Chiral approach to massive higher spins

based on 2207.14597 with Evgeny Skvortsov

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Why massive higher spins?

- Mathematical interest
- Quantum gravity
  (e.g. string theory spectrum contains higher spins)
- Higher-spin resonances exist in nature
  (e.g. Δ-baryons w/ lifetimes $\approx 5 \times 10^{-24}$ s)
- Model celestial objects' dynamics via classical limit

Guevara, AO, Vines '18, '19
Maybee, O'Connell, Vines '19
Bern, Luna, Roiban, Shen, Zeng '20
Aoude, AO '21

- QFT applications to other composite particles
  (e.g. atoms, atoms' excited states?)

Issues with higher spins

Textbook QFT:

- scalars 😄, gauge th. 🙂, gravity 😐, higher spins ☹
  Proca th. 🙂, massive gravity 😐 Bergshoeff, Hohm, Townsend '09; de Rham, Gabadadze, Tolley '10; Hassan, Rosen '11
- spin $s\uparrow \quad\Rightarrow\quad$ unphysical d.o.f.
- massless higher-spins: no-go theorems Weinberg '64, Coleman, Mandula '67
- massive higher-spins: no no-go theorems
- claims of inconsistency, causality violations
  (e.g. EM interaction via $\partial_\mu \to \partial_\mu - iQA_\mu$)
  e.g. Johnson, Sudarshan '60; Velo, Zwanziger '69, '72
  - artificial in EFT approach (natural for composite particles)
  - also related to unphysical d.o.f.
- need a host of auxiliary fields (even in free theory)
  Fierz, Pauli '39; Singh, Hagen '74
  - tamed by increasing amount of gauge symmetry Zinoviev '01

This talk:

- Chiral approach w/o auxiliary fields
  ($\Rightarrow$ no massive gauge symmetry needed)

Outline

0. Why massive higher spins?

1. Managing unphysical d.o.f.
2. Amplitudes hint at simplicity
3. Gauge interactions
4. Gravitational interactions

5. Summary & outlook

Managing unphysical d.o.f.

Symmetric tensors need transversality

Standard (non-chiral) choice — sym. traceless tensors $\Phi_{\mu_1\ldots\mu_s}$

Recall Lorentz group homomorphism:

$$\mathrm{SL}(2,\mathbb{C}) \quad\rightarrow\quad \mathrm{SO}(1,3)$$

$$\underbrace{V_\mu \sigma^\mu_{\alpha\dot\beta} =: V_{\alpha\dot\beta}}_{\text{SPINOR MAP}} \rightarrow S_\alpha{}^\gamma V_{\gamma\dot\delta}\big(S_\beta{}^\delta\big)^* \;\Rightarrow\; V^\mu \rightarrow L^\mu{}_\nu V^\nu, \quad L^\mu{}_\nu = \frac{1}{2}\,\mathrm{tr}\big(\bar\sigma^\mu S \sigma_\nu S^\dagger\big)$$

Also: $\Phi_{\alpha_1\ldots\alpha_s\dot\beta_1\ldots\dot\beta_s} := \Phi_{\mu_1\ldots\mu_s}\sigma^{\mu_1}_{\alpha_1\dot\beta_1}\cdots\sigma^{\mu_s}_{\alpha_s\dot\beta_s}$

$\Rightarrow$ denote as (s,s) rep. of Lorentz group $\mathrm{SL}(2,\mathbb{C})$

Problem: too many d.o.f.!

i.e. highly reducible under Wigner's little group $\mathrm{SU}(2)\subset\mathrm{SL}(2,\mathbb{C})$

(decomp. into sym. $\mathrm{SU}(2)$ tensors of rank $0,2,\ldots,2s$)

$\Rightarrow$ transversality constraint for irreducibility (also for energy positivity):

$$(\partial^2+m^2)\Phi_{\mu_1\ldots\mu_s}=0, \qquad \partial^\mu\Phi_{\mu\mu_2\ldots\mu_s}=0$$

indeed, # of d.o.f.: $\underbrace{\tfrac{1}{2}(s+2)(s+1)}_{\text{3d sym. tensor components}} - \underbrace{\tfrac{1}{2}s(s-1)}_{\text{traces}} = 2s+1$

Auxiliary fields & massive gauge invariance

Lagrangian descr. of constr. eqns $(\partial^2 + m^2)\Phi_{\mu_1\ldots\mu_s} = 0$, $\partial^\mu \Phi_{\mu\mu_2\ldots\mu_s} = 0$ requires aux. fields; originally:

Fierz, Pauli '39; Singh, Hagen '74

$$\text{sym. traceless } \Phi_{\mu_1\ldots\mu_s}, \underbrace{\Phi_{\mu_1\ldots\mu_{s-2}}, \Phi_{\mu_1\ldots\mu_{s-3}}, \ldots, \Phi_\mu, \Phi}_{\text{auxiliary}}$$

More recently:

