New probes of the string spectrum

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Generalities of the spectrum

- two universal string parameters: α' and g_S
- infinitely many *physical* states

$$M_N^2 = N \frac{1}{\alpha'}$$

that are thought of as:

1. mass eigenstates $\Rightarrow$ on–shell mass
2. irreps of $SO(D-1)$ or $SO(D-2)$ $\Rightarrow$ TT

1–particle states à la Bargmann and Wigner ?

- state construction: light–cone gauge or DDF

$\Rightarrow$ *non*–covariant, *superposition* or *fraction* of states

Goddard, Thorn 1972
Goddard, Goldstone, Rebbi, Thorn 1973
Del Giudice, Di Vecchia, Fubini 1972
Brower 1972

Visualisation: Regge trajectories

- state–operator correspondence: e.g. for open bosonic strings

$$\alpha^{\mu_1}_{-n_1} \dots \alpha^{\mu_k}_{-n_k} |0; p\rangle \quad \leftrightarrow \quad \partial^{n_1} X^{\mu_1} \dots \partial^{n_k} X^{\mu_k} \, e^{ip\cdot X}$$

- *leading* Regge: highest spins ∀ level
  1. bosonic strings:

$$V(p,z) = \sum_s f_{\mu_1 \dots \mu_s}(p)\, \partial X^{\mu_1} \dots \partial X^{\mu_s} \, e^{ip\cdot X} \quad , \quad p^2 = -\frac{s-1}{\alpha'}$$

$$p^\mu f_{\mu\mu_2\dots\mu_s} = 0 \quad , \quad f^\mu{}_{\mu\mu_3\dots\mu_s} = 0 \quad \text{or} \quad p \cdot \frac{\delta F_1}{\delta \partial X} = 0 \quad , \quad \frac{\delta^2 F_1}{\delta \partial X \cdot \delta \partial X} = 0$$

$\Rightarrow \mathcal{A}_3, \mathcal{A}_4$ and s_1–s_2–s_3 couplings

Sagnotti, Taronna '10

  2. superstrings, heterotic strings

Schlotteter '10
Bianchi, Lopez, Richter '10
Bianchi, Teresi '11

- disparate decays of states at very high N, *chaos*?

Gross, Rosenhaus '21
Rosenhaus '21
Firrotta, Rosenhaus '22
Bianchi, Firrotta, Sonnenschein, Weissman '22–'23

Lightest (open) bosonic states

- generic vertex operator:

$$V_F(z) = F\Big(\partial X^\mu(z), \partial^2 X^\mu(z), \ldots, \partial^k X^\mu(z)\Big)\, e^{ip\cdot X(z)}$$

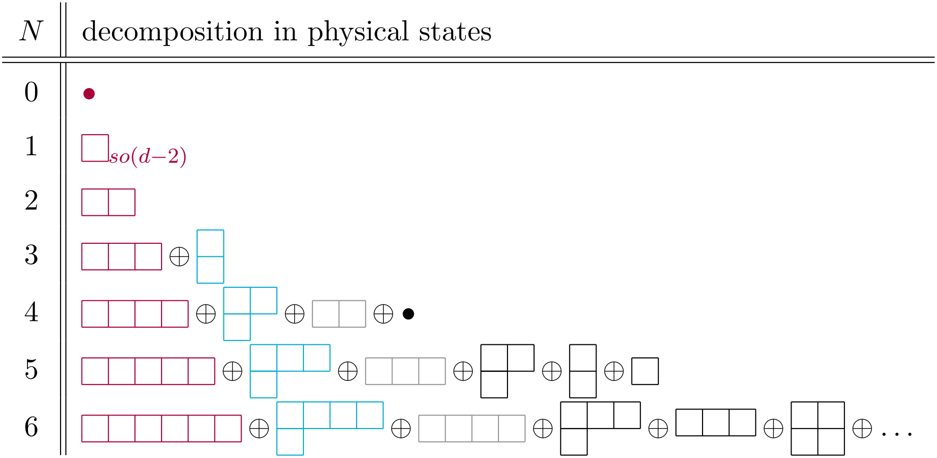
- physical state condition:

$$[Q, V_F] \overset{!}{=} \text{tot. deriv.} \quad \Rightarrow \quad h_V = 1 \quad , \quad \alpha' p^2 = N - 1 \quad , \quad h_F = N$$

