Localizing Romans SUGRA

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Introduction

- Solving Einstein's equations is hard!
- But sometimes it is not necessary!
- Some AdS/CFT observables are just integrals over (sub-)spaces of spacetime.
- E.G. the on-shell action should give the large N index in field theory.

$$I \sim \int_M \mathrm{vol}(M)\Big(R + \dots\Big)$$

- Looks very dependent on knowing the metric.....

Introduction

- Equivariant Localization gives a way to compute these without knowing the metric! [Benetti-Genolini, Gauntlett, Sparks]
- Rough Idea:
  - Spacetime with a symmetry.
  - Integrals only receive contributions from symmetry fixed points.
  - $$\text{Obs} = \int_M \Phi = \sum_{\text{f.p.}} \Phi_0$$

Remarks:

- Don't need to know explicit solution.
- Result depends on topology.
- Uniform method for obtaining results.

Equivariant Localization

Equivariant Cohomology

- Interested in computing integrals over manifolds with a symmetry group.
- Consider a $2n$-dimensional manifold without boundary.
- ξ a Killing vector field:

$$\mathscr{L}_\xi g = 0 \quad \Leftrightarrow \quad \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$$

- Assume that it generates a $U(1)$ isometry.
- We are interested in integrating forms, Φ which satisfy

$$\mathscr{L}_\xi \Phi = 0$$

Equivariant Cohomology

- We replace cohomology with equivariant cohomology.
- Exterior derivative replaced by twisted derivative

$$\mathrm{d} \to \mathrm{d}_\xi = \mathrm{d} - \xi \lrcorner$$

- Forms are replaced by poly-forms, these are just forms of mixed degree.

$$\Phi = \Phi_n + \Phi_{n-2} + \ldots + \Phi_0$$

- In general d_ξ is not nilpotent: $\mathrm{d}_\xi^2 = -\mathscr{L}_\xi$
- Restrict to equivariant polyforms: $\mathscr{L}_\xi \Phi = 0$, then d_ξ is nilpotent.

Equivariant Cohomology

- Can define ξ-equivariant de Rham cohomology.
- Equivariantly closed if $\mathrm{d}_\xi \Phi = 0$.

$$\mathrm{d}\Phi_n = 0\,, \quad \xi \lrcorner \Phi_n = \mathrm{d}\Phi_{n-2}\,, \quad \ldots \quad \xi \lrcorner \Phi_2 = \mathrm{d}\Phi_0$$

- Equivariantly exact if $\Phi = \mathrm{d}_\xi \Psi$.
- n'th ξ-equivariant cohomology group is

$$H^n_\xi(M) = \frac{\ker(\mathrm{d}_\xi) \,|\, \bigwedge^n_\xi(M)}{\mathrm{Im}(\mathrm{d}_\xi) \,|\, \bigwedge^{n-1}_\xi(M)}$$

- For freely acting groups is ordinary cohomology on M/G.

Equivariant Cohomlogy

- Equivariant integrals over M are defined as integrals over the top form of equivariantly closed form Φ:

$$\int_M \Phi \equiv \int_M \Phi_{2n}$$

- Top form is closed but not exact.
- Can add an equivariantly exact poly-form without changing result.

$$\int_M (\Phi + \mathrm{d}_\xi \Phi) = \int_M \Phi$$

- Integrals depend only on the equivariant cohomology class.

Equivariant integrals localise

- Integrals localise to fixed points of the symmetry.

$$M_\xi = \{x \in M \,|\, \xi|_x = 0\}$$

- We are free to modify intervals by equivariantly exact pieces.

$$\int_M \Phi = \int_M \Phi_t \equiv \int_M \Phi e^{t d_\xi \beta}$$

- Where β is some ξ-equivariant polyform: $\mathscr{L}_\xi \beta = 0$.

$$\frac{d}{dt}\Phi_t = \Phi(d_\xi \beta)e^{t d_\xi \beta} = d_\xi(\beta \wedge \Phi_t)$$

- Difference is equivariantly exact! Integrals are the same for all t.

Equivariant integrals localise

- For $t = 0$ we have the original integral, but we can evaluate for any t.
- If $\mathrm{d}_\xi \beta|_{0\text{–form}}$ is semi-negative definite with maximum equal to 0 and we take $t \to \infty$ limit the integral localises to the minima.
- Take $\beta = \eta \equiv g(\xi, \bullet)$ then we have:

$$\int_M \Phi = \lim_{t\to\infty} \int_M \Phi \mathrm{e}^{t\mathrm{d}\eta} \mathrm{e}^{-t|\xi|^2} = \lim_{t\to\infty} \int_M \mathrm{e}^{-t|\xi|^2} \Phi \wedge \sum_k (t\mathrm{d}\eta)^k \frac{1}{k!}$$

- The $\mathrm{e}^{-t|\xi|^2}$ acts as a delta function onto M_ξ!

