# Even photons break de Sitter symmetry

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ITMP, 11.12.2024.

[1] Glavan, *Photon quantization in cosmological spaces*, Phys.Rev.D 109 (2024) 8, 8, arXiv:2212.13975
[2] Glavan, Prokopec, *Photon propagator in de Sitter space in the general covariant gauge*, JHEP 05 (2023) 126, arXiv:2212.13982
[3] Glavan, Prokopec, *Even the photon propagator must break de Sitter symmetry*, Phys.Lett.B 841 (2023) 137928, arXiv:2212.13997

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# QFT in inflation

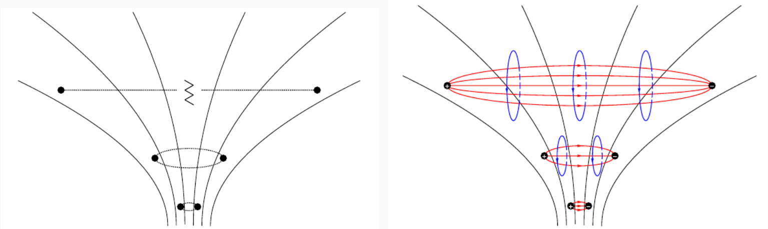
Cosmological space (Friedmann–Lemaître–Robertson–Walker)

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^{\,2} = a^2(\eta)\left[-d\eta^2 + d\vec{x}^{\,2}\right], \qquad dt = a d\eta$$

Accelerated expansion (inflation):

$$H = \frac{\dot{a}}{a} > 0\,, \qquad \epsilon = -\frac{\dot{H}}{H^2} \ll 1$$

Gravitational particle creation:



Parker, Phys.Rev.Lett. 21 (1968) 562-564; Phys.Rev. 183 (1969) 1057-1068; Phys.Rev.D 3 (1971) 346-356

# Photons in inflation

Conformal coupling

$\Longrightarrow$ Photons see no cosmological expansion

Action for photons,

$$S[A_\mu] = \int d^D x \sqrt{-g} \left[ -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right], \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

in $D=4$ FLRW

$$S[A_\mu] = \int d^4 x \left[ -\frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right],$$

is the same as in flat space

BUT, photons couple to other fields!

## Loop corrections

Scalars and gravitons mediate effects of expansion to photons

$\longrightarrow$ quantify interaction effects by quantum loops

$\longrightarrow$ idealizations:

de Sitter space as model for inflation (expanding Poincaré patch)

$$H = \texttt{const.} \qquad \epsilon = 0 \qquad [a(t) = e^{Ht}]$$

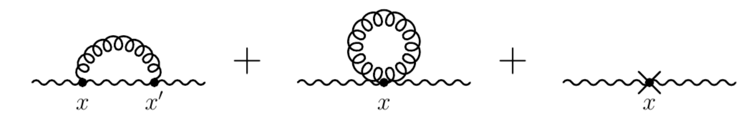
Often close enough to slow-roll inflation for practical purposes

Maximally symmetric space $\rightarrow$ 10 isometries in $D{=}4$

Large number of background symmetries simplifies the problem conceptually and practically (think of Poincaré symmetries in flat space)

# Loop corrections

Gravity + EM




Leonard, Woodard, arXiv:1304.7265
Glavan, Miao, Prokopec, Woodard, arXiv:1308.3453
Wang, Woodard, arXiv:1408.1448
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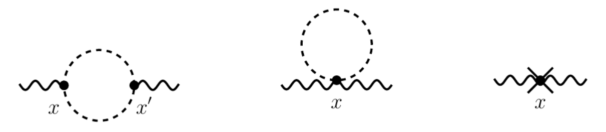
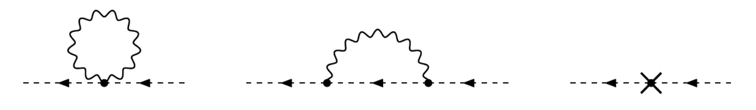
Glavan, Miao, Prokopec, Woodard, arXiv:1609.00386
Miao, Prokopec, Woodard, arXiv:1806.00742

# Loop corrections

## Scalar electrodynamics



Kahya, Woodard, Phys. Rev. D **72** (2005), 104001 [arXiv:gr-qc/0508015].