- first few levels:
  1. $N = 0$: $V_{\text{tachyon}}(p, z) = e^{ip\cdot X(z)}$
  2. $N = 1$: $V_\epsilon(p, z) = \epsilon_\mu(p)\, i\partial X^\mu(z)\, e^{ip\cdot X(z)}$
  3. $N = 2$: $V_B(p, z) = B_{\mu\nu}(p)\, i\partial X^\mu(z) i\partial X^\nu(z)\, e^{ip\cdot X(z)}$

[$g_{\rm o}$, T^a and normalizations in front of v.o. omitted for simplicity]

Lightest (open) bosonic states



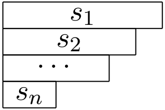
see e.g. Weinberg 1985, Mañes, Vozmediano 1989

the spectrum seems repetitive, but is there a certain *pattern*?

How does a state look like?

- *any* state is associated with a polarization and a Young diagram

$$\epsilon^{\mu_1(s_1),\,\mu_2(s_2),\ldots,\,\mu_k(s_n)}(p):$$



$$,\quad s_1 \geq s_2 \geq \ldots \geq s_n$$

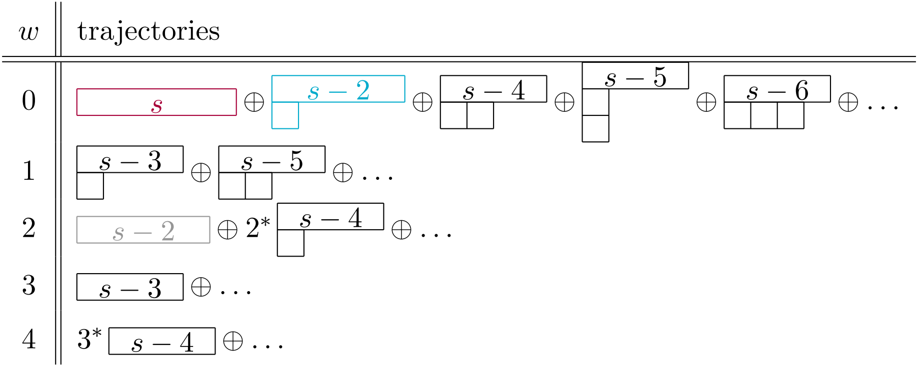
- conversely, there are *infinitely* many ways to embed a given diagram into the spectrum
  - lowest possible level: $N_{\text{min}} = \sum_{i=1}^{n} s_i\, i$
    example: leading Regge
  - higher possible levels: $N = N_{\text{min}} + w$
    example: *clone* of (massive) leading Regge that starts at $N = 4$

$\Rightarrow$ use "depth" w, instead of level N, to organize the spectrum

CM, Skvortsov '23

Let's reorganize the spectrum

- let's call trajectory a Young shape with a *given* number of rows
  - leading Regge: 1 row, spin–s, $N = s$, $w = 0$
  - clone of leading Regge: 1 row, spin–$(s-2)$, $N = s$, $w = 2$
  - $\exists$ *infinitely* many clones, *infinitely* many other trajectories & clones



CM, Skvortsov '23

All trajectories at once

- let's consider the *most general* vertex operator:

$$\mathbb{V}_F(z,p) = F\left[X^{(1)}, X^{(2)}, ...\right] e^{ip\cdot X^{(0)}} \quad , \quad X^{(k)}_\mu \equiv \partial^k X_\mu$$