Zinoviev '01

$$\text{sym. double-traceless } \Phi_{\mu_1\ldots\mu_s}, \underbrace{\Phi_{\mu_1\ldots\mu_{s-1}}, \Phi_{\mu_1\ldots\mu_{s-2}}, \ldots, \Phi_\mu, \Phi}_{\text{auxiliary}}$$

$$\begin{cases} \delta\Phi_{\mu_1\ldots\mu_s} = s\partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)} + \# m\eta_{(\mu_1\mu_2}\xi_{\mu_3\ldots\mu_s)} + \ldots \\ \delta\Phi_{\mu_1\ldots\mu_{s-1}} = (s-1)\partial_{(\mu_1}\xi_{\mu_2\ldots\mu_{s-1})} + m\xi_{\mu_1\ldots\mu_{s-1}} + \# m\eta_{(\mu_1\mu_2}\xi_{\mu_3\ldots\mu_{s-1})} + \ldots \\ \vdots \\ \delta\Phi_\mu = \partial_\mu\xi + m\xi_\mu + \ldots \\ \delta\Phi = m\xi, \qquad \qquad \Phi^{\lambda\mu}{}_{\lambda\mu\mu_5\ldots\mu_k} = \xi^\mu{}_{\mu\mu_3\ldots\mu_k} = 0 \end{cases}$$

- 2nd-class constraints traded for 1st-class (more aux. fields streamline analysis)
- allows for systematic introduction of interactions
- still highly non-trivial, order by order
- no results beyond cubic level (trivalent vertices)

Spin-1 example

- marginal spin $\Rightarrow$ Lagrangian w/o aux. fields exists

$$\mathcal{L}_{\text{Proca}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{m^2}{2} B_\mu B^\mu$$

$$\partial_\mu B^{\mu\nu} + m^2 B^\nu = 0 \;\Big|\, \cdot \partial_\nu \qquad \Rightarrow \qquad \begin{cases} (\partial^2 + m^2) B^\nu = 0 \\ \partial_\nu B^\nu = 0 \end{cases}$$

- aux. field à la Zinoviev well-known: Stückelberg '38

$$\mathcal{L}_{s=1} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} (m B_\mu - \partial_\mu \varphi)(m B^\mu - \partial^\mu \varphi)$$

massive gauge freedom: $\begin{cases} B_\mu \mapsto B_\mu + \partial_\mu \xi \\ \varphi \mapsto \varphi + m\xi \end{cases}$

Familiar setting: complex Stück. field $=$ 2 Goldstone bosons after sym. breaking $\mathrm{SO}(3) \to \mathrm{SO}(2)$ and Higgs decoupling

$$\mathcal{L}_{\mathrm{SO}(3)} = \sum_{i=1}^{3} \left\{ -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \frac{1}{2} (D_\mu \phi)_i (D^\mu \phi)_i \right\} + \frac{\mu^2}{2} \|\phi\|^2 - \frac{\lambda}{4!} \|\phi\|^4$$

$$\Rightarrow -\frac{1}{2} B^*_{\mu\nu} B^{\mu\nu} + (m B_\mu - \partial_\mu \varphi)^* (m B^\mu - \partial^\mu \varphi)$$

$$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i g B^*_\mu F^{\mu\nu} B_\nu + \text{self- \& more EM interactions}$$

Amplitudes hint at simplicity

Massless vs massive spinor helicity

Arkani-Hamed, Huang, Huang '17

Spinor map: $p_{\alpha\dot{\beta}} = p_\mu \sigma^\mu_{\alpha\dot{\beta}}$

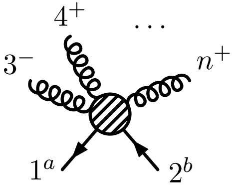
MASSLESS	MASSIVE
$\det\{p_{\alpha\dot{\beta}}\} = 0$	$\det\{p_{\alpha\dot{\beta}}\} = m^2$
$p_{\alpha\dot{\beta}} = \vert p\rangle_\alpha [p\vert_{\dot{\beta}}$	$p_{\alpha\dot{\beta}} = \vert p^a\rangle_\alpha [p_a\vert_{\dot{\beta}}$ $\det\{\vert p^a\rangle_\alpha\} = \det\{[p^a\vert_{\dot{\alpha}}\} = m$
$p^\mu = \frac{1}{2}\langle p\vert\sigma^\mu\vert p]$	$p^\mu = \frac{1}{2}\langle p^a\vert\sigma^\mu\vert p_a]$
$p_{\alpha\dot{\beta}}\vert p]^{\dot{\beta}} = 0$	$p_{\alpha\dot{\beta}}\vert p^a]^{\dot{\beta}} = m\vert p^a\rangle_\alpha$
$\langle p\, q\rangle = -\langle q\, p\rangle \quad \Rightarrow \quad \langle p\, p\rangle = 0$ $[p\, q] = -[q\, p] \quad \Rightarrow \quad [p\, p] = 0$ $\langle p\, q\rangle [q\, p] = 2p\cdot q$	$\langle p^a q^b\rangle = -\langle q^b p^a\rangle$ e.g. $\langle p^a p^b\rangle = -m\epsilon^{ab}$ $[p^a q^b] = -[q^b p^a]$ e.g. $[p^a p^b] = m\epsilon^{ab}$ $\langle p^a q^b\rangle [q_b\, p_a] = 2p\cdot q$

Why does spinor helicity help?