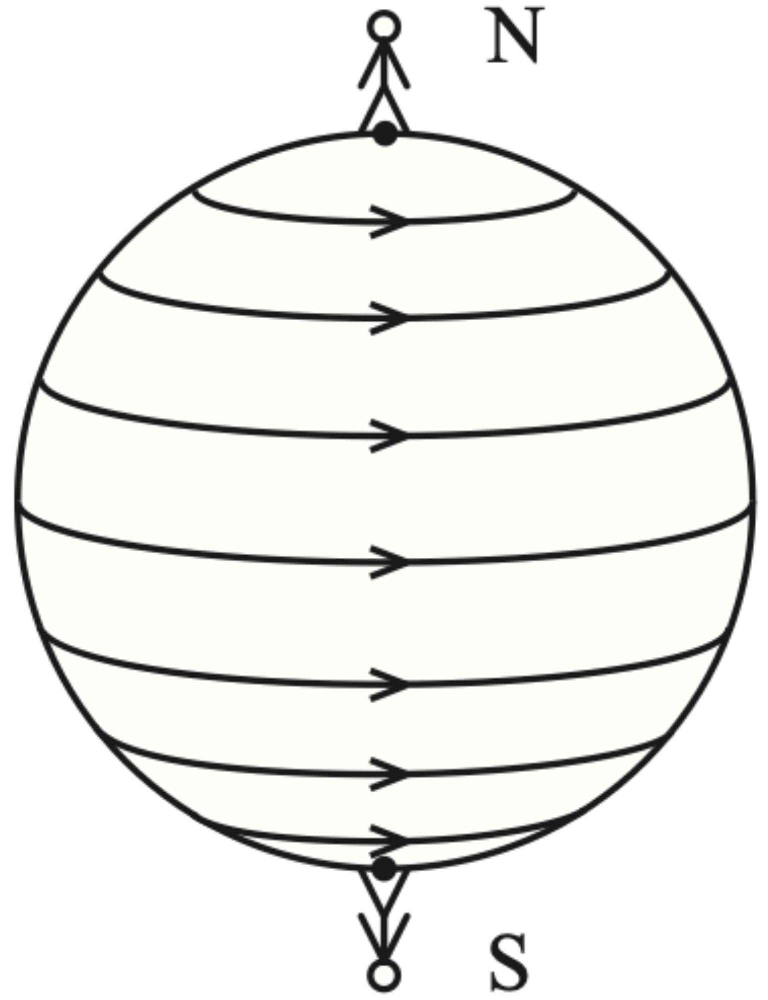
Isolated fixed points

- Integrals localise, but how do we compute the contributions?
- Assume the localisation locus M_ξ is a set of fixed points: $M_\xi = \{x_k\}$. E.g. S^2.
- Zoom in near such a fixed point, p using Cartesian coordinates $x_i = r_i \cos\phi_i$, $y_i = r_i \sin\phi_i$ $i \in \{1,..,n\}$ with origin at p.
- Locally metric reads:

$$\mathrm{d}s^2 \simeq \sum_{i=1}^{n} (\mathrm{d}x_i^2 + \mathrm{d}y_i^2) = \sum_{i=1}^{n} (\mathrm{d}r_i^2 + r_i^2 \mathrm{d}\phi_i^2)$$

- Killing vector takes the form

$$\xi \simeq \sum_{i=1}^{n} b_i^p \left(x_i \frac{\partial}{\partial y_i} - y_i \frac{\partial}{\partial x_i} \right) = \sum_{i=1}^{n} b_i^p \frac{\partial}{\partial \phi_i}$$



Isolated fixed points

- The circle action generated by ξ acts on the i'th eigenspace as:

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{pmatrix} \cos(b_i^p \phi_i) & \sin(b_i^p \phi_i) \\ -\sin(b_i^p \phi_i) & \cos(b_i^p \phi_i) \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix}$$

- Can also compute η, for us $\mathrm{d}_\xi \eta$ is locally given by:

$$\mathrm{d}_\xi \eta \simeq \sum_{i=1}^{n} b_i^p \mathrm{d}(r_i^2) \wedge \mathrm{d}\phi_i - \sum_{i=1}^{n} (b_i^p)^2 r_i^2$$

- Remains to plug all this into the integral for Φ_t.

Isolated fixed points

- The final result is:

$$\lim_{t\to\infty}\int_{\mathcal{N}_p}\Phi_t = \lim_{t\to\infty}\Phi_0(p)\prod_{i=1}^{n} t b_i^p \int_0^{2\pi} \mathrm{d}\phi_i \int_0^{\infty} \mathrm{d}r_i^2 \mathrm{e}^{-t(b_i^p)^2 r_i^2}$$

- This is just a Gaussian integral!

$$\int_{N_p}\Phi = \Phi_0(p)\frac{(2\pi)^n}{\prod_{i=1}^{n} b_i^p}$$

- We now just add up all the contributions from fixed points!

Beyond isolated fixed points

- One does not need to have just isolated fixed points.
- E.g. The Schwarzschild metric.

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}s^2(S^2)$$

- At the horizon $f(r_h) = 0$ and it locally looks like

$$\mathrm{d}s^2 \simeq \mathrm{d}s^2(\mathbb{R}^{1,1}) + r_h^2\mathrm{d}s^2(S^2)$$

- The Killing vector $\xi = \partial_t$ has a *bolt* at the horizon. A whole S^2 is fixed by the action.
- Need to take into account other fixed point loci.
- Can have fixed point sets of dimension 0,2,4,….