- let's impose $[Q, \mathbb{V}_F(z,p)] =$ tot. deriv. $\Rightarrow$ obtain:
  - 1 on–shell condition

$$\left(\sum_{n=0} n\, X^{(n)} \cdot \frac{\delta}{\delta X^{(n)}} + \alpha' p^2 - 1 \right) F = 0$$

  - n differential constraints

$$\Bigg[2\alpha'\, n!\, ip \cdot \tfrac{\delta}{\delta X^{(n)}} + \alpha' \sum_{m=1}^{m=n-1} m!(n-m)! \frac{\delta^2}{\delta X^{(m)} \cdot \delta X^{(n-m)}}$$

$$- \sum_{m=0} \frac{(n+m+1)!}{m!} X^{(m+1)} \cdot \frac{\delta}{\delta X^{(n+m+1)}} \Bigg] F = 0 \quad , \quad \forall\, n \in \mathbb{N}^*$$

- now leading Regge is the special case $F = F_1$

CM, Skvortsov '23

Howe duality and a bigger organizing symmetry

- let's define the operators

$$T^k{}_l \equiv X^{(k)} \cdot \frac{\delta}{\delta X^{(l)}} \quad , \quad T_{kl} \equiv \frac{\delta^2}{\delta X^{(k)} \cdot \delta X^{(l)}} \quad , \quad T^{kl} \equiv X^{(k)} \cdot X^{(l)}$$

- can further define

$$T^0{}_l \quad \equiv p \cdot \frac{\delta}{\delta X^{(l)}}$$

  so Young symmetry & TT constraints now become

$$T^k{}_l\, f \quad = 0\,,\, 0 < k < l \quad , \quad T^0{}_l\, f \quad = 0\,,\, l > 0 \quad , \quad T_{kl} f = 0\,,\, k,l > 0$$

- can further define

$$T_n \equiv \sum_{m=0} \frac{(n+m+1)!}{m!} T^{m+1}{}_{n+m+1} \quad , \quad n \in \mathbb{N}^*$$

CM, Skvortsov '23

- $T^0{}_l\,, T^k{}_l\,, T_{kl}, T^{kl}$ $(k, l \neq 0)$: subalgebra of $sp\big(2(\bullet + 1)\big)$
  $\Rightarrow$ *Howe dual* to $iso(1, D-1)$

Howe 1989

Howe duality and a bigger organizing symmetry

- idea: instead of Virasoro, use the *bigger* symmetry *sp* to construct trajectories

CM, Skvortsov '23

- simplification: transverse subspace is sufficient
$\Rightarrow$ ignore terms *linear* in p, that are BRST–exact

see e.g. Mañes, Vozmediano 1989

- Howe duality then implies

$$\text{lowest weight states of } sp(2\bullet) \quad \Longleftrightarrow \quad \text{irreps of } so(d-1,1)$$

- can now rewrite our constraints as

$$(L_0 - 1)F = \Big(\textstyle\sum_{n=0} n\, T^n{}_n + \alpha' p^2 - 1 \Big) F = 0$$

$$-L^{\perp}_{n>0} F = \Big[\alpha' \textstyle\sum_{m=1}^{m=n-1} m!(n-m)!\, T^{\perp}_{m,n-m} - T_n \Big] F = 0$$

- on any function F need only three conditions:

see e.g. Sasaki, Yamanaka 1985

$$L_0 F = 0 \quad , \quad L_1^{\perp} F = 0 \quad , \quad L_2^{\perp} F = 0$$

CM, Skvortsov '23

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Embedding trajectories into the spectrum

1. *Principal* embedding: $w = 0$

$$F_\epsilon = \epsilon^{\mu(s_1),\ldots,\nu(s_K)}\, X^{(1)}_{\mu_1}\ldots X^{(1)}_{\mu_{s_1}}\ldots X^{(K)}_{\nu_1}\ldots. X^{(K)}_{\nu_{s_K}}$$