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Prokopec, Tsamis, Woodard, Phys. Rev. D **78** (2008), 043523 [arXiv:0802.3673].

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# Photon 2-point functions

Understand the basic ingredients of perturbation theory

— 2-point functions (propagators)

Nonequilibrium QFT – Schwinger-Keldysh formalism

$$
\begin{aligned}
\texttt{Feynman}: &\quad i\big[{}^{+}_{\mu}\Delta^{+}_{\nu}\big](x;x') = \big\langle \Omega \big| \mathcal{T} \hat{A}_\mu(x) \hat{A}_\nu(x') \big| \Omega \big\rangle \\
\texttt{Wightman}: &\quad i\big[{}^{-}_{\mu}\Delta^{+}_{\nu}\big](x;x') = \big\langle \Omega \big| \hat{A}_\mu(x) \hat{A}_\nu(x') \big| \Omega \big\rangle \\
\texttt{Dyson}: &\quad i\big[{}^{-}_{\mu}\Delta^{-}_{\nu}\big](x;x') = \Big\{ i\big[{}^{+}_{\mu}\Delta^{+}_{\nu}\big](x;x') \Big\}^* \\
\texttt{anti-Wightman}: &\quad i\big[{}^{+}_{\mu}\Delta^{-}_{\nu}\big](x;x') = \Big\{ i\big[{}^{-}_{\mu}\Delta^{+}_{\nu}\big](x;x') \Big\}^*
\end{aligned}
$$

# Photon propagator: gauge-fixing

Computations require gauge-fixing.

General covariant gauge:

$$S_\star[A_\mu] = \int d^Dx\,\sqrt{-g}\left[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2\xi}\big(\nabla^\mu A_\mu\big)^2\right]$$

Respects all isometries of de Sitter $\longrightarrow$ simplifications!

Fee gauge-fixing parameter $\xi$ allows to check observables.

Gauge-fixed equation of motion:

$$\left[\delta^\mu_\rho\,\square - \Big(1-\frac{1}{\xi}\Big)\nabla_\rho\nabla^\mu - R_\rho{}^\mu\right] i\big[{}_\mu\Delta_\nu\big](x;x') = g_{\rho\nu}\frac{i\delta^D(x-x')}{\sqrt{-g}}$$

# Photon propagator: dS invariant ansatz

Allen, Jacobson, *Vector Two Point Functions in Maximally Symmetric Spaces*,

Commun.Math.Phys. 103 (1986) 669

$\xi = 1,\ D$

Photon in dS $\longrightarrow$ respects dS isometries

de Sitter invariant distance (related to geodesic length)

$$y(x;x') = H^2 a a' \Big[ \|\vec{x} - \vec{x}\,'\|^2 - \big( |\eta - \eta'| - i\varepsilon \big)^2 \Big]$$

dS invariant ansatz:

$$i\big[{}_\mu \Delta_\nu\big](x;x') = \big(\partial_\mu \partial'_\nu y\big)\, \mathcal{C}_1(y) + \big(\partial_\mu y\big)\big(\partial'_\nu y\big)\, \mathcal{C}_2(y)$$

Plug it into equations of motion...

## Photon propagator: dS invariant solution

Solve for structure functions (with correct singular structure):

$$\mathcal{C}_1(y) = \frac{1}{2\nu H^2}\left[-\Big(\nu+\frac{1}{2}\Big)\mathcal{F}_\nu(y) - \Big(1-\frac{\xi}{\xi_s}\Big)\frac{\partial}{\partial y}\frac{\partial}{\partial \nu}\mathcal{F}_{\nu+1}(y)\right]$$

$$\mathcal{C}_2(y) = \frac{1}{2\nu H^2}\left[-\frac{1}{2}\frac{\partial}{\partial y}\mathcal{F}_\nu(y) - \Big(1-\frac{\xi}{\xi_s}\Big)\frac{\partial^2}{\partial y^2}\frac{\partial}{\partial \nu}\mathcal{F}_{\nu+1}(y)\right]$$

where parameters are,

$$\nu = \frac{D-3}{2} \xrightarrow{D\to 4} \frac{1}{2}\,, \qquad \xi_s = \frac{D-1}{D-3} \xrightarrow{D\to 4} 3$$