BVAB theorem

- Can apply a similar logic to work out these contributions, result is the
- **BVAB theorem**: [Berline, Vergne 82, Atiyah, Bott 84].

The integral of an equivariantly closed form localises to fixed points of symmetry.

$$\int_M \Phi = \sum_{\Sigma} \int_{\Sigma} \frac{f^*\Phi}{e_\xi(\mathcal{N})}$$

What we care about

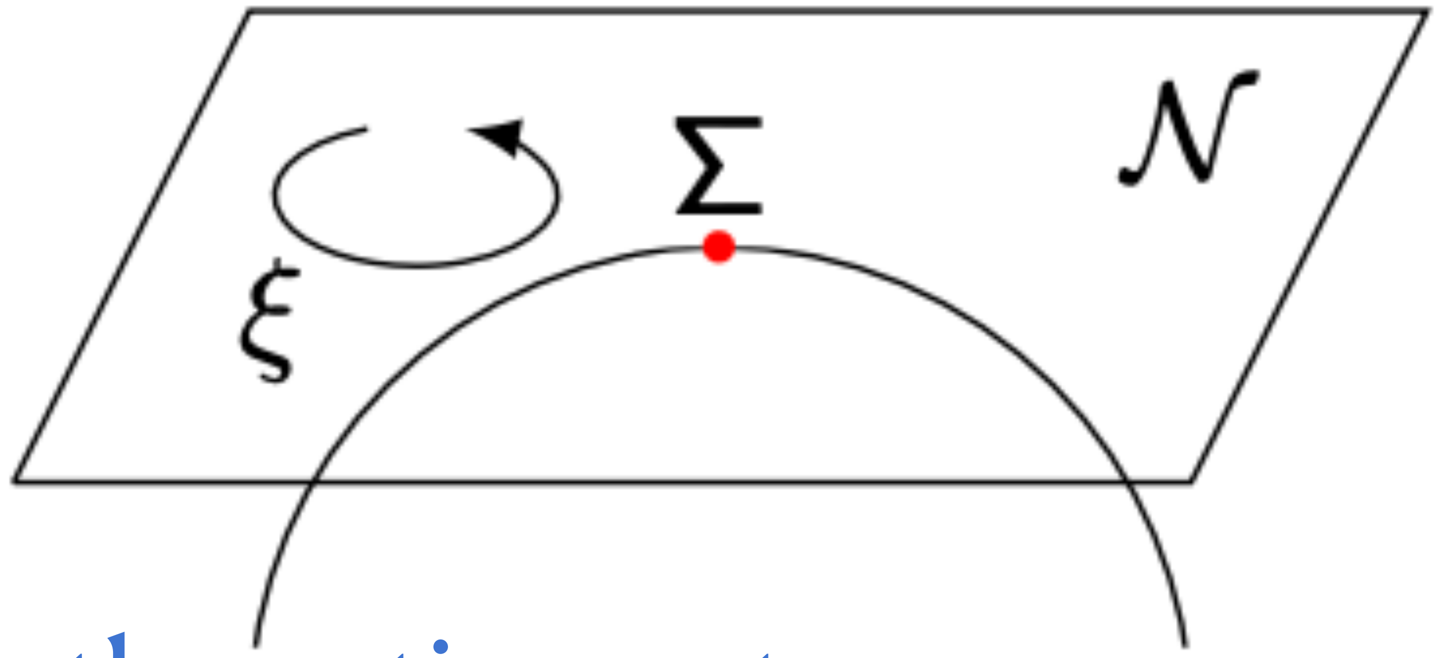
Pullback to Σ of Φ

Euler form of normal bundle

Sum over fixed point set

- This looks a bit scary but it is not.

BVAB Theorem



- What is a fixed point set?
- What is the normal bundle?
- $e_\xi(\mathscr{N})$?

- Somewhere where the action acts trivially, $\xi = 0$ there.
- Bundle of points normal to fixed point set: $\mathscr{N} = \sum_{i=1}^{k} \mathscr{L}_i \cong \mathbb{R}^{2k}$
- The Euler class of the normal bundle.

$$e(\xi)(\mathscr{N}) = \prod_{i=1}^{k} \left[c_1(\mathscr{L})_i + \frac{\epsilon_i}{2\pi} \right]$$

First Chern class

Weights of action

BVAB

The full gory details:

$$\int_{M_{2n}} \Phi = \sum_{\Sigma} \frac{(2\pi)^k}{\prod_{i=1}^k \epsilon_i} \int_{\Sigma} \frac{f^*\Phi}{\prod_{i=1}^k \left[1 + \frac{2\pi}{\epsilon_i} c_1(\mathscr{L}_i)\right]}$$

$$= \sum_{\text{dim} 0} \frac{1}{d_{F_0}} \frac{(2\pi)^n}{\epsilon_1 \ldots \epsilon_n} \Phi_0 + \sum_{\text{dim } 2} \frac{1}{d_{F_2}} \frac{(2\pi)^{n-1}}{\epsilon_1 \ldots \epsilon_{n-1}} \int \left[\Phi_2 - \Phi_0 \sum_{1 \le i \le n-1} \frac{2\pi}{\epsilon_i} c_1(\mathscr{L}_i) \right]$$