Consider QFT amplitude $\mathcal{A}(\underline{1}^a, 3^-, 4^+, \ldots, n^+, \overline{2}^b)$

Feynman rules give function of

- momenta p_i^μ
- pol. tensors $\varepsilon_\pm^\mu(p_i)$, $\varepsilon_\pm^{\mu\nu}(p_i)$ — gauge-dep.!
- external spinors $\bar{v}^a(p_1)$, $u^b(p_2)$



But all vector, spinor indices must be contracted

Remaining indices $\Leftrightarrow$ physical quantum numbers:

- helicities $\pm$ $\Leftrightarrow$ spins $\{\pm 1/2\}_p$, $\{\pm 1\}_p$, etc.
- $\mathrm{SU}(2)$ labels a, b $\Leftrightarrow$ spins $\{\pm 1/2\}_q$, $\{\pm 1, 0\}_q$, etc.

Crucial on-shell notion — LITTLE GROUP

Little groups

- Quantum fields $\Leftarrow$ reps of $\mathrm{SO}(1,3)$
- Quantum states $\Leftarrow$ reps of LITTLE GROUP
  - massless states $\Leftarrow$ $\mathrm{SO}(2)$
  - massive states $\Leftarrow$ $\mathrm{SO}(3)$

Little groups

- Quantum fields $\Leftarrow$ reps of $\mathrm{SO}(1,3)$ $\subset$ $\mathrm{SL}(2,\mathbb{C})$
- Quantum states $\Leftarrow$ reps of LITTLE GROUP's dbl cover
  - massless states $\Leftarrow$ $\mathrm{SO}(2)$ $\subset$ $\mathbf{U(1)}$
  - massive states $\Leftarrow$ $\mathrm{SO}(3)$ $\subset$ $\mathbf{SU(2)}$

Minor complication: spinorial reps use groups' double covers

$\mathrm{U}(1)$ and $\mathrm{SU}(2)$ arise naturally in spinor helicity

Wavefunctions from helicity spinors

Massless:

$$\varepsilon^\mu_{p+} = \frac{\langle q|\sigma^\mu|p]}{\sqrt{2}\langle q\,p\rangle} \qquad \varepsilon^\mu_{p-} = \frac{\langle p|\sigma^\mu|q]}{\sqrt{2}[p\,q]} \qquad \Rightarrow \qquad \begin{cases} \varepsilon^\pm_p \cdot p = \varepsilon^\pm_p \cdot q = 0 \\ \varepsilon^\mu_{p+}\varepsilon^\nu_{p-} + \varepsilon^\mu_{p-}\varepsilon^\nu_{p+} = -\eta^{\mu\nu} + \dfrac{p^\mu q^\nu + q^\mu p^\nu}{p\cdot q} \\ \varepsilon^{h_1}_p \cdot \varepsilon^{h_2}_p = -\delta^{h_1(-h_2)} \end{cases}$$

Xu, Zhang, Chang '85

Massive:

$$\varepsilon^{ab}_{p\mu} = \frac{i\langle p^{(a}|\sigma_\mu|p^{b)}]}{\sqrt{2}m} \qquad \Rightarrow \qquad \begin{cases} p \cdot \varepsilon^{ab}_p = 0 \\ \varepsilon^{ab}_{p\mu}\varepsilon_{p\nu ab} = -\eta_{\mu\nu} + \dfrac{p_\mu p_\nu}{m^2} \\ \varepsilon^{ab}_p \cdot \varepsilon_{pcd} = -\delta^{(a}_{(c}\delta^{b)}_{d)} \end{cases}$$

Guevara, AO, Vines '18
Chung, Huang, Kim, Lee '18

and (symmetrized) tensor products thereof

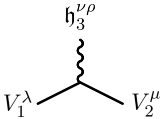
- sym. rank-$2s$ tensors — irreps of $\mathrm{SU}(2)$ w/ $(2s+1)$ d.o.f.

Off-shell to on-shell: spin-1 in gravity

Expand metric e.g. as $\sqrt{-g}g^{\mu\nu} = \eta^{\mu\nu} - \kappa \mathfrak{h}^{\mu\nu}$

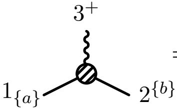
$$(\sqrt{-g}\mathcal{L}_{\mathsf{Proca}})_{VV\mathfrak{h}} = \frac{\kappa}{2}\mathfrak{h}^{\mu\nu}\big(V_{\mu\sigma}V_{\nu}{}^{\sigma} - m^2 V_\mu V_\nu\big) - \frac{\kappa}{8}\mathfrak{h}V_{\mu\nu}V^{\mu\nu},$$