Scalar propagator function:

$$\mathcal{F}_\nu(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma\big(\frac{D-1}{2}+\nu\big)\,\Gamma\big(\frac{D-1}{2}-\nu\big)}{\Gamma\big(\frac{D}{2}\big)} \times {}_2F_1\Big(\big\{\tfrac{D-1}{2}+\nu, \tfrac{D-1}{2}-\nu\big\}, \big\{\tfrac{D}{2}\big\}, 1-\tfrac{y}{4}\Big)$$

# Photon propagators in dS

Tsamis, Woodard, *A Maximally symmetric vector propagator*, J.Math.Phys. 48 (2007) 052306, arXiv:gr-qc/0608069

$\xi \to 0$, $D$

Youssef, *Infrared behavior and gauge artifacts in de Sitter spacetime: The photon field*, Phys.Rev.Lett. 107 (2011) 021101, arXiv:1011.3755

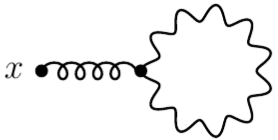
$\xi$, $D = 4$

Fröb, Higuchi, *Mode-sum construction of the two-point functions for the Stueckelberg vector fields in the Poincaré patch of de Sitter space*, J.Math.Phys. 55 (2014) 062301 arXiv:1305.3421

$\xi$, $D$

# Energy-momentum tensor

Simplest one-loop observable:




Two possible definitions:

$$\boldsymbol{T_{\mu\nu}} = \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = \left(\delta^\rho_\mu \delta^\sigma_\nu - \tfrac{1}{4} g_{\mu\nu} g^{\rho\sigma}\right) g^{\alpha\beta} F_{\rho\alpha} F_{\sigma\beta}$$

$$T^\star_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_\star}{\delta g_{\mu\nu}} = \boldsymbol{T_{\mu\nu}} + \boldsymbol{T^{\mathrm{gf}}_{\mu\nu}}$$

$$\boldsymbol{T^{\mathrm{gf}}_{\mu\nu}} = -\frac{2}{\xi} A_{(\mu} \nabla_{\nu)} \nabla^\rho A_\rho + \frac{g_{\mu\nu}}{\xi} \left[ A_\rho \nabla^\rho \nabla^\sigma A_\sigma + \tfrac{1}{2} (\nabla^\rho A_\rho)^2 \right]$$

# Energy-momentum tensor: problem

Different result for two definitions:

$$\boxed{\left\langle \hat{T}_{\mu\nu} \right\rangle = 0\,, \qquad \left\langle \hat{T}^{\star}_{\mu\nu} \right\rangle = -\frac{3H^4}{16\pi^2} g_{\mu\nu}}$$

$\Longrightarrow$ Take a closer look at the propagator

# Propagators vs Green's functions

Not every solution to the propagator equation is a two-point function (propagator).

Janssen, Miao, Prokopec, Woodard, *Infrared Propagator Corrections for Constant Deceleration*, Class.Quant.Grav. 25 (2008) 245013, 0808.2449 [gr-qc]

E. g. Harmonic oscillator

$$-m\Big[\frac{d^2}{dt^2}+\omega^2\Big]\big\langle\mathcal{T}[\hat{X}(t)\hat{X}(t')]\big\rangle = i\delta(t-t')$$

Solution for arbitrary $\alpha,\beta,\gamma$:

$$\big\langle\mathcal{T}[\hat{X}(t)\hat{X}(t')]\big\rangle = \frac{-i}{2m\omega}\sin(\omega|t-t'|) + \alpha\cos(\omega t)\cos(\omega t') + \beta\sin[\omega(t+t')] + \gamma\sin(\omega t)\sin(\omega t)$$

