First 40 lines of 3-leg vertex in Stelle gravity. Total is 790 lines, txt file 120 kB

4-leg vertex txt file is 4 MB

5-leg vertex txt file is 165 MB

Why not doing directly Stelle gravity instead of toy models?

```
- d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*i_*Lambda + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp2*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*
    pp2.pp2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp2.pp3*i_*Aa + 2/3*
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp1*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*
    d_(mu3,nu3)*pp1.pp1*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*pp3.pp3*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp2^2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*
    pp2.pp2*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp1.pp3*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*
    d_(mu3,nu3)*pp1.pp3^2*i_*Aa + 2/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*pp3.pp3*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*pp3.pp3*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp2*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp3*i_*Cc - d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp2.pp3^2*i_*Aa -
    d_(mu1,nu1)*d_(mu2,nu2)*d_(mu3,nu3)*pp3.pp3*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*i_*Cc - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp2*
    i_*Bb - 8/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp1(nu3)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*
    pp1(mu3)*pp1(nu3)*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp1.pp2*i_*Aa + d_(mu1,nu1)*
    d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp1.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp2(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*i_*Cc +
    d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*pp1.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*pp3(nu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(mu3)*
    pp3(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*i_*Cc + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*pp1.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)
    *pp1(nu3)*pp2(mu3)*pp1.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp2(mu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*i_*Cc + d_(mu1,nu1)*
    d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*pp1.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp1(nu3)*pp3(mu3)*
    pp2.pp2*i_*Aa + 2*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*i_*Cc - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp1.pp1*i_*Bb - 8/3*d_(mu1,nu1)*d_(mu2,nu2)*
    pp2(mu3)*pp2(nu3)*pp1.pp1*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp1.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp2(nu3)*pp3.pp3*i_*Aa +
    d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*
    pp1.pp1*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(mu3)*pp3(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*
    pp2(nu3)*pp3(mu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*pp1.pp1*i_*Aa + d_(mu1,nu1)*d_(mu2,nu2)*pp2(nu3)*pp3(mu3)*pp2.pp3*i_*Aa +
    d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*i_*Cc - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp1*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*
    pp1.pp1*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp1.pp2*i_*Aa - 2/3*d_(mu1,nu1)*
    d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,nu2)*pp3(mu3)*pp3(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*i_*Lambda - 2/
    3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp2*i_*Bb - 4/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*
    d_(nu2,nu3)*pp1.pp1*pp2.pp3*i_*Bb - 2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp2.pp3*i_*Aa - 2/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp3.pp3*i_*
    Bb - 4/3*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp1*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*
    pp1.pp2*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2*i_*Cc + d_(mu1,nu1)*
    d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp2^2*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3*i_*Cc +
    d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp1.pp3^2*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp2.pp2*pp3.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*
    pp2.pp2*i_*Cc + 3/2*d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp2.pp3*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*d_(nu2,nu3)*pp3.pp3*i_*Cc - 2*d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)
    *pp1(nu3)*i_*Cc + d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp1(nu3)*pp2.pp2*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp1(nu3)*pp2.pp3*i_*Aa + d_(mu1,nu1)*d_(mu2,mu3)*
    pp1(nu2)*pp1(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*pp1.pp2*i_*Aa + 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp1(nu2)*pp2(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp1.pp2*i_*Aa -
    d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu2)*pp3(nu3)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*
    pp2(nu2)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp1.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp1.pp3*i_*Aa - 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp1(nu3)*pp2(nu2)*pp2.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*i_*Cc - d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*pp1.pp3*i_*Aa + 1/2*
    d_(mu1,nu1)*d_(mu2,mu3)*pp1(nu3)*pp3(nu2)*pp2.pp2*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*
    pp1.pp1*i_*Bb + 4/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp1.pp1*i_*Aa + 1/2*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp1.pp3*i_*Aa + 1/2*d_(mu1,nu1)*
    d_(mu2,mu3)*pp2(nu2)*pp2(nu3)*pp3.pp3*i_*Aa - d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp3(nu3)*i_*Cc + 2/3*d_(mu1,nu1)*d_(mu2,mu3)*pp2(nu2)*pp3(nu3)*pp1.pp1*i_*Bb +
```

First 40 lines of 3-leg vertex in Stelle gravity. Total is 790 lines, txt file 120 kB

4-leg vertex txt file is 4 MB

5-leg vertex txt file is 165 MB

6-leg vertex txt file is 5.7 GB.