   $\exists$ new trajectory $\forall$ value of K, e.g. leading Regge for $K = 1$

2. *Non–principal* embedding: $w > 0$
   - use $sp(2\bullet)$ creation operators to construct trajectories

$$F^f_\epsilon \equiv f(T^{mn}_\perp, T^k{}_l)\, F_\epsilon \quad , \quad k > l\,,$$

     where f: trajectory–shifting operator of weight w
   - example: a subleading family at $w = 2$

$$F^f_\epsilon = \left[\frac{\beta_1}{\alpha'} T^{11}_\perp + \beta_2 T^3{}_1 + \beta_3 (T^2{}_1)^2\right] F_\epsilon$$

$$\boxed{s-2} = f \times \boxed{s-2}$$

$$F_\epsilon \equiv \epsilon^{a(s-2)} X^{(1)}_{a(s-2)} \quad , \quad T^\perp_{11} F_\epsilon = 0 \quad , \quad T^1{}_1\, F_\epsilon = (s-2) F_\epsilon$$

CM, Skvortsov '23

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Examples of interactions

- solve BRST ⇒ only one free β–parameter (along with spin)
  - example: massive spin–2 at $N = 4$, $w = 2$

$$F_{\widetilde{B}} = \widetilde{B}_{\mu\nu}\left[-\frac{3}{\alpha'}\, i\partial X^{\mu} i\partial X^{\nu} i\partial X^{\kappa} i\partial X_{\kappa} + 29\, i\partial X^{\mu} i\partial^{3} X^{\nu} - 87\, i\partial^{2} X^{\mu} i\partial^{2} X^{\nu}\right]$$

  it is a trajectory–shifted massive spin–2 from $N = 2$, $w = 0$

$$F_{B} = B_{\mu\nu}\, i\partial X^{\mu} i\partial X^{\nu}$$

  its full trajectory is now known!

  - other subleading trajectories already accessible

- N–point amplitudes for entire subleading trajectories accessible!
  - example: scattering of tachyon off a spin–s

$$\mathcal{A}^{s00}_{w=2} \sim \mathcal{A}^{s00}_{w=0}(-d + 4s + 31) \sim (p_2 \cdot \epsilon)^{\,s}(-d + 4s + 31)$$

CM, Skvortsov ’23

Subleading examples of light superstring states

- lightest brane massive spin–2 multiplet:

$$\text{e.g.}\quad \mathcal{N}=1: \quad \big(2, 2(3/2), 1\big) \quad , \quad \mathcal{N}=2: \quad \big(2, 4(3/2), 6(1), 4(1/2), 0\big)$$

Feng, Lüst, Schlotterer, Stieberger, Taylor '10
Feng, Lüst, Schlotterer '12

  - spin–1: Kac–Moody current due to internal fermions

Banks, Dixon 1988
Ferrara, Lüst, Theisen 1989
Dixon, Kaplunovsky, Louis 1989

$$V_{\boldsymbol{A}}^{(-1)}(z,a,p) = g_{\rm o} T^a\, e^{-\phi}\, \psi^\mu\, e^{ip\cdot X} \alpha_\mu\, J \quad , \quad p\cdot a = 0 \quad , \quad p^2 = -\frac{1}{\alpha'}$$

- (some of the) lightest bulk massive spin–2 of *subleading* Regge:

$$V_{M,\text{closed}}^{(-1,-1)}(z,\overline{z},\alpha,k) = g_{\rm c}\, e^{-\phi-\widetilde{\phi}}\, \psi^\mu\, \widetilde{\psi}^\nu\, e^{ik\cdot X} \alpha_{\mu\nu}\, J\, \widetilde{J}$$

$$\alpha_{\mu\nu}k^\mu = 0 \quad , \quad k^2 = -\frac{4}{\alpha'} \quad , \quad \alpha_{\mu\nu} = \alpha_{\nu\mu} \quad , \quad \alpha_{\mu\nu}\eta^{\mu\nu} = 0$$