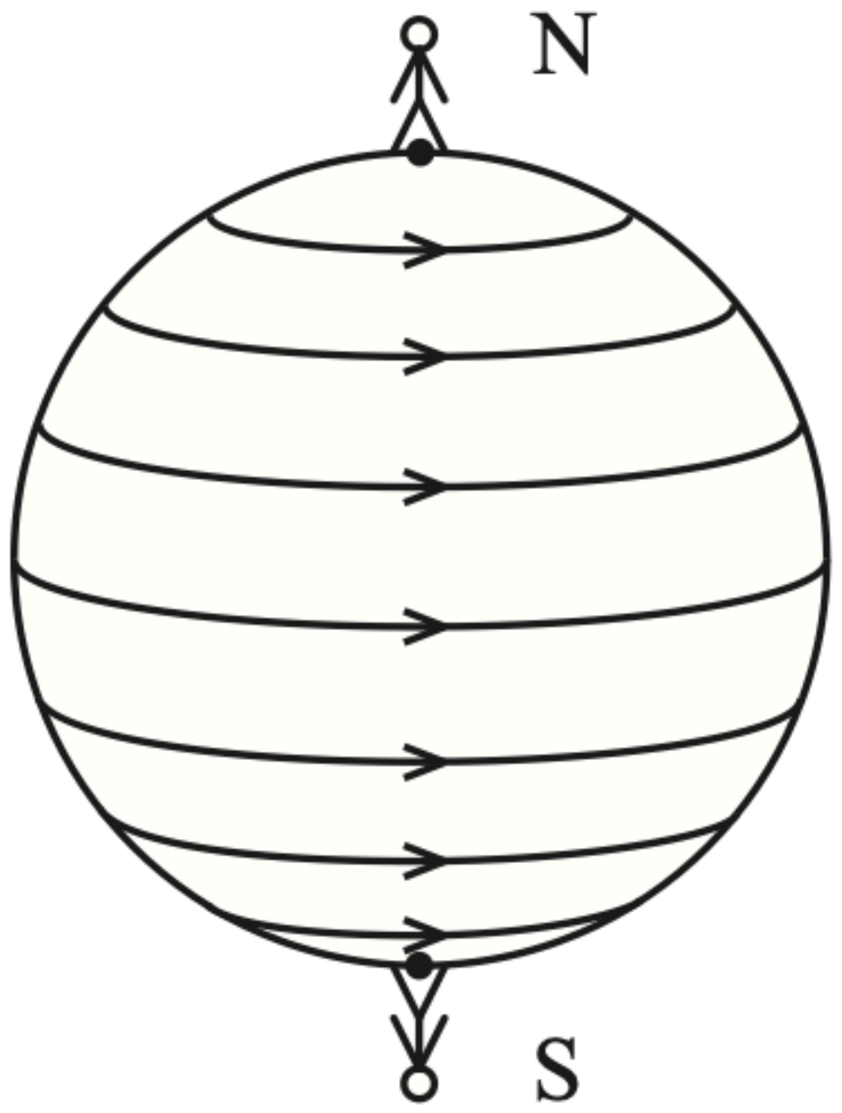
$$+ \sum_{\text{dim } 4} \frac{1}{d_{F_4}} \frac{(2\pi)^{n-2}}{\epsilon_1 \ldots \epsilon_{n-2}} \int \left[\Phi_4 - \Phi_2 \wedge \sum_{1 \le i \le n-2} \frac{2\pi}{\epsilon_i} c_1(\mathscr{L}_i) + \Phi_0 \sum_{1 \le i \le j \le n-2} \frac{(2\pi)^2}{\epsilon_i \epsilon_j} c_1(\mathscr{L}_i) \wedge c_1(\mathscr{L}_j) \right] + \cdots$$