<u>BUT</u>, uncertainty principle requires:

$$\alpha\gamma \geq \frac{1}{4m^2\omega^2}$$

# Gauge fields propagators

Canonical quantization requires that the two-point function

$$i\big[{}_{\mu}^{-}\Delta_{\nu}^{+}\big](x;x') = \big\langle \hat{A}_\mu(x)\hat{A}_\nu(x')\big\rangle$$

satisfies the equation of motion,

$$\bigg[\delta_\rho^\mu\,\square - \Big(1-\frac{1}{\xi}\Big)\nabla_\rho\nabla^\mu - R_\rho{}^\mu\bigg] i\big[{}_{\mu}^{-}\Delta_{\nu}^{+}\big](x;x') = 0$$

AND subsidiary conditions:

$$\begin{aligned}
\big\langle \hat{\Pi}_0\hat{\Pi}_0\big\rangle = 0 \qquad &\Rightarrow \qquad \nabla^\mu\nabla'^\nu i\big[{}_{\mu}^{-}\Delta_{\nu}^{+}\big](x;x') = 0\,,\\
\big\langle \partial_i\hat{\Pi}_i\,\hat{\Pi}_0\big\rangle = 0 \qquad &\Rightarrow \qquad \big(\delta_{[i}^{\mu}\partial_{0]}\partial_i\big)\nabla'^\nu i\big[{}_{\mu}^{-}\Delta_{\nu}^{+}\big](x;x') = 0\\
\big\langle \partial_i\hat{\Pi}_i\,\partial_j\hat{\Pi}_j\big\rangle = 0 \qquad &\Rightarrow \qquad \big(\delta_{[i}^{\mu}\partial_{0]}\partial_i\big)\big(\delta_{[j}^{\mu}\partial'_{0]}\partial'_j\big) i\big[{}_{\mu}^{-}\Delta_{\nu}^{+}\big](x;x') = 0
\end{aligned}$$

These are quantum analogues classical first-class constraints

# Checks of subsidiary conditions

| dS propagator | $\langle \hat{\Pi}_0 \hat{\Pi}_0 \rangle$ | $\langle \partial_i \hat{\Pi}_i \, \hat{\Pi}_0 \rangle$ | $\langle \partial_i \hat{\Pi}_i \, \partial_j \hat{\Pi}_j \rangle$ |
|---|---|---|---|
| [1] $\xi = 1, \quad D$ | ✗ | ✓ | ✓ |
| [2] $\xi = 0, \quad D$ | ✓ | ✓ | ✓ |
| [3] $\xi, \quad D = 4$ | ✗ | ✓ | ✓ |
| [4] $\xi, \quad D$ | ✗ | ✓ | ✓ |


[1] Allen, Jacobson, *Vector Two Point Functions in Maximally Symmetric Spaces*, Commun.Math.Phys. 103 (1986) 669

[2] Tsamis, Woodard, *A Maximally symmetric vector propagator*, J.Math.Phys. 48 (2007) 052306, arXiv:gr-qc/0608069

[3] Youssef, *Infrared behavior and gauge artifacts in de Sitter spacetime: The photon field*, Phys.Rev.Lett. 107 (2011) 021101, arXiv:1011.3755

[4] Fröb, Higuchi, *Mode-sum construction of the two-point functions for the Stueckelberg vector fields in the Poincaré patch of de Sitter space*, J.Math.Phys. 55 (2014) 062301 arXiv:1305.3421

# Energy-momentum tensor & subsidiary conditions

Problematic subsidiary condition:

$$\nabla^{\mu}\nabla'^{\nu} i\big[{}^{-}_{\mu}\Delta^{+}_{\nu}\big](x;x') = -\frac{\xi H^{D}\,\Gamma(D)}{(4\pi)^{\frac{D}{2}}\,\Gamma\big(\frac{D}{2}\big)}$$

It accounts for the issue with energy-momentum tensor:

$$\big\langle T^{\star}_{\mu\nu}\big\rangle \sim \frac{g_{\mu\nu}}{2\xi}\nabla^{\rho}\nabla'^{\sigma} i\big[{}_{\rho}\Delta_{\sigma}\big](x;x') = \frac{g_{\mu\nu}}{2\xi}\times\left[-\frac{3\xi H^{4}}{8\pi^{2}}\right] = -\frac{3H^{4}}{16\pi^{2}}g_{\mu\nu}$$

$\Rightarrow$ fix the propagator!