$\Rightarrow$

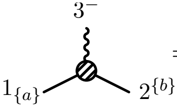


$$= -i\kappa\Big[\big((p_1\cdot p_2) + m^2\big)\eta^{\lambda(\nu}\eta^{\rho)\mu} + \eta^{\lambda\mu}p_1^{(\nu}p_2^{\rho)} - p_1^\mu\eta^{\lambda(\nu}p_2^{\rho)} - p_2^\lambda\eta^{\mu(\nu}p_1^{\rho)} - \frac{1}{2}\eta^{\nu\rho}\big(\eta^{\lambda\mu}(p_1\cdot p_2) - p_2^\lambda p_1^\mu\big)\Big]$$

Plug in pol. vectors:



$$= \frac{\langle 1_{(a_1}2^{(b_1}\rangle\langle 1_{a_2)}2^{b_2)}\rangle}{m^2}\mathcal{M}_3^{(0,+)},$$



$$= \frac{[1_{(a_1}2^{(b_1}][1_{a_2)}2^{b_2)}]}{m^2}\mathcal{M}_3^{(0,-)},$$

where $\mathcal{M}_3^{(0,\pm)} = -i\kappa(p_1\cdot\varepsilon_3^\pm)^2$

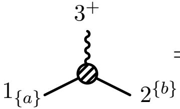
Physical patterns make sense on shell

e.g. double-copy aspects in Johansson, AO '19

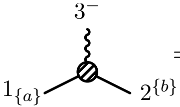
AHH amplitudes & black holes

3-pt amplitudes singled out (and misnamed as "minimal coupling"):

Arkani-Hamed, Huang, Huang '17



$$= \frac{\langle 1_a 2^b \rangle^{\odot 2s}}{m^{2s}} \mathcal{M}_3^{(0,+)}$$



$$= \frac{[1_a 2^b]^{\odot 2s}}{m^{2s}} \mathcal{M}_3^{(0,-)}$$

$$\xrightarrow[\text{class. limit}]{} \mathcal{M}_3^{(0,\pm)} \exp\left(\mp \frac{p_3 \cdot S}{m} \right)$$

Guevara, AO, Vines '18, '19

Kerr's spin exp. from Newman-Janis shift, e.g. Arkani-Hamed, Huang, O'Connell '19

- high interest e.g. in view of GW applications
  - higher-point obstacles (unphysical pole at 4 pts, identifying BH vs NS etc., NLO grav. interactions of Kerr)
- multitude of genuine higher-spin theories still to discover

All-plus amplitudes

Some higher-spin amplitudes are not problematic at all!

(healthy from naive on-shell recursion, 2 distinct shifts) Britto, Cachazo, Feng, Witten '05
w/ masses by Badger, Glover, Khoze, Svrcek '05

- Color-ordered amplitudes in gauge theory:

$$A(1,2^+,3^+,\ldots,(n-1)^+,n) = \frac{im^2[2|\prod_{j=2}^{n-3}\{\not{P}_{12\ldots j}\,\not{p}_{j+1}+(s_{12\ldots j}-m^2)\}|n-1]}{\prod_{j=2}^{n-2}\langle j|j+1\rangle(s_{12\ldots j}-m^2)}$$

$$A(1_{\{a\}},2^+,3^+,\ldots,(n-1)^+,n^{\{b\}}) = \frac{\langle 1_a n^b\rangle^{\odot 2s}}{m^{2s}}A(1,2^+,3^+,\ldots,(n-1)^+,n)$$

spin-0: Ferrario, Rodrigo, Talavera '06
spin-1/2: Schwinn, Weinzierl '07; AO '18
spin-1: Ballav, Manna '21
spin-s: Lazopoulos, AO, Shi '21

- EM from QCD-to-QED projection:

$$\mathcal{A}(1_{\{a\}},2,3,\ldots,n-1,n^{\{b\}}) = \left(\sqrt{2}Q\right)^{n-2}\sum_{\sigma\in S_{n-2}(\{2,3,\ldots,n-1\})}A(1_{\{a\}},\sigma,n^{\{b\}})$$

- Grav. amplitudes either from BCFW or KLT double copy (massive)

$$\mathcal{M}(1_{\{a\}},2^{\{b\}},3^+,\ldots,n^+) = \frac{\langle 1_a 2^b\rangle^{\odot 2s}}{m^{2s}}\mathcal{M}(1,2,3^+,\ldots,n^+)$$

Lazopoulos, AO, Shi '21; see also Aoude, Haddad, Helset '20
Double copy: Kawai, Lewellen, Tye; Bern, Carrasco, Johansson; Bjerrum-Bohr, Donoghue, Vanhove; Naculich; Johansson, AO; Bautista, Guevara

Hidden gems among massive higher-spin theories?