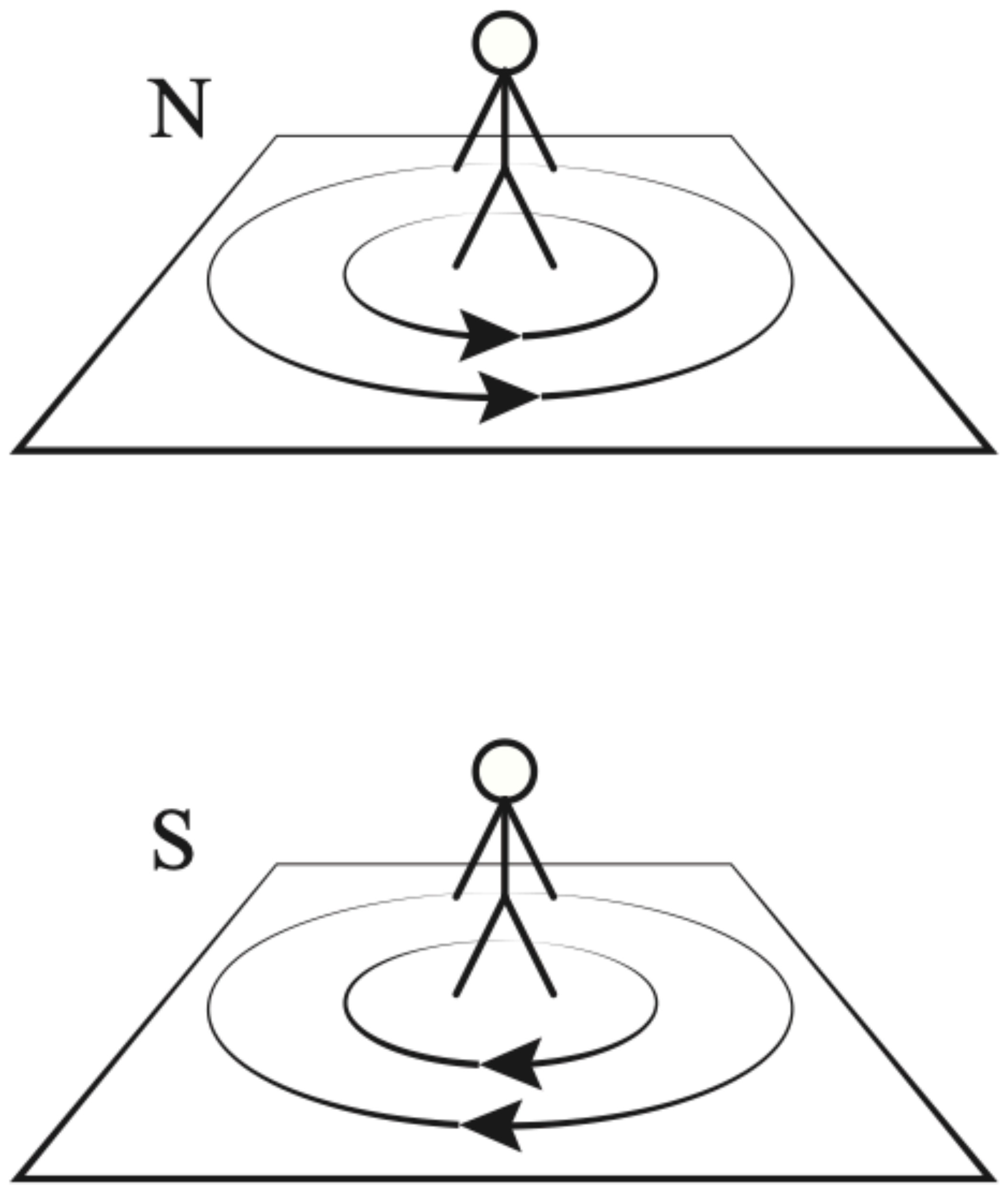
This is cheating a bit, we know the metric.
This is a problem with the example.

S^2 example

- $\mathrm{d}s^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2$
- $\mathrm{Vol} = \int_{S^2} \sin\theta\,\mathrm{d}\theta \wedge \mathrm{d}\phi \equiv \int_{S^2} \Phi$
- $\xi = \partial_\phi$. Need polyform:
- $\xi \lrcorner \Phi_2 = \mathrm{d}\Phi_0 = \mathrm{d}\cos\theta$.
- Two fixed points at poles of sphere.
- $\mathrm{Vol} = \dfrac{2\pi}{\epsilon_N}\Phi_0|_N + \dfrac{2\pi}{\epsilon_S}\Phi_0|_S$

$$= 2\pi(\cos(0) - \cos(\pi)) = 4\pi$$





Same cheating issue as the S^2

$\mathbb{CP}^2$ Example

- Consider the metric

$$\mathrm{d}s^2 = \mathrm{d}\zeta^2 + \frac{1}{4}\sin^2\zeta\cos^2\zeta(\mathrm{d}\psi + \cos\theta\mathrm{d}\phi)^2 + \frac{1}{4}\sin^2\zeta(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)$$

- Choice of Killing vector $\xi = b_1\partial_\psi + b_2\partial_\phi$. Different types of fixed point sets!
- For ∂_ψ fixed point at $\zeta = 0$ and bolt at $\zeta = \frac{\pi}{2}$.
- For ∂_ϕ, 3 fixed points at $(\zeta = 0)$, $(\zeta = \frac{\pi}{2}, \theta = 0)$ and $(\zeta = \frac{\pi}{2}, \theta = \pi)$.
- Polyforms different and different types of fixed point locus but give same results!

(Useful) Applications

Romans SUGRA

Euclidean Romans SUGRA

- 6d gauged supergravity theory.
- Bosonic content: metric, dilaton X, $\mathrm{SU}(2)$ gauge fields + 2-form potential.
- Preserve supersymmetry, need to solve Killing spinor equations $\nabla_\mu \epsilon + \ldots . = 0$.
- Places constraints on metric and fields. "Torsion conditions" from spinor bilinears: $P = -\bar{\epsilon}\gamma_7\epsilon$. [Alday, Fluder, Gregory, Richmond, Sparks]
- Implies existence of a Killing vector ξ + conditions like $\mathrm{d}(XP) = -\frac{1}{\sqrt{2}}\xi \lrcorner F$.
- Construct Polyforms using torsion conditions, e.g. $\Phi^F = F - \sqrt{2}XP$.