## Propagator as sum-over-modes (1)

Momentum space construction: solve for field operators in terms of momentum space mode functions, and "creation/annihilation" operators that act on the space of states.

The physical state has to satisfy subsidiary conditions — they must be annihilated by a **non**-Hermitian linear combination of first class constraints ($\longrightarrow$ *derivation of Gupta-Bleuler quantization*),

$$\hat{\mathcal{K}}(\vec{k})\big|\Omega\big\rangle = 0 \,, \qquad \hat{\mathcal{K}}^\dagger \neq \hat{\mathcal{K}}$$

which guarantees correlators of Hermitian constraints vanish,

$$\big\langle \hat{\Pi}_0 \hat{\Pi}_0 \big\rangle = 0 \,, \qquad \big\langle \partial_i \hat{\Pi}_i \, \hat{\Pi}_0 \big\rangle = 0 \,, \qquad \big\langle \partial_i \hat{\Pi}_i \, \partial_j \hat{\Pi}_j \big\rangle = 0$$

dS symmetry generators $\hat{G}_I$ from gauge-invariant action fix the transverse part only (physical symmetries),

$$\hat{G}_I \big|\Omega\big\rangle_T = 0 \,. \qquad (\mathtt{T} = \mathtt{transverse})$$

# Propagator as sum-over-modes (2)

dS symmetry generators $\hat{G}_I^\star$ descending from the gauge-fixed action fixes the complete state (gauge-fixed symmetries),

$$\hat{G}_I^\star \big| \Omega \big\rangle = 0 \, ,$$

while being compatible with the non-Hermitian constraint from the subsidiary condition,

$$\big[ \hat{G}_I , \hat{\mathcal{K}}(\vec{k}) \big] \propto \hat{\mathcal{K}}(\vec{k})$$

Does such a state exist?

— Yes, but it **does not** give a dS invariant propagator:

$$i \big[ {}_\mu \Delta_\nu \big](x;x') = \big( \partial_\mu \partial'_\nu y \big) \, \mathcal{C}_1(y) + \big( \partial_\mu y \big) \big( \partial'_\nu y \big) \, \mathcal{C}_2(y) + \big( a a' \delta_\mu^0 \delta_\nu^0 \big) \times \frac{\xi H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{(D-1) \, \Gamma\big(\frac{D}{2}\big)}$$

Extra piece — homogeneous term of e.o.m.

# BRST explanation (1)

Add another piece to the gauge-fixed action

$$S\big[A_\mu, c, \overline{c}\,\big] = \int d^D x\,\sqrt{-g}\,\left[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2\xi}\big(\nabla^\mu A_\mu\big)^2 - \overline{c}\,\Box c\right],$$

containing anti-commuting Faddeev-Popov ghost fields, $\overline{c}c = -c\overline{c}$, that satisfy equations of motion,

$$\Box c = 0\,, \qquad\qquad \Box\overline{c} = 0$$

Now there is a global BRST symmetry,

$$A_\mu \to A_\mu + \theta\xi\partial_\mu c\,, \qquad \overline{c} \to \overline{c} - \theta\nabla^\mu A_\mu\,, \qquad c \to c$$

generated by the BRST charge,

$$\mathrm{Q} = \int d^{D-1}x\, a^{D-4}\Big[a^2\big(\nabla^\mu A_\mu\big)\partial_0 c + \xi F_{0i}\,\partial_i c\Big]\,.$$

Observables commute with the BRST charge, $\big\{Q, \mathscr{O}\big\} = 0$.