Free massive higher-spin theory

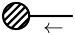
henceforth AO, Skvortsov '22

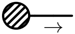
Start chiral, no need to remove d.o.f.!

$$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu \Phi^{\alpha_1\ldots\alpha_{2s}})(\partial^\mu \Phi_{\alpha_1\ldots\alpha_{2s}}) - \frac{m^2}{2}\Phi^{\alpha_1\ldots\alpha_{2s}}\Phi_{\alpha_1\ldots\alpha_{2s}}$$

Free field expansion for KFG eqn:

$$\Phi_{\alpha_1\ldots\alpha_{2s}}(x) = \int \frac{\hat{d}^3p}{2p^0}\Bigg[\frac{|p^{(a_1}\rangle_{\alpha_1}\cdots|p^{a_{2s})}\rangle_{\alpha_{2s}}}{m^s}\, a_{a_1\ldots a_{2s}}(\vec{p})\, e^{-ip\cdot x} + (-1)^{2s}\frac{|p_{(a_1}\rangle_{\alpha_1}\cdots|p_{a_{2s})}\rangle_{\alpha_{2s}}}{m^s}\, a^{\dagger a_1\ldots a_{2s}}(\vec{p})\, e^{ip\cdot x}\Bigg]\Bigg|_{p^0=\sqrt{\vec{p}^2+m^2}}$$

 $$p, a_1, \ldots, a_{2s} \;=\; \frac{1}{m^s}|p^{(a_1}\rangle_{\alpha_1}\cdots|p^{a_{2s})}\rangle_{\alpha_{2s}}$$

 $$p, a_1, \ldots, a_{2s} \;=\; \frac{(-1)^{2s}}{m^s}|p_{(a_1}\rangle_{\alpha_1}\cdots|p_{a_{2s})}\rangle_{\alpha_{2s}}$$

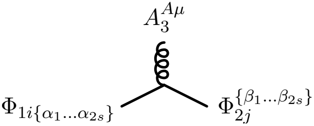
Massive spinor helicity ideal for ext. wavefunctions

Gauge interactions

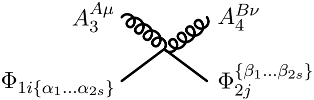
Gauge interactions

$$\mathcal{L}_g = \frac{1}{2}(D_\mu \Phi^{\{\alpha\}})_i (D^\mu \Phi_{\{\alpha\}})_i - \frac{m^2}{2}\Phi_i^{\{\alpha\}}\Phi_{i\{\alpha\}} - \frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu},$$

$$D_\mu := \partial_\mu + g A_\mu, \quad A_\mu = A^A_\mu t^A, \quad [t^A, t^B] = f^{ABC}t^C, \quad t^A_{ij} = -t^A_{ji}$$



$$= g t^A_{ij}\delta^{(\beta_1}_{\alpha_1}\cdots\delta^{\beta_{2s})}_{\alpha_{2s}}(p_1 - p_2)^\mu$$



$$= -ig^2[t^A_{ik}t^B_{kj} + t^B_{ik}t^A_{kj}]\delta^{(\beta_1}_{\alpha_1}\cdots\delta^{\beta_{2s})}_{\alpha_{2s}}\eta^{\mu\nu}$$

- Amplitudes are simply (not only all-plus!):

$$\mathcal{A}(1_{\{a\}}, 2^{\{b\}}, 3^{h_3}, \ldots, n^{h_n}) = \frac{\langle 1_a 2^b\rangle^{\odot 2s}}{m^{2s}}\mathcal{A}(1, 2, 3^{h_3}, \ldots, n^{h_n})$$

$$\mathcal{A}(1_{\{a\}}, 2^{\{b\}}, 3_{\{c\}}, 4^{\{d\}}, 5^{h_5}, \ldots, n^{h_n}) = \frac{\langle 1_a 2^b\rangle^{\odot 2s}\langle 3_c 4^d\rangle^{\odot 2s}}{m^{4s}}\mathcal{A}(1, 2, 3, 4, 5^{h_5}, \ldots, n^{h_n})$$
$$+(-1)^{2s}\frac{\langle 1_a 4^d\rangle^{\odot 2s}\langle 3_c 2^b\rangle^{\odot 2s}}{m^{4s}}\mathcal{A}(1, 4, 3, 2, 5^{h_5}, \ldots, n^{h_n})$$

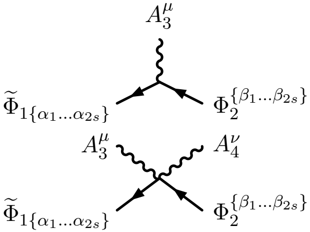
Electromagnetic interactions

Restrict to $\mathrm{SO}(2)$: $\quad f^{ABC} = 0, \quad t_{ij}^{A=1} = \epsilon^{ij}$

$$\Phi^{\{\alpha\}} := \Phi_{j=1}^{\{\alpha\}} + i\Phi_{j=2}^{\{\alpha\}}$$
$$\widetilde{\Phi}^{\{\alpha\}} := \Phi_{j=1}^{\{\alpha\}} - i\Phi_{j=2}^{\{\alpha\}}$$

$$D_\mu := \partial_\mu - iQA_\mu$$

$$\underline{\mathcal{L}_Q = (\widetilde{D_\mu \Phi^{\{\alpha\}}})(D^\mu \Phi_{\{\alpha\}}) - m^2 \widetilde{\Phi}^{\{\alpha\}} \Phi_{\{\alpha\}} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$



$$= iQ\delta_{\alpha_1}^{(\beta_1} \cdots \delta_{\alpha_{2s}}^{\beta_{2s})} (p_2 - p_1)^\mu$$

$$= 2iQ^2 \delta_{\alpha_1}^{(\beta_1} \cdots \delta_{\alpha_{2s}}^{\beta_{2s})} \eta^{\mu\nu}$$