$\Rightarrow$ *mimics* graviton but is *not* highest spin of supermultiplet

Lüst, CM, Mazloumi, Stieberger '23

[normalisations in front of v.o. omitted for clarity]

A stringy massive DC

3–point string amplitudes:

$$\begin{aligned}
\mathcal{M}_{GGG} &= \varepsilon_{1\mu\rho}\,\varepsilon_{2\nu\sigma}\,\varepsilon_{3\lambda\kappa}\,E_{GGG}\,\mathcal{B}^{\mu\nu\lambda}\,\widetilde{\mathcal{B}}^{\rho\sigma\kappa} \\
\mathcal{M}_{GGM} &= \varepsilon_{1\mu\rho}\,\varepsilon_{2\nu\sigma}\,\alpha_{\lambda\kappa}\,E_{GGM}\,\mathcal{B}^{\mu\nu\lambda}\,\widetilde{\mathcal{B}}^{\rho\sigma\kappa}\,\langle\mathcal{J}_3\rangle\,\langle\widetilde{\mathcal{J}}_3\rangle \\
\mathcal{M}_{MMG} &= \alpha_{1\mu\rho}\,\alpha_{2\nu\sigma}\,\varepsilon_{\lambda\kappa}\,E_{MMG}\,\mathcal{B}^{\mu\nu\lambda}\,\widetilde{\mathcal{B}}^{\rho\sigma\kappa}\,\langle\mathcal{J}_1\,\mathcal{J}_2\rangle\,\langle\widetilde{\mathcal{J}}_1\,\widetilde{\mathcal{J}}_2\rangle \\
\mathcal{M}_{MMM} &= \alpha_{1\mu\rho}\,\alpha_{2\nu\sigma}\,\alpha_{3\lambda\kappa}\,E_{MMM}\,\mathcal{B}^{\mu\nu\lambda}\,\widetilde{\mathcal{B}}^{\rho\sigma\kappa}\,\langle\mathcal{J}_1\,\mathcal{J}_2\,\mathcal{J}_3\rangle\,\langle\widetilde{\mathcal{J}}_1\,\widetilde{\mathcal{J}}_2\,\widetilde{\mathcal{J}}_3\rangle
\end{aligned}$$

$$E = |z_{12}|^{\alpha' p_1\cdot p_2}\,|z_{13}|^{\alpha' p_3\cdot p_3}\,|z_{23}|^{\alpha' p_2\cdot p_3}\quad,\quad \mathcal{B}^{\mu\nu\lambda} = 2\alpha'\,(\eta^{\mu\nu}p_1^{\lambda} + \eta^{\nu\lambda}p_2^{\mu} + \eta^{\lambda\mu}p_3^{\nu})$$

- manifest DC structure, e.g. $\mathcal{M}_{MMG} = \overline{\mathcal{A}}_{\boldsymbol{AA}A}\cdot\overline{\mathcal{A}}_{\boldsymbol{AA}A}$
- WS location dependence: Koba–Nielsen factors vs $\mathcal{J}$ correlators
- $\langle\mathcal{J}\rangle = 0$ always , $\langle\mathcal{J}_1\mathcal{J}_2\mathcal{J}_3\rangle \neq 0$ only in the non–abelian case!

