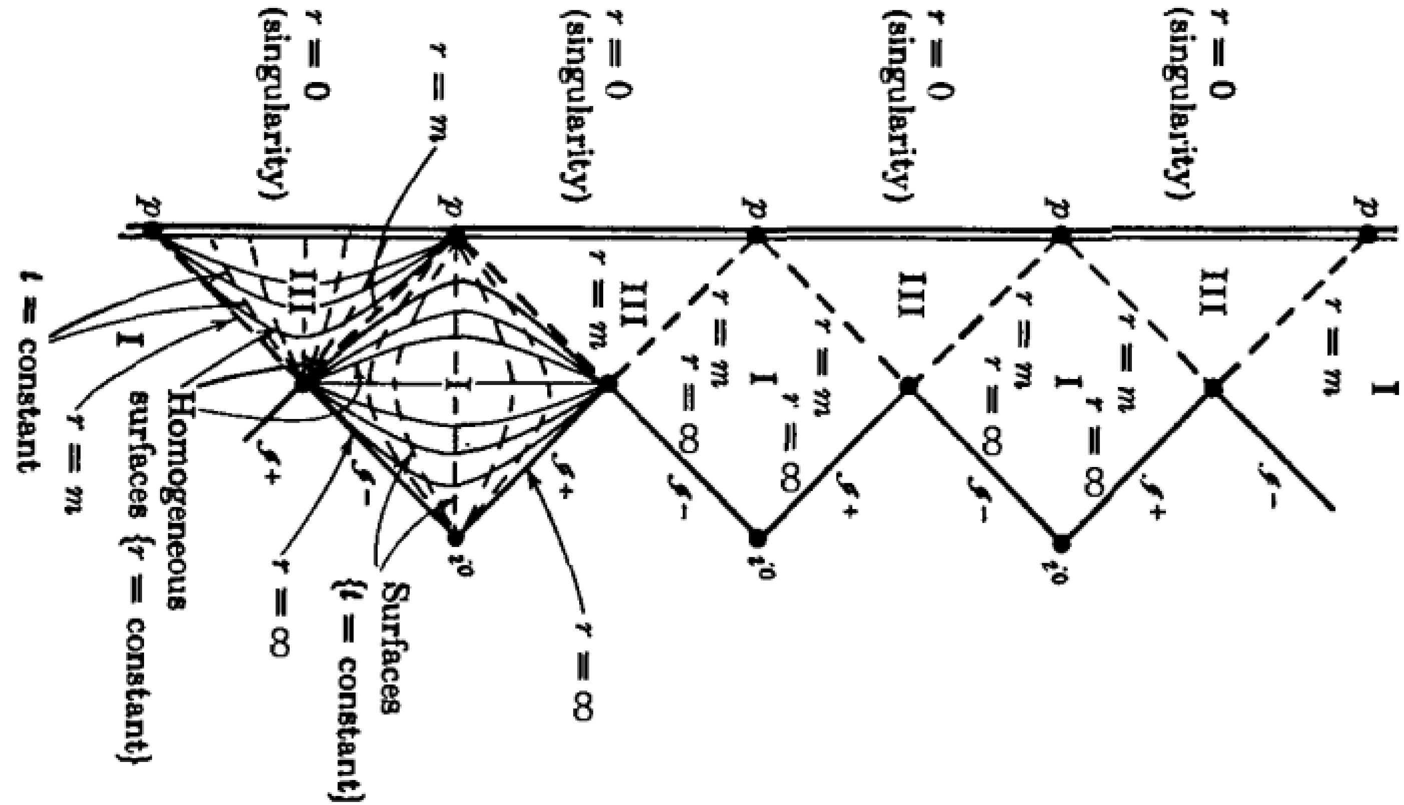
Quantum Gravity and Extremal Black Holes



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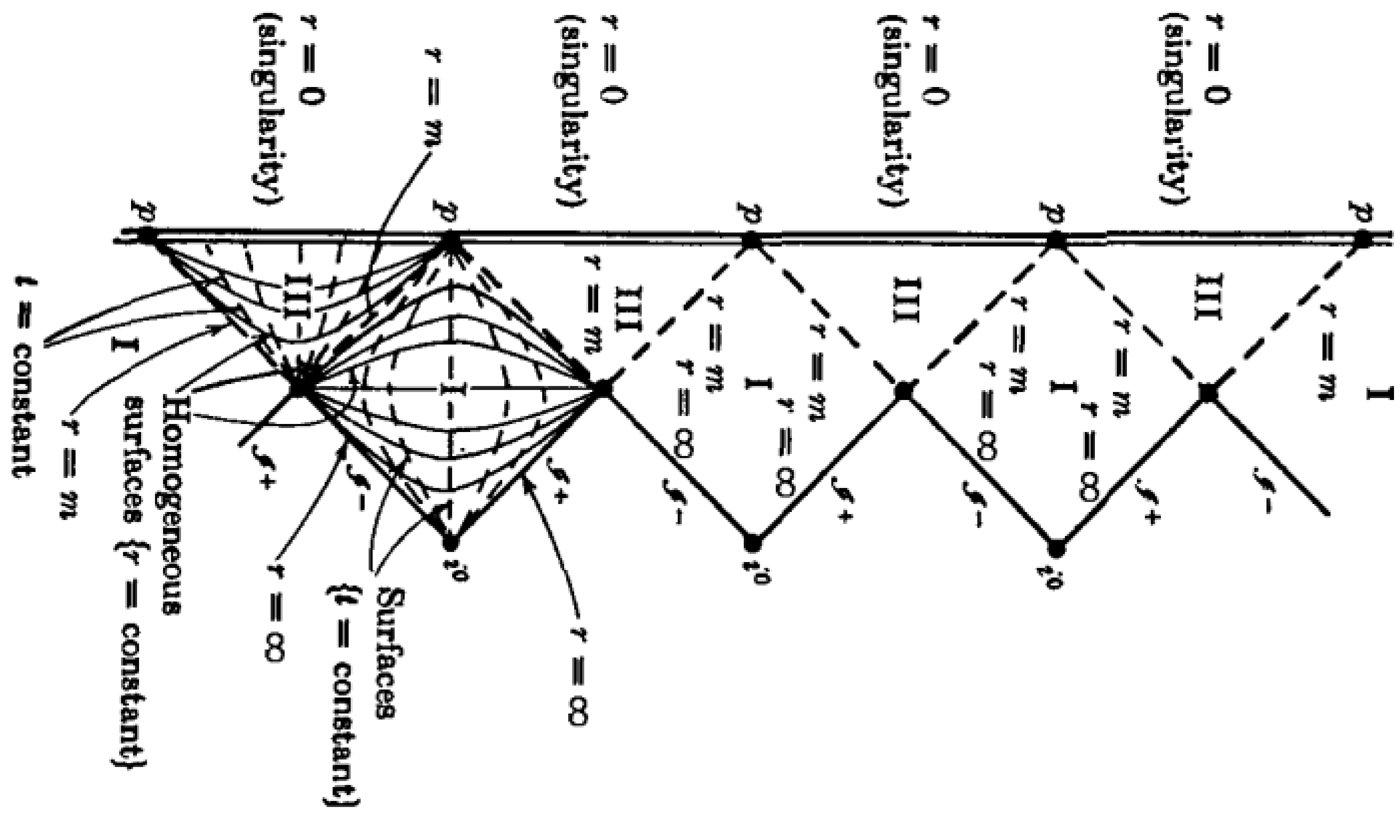
18th Dec 2024
ITMP Seminar