# BRST explanation (2)

Quantized theory contains two propagators, for the vector field,

$$\left[\delta^{\mu}_{\rho}\,\Box - \Big(1-\frac{1}{\xi}\Big)\nabla_{\rho}\nabla^{\mu} - R_{\rho}{}^{\mu}\right] i\big[{}_{\mu}\Delta_{\nu}\big](x;x') = g_{\rho\nu}\frac{i\delta^{D}(x-x')}{\sqrt{-g}}$$

and for the FP ghost,

$$i\Delta(x;x') \equiv \big\langle \hat{\bar{c}}(x)\,\hat{c}(x')\big\rangle$$

that satisfies,

$$\Box\, i\Delta(x;x') = \frac{i\delta^{D}(x-x')}{\sqrt{-g}}\,.$$

Physical states are required to be annihilated by the BRST charge,

$$\hat{\mathrm{Q}}\big|\Omega\big\rangle = 0\,,$$

which implies a linear Ward-Takahashi identity,

$$\boxed{\nabla^{\mu} i\big[{}_{\mu}\Delta_{\nu}\big](x;x') = -\xi\partial'_{\nu} i\Delta(x;x')}$$

# Violated Ward-Takahashi identity

Ward-Takahashi identity is violated for de Sitter invariant solutions of the propagator equation!

$$\nabla^{\mu} i\big[{}_{\mu}\Delta_{\nu}\big](x;x') = -\xi \partial'_{\nu} i\Delta(x;x') + \xi \frac{H^{D-1}\,\Gamma(D-1)}{(4\pi)^{\frac{D}{2}}\,\Gamma\big(\frac{D}{2}\big)} (\delta^{0}_{\nu} a')$$

(Flat space experience: assuming symmetries and solving equations of motion automatically satisfies W-T)

# Consistent photon propagator

Photon propagator must satisfy both the equation of motion

$$\left[\delta^{\mu}_{\rho}\Box - \left(1-\frac{1}{\xi}\right)\nabla_{\rho}\nabla^{\mu} - R_{\rho}{}^{\mu}\right] i\big[{}_{\mu}\Delta_{\nu}\big](x;x') = \frac{i\delta^{D}(x-x')}{\sqrt{-g}}$$

**AND**

the Ward-Takahashi identity:

$$\nabla^{\mu}\, i\big[{}_{\mu}\Delta_{\nu}\big](x;x') = -\xi\partial'_{\nu}\, i\Delta(x;x')$$

**AND**

the ghost propagator must satisfy MMCS equation of motion

$$\Box\, i\Delta(x;x') = \frac{i\delta^{D}(x-x')}{\sqrt{-g}}$$

$\Longrightarrow$ photon propagator must break de Sitter

# de Sitter breaking: scalar propagator

There is no de Sitter invariant solution for the MMCS propagator

Allen, *Vacuum states in de Sitter space*, Phys. Rev. D **32** (1985), 3136
Allen, Folacci, *The Massless Minimally Coupled Scalar Field in De Sitter Space*, Phys. Rev. D **35** (1987), 3771

Consider a massive, minimally coupled scalar,

$$S = \int d^D x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{m^2}{2} \phi^2 \right] .$$

Its propagator $i\Delta_m(x;x') = \langle \mathcal{T} \hat{\phi}(x) \hat{\phi}(x') \rangle$ satisfies,

$$\Box \, i\Delta_m(x;x') - m^2 i\Delta_m(x;x') = \frac{i\delta^D(x-x')}{\sqrt{-g}} \, .$$

Massive propagator *does* admit dS invariant solution

# de Sitter breaking: scalar propagator

dS invariant ansatz,

$$i\Delta_m(x;x') = \mathcal{F}_m(y)\,, \qquad y = aa'H^2\Big[\|\vec{x}-\vec{x}\,'\|^2 - \big(|\eta-\eta'|-i\varepsilon\big)^2\Big]$$

leads to ordinary diff. equation,

$$\bigg[(4y-y^2)\frac{\partial^2}{\partial y^2} + D(2-y)\frac{\partial}{\partial y} - \frac{m^2}{H^2}\bigg]\mathcal{F}_m(y) = 0$$

Solution:

$$\mathcal{F}_m = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma\big(\frac{D-1}{2}+\nu_m\big)\,\Gamma\big(\frac{D-1}{2}-\nu_m\big)}{\Gamma\big(\frac{D}{2}\big)} \times {}_2F_1\Big(\big\{\tfrac{D-1}{2}+\nu_m, \tfrac{D-1}{2}-\nu_m\big\}, \big\{\tfrac{D}{2}\big\}, 1-\tfrac{y}{4}\Big)$$

where,

$$\nu_m^2 = \Big(\frac{D-1}{2}\Big)^2 - \frac{m^2}{H^2} \xrightarrow{m\to 0} (\nu+1)^2$$

# de Sitter breaking: scalar propagator

Singular limit $m^2 \to 0$,

$$\mathcal{F}_m(y) \sim \# \frac{H^4}{m^2} \xrightarrow{m\to 0} \infty$$

The propagator cannot respect all de Sitter symmetries.

Solution: propagator appropriate for cosmology preserves spatial homogeneity and isotropy,

$$i\Delta(x;x') = \mathcal{F}_{\nu+1} - \frac{2H^{D-2}\,\Gamma(2\nu+2)\,\Gamma(\nu+1)}{(4\pi)^{\frac{D}{2}}\,\Gamma\big(\nu+\frac{3}{2}\big)\,\Gamma\big(\frac{D-1}{2}\big)} \frac{\left(\frac{k_0^2}{aa'H^2}\right)^{\frac{D-3}{2}-\nu}}{D-3-2\nu}\Bigg|_{\nu\to\frac{D-3}{2}}$$

dS breaking survives in the derivative too,

$$\partial_\mu i\Delta(x;x') = \partial_\mu \mathcal{F}_{\nu+1}(y) + a\delta^0_\mu \times \frac{H^{D-1}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma\big(\frac{D}{2}\big)}$$

# dS breaking: photon propagator

Derivative of MMCS propagator breaks de Sitter

$\Longrightarrow$ Ward-Takahashi identity explicitly breaks de Sitter

$\Longrightarrow$ photon propagator must break de Sitter

Solve simultaneously both equations the photon must satisfy:

$$i\big[{}_\mu\Delta_\nu\big](x;x') = \big(\partial_\mu\partial'_\nu y\big)\,\mathcal{C}_1(y) + \big(\partial_\mu y\big)\big(\partial'_\nu y\big)\,\mathcal{C}_2(y)$$
$$+(aa'\delta^0_\mu\delta^0_\nu) \times \frac{\xi H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{(D-1)\,\Gamma\big(\frac{D}{2}\big)}$$

Propagator picks up a homogeneous dS breaking piece.

Preserves dilations, but breaks spatial special conformal symmetry

# Problems resolved

Energy-momentum tensor discrepancy removed:

$$\langle \hat{T}^{\star}_{\mu\nu} \rangle = \frac{g_{\mu\nu}}{2\xi} \nabla^{\rho} \nabla'^{\sigma} \, i\big[{}_{\mu}\Delta_{\nu}\big](x;x') \bigg|_{x'\to x} = \frac{g_{\mu\nu}}{2\xi} \Box \, i\Delta(x;x') \bigg|_{x'\to x} = 0 \quad \checkmark$$

Infrared behaviour and cluster decomposition...

Youssef, Phys.Rev.Lett. 107 (2011) 021101, arXiv:1011.3755

Rendell, Int.J.Mod.Phys.D 27 (2018) 11, 1843005, arXiv:1802.00687

dS invariant solution:

$$i\big[{}_{\mu}\Delta_{\nu}\big](x;x') \overset{|y|\to\infty}{\sim} -\boldsymbol{\xi} \times \frac{H^{D-2}\,\Gamma(D-1)}{(4\pi)^{\frac{D}{2}}(D-1)\,\Gamma\big(\frac{D}{2}\big)} \times \big(\boldsymbol{aa'\delta^0_{\mu}\delta^0_{\nu}}\big)$$

Consistent dS breaking solution:

$$i\big[{}_{\mu}\Delta_{\nu}\big](x;x') \xrightarrow{|y|\to\infty} 0 \qquad \checkmark$$

# Conclusions

- Photon propagator breaks de Sitter symmetries becasue of Ward-Takahashi identity
- dS breaking pertains to gauge sector, but important for interactions
- Symmetries can be misleading in de Sitter space
- de Sitter group does not play the same role of a convenient organizational principle for computations, as does Poincaré group in flat space (better gauges...)

# Energy-momentum tensor: gauge-fixing contribution

FP ghosts account for proper operator ordering in the definition of observables.

$$T^{\star}_{\mu\nu} = T_{\mu\nu} - \frac{2}{\xi} A_{(\mu} \nabla_{\nu)} \nabla^{\rho} A_{\rho} + \frac{g_{\mu\nu}}{\xi} \Big[ A_{\rho} \nabla^{\rho} \nabla^{\sigma} A_{\sigma} + \tfrac{1}{2} (\nabla^{\rho} A_{\rho})^2 \Big]$$
$$- 2 \partial_{(\mu} \overline{c} \, \partial_{\nu)} c + g_{\mu\nu} \nabla^{\rho} \overline{c} \, \nabla_{\rho} c$$

$$\big\langle T^{\star}_{\mu\nu} \big\rangle - \big\langle T_{\mu\nu} \big\rangle = \bigg\{ \frac{g_{\mu\nu}}{2\xi} \nabla^{\rho} \nabla'^{\sigma} i \big[ {}_{\rho}\Delta_{\sigma} \big](x;x')$$
$$- \frac{1}{\xi} \Big[ \delta^{\rho}_{(\mu} \nabla'_{\nu)} \nabla'^{\sigma} + \delta^{\sigma}_{(\mu} \nabla_{\nu)} \nabla^{\rho} \Big] i \big[ {}_{\rho}\Delta_{\sigma} \big](x;x')$$
$$+ \frac{g_{\mu\nu}}{2\xi} \Big[ \nabla'^{\rho} \nabla'^{\sigma} + \nabla^{\sigma} \nabla^{\rho} \Big] i \big[ {}_{\rho}\Delta_{\sigma} \big](x;x')$$
$$- 2 \nabla_{\mu} \nabla'_{\nu} \, i\Delta(x;x') + g_{\mu\nu} \nabla^{\rho} \nabla'_{\rho} \, i\Delta(x;x') \bigg\} \bigg|_{x' \to x}$$

# Energy-momentum tensor: gauge-fixing contribution

$$\langle T^{\star}_{\mu\nu}\rangle - \langle T_{\mu\nu}\rangle = \bigg\{ \frac{g_{\mu\nu}}{2\xi} \nabla^{\rho}\nabla'^{\sigma} i\big[{}_{\rho}\Delta_{\sigma}\big](x;x')$$

$$- \frac{1}{\xi}\Big[\delta^{\rho}_{(\mu}\nabla'_{\nu)}\nabla'^{\sigma} + \delta^{\sigma}_{(\mu}\nabla_{\nu)}\nabla^{\rho}\Big] i\big[{}_{\rho}\Delta_{\sigma}\big](x;x')$$

$$+ \frac{g_{\mu\nu}}{2\xi}\Big[\nabla'^{\rho}\nabla'^{\sigma} + \nabla^{\sigma}\nabla^{\rho}\Big] i\big[{}_{\rho}\Delta_{\sigma}\big](x;x')$$

$$- 2\nabla_{\mu}\nabla'_{\nu} i\Delta(x;x') + g_{\mu\nu}\nabla^{\rho}\nabla'_{\rho} i\Delta(x;x') \bigg\}\bigg|_{x'\to x}$$

Ward-Takahashi identity relates propagators,

$$\boxed{\nabla^{\mu} i\big[{}_{\mu}\Delta_{\nu}\big](x;x') = -\xi \partial'_{\nu} i\Delta(x;x')}$$

$$\Longrightarrow \qquad \nabla_{\rho}\nabla^{\mu} i\big[{}_{\mu}\Delta_{\nu}\big](x;x') = -\xi \partial_{\rho}\partial'_{\nu} i\Delta(x;x') \qquad \checkmark$$

$$\Longrightarrow \qquad \nabla'^{\nu}\nabla^{\mu} i\big[{}_{\mu}\Delta_{\nu}\big](x;x') = -\xi \Box\, i\Delta(x;x') = 0 \qquad \times$$