Results

Cross sections

For $\varphi^a\varphi^a \to \varphi^b\varphi^b$ with $a \neq b$

- Tree-level cross section

$$\sigma(s) = \frac{a_0^2}{16\pi s}\left(\lambda_0 - \lambda_1 \frac{s}{M^2}\right)^2 \sim \frac{s}{M^4} \text{ at high energies,} \tag{38}$$

- Resummed cross sections

$$\sigma(s,\tau) = \frac{a_0^2}{16\pi s}|D(s,\tau)|^2 \tag{39}$$

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Ghost vs PVP

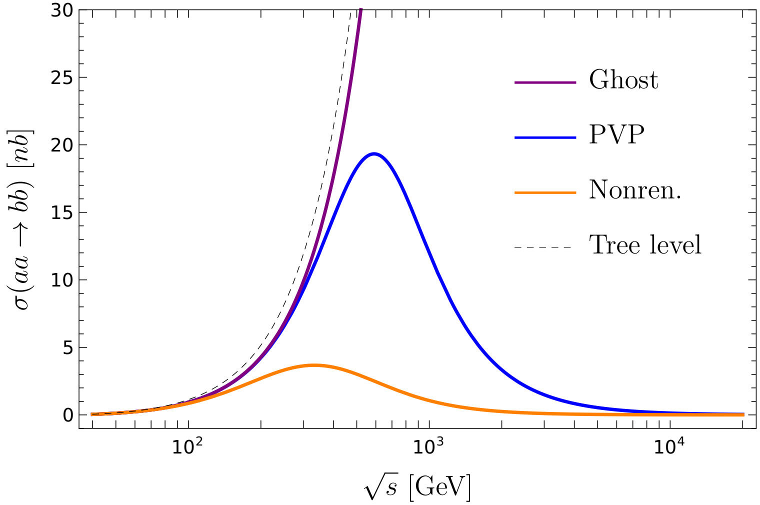
$$\sigma_{\text{gh}}(s) \equiv \sigma(s,1) \sim \frac{\tilde{\lambda}_1^2 s}{16\pi(M^2-m^2)^2 N^2}, \qquad \sigma_{\text{PVP}}(s) \equiv \sigma(s,0) \sim \frac{16\pi(M^2-m^2)^2}{M^4 N^2}\frac{1}{s} \tag{40}$$

Nonrenormalizable case

$$F_0(z) = 1, \qquad G_1(z) = \lambda_0 - \lambda_1 z \tag{41}$$

$$\sigma_{\text{nr}}(s) \sim \frac{16\pi}{s\left(1+\ln^2\frac{s}{m^2}\right)N^2} \tag{42}$$

Cross sections (plot)

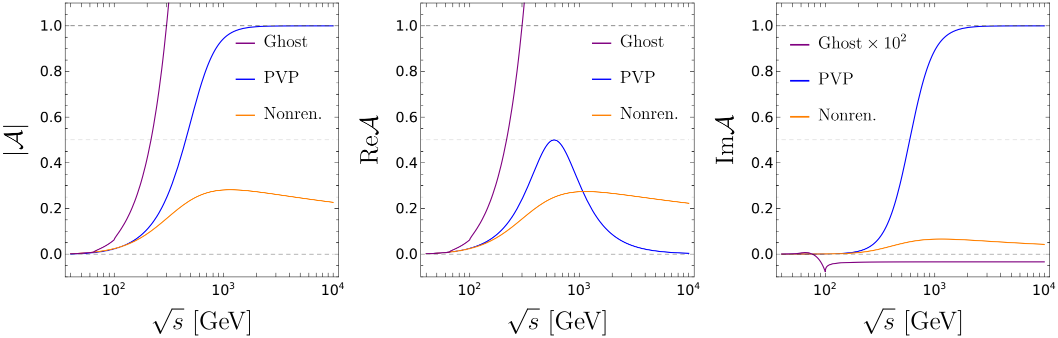


$m = 15$ GeV, $M = 50$ GeV, $\lambda_0 = 0.3$, $\lambda_1 = 0.1$

Positivity bounds

$$\mathcal{A}_{\rm gh}(s) = -\frac{a_0^2}{16\pi}\left[\sqrt{1-\frac{4m^2}{s}} + 2\sqrt{\frac{\kappa(s,m^2,M^2)}{s^2}} + \sqrt{1-\frac{4M^2}{s}}\right] D(s,1) \tag{43}$$

$$\mathcal{A}_{\rm PVP}(s) = -\frac{a_0^2}{16\pi}\sqrt{1-\frac{4m^2}{s}}\,D(s,0), \qquad \mathcal{A}_{\rm nr}(s) = -\frac{1}{16\pi}\sqrt{1-\frac{4m^2}{s}}\,D_{\rm nr}(s) \tag{44}$$



Conclusions

- Nonperturbative resummations can cure appartent violations of unitarity bounds in renormalizable theories with PVP, as well as in unitary nonrenormalizable theories
- Renormalizable theories with ghosts cannot be cured in this way
- The resummations add new poles in the cross sections that could be interpreted as resonances

Difficulties with gravity

- Resum quartic vertices
- Justify the resummation (could be just a prescription)
- Long computations...