Fixed points

- What are the possible fixed points?

$$F = \{\xi = 0\} \subset M_6$$

- Even dimensional sub manifolds of M_6

$$F = F_0 \cup F_2 \cup F_4$$

- On fixed point set the Killing spinor is chiral! $\epsilon\,|_F = \epsilon_\pm$.
- In fact it is stronger:

$$-i\gamma^{(2i-1)(2i)}\epsilon = \sigma^{(i)}\epsilon, \quad \sigma^{(i)} = \pm 1$$

Master formula

$$I = \frac{F_{S^5}}{27}\Bigg[\sum_{\text{dim}0} \chi \frac{(\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2 + \sigma^{(3)}\epsilon_3)^3}{\epsilon_1\epsilon_2\epsilon_3}$$

$$- \sum_{\text{dim } 2} \frac{\chi(\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2)^2}{\epsilon_1\epsilon_2} \int_{F_2} \left[3c_1(F) + (\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2)\left(\frac{c_1(L_1)}{\epsilon_1} + \frac{c_1(L_2)}{\epsilon_2}\right)\right]$$

$$+ \sum_{\text{dim } 4} \chi\sigma^{(1)} \int_{F_4} \left[3c_1(F) \wedge c_1(F) + 3\sigma^{(1)} c_1(F) \wedge c_1(L_1) + c_1(L_1) \wedge c_1(L_1)\right]$$

- $\sigma^{(1)}\sigma^{(2)}\sigma^{(3)} = \chi$
- The on-shell action for ***any*** solution is given by the above!
- Need to specify the weights, σ's and $c_1(L)$'s.

Some technical stuff

- There are also conditions for the spinor to be well defined that one needs to impose. For two-dim fixed point set

$$\int_{\Sigma_g} c_1(F) = \sigma^{(1)} \int_{\Sigma_g} c_1(L_1) + \sigma^{(2)} \int_{\Sigma_g} c_1(L_2) - \sigma^{(3)} \chi(\Sigma_g)$$

- Imposes the type of twist to preserve supersymmetry e.g a “topological twist”.
- Similar condition for four-dim fixed point set but more complicated.

$$2\eta\chi(B_4) + 3\tau(B_4) = \int_{B_4} \left(\sigma^{(1)} c_1(L_1) + c_1(F)\right)^2$$

- Plug into on-shell action. For dim 0 and dim 2 purely topological result. For dim 4 need information about magnetic charges.

Example 1

5d SCFTs on a Riemann surface

- Take $M_6 = \mathbb{R}^4 \times \Sigma_g$.
- Conformal boundary is $S^3_b \times \Sigma_g$.
- Dual to twisted compactification of 5d SCFTs on Σ_g, placed on squashed S^3_b.
- Write $\mathbb{R}^4 = \mathbb{R}^2 \oplus \mathbb{R}^2$ with ∂_{ϕ_i} rotating the two copies, take $c_1(L_i) = 0$.
- Take $\xi = b_1\partial_{\phi_1} + b_2\partial_{\phi_2}$. Fixed point set: $F_2 = \Sigma_g$ at centre of $\mathbb{R}^4$.

Plug everything in!

$$I = \frac{F_{S^5}}{27}\Bigg[\sum_{\text{dim}0} \chi \frac{(\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2 + \sigma^{(3)}\epsilon_3)^3}{\epsilon_1\epsilon_2\epsilon_3}$$

$$- \sum_{\text{dim }2} \frac{\chi(\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2)^2}{\epsilon_1\epsilon_2}\int_{F_2}\left[3c_1(F) + (\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2)\left(\frac{c_1(L_1)}{\epsilon_1} + \frac{c_1(L_2)}{\epsilon_2}\right)\right]$$

$$+ \sum_{\text{dim }4} \chi\sigma^{(1)}\int_{F_4}\left[3c_1(F)\wedge c_1(F) + 3\sigma^{(1)}c_1(F)\wedge c_1(L_1) + c_1(L_1)\wedge c_1(L_1)\right]$$

$$\epsilon_i = b_i$$

$$\int_{\Sigma_g} c_1(F) = \sigma^{(1)}\int_{\Sigma_g} c_1(L_1) + \sigma^{(2)}\int_{\Sigma_g} c_1(L_2) - \sigma^{(3)}\chi(\Sigma_g)$$

Example 1

5d SCFTs on a Riemann surface

$$I = \frac{F_{S^5}}{9} \chi(\Sigma_g) \sigma^{(1)} \sigma^{(2)} \frac{\left(b_1 + \sigma^{(1)} \sigma^{(2)} b_2\right)^2}{b_1 b_2}$$

- Matches field theory results!
- Easy in the end, just plugging things into the mater formula.
- No solving equations of motion!

Example 2

Black saddle

- $M_6 = \mathbb{R}^2 \times B_4$ "Schwarzschild like solution" a.k.a Black Saddle
- Conformal boundary is $S^1 \times B_4$.
- Dual to 5d SCFT on B_4.
- $\xi = b\partial_\phi$ with ∂_ϕ rotating $\mathbb{R}^2$
- Fixed point set $F = B_4$ at centre of $\mathbb{R}^2$. Set $c_1(L) = 0$ again.