Lüst, CM, Mazloumi, Stieberger ’23

[normalisations of amplitudes omitted for clarity]

A stringy massive DC

- $\mathcal{M}_{GGM}^{\mathcal{N}=2,4,8} = 0$
  - true for both abelian and non–abelian brane vectors
  - extension of Landau–Yang theorem? see also

Arkani–Hamed, Huang, Huang '17

- $\mathcal{M}_{MMG}$ same in all cases:

$$\mathcal{M}_{MMG}^{\mathcal{N}=2,4,8} = g_c\Big[(k_1 \cdot \varepsilon \cdot k_1)\,\mathrm{Tr}(\alpha_1 \cdot \alpha_2) + 2\,k_2 \cdot \alpha_1 \cdot \alpha_2 \cdot \varepsilon \cdot k_1\Big]$$
$$+ \text{cyclic permutations}$$

- $\mathcal{M}_{MMM}^{\mathcal{N}=2} = 0$ for abelian but non–zero for non–abelian:

$$\mathcal{M}_{MMM}^{\mathcal{N}=\tilde{4},\tilde{8}} = g_c\Big[(k_1 \cdot \alpha_3 \cdot k_1)\,\mathrm{Tr}(\alpha_1 \cdot \alpha_2) + 2\,k_2 \cdot \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot k_1\Big]$$
$$+ \text{cyclic permutations}$$

$\Rightarrow$ same form as universal 3–graviton string amplitude

Gross, Sloan 1987
Lüst, Theisen, Zoupanos 1988
Stieberger, Taylor '14 – '16

- *finite cubic* amplitudes: can we compare with field theory?

Lüst, CM, Mazloumi, Stieberger '23

"Effective" cubic vertices

- bigravity around Minkowski: massless and massive spin–2 states

Hassan, Rosen '11
Hassan, Schmidt–May, von Strauss '12
Babichev, Marzola, Raidal, Schmidt–May, Urban, Veermäe, von Strauss '16

- our "effective" vertices from $\mathcal{M}_{MMG}$ and $\mathcal{M}_{MMM}$:

$$\mathcal{L}^{\text{eff}}_{\text{GM}^2} = g_c \Big[G^{\mu\nu} (\partial_\mu M_{\rho\sigma} \partial_\nu M^{\rho\sigma} - 4 \partial_\nu M_{\rho\sigma} \partial^\sigma M^\rho_\mu) + z\, M^{\mu\nu} (\partial_\mu G_{\rho\sigma} \partial_\nu M^{\rho\sigma} - \partial_\rho G_{\mu\sigma} \partial_\nu M^{\rho\sigma}) \Big]$$

$$\mathcal{L}^{\text{eff}}_{\text{M}^3} = \frac{g_c}{\alpha'} \Big\{ y\, [M^3] + 2\alpha' M^{\mu\nu} [\partial_\mu M_{\rho\sigma} \partial_\nu M^{\rho\sigma} - x\, \partial_\nu M_{\rho\sigma} \partial^\sigma M^\rho_\mu] \Big\}$$

$$(x, y, z) = \begin{cases} (2,0,2), & \text{if} \quad M_{\mu\nu} \quad \text{is a bulk state of} \quad \mathcal{N} = \widetilde{4}, \widetilde{8} \\ (3,1,1), & \text{if} \quad M_{\mu\nu} \quad \text{is a brane state} \end{cases}$$

$\Rightarrow$ upon comparing with bigravity we obtain $\beta_1 = \beta_3$

Lüst, CM, Mazloumi, Stieberger '21 and '23

curiously same family as:
Bonifacio, Hinterbichler, Joyce, Rosen '17
Momeni, Rumbutis, Tolley '20 and Johnson, Jones, Paranjape '20
Engelbrecht, Jones, Paranjape '22

Concluding remarks

- fields and strings: only cubic level similarities

  Sagnotti, Taronna '10
  Lüst, CM, Mazloumi, Stieberger '21, '23

- open bosonic string: using a symmetry bigger than the Virasoro, in principle we now have access to any trajectory
  - complexity of states *doesn't* necessarily increase with spin
  - can we treat the *entire* spectrum? where do new trajectories start and with what multiplicity?

  CM, Skvortsov '23

- leading Regge: no BH features

  Pichini, Cangemi '22

  Can other trajectories, or more generally string theory, yield Kerr amplitudes?

- other open questions: chaos? AdS/CFT and the tensionless limit of string theory?