Gravitational interactions

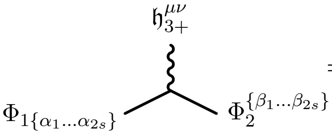
Gravitational interactions

$$\underline{\mathcal{L}_G = \sqrt{-g}\left\{\frac{1}{2}(\nabla_\mu \Phi^{\{\alpha\}})(\nabla^\mu \Phi_{\{\alpha\}}) - \frac{m^2}{2}\Phi^{\{\alpha\}}\Phi_{\{\alpha\}} + R\right\}},$$

$$\nabla_\mu \Phi_{\alpha_1 \ldots \alpha_{2s}} = \partial_\mu \Phi_{\alpha_1 \ldots \alpha_{2s}} + 2s\, \omega_{\mu,(\alpha_1}{}^{\beta} \Phi_{\alpha_2 \ldots \alpha_{2s})\beta}$$

Anti-self-dual spin connection $\omega_{\mu,\alpha}{}^{\beta} := \frac{i}{2}\omega_\mu{}^{\hat\nu\hat\rho}\sigma_{\hat\nu,\alpha\dot\gamma}\bar\sigma_{\hat\rho}^{\dot\gamma\beta} \underset{\text{SDGR}}{\Rightarrow} 0$

e.g. Penrose '76

$\Rightarrow$ 

$= i\kappa \delta^{(\beta_1}_{\alpha_1} \cdots \delta^{\beta_{2s})}_{\alpha_{2s}} \left[p_2^{(\mu} p_2^{\nu)} + \frac{m^2}{2}\eta^{\mu\nu}\right]$, etc.

- All-plus amplitudes satisfy

$$\mathcal{M}(1_{\{a\}}, 2^{\{b\}}, 3^+, \ldots, n^+) = \frac{\langle 1_a 2^b \rangle^{\odot 2s}}{m^{2s}} \mathcal{M}(1, 2, 3^+, \ldots, n^+)$$

Summary & outlook

- First consistent interacting higher-spin theories in $d = 4$
- Straightforward to introduce further interactions, e.g. in gauge theory:

$$(D_{\alpha_1\dot{\gamma}_1}\cdots D_{\alpha_k\dot{\gamma}_k}\Phi^{\alpha_1\ldots\alpha_{2s}})\epsilon_{\alpha_{k+1}\beta_{k+1}}\cdots\epsilon_{\alpha_{2s-1}\beta_{2s-1}}F_{\alpha_{2s}\beta_{2s}} \\ \times(D_{\beta_1}{}^{\dot{\gamma}_1}\cdots D_{\beta_k}{}^{\dot{\gamma}_k}\Phi^{\beta_1\ldots\beta_{2s}})$$

  - Known chiral action for massive quark in QCD: Chalmers, Siegel '97

$$\mathcal{L}_{\mathsf{QCD}} = \mathcal{L}_{\mathsf{YM}} + \frac{1}{2}(D_\mu\Phi^\alpha)(D^\mu\Phi_\alpha) - \frac{m^2}{2}\Phi^\alpha\Phi_\alpha - \frac{g}{2}\Phi^\alpha F_\alpha{}^\beta\Phi_\beta$$

- Restore parity for higher spins
  in progress w/ Cangemi, Chiodaroli, Johansson, Pichini, Skvortsov
- Applications to spinning BHs?
  in progress w/ Cangemi, Chiodaroli, Johansson, Pichini, Skvortsov
- Multitude of theories to construct and explore!

Thank you!

Backup slides

Spinor map

Basis for spinor helicity

- Minkowski space isomorphism:*

$$
\begin{aligned}
\mathrm{M}^{2\times 2,\mathbb{C}}_{\text{Hermitian}} \quad &\leftrightarrow \quad \mathbb{R}^{1,3} \\
p_{\alpha\dot\beta} = p_\mu \sigma^\mu_{\alpha\dot\beta} &= \begin{pmatrix} p^0 - p^3 & -p^1 + ip^2 \\ -p^1 - ip^2 & p^0 + p^3 \end{pmatrix} \\
\det\{p_{\alpha\dot\beta}\} \quad &= \quad m^2
\end{aligned}
$$

- Lorentz group homomorphism:

$$
\begin{aligned}
\mathrm{SL}(2,\mathbb{C}) \quad &\rightarrow \quad \mathrm{SO}(1,3) \\
p_{\alpha\dot\beta} \rightarrow S_\alpha{}^\gamma p_{\gamma\dot\delta} \big(S_\beta{}^\delta\big)^* \quad &\Rightarrow \quad p^\mu \rightarrow L^\mu{}_\nu p^\nu, \quad L^\mu{}_\nu = \frac{1}{2}\,\mathrm{tr}\big(\bar\sigma^\mu S \sigma_\nu S^\dagger\big)
\end{aligned}
$$

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*$\sigma^0 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$, $\sigma^1 = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$, $\sigma^2 = \left(\begin{smallmatrix} 0 & -i \\ i & 0 \end{smallmatrix}\right)$, $\sigma^3 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$, $\epsilon^{\alpha\beta} = -\epsilon_{\alpha\beta} = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$.