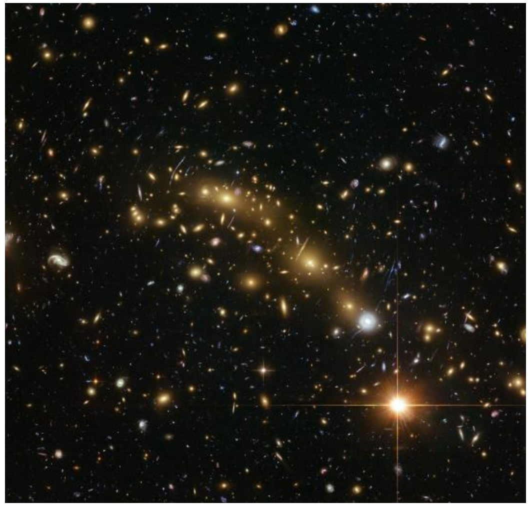
Quantum Gravity and Extremal Black Holes

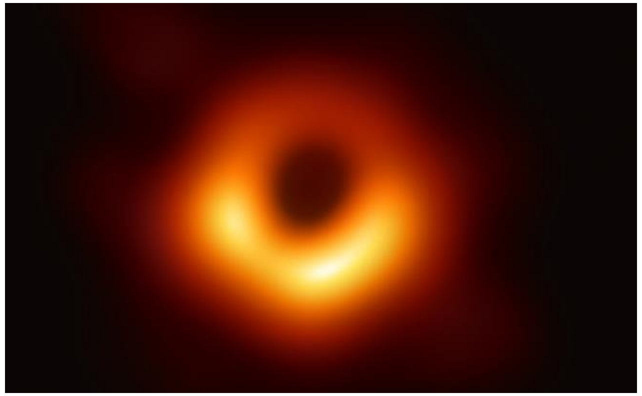


mostly based on 2408.05549 in collaboration with C. de Rham and A. J. Tolley

Effective Field Theory of Gravity

General relativity accurately describes gravity across various scales...





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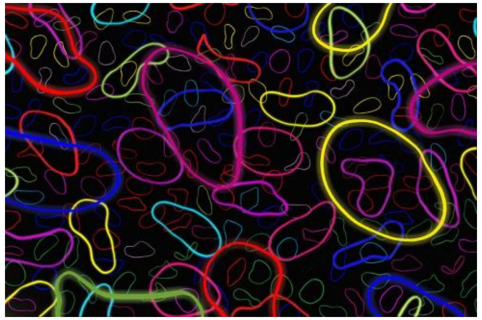
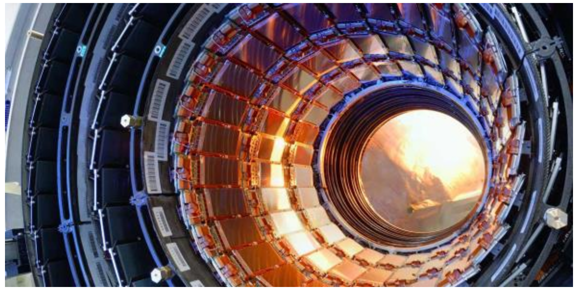


Energy

$M_{\rm Pl}$



?

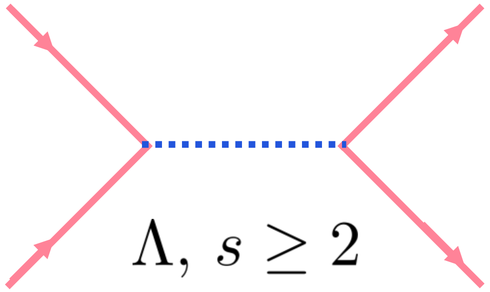
...but predicts its own breakdown: Need **UV completion**!