Plug everything in!

$$I = \frac{F_{S^5}}{27}\left[\sum_{\text{dim}0} \chi \frac{(\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2 + \sigma^{(3)}\epsilon_3)^3}{\epsilon_1\epsilon_2\epsilon_3}\right.$$

$$- \sum_{\text{dim }2} \frac{\chi(\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2)^2}{\epsilon_1\epsilon_2}\int_{F_2}\left[3c_1(F) + (\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2)\left(\frac{c_1(L_1)}{\epsilon_1} + \frac{c_1(L_2)}{\epsilon_2}\right)\right]$$

$$\left. + \sum_{\text{dim }4} \chi\sigma^{(1)}\int_{F_4}\left[3c_1(F)\wedge c_1(F) + 3\sigma^{(1)}c_1(F)\wedge c_1(L_1) + c_1(L_1)\wedge c_1(L_1)\right]\right]$$

$$2\eta\chi(B_4) + 3\tau(B_4) = \int_{B_4}\left(\sigma^{(1)}c_1(L_1) + c_1(F)\right)^2$$

Example 2

Black saddle

$$I = \frac{F_{S^5}}{9}\left(2\chi(B_4) + 3\eta\tau(B_4)\right)$$

- For any choice of B_4. Result is purely topological!
- Result not noticed before.
- To compare with literature let $B_4 = \Sigma_{g_1} \times \Sigma_{g_2}$.
- $\tau(\Sigma_{g_1} \times \Sigma_{g_2}) = 0$ and $\chi(\Sigma_{g_1} \times \Sigma_{g_2}) = \chi(\Sigma_{g_1})\chi(\Sigma_{g_2})$