Little group transformations

Consider Lorentz transform $p^\mu \to L^\mu{}_\nu p^\nu \quad \leftrightarrow \quad \underset{\in SO(1,3)}{L^\mu{}_\nu} = \frac{1}{2}\,\mathrm{tr}\big(\bar\sigma^\mu \underset{\in SL(2,\mathbb{C})}{S}\sigma_\nu S^\dagger\big)$

MASSLESS:

$$|p\rangle \to S|p\rangle = e^{i\phi/2}|Lp\rangle \qquad \langle p| \to \langle p|S^{-1} = e^{i\phi/2}\langle Lp|$$
$$|p] \to S^{\dagger\,-1}|p] = e^{-i\phi/2}|Lp] \qquad [p| \to [p|S^\dagger = e^{-i\phi/2}[Lp|$$

$e^{ih\phi} \in \mathrm{U}(1)$ encode $2d$ rotations in frame where $p = (E, 0, 0, E)$

MASSIVE:

$$|p^a\rangle \to S|p^a\rangle = \omega^a{}_b|Lp^b\rangle \qquad |p^a\rangle \to |p^a\rangle S^{-1} = \omega^a{}_b|Lp^a\rangle$$
$$|p^a] \to S^{\dagger\,-1}|p^a] = \omega^a{}_b[Lp^b| \qquad [p^a| \to [p^a|S^\dagger = \omega^a{}_b[Lp^b|$$

$\omega \in \mathrm{SU}(2)$ encode $3d$ rotations in rest frame where $p = (m, 0, 0, 0)$

Helicity basis

Arkani-Hamed, Huang, Huang '17

Take $p^\mu = (E, P\cos\varphi\sin\theta, P\sin\varphi\sin\theta, P\cos\theta)$

$$|p^a\rangle = \lambda_{p\alpha}^{\ a} = \begin{pmatrix} \sqrt{E-P}\cos\frac{\theta}{2} & -\sqrt{E+P}e^{-i\varphi}\sin\frac{\theta}{2} \\ \sqrt{E-P}e^{i\varphi}\sin\frac{\theta}{2} & \sqrt{E+P}\cos\frac{\theta}{2} \end{pmatrix}$$

$$[p^a| = \tilde{\lambda}_{p\dot{\alpha}}^{\ a} = \begin{pmatrix} -\sqrt{E+P}e^{i\varphi}\sin\frac{\theta}{2} & -\sqrt{E-P}\cos\frac{\theta}{2} \\ \sqrt{E+P}\cos\frac{\theta}{2} & -\sqrt{E-P}e^{-i\varphi}\sin\frac{\theta}{2} \end{pmatrix}$$

Then spin quant. axis:

$$s^\mu(u_p^a) = \frac{1}{2m}\bar{u}_{pa}\gamma^\mu\gamma^5 u_p^a = (-1)^{a-1}s_p^\mu$$

$$s_p^\mu = \frac{1}{m}(P, E\cos\varphi\sin\theta, E\sin\varphi\sin\theta, E\cos\theta)$$

Spin quantization

Define Pauli-Lubanski vector operator $\Sigma_\lambda = \frac{1}{2m}\epsilon_{\lambda\mu\nu\rho}\Sigma^{\mu\nu}p^\rho$

Its 1-particle matrix elements are

$$\begin{aligned} S_{p\mu}^{\{a\}\{b\}} &= (-1)^s \varepsilon_p^{\{a\}} \cdot \Sigma_\mu \cdot \varepsilon_p^{\{b\}} \\ &= -\frac{s}{2m}\big\{\langle p^{(a_1}|\sigma_\mu|p^{(b_1}] + [p^{(a_1}|\bar{\sigma}_\mu|p^{(b_1}\rangle\big\}\epsilon^{a_2 b_2}\dots\epsilon^{a_{2s})b_{2s})} \end{aligned}$$