Effective Field Theory perspective: Use most general **local** action

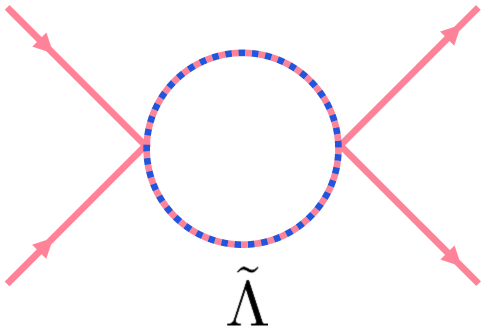
1) consistent with **symmetries**,

2) organised in **derivative expansion**, and

3) with **coefficients** fixed by dimensional analysis.



$$S_{\rm EFT} = M_{\rm Pl}^{D-2} \int d^D x \sqrt{-g} \left[\frac{1}{2} R + \Lambda^2 \sum_{m \geq 0, n \geq 2} c_{mn} \left(\frac{\nabla}{\Lambda} \right)^m \left(\frac{\text{Riemann}}{\Lambda^2} \right)^n \right]$$

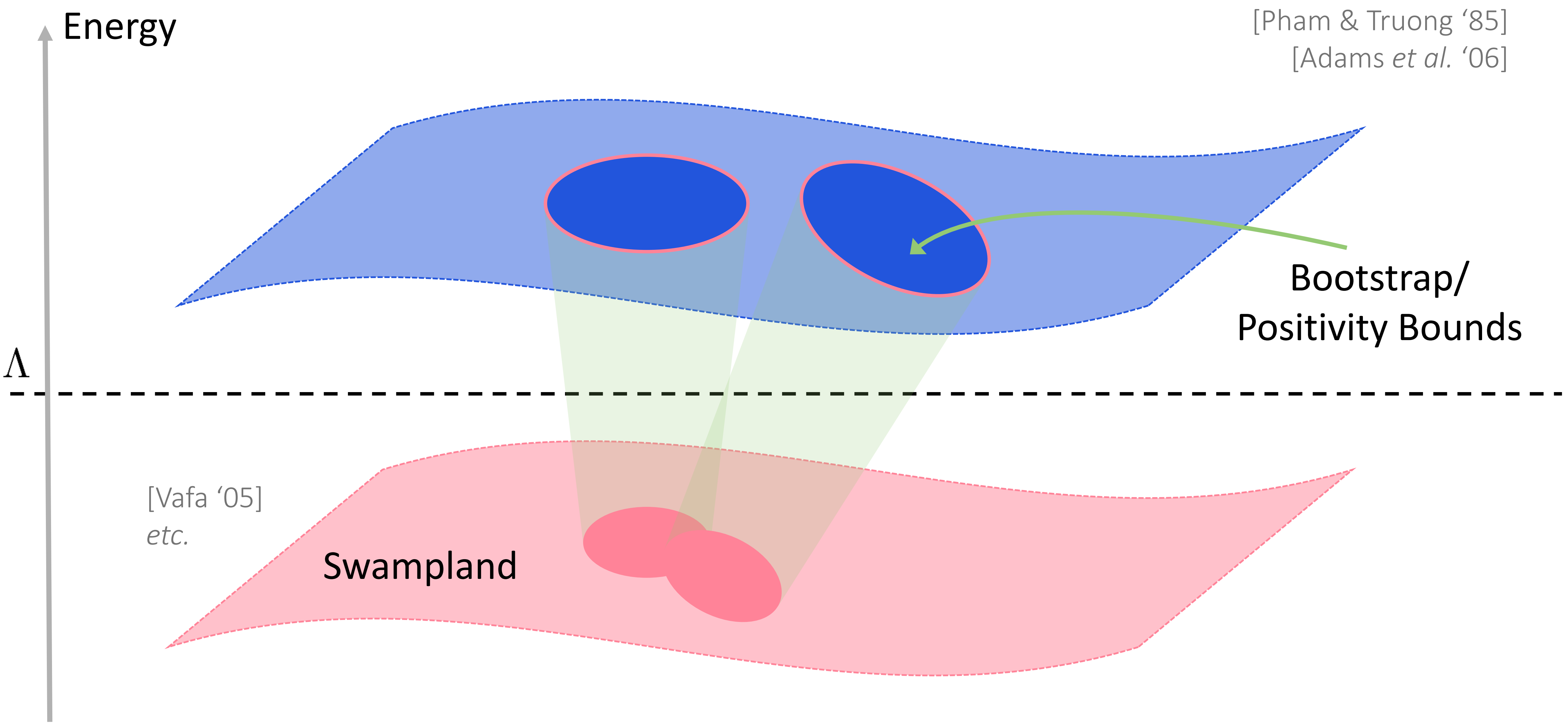


Suppressed by loops

→ Treat as **standard QFT** (careful about breakdown!)

Challenge: Decoupling/separation of scales cause suppression of UV effects/EFT corrections!

Successful efforts using knowledge about UV to constrain possible EFT operators.



Goal: Study set-up where leading-order effect accidentally vanishes!

Charged Black Holes

Charged Black Holes

- **Gravity + Maxwell** in D dimensions

$$S_{\rm EM} = \int \mathrm{d}^D x \sqrt{-g}\left[\frac{1}{2\kappa}\left(R + \frac{(D-2)(D-1)}{L^2}\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

→ Asymptotically flat limit when $L \to \infty$.

- Spherically symmetric and static **background solutions**

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B(r)} + r^2 \mathrm{d}\Omega_d^2, \qquad F = \Psi'(r)\mathrm{d}t \wedge \mathrm{d}r$$

$d = D - 2$

→ Anti-de Sitter **Reissner-Nordström** (AdS RN)

$Q^2 = \frac{D-3}{D-2}\kappa q^2$

$$A(r) = B(r) = f(r) := 1 + \frac{r^2}{L^2} - \frac{M}{r^{D-3}} + \frac{Q^2}{r^{2(D-3)}}, \qquad \Psi(r) = \frac{q}{r^{D-3}}$$

Extremal Black Holes

- Solution possesses two real **horizons.** For AF RN:

$$r_{\pm} = \frac{M}{2} - \sqrt{\left(\frac{M}{2}\right)^2 - Q^2}$$

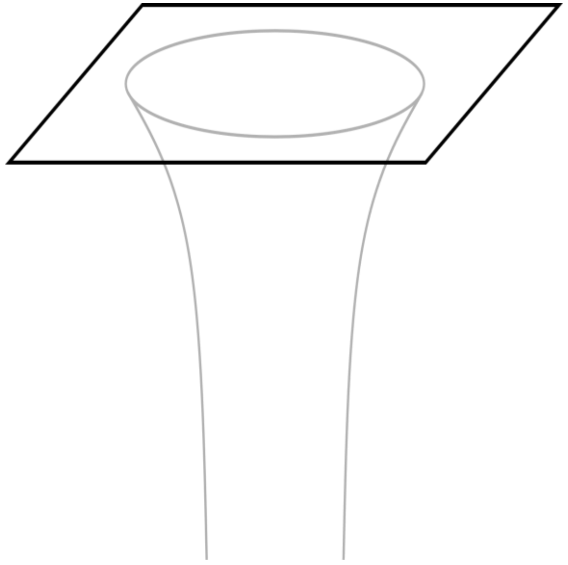
→ Degenerate to extremal horizon $r_H := r_+ = r_-$ in **extremal limit**.

$\rho = r - r_+$

- **Extremal near-horizon geometry** is leading-order in ρ / r_H :

$$ds^2 = \frac{2}{f''(r_H)} \left[-\rho^2 \left(\frac{f''(r_H)}{2} \mathrm{d}t \right)^2 + \frac{\mathrm{d}\rho^2}{\rho^2} \right] + r_H^2 \mathrm{d}\Omega_d^2$$

i.e. $\mathrm{AdS}_2 \times S^d$!



[Bertotti '59; Robinson '59]

→ **Generic** to extremal black holes!

[Kunduri, Lucietti, Reall '07]

Smooth Horizons

- Degenerate horizon of extremal AdS-RN is **smooth.**

→ Is this a generic feature of extremal black holes?

No.

- Consider **multi-black hole solutions**

[Majumdar ‘47; Papapetrou ‘47]

$$\mathrm{d}s^2 = -H^{-2}\mathrm{d}t^2 + H^{2/(D-3)}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \quad H = 1 + \sum_{n=0}^{N} \frac{M_n}{|\mathbf{x} - \mathbf{x}_n|^{D-3}}$$

- Recover AF RN in isotropic coordinates for $N = 0$.

- Take $\mathbf{x}_0 = \mathbf{0}$ as reference and zoom in on $\hat{\rho} := |\mathbf{x}| = 0$ (its horizon).

$$\rho \propto \hat{\rho}^{D-3}$$

Smooth Horizons

- Metric at leading order:

$$ds^2 = \left(\frac{\rho}{L}\right)^2 dt^2 + \left(\frac{L}{\rho}\right)^2 \left[1 + (D-3)g(\rho) + (D-3)^2 \frac{dg(\rho)}{d\log\rho}\right] d\rho^2$$
$$+ r_0^2 \left[1 + 2(D-3)g(\rho)\right] d\Omega_{D-3}^2$$

with **perturbation**

$$g(\rho) \sim \sum_{m=1}^{\infty} c_m \rho^{1+\frac{m}{D-3}}$$

→ View these as static deformations of single extremal black hole.

- In $D = 4$: Scaling in integer powers, so horizon is **smooth**. [Welch '95]

- In $D \geq 5$: Scaling exponents generically non-integer. Curvature invariants don't diverge, but **tidal forces** on infalling observers diverge → **mildly singular**!

[Candish & Reall '07; Gowdigere, Kumar, Raj, Srivastava '14]

Can we study these more systematically?

Extremal Black Holes as Deformations

Deformations

- **Decompose** metric $\sqrt{\kappa}h := g - \bar{g}$ and gauge field $\delta F := F - \bar{F}$ perturbations with respect to spherical symmetry.

→ Decoupled **master equations** for gauge-invariant "master variables" labelled by harmonic ℓ universally take the form

$$2\partial_v \partial_r \phi + \partial_r (f \partial_r \phi) - U(r)\phi = 0$$

with effective potentials U.

[Kodama, Ishibashi, Seto]

→ Interested in ***stationary perturbations*** to ***near-horizon*** regions of ***extremal AdS-RN***!

[Horowitz, Kolanowski, Santos '22 & '23]
[Gralla & Zimmermann '18]

- Focus on stationary solutions with

$$\partial_v \phi = 0$$

Deformations

- In the near-horizon limit, the equation of motion takes form

$$(\rho^2 + \beta\varepsilon\rho)\phi'' + (2\rho + \beta\varepsilon)\phi' - U_*\phi = 0, \quad U_* = \frac{2U(r)}{f''(r)}\bigg|_{r=r_+} = m_{\text{eff}}^2 L_2^2 + \mathcal{O}\left(\varepsilon\right)$$

with

$$\varepsilon := \frac{r_+ - r_-}{r_+ + r_-}, \quad \beta := \frac{\varepsilon f''(r_+)}{2f'(r_+)} = \mathcal{O}\left(\varepsilon^0\right)$$

- In extremal limit, $\varepsilon = 0$ and solutions scale

$$h = c_-\rho^{\gamma_-} + c_+\rho^{\gamma_+}, \quad \gamma_\pm = \frac{1}{2}\big(-1 \pm \sqrt{1 + 4m_{\text{eff}}^2 L^2}\big)$$

→ Boundary conditions at $\rho = 0$ set $c_- = 0$!

- Tidally sourced asymptotically far away.

The Extremal Limit

Caveat: The **extremal** limit of the master equation is subtle. Tension with...

- **Stationary** limit: Classification of singular points changes.

  → **Stationary limit first**: Dynamical perturbations behave qualitatively different; deformations highly **non-generic**!

	$\omega \neq 0$	$\omega = 0$
sub.	$r_\pm$ reg. ∞ irreg.	$r_\pm$ reg. ∞ irreg.
ext.	r_H irreg. ∞ irreg.	r_H reg. ∞ irreg.

- **Near-Horizon** limit: Non-uniform convergence of $f^{-1}(\rho, \varepsilon)$ near origin

$$\lim_{\varepsilon \to 0} \left(\lim_{\rho/r_H \to 0} f^{-1} \right) \neq \lim_{\rho/r_H \to 0} \left(\lim_{\varepsilon \to 0} f^{-1} \right)$$

  i.e. order of limits ambiguous.

  → **Extremal limit last**: View extremal solutions as extremal limit of sub-extremal solutions!

[Gubser '00]

The Extremal Limit

Can we recover the scaling behaviour from **extremal limit** of sub-extremal solutions?

- Solutions in the sub-extremal case

$$\phi = AP_\alpha(z) + BQ_\alpha(z) \qquad z = 1 + \frac{2\rho}{\beta\varepsilon}, \quad \alpha = \sqrt{1+4U_*}$$

- Expanding near horizon

$$\phi \sim A\left[1 + \frac{1}{2}\alpha(\alpha-1)(z-1) + \dots\right] + B\left[c - \frac{1}{2}\log(z-1) + \dots\right]$$

→ Regularity requires $B = 0$.

- **Extremal limit**

$$\tilde{\alpha} = \alpha(\varepsilon = 0)$$

$$\phi \sim A\left[\frac{\Gamma(-1-2\tilde{\alpha})}{\Gamma(-\tilde{\alpha})^2}\left(\frac{\beta\varepsilon}{\rho}\right)^{-\gamma_-} + \dots + \frac{\Gamma(1+2\tilde{\alpha})}{\Gamma(\tilde{\alpha})^2}\left(\frac{\beta\varepsilon}{\rho}\right)^{-\gamma_+} + \dots\right]$$

→ Indeed picks out γ_+ solution.

Singularities

Scaling in terms of original metric perturbations is $h_{..} \sim \rho^{\gamma}$.

- **Scalar invariants** on deformed geometry scale as:

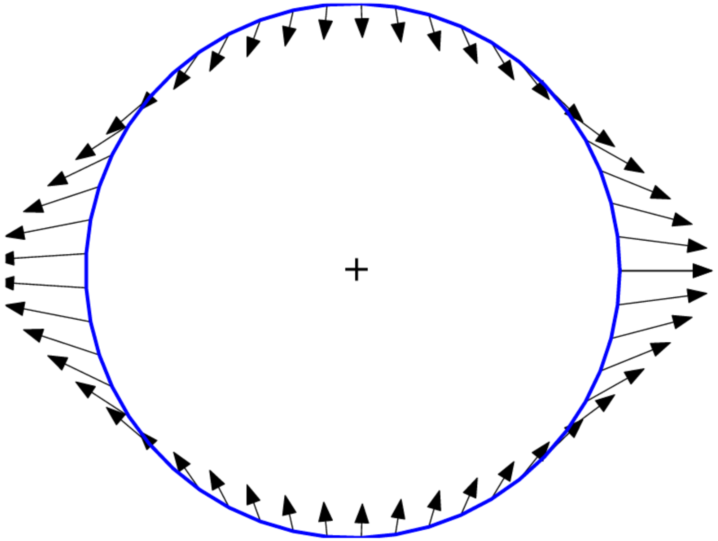
$$S \sim \rho^{n\gamma}, \quad n \in \mathbb{N}^{+}$$

→ Scalar polynomial **singularity** when $\gamma < 0$.

- Perturbations to the **Weyl tensor** scale as:

$$\delta C_{....} \sim \rho^{\gamma-2}$$

→ Parallel-propagated **singularity** when $\gamma < 2$.



Tidal force on particles travelling along geodesics of deformed background.
Artefact of **geodesic approximation**/breakdown of wordline EFT.

[Horowitz, Kolanowski, Remmen, Santos '24]

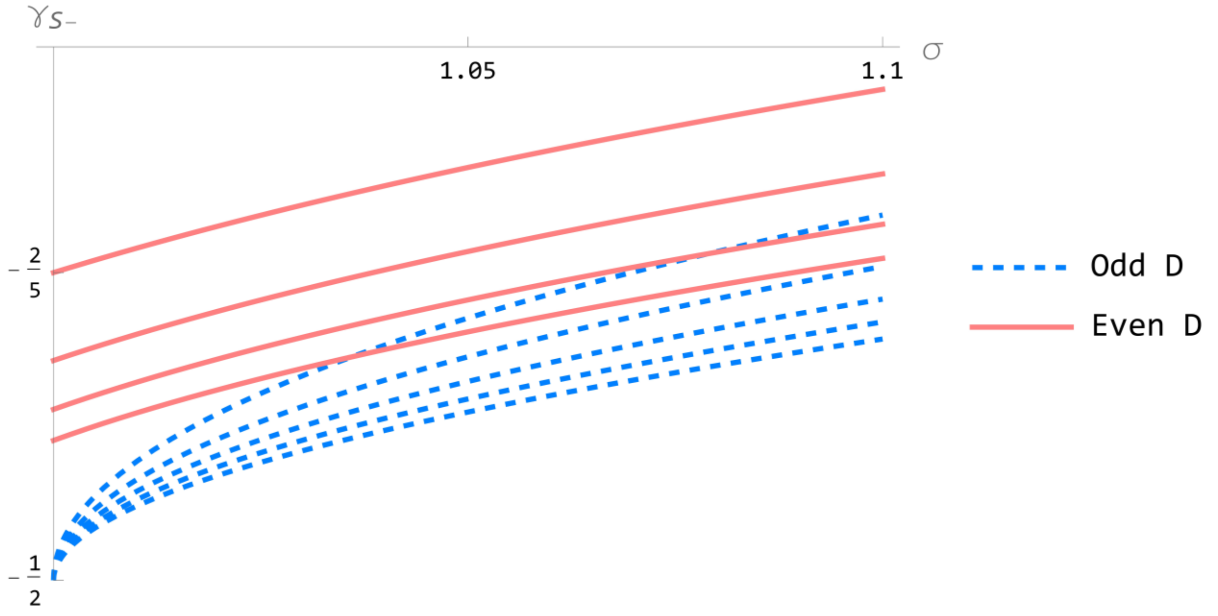
Extremal Black Holes in GR

We will focus on **curvature singularities**.

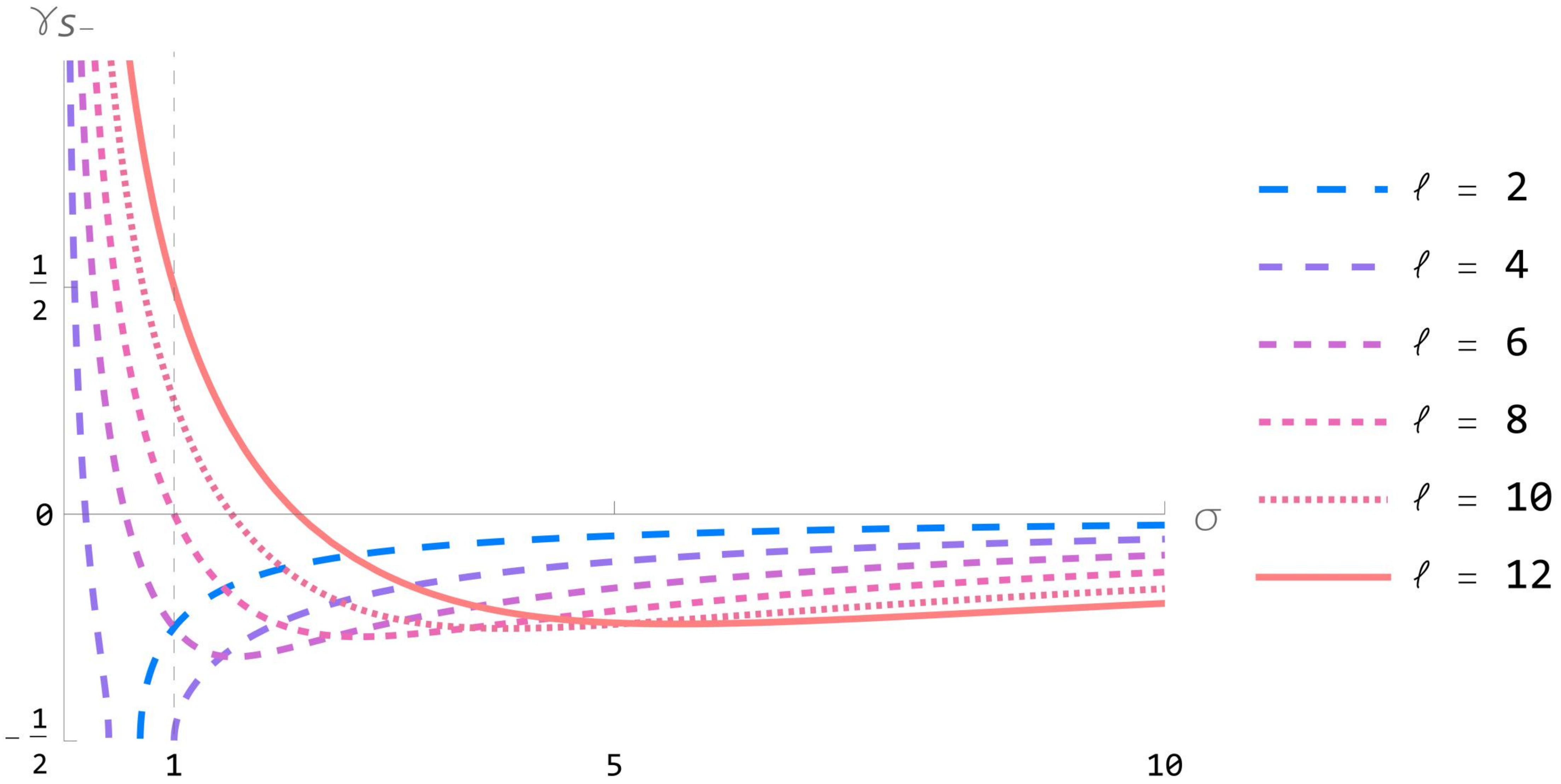
- Can find expressions for scaling exponents of **gravito-electromagnetic perturbations** in GR. In $D \geq 5$:
  - Tensor, vector, and electromagnetic scalar modes **positive-definite.**
  - **Gravitational scalar modes** are singular down to $\gamma = -1/2$ for either

    a) any r_H/L and $\ell < D-3$,

    b) any fixed ℓ provided sufficiently large r_H/L.



E.g. in $D = 11$



γ_{S-}
$\frac{1}{2}$
0
$-\frac{1}{2}$
1
5
10
σ
$\ell = 2$
$\ell = 4$
$\ell = 6$
$\ell = 8$
$\ell = 10$
$\ell = 12$

Marginal Deformations

Scaling exponents cross through zero!

→ **Marginal deformations:** Naïvely, presence of singularity sensitive to EFT corrections.

- For gravitational scalar modes: Tower of **marginal deformations** at

$$\frac{r_H^2}{L^2} = \frac{D-2}{D-4}\left[-1 + \frac{1}{2(D-3)^2}\ell(\ell + D - 3)\right]$$

  - *E.g.* for AF RN $\ell = D - 3$ is marginal.

- Non-linearly, all modes are generically excited!

  - $D = 5$ AF RN is non-linearly marginal: Lowest non-trivial mode $\ell = 2$ marginal and higher modes non-singular!

EFT Corrections

Deformations in the EFT of Gravity

Parameterise corrections from UV with higher-derivative **EFT corrections**

$$S = S_{\rm EM} + S_{\rm EFT}$$

- Due to rigidity of near-horizon geometry:

$$h_{..} \sim \rho^{\gamma}, \qquad \gamma = \gamma_{\rm GR} + \gamma_{\rm EFT}$$

→ EFT correction **resums** into exponent

Specifically:

$$S_{\rm EFT} = \int {\rm d}^D x \sqrt{-g} \sum_{\mathcal{O}} \frac{c_{\mathcal{O}}}{\Lambda^{[\mathcal{O}]-D}} \mathcal{O} \qquad \longrightarrow \qquad \gamma_{\rm EFT} = \sum_{\mathcal{O}} c_{\mathcal{O}} \gamma_{\mathcal{O}}$$

- **Marginal case**: When $\gamma_{\rm GR} = 0$, singularity of horizon sensitive to sign of

$$\hat{\gamma} := \gamma_{\rm EFT} \big|_{\gamma_{\rm GR}=0}$$

Example: EFT Correction

Specific yet generic **EFT correction**

$$S = S_{\mathrm{EM}} + \frac{\kappa c}{\Lambda^2} \int \mathrm{d}^5 x \sqrt{-g} (F_{\mu\nu} F^{\mu\nu})^2 .$$

- Static and spherically symmetric **background** solutions with

$$A(r) = B(r) = f(r) - \frac{c}{\Lambda^2} \frac{12 Q^4}{r^{10}}, \quad \Psi(r) = \frac{q}{r^2} - 16 c \frac{\kappa}{\Lambda^5} \frac{q^3}{r^8}$$

  - Changes to: Extremality relation and near-horizon geometry (same isometry)

→ Perturbation equations take same form (with shifted background) – shift in effective mass!

  - Marginal deformations for every harmonic

$$\hat{\gamma} = -\frac{c}{\Lambda^2 r_H^2} \frac{72 k_S^2 (k_S^2 - 4)^2}{15 k_S^4 - 128 k_S^2 + 256}$$

Presence of curvature singularities on horizon depends on UV physics!

Is this a breakdown of EFT?

Breakdown of Breakdowns

- Can estimate the ranges of validity for different approximations used.

  - **EFT expansion** (at quadratic order in metric perturbation)

$$S \supset \frac{1}{\kappa}\int \mathrm{d}^D x\sqrt{-g}\sum_{p=0,q=0}^{\infty}\left(\sqrt{\kappa}h\right)^2\left(\frac{\nabla}{\Lambda}\right)^p\left(\frac{\mathrm{Rie}}{\Lambda^2}\right)^q$$

under control when

$$r_H\Lambda \gg 1, \quad h \sim h_0\rho^\gamma \ll \frac{1}{\gamma}(\Lambda r_H)^2$$

  - **Metric perturbation series**

$$S \supset \frac{1}{\kappa}\mathrm{d}^D x\sqrt{-g}\sum_{m=2}^{\infty}\nabla^2\left(\sqrt{\kappa}h\right)^m$$

under control when

$$h \sim h_0\rho^\gamma \ll 1$$

→ **Metric perturbation theory out of control before EFT breaks down!**

Example: UV Avatar

Illustrate example: **Einstein-Maxwell-Dilaton** system

$$S_{\mathrm{EMD}} = \int \mathrm{d}^D x \sqrt{-g} \left(\frac{1}{2\kappa}(R - 2\Lambda) - \frac{1}{4} e^{\alpha\phi} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{1}{2} m^2 \phi^2 \right)$$

- Near-horizon geometry in previous example is exact background solution in this theory!
- Perturbation equations take same form with shifted mass → corrections to scaling exponents.
    - For **marginal** deformations

$$\hat{\gamma} = -\frac{3k_S^2 (k_S^2 - 4)^2}{4(15k_S^4 - 128k_S^2 + 256)} \left(\frac{4\alpha^2/\kappa}{r_H^2 m^2 + k_S^2} - \frac{1}{r_H^2 m^2} \right)$$

→ Possesses interesting features!

Example: UV Avatar

- Good **UV behaviour**: Two-derivative theory, arises universally in **supergravity!**

- When $m^2 r_H^2 \gg 1$, tree-level effective action includes F^{2n}-terms ($n > 1$). At leading order, reproduce F^4-correction from previous example with

$$c = \frac{\alpha^2}{32\kappa}, \qquad \Lambda = m$$

→ This presents a **partial UV completion** of the EFT before!

- In EFT expansion, marginal scaling exponents **manifestly negative** at leading-order

$$\hat{\gamma} = -\frac{\alpha^2/\kappa}{r_H^2 m^2} \frac{9k_S^2(k_S^2-4)^2}{4\left(15k_S^4 - 128k_S^2 + 256\right)} + \dots$$

→ Singularity already present in the UV! If EFT had broken down, would have expected details of UV completion to resolve this.

EFT doesn't break down

→ Still accurately describes UV theory!

Exactly when metric perturbations go out of control is UV sensitive.

A Conjecture

Leading EFT

EFT corrections in $D = 5$ up to four-derivatives

$$S_{\text{EFT}} = \int \mathrm{d}^5x\sqrt{-g}\left[\frac{c_1}{\kappa\Lambda^2}R^2 + \frac{c_2}{\kappa\Lambda^2}R_{\mu\nu}R^{\mu\nu} + \frac{c_3}{\kappa\Lambda^2}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \frac{c_4}{\Lambda^2}RF^2 \right.$$
$$\left. + \frac{c_5}{\Lambda^2}R_{\mu\nu}F^{\mu}{}_{\lambda}F^{\nu\lambda} + \frac{c_6}{\Lambda^2}R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \frac{\kappa c_7}{\Lambda^2}(F^2)^2 + \frac{\kappa c_8}{\Lambda^2}F_{\mu}{}^{\nu}F_{\nu}{}^{\rho}F_{\rho}{}^{\sigma}F_{\sigma}{}^{\mu}\right]$$

- Field-redefinition **invariant** combinations are c_3, c_6, and

$$c_0 = \frac{1}{2}\left[c_1 + 11c_2 + 31c_3 + 6c_4 + 12\left(c_5 + c_6\right) + 18\left(2c_7 + c_8\right)\right]$$
$$c_9 = c_2 + c_5 + c_8$$

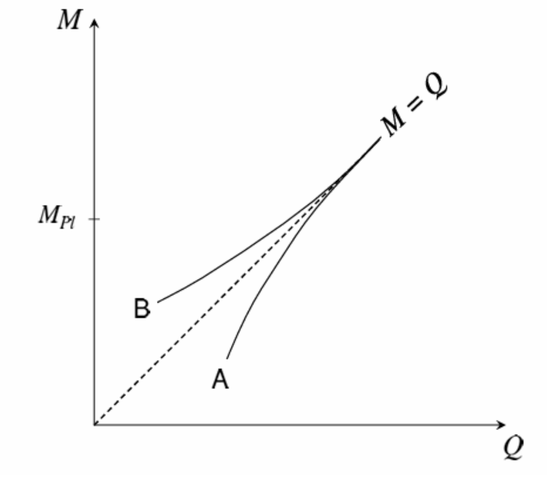
- Previous **near-horizon geometry** is still exact solution (perturbatively in EFT).

→ Shift in effective mass and scaling exponents.

Weak Gravity Conjecture

When $L \to \infty$, this is constrained by the Weak Gravity Conjecture (WGC).

- **Super-extremal** black holes in tension with Weak Cosmic Censorship**.**



  - **WGC**: UV physics should allow super-extremal states to allow for decay without super-extremality:

$$\frac{M/2}{|Q|} < 1$$

- For **small** black holes: Extremal charge-to-mass ratio should **decrease** with decreasing mass

  - At leading order (four derivatives), WGC implies

$$\frac{M/2}{|Q|} = 1 - \frac{1}{3\Lambda^2 r_H^2} c_0 \longrightarrow c_0 > 0$$

[Arkani-Hamed, Motl, Nicolis, Vafa ‘06]

Near-Horizon Negativity

Examples (Einstein-Maxwell-Dilaton and scalar toy model) suggest following **speculative conjecture**...

- **Near-Horizon Negativity**: EFTs **consistent** with UV completions have

$$\hat{\gamma} \lesssim 0$$

- For four-derivative corrections to Einstein-Maxwell, marginal scaling exponent is

$$\hat{\gamma} = -\frac{1}{\Lambda^2 r_H^2} \frac{64 k_S^2}{256 - 128 k_S^2 + 15 k_S^4} \tilde{c}_0$$

with

$$\tilde{c}_0 = \frac{\left(k_S^2 - 4\right)^2}{16} c_0 + \frac{1}{64}\left(k_S^2 - 8\right)\left[c_3\left(72 - 21 k_S^4\right) + 8\left(k_S^2 - 4\right) c_6\right]$$

Near-Horizon Negativity

→ **Near-Horizon Negativity** implies

$$\tilde{c}_0(\ell) > 0, \quad \forall \ell$$

- For $L \to \infty$, marginal mode is $\ell = 2$

$$\tilde{c}_0(\ell = 2) = c_0 > 0$$

→ Reproduces (and hence **strictly stronger** than) **AF WGC**.

[Kats, Motl, Padi '07]
[Horowitz, Kolanowski, Remmen, Santos '23]

- **Other bounds** obtained from $\ell > 2$.
  - For instance, as $\ell \to \infty$

$$\tilde{c}_\infty = \lim_{k_S \to \infty} \frac{\tilde{c}_0}{k_S^4} = \frac{4c_0 - 21c_3 + 8c_6}{64} > 0$$

Deformations of Extremal Black Branes

WIP in collaboration with A. D. Kovacs

Charged Black Branes

Can generalise the discussion to black branes in supergravity.

- Consider **Gravity + form field** $F_{(d+1)} = \mathrm{d}A_{(d)}$:

$$S = \frac{1}{2\kappa}\int \mathrm{d}^D x\sqrt{-g}\left[R - \frac{1}{2(d+1)!}F^2_{(d+1)}\right]$$

- Branes are solutions with isometry

$$\tilde{d} = D - d - 2$$

$$\mathrm{ISO}(1, d-1)\times \mathrm{SO}(\tilde{d}+2) \subset \mathrm{ISO}(1, D-1)$$

describing $d = p + 1$-dimensional world volumes.

- Metric for **extremal black branes**

$$\{\alpha, \beta, \ldots, \rho\} \in \{0, \ldots, d\}$$

$$\mathrm{d}s^2 = H^{-2/d}\eta_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta + H^{2/\tilde{d}}\left(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{\tilde{d}+1}\right), \quad H(r) = 1 + \left(\frac{r_0}{r}\right)^{\tilde{d}}$$

→ Reproduce AF RN in isotropic coordinates for $p = 0$.

Freund-Rubin Compactifications

- Near-horizon limit: Define $r = r_0(\rho/L)^{d/\tilde{d}}$, and zoom in on $\rho = 0$

$$\mathrm{d}s^2_{\mathrm{NH}} = \frac{\rho^2}{L^2}\eta_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta + \frac{L^2}{\rho^2}\mathrm{d}\rho^2 + \mathrm{d}\Omega^2_{D-p-2}, \quad L/d = r_0/\tilde{d}$$

→ **Freund-Rubin compactifications** ($\mathrm{AdS}_{d+1} \times S^{\tilde{d}+1}$): These are exact solutions to background equations of motion!

  - **Regular** radial coordinates still linearly related to ρ.

- **Perturbation equations**: Decomposition on sphere results in decoupled AdS_{d+1} **wave equations** labelled by harmonic ℓ with shifted effective masses.

$$\Box_{\mathrm{AdS}}\phi - m^2_{\mathrm{eff}}\phi = 0$$

→ Effectively **Kaluza-Klein reduction** onto sphere.

Deformations of Extremal Black Branes

- Analogous class of **deformations** to near-horizon geometry have

$$\eta^{\alpha\beta