$$I = \frac{8F_{S^5}}{9}(1 - g_1)(1 - g_2)$$

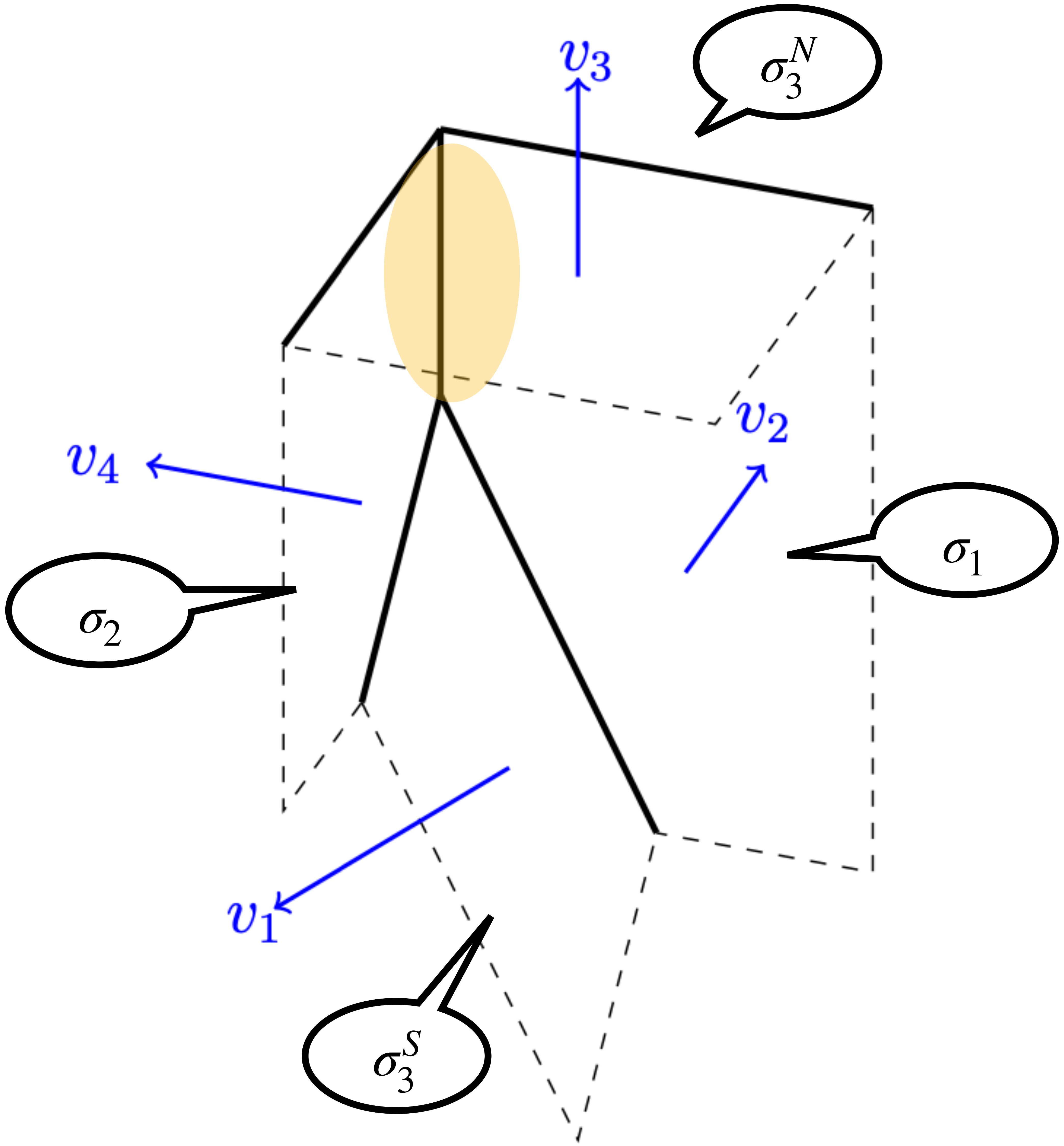
Example 3

$\mathcal{O}(-p_1) \oplus \mathcal{O}(-p_2) \to S^2$

- Chemical potential for angular momentum.
- $\mathcal{O}(-p)$ is fancy for $\mathbb{R}^2$ plus $\mathrm{d}\phi \to \mathrm{d}\phi + A$ with magnetic charge $-p$ for A over S^2.
- $\xi = b_1\partial_{\phi_1} + b_2\partial_{\phi_2} + b\partial_\psi$
- Fixed point set at centre of each $\mathbb{R}^2$ and poles of $S^2 \Rightarrow$ 2 fixed points.
- $\displaystyle\int_{S^2} c_1(L_i) = -\,p_i$
- Weights a bit more difficult now but easy to compute with toric geometry.

Some toric geometry

- Toric diagram for $\mathcal{O}(-p_1) \oplus \mathcal{O}(-p_2) \to S^2$
- Faces label where a circle shrinks.
- Each face has an associated vector
- Also associated to each face the sign σ.
- The S^2 is in orange.
- v_2, v_4 are the vectors for the $\mathcal{O}(-p_i)$ factors.



$$I = \frac{F_{S^5}}{27}\Bigg[\sum_{\text{dim}0} \chi \frac{(\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2 + \sigma^{(3)}\epsilon_3)^3}{\epsilon_1\epsilon_2\epsilon_3}$$

$$- \sum_{\text{dim } 2} \frac{\chi(\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2)^2}{\epsilon_1\epsilon_2}\int_{F_2}\left[3c_1(F) + (\sigma^{(1)}\epsilon_1 + \sigma^{(2)}\epsilon_2)\left(\frac{c_1(L_1)}{\epsilon_1} + \frac{c_1(L_2)}{\epsilon_2}\right)\right]$$

$$+ \sum_{\text{dim } 4} \chi\sigma^{(1)}\int_{F_4}\left[3c_1(F)\wedge c_1(F) + 3\sigma^{(1)}c_1(F)\wedge c_1(L_1) + c_1(L_1)\wedge c_1(L_1)\right]$$

$$\{\epsilon_1, \epsilon_2, \epsilon_3\} = \begin{cases} \{b_1, b_2, b_3\} & \text{North pole} \\ \{b_1 + p_1 b_3, b_2 + p_2 b_3, -b_3\} & \text{South pole} \end{cases}$$

Example 3

$\mathcal{O}(-p_1) \oplus \mathcal{O}(-p_2) \to S^2$

$$I = \frac{F_{S^5}}{27}\sigma^{(1)}\sigma^{(2)}\left(\frac{\sigma_N^{(3)}\left(\sigma^{(1)}b_1 + \sigma^{(2)}b_2 + \sigma_N^{(3)}b_3\right)^3}{b_1b_2b_3} - \frac{\sigma_S^{(3)}\left(\sigma^{(1)}(b_1 + p_1b_3) + \sigma^{(2)}(b_2 + p_2b_3) - \sigma_S^{(3)}b_3\right)^3}{(b_1 + p_2b_3)(b_2 + p_2b_3)b_3}\right)$$

- Completely new result! No SUGRA solution nor field theory results!
- Has the form of gravitational blocks, like holomorphic blocks from SUSY localisation.
- "Twist" or "anti-twist" depending on whether $\sigma_N^{(3)} = \pm\, \sigma_S^{(3)}$.
- Reduces to Example 1 for $p_i = 0, \quad b_3 = 0, \quad g = 2$.

Conclusion

- Equivariant localisation gives a powerful method for computing certain holographic observables.
- Only requires an isometry to apply, for SUSY solutions this is often present (R-sym).
- Applied to Romans SUGRA, recovered old results and found new predictions.

- Hidden subtleties: Existence of actual solutions? Odd-dim works slightly different (need 2 U(1)'s). SUSY needed to construct polyforms.
- Many ways to extend: Higher derivative corrections, different matter content, boundaries, exact matches with field theory?

Thank you!

$\mathbb{CP}^2$ more details

- For ∂_ψ the polyform is:

$$\Phi = \text{vol}(\mathbb{CP}^2) - \frac{1}{8}\sin^3\zeta\cos\zeta \mathrm{d}\zeta \wedge D\psi + \frac{\sin^4\zeta + c}{32}$$

- Here c is an arbitrary constant, it will drop out!
- Fixed point has weights: $b_1 = b_2 = \frac{1}{2}$
- Bolt has weights: $b_1 = \frac{1}{2}$.

$$\text{Vol} = \frac{(2\pi)^2}{(1/2)^2}(\Phi_0|_{\zeta=\frac{\pi}{2}}) + \frac{2\pi}{1/2}\left(\int_{S^2}\Phi_2 - \Phi_0|_{\zeta=0}\int_{S^2}c_1(\mathscr{L})\right)$$

$$= \frac{\pi^2}{2}$$