Spin quantized explicitly:

$$\frac{\varepsilon_{p\{a\}} \cdot \Sigma^\mu \cdot \varepsilon_p^{\{a\}}}{\varepsilon_{p\{a\}} \cdot \varepsilon_p^{\{a\}}} = \begin{cases} s s_p^\mu, & a_1 = \ldots = a_{2s} = 1, \\ (s-1)s_p^\mu, & \sum_{j=1}^{2s} a_j = 2s+1, \\ (s-2)s_p^\mu, & \sum_{j=1}^{2s} a_j = 2s+2, \\ \quad\dots & \\ -s s_p^\mu, & a_1 = \ldots = a_{2s} = 2, \end{cases}$$

in terms of unit spin vector

$$\begin{aligned} s_p^\mu &= -\frac{1}{2m}\big\{\langle p_1|\sigma^\mu|p^1] + [p_1|\bar{\sigma}^\mu|p^1\rangle\big\} \\ &= \frac{1}{2m}\bar{u}_{p1}\gamma^\mu\gamma^5 u_p^1 = -\frac{1}{2m}\bar{u}_{p2}\gamma^\mu\gamma^5 u_p^2 \end{aligned}$$

$$p \cdot s_p = 0 \qquad s_p^2 = -1$$

Kerr $\Leftarrow$ minimal coupling to gravity (of higher spins)

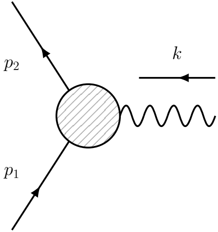
Guevara, AO, Vines '18

$$h_{\mu\nu}(k)T^{\mu\nu}_{\text{BH}}(-k) = \hat{\delta}(k^2)\hat{\delta}(p\cdot k)(p\cdot\varepsilon)^2\exp\left(-i\frac{k_\mu\varepsilon_\nu S^{\mu\nu}}{p\cdot\varepsilon}\right)$$

Compare to

$$\mathcal{M}_3^{(s,+)} = \frac{\mathcal{M}_3^{(0,+)}}{m^{2s}}[2|^{\odot 2s}\exp\left(-i\frac{k_\mu\varepsilon_\nu^+\bar{\sigma}^{\mu\nu}}{p_1\cdot\varepsilon^+}\right)|1]^{\odot 2s}$$

$$\mathcal{M}_3^{(s,-)} = \frac{\mathcal{M}_3^{(0,-)}}{m^{2s}}\langle 2|^{\odot 2s}\exp\left(-i\frac{k_\mu\varepsilon_\nu^-\sigma^{\mu\nu}}{p_1\cdot\varepsilon^-}\right)|1\rangle^{\odot 2s}$$



Matching spin-induced multipole structure!

complementary picture: 1-body EFT of Kerr by Levi, Steinhoff '15
match to Wilson coeffs by Chung, Huang, Kim, Lee '18

Various spin exponentials

originally in Guevara, AO, Vines '18:

$$\mathcal{M}_3^{(s,+)} = \frac{\mathcal{M}_3^{(0)}}{m^{2s}}[2|^{\odot 2s}\exp\left(-i\frac{k_\mu \varepsilon_\nu^+ \bar{\sigma}^{\mu\nu}}{p_1\cdot\varepsilon^+}\right)|1]^{\odot 2s}, \qquad \bar{\sigma}^{\mu\nu} = \tfrac{i}{2}\bar{\sigma}^{[\mu}\sigma^{\nu]}$$

covariantly in Bautista, Guevara '19:

$$\mathcal{M}_3^{(s)} = \mathcal{M}_3^{(0)}\varepsilon_2\cdot\exp\left(-i\frac{k_\mu\varepsilon_\nu\Sigma^{\mu\nu}}{p_1\cdot\varepsilon}\right)\cdot\varepsilon_1, \qquad \Sigma^{\mu\nu,\sigma}{}_\tau = 2i\eta^{\sigma[\mu}\delta_\tau^{\nu]}$$

w/ spin vector and boosts in Guevara, AO, Vines '19:

$$\mathcal{M}_3^{(s,+)} = \frac{\mathcal{M}_3^{(0)}}{m^{2s}}\langle 21\rangle^{\odot 2s} = \frac{\mathcal{M}_3^{(0)}}{m^{2s}}\left\{U_{12}\langle 1|\right\}^{\odot 2s}e^{-k\cdot a}|1\rangle^{\odot 2s}$$
$$= \frac{\mathcal{M}_3^{(0)}}{m^{2s}}[2|^{\odot 2s}e^{-2k\cdot a}|1]^{\odot 2s} = \frac{\mathcal{M}_3^{(0)}}{m^{2s}}\left\{U_{12}[1|\right\}^{\odot 2s}e^{-k\cdot a}|1]^{\odot 2s}$$

from coherent spin states in Aoude, AO '21:

$$\mathcal{M}_3^{\text{min}}(p_2;\beta|p_1,\alpha;k^+) = \mathcal{M}_3^{(0)}(p_2|p_1;k^+)\langle\beta|\alpha\rangle\exp\left\{-\frac{\bar{k}_\mu\langle\beta|S_{p_a}^\mu|\alpha\rangle}{m\langle\beta|\alpha\rangle}\right\}$$

Pauli-Lubanski pseudovector is $S_p^\lambda = m a_{(p)}^\lambda = \frac{1}{2m}\epsilon^{\lambda\mu\nu\rho}S_{\mu\nu}p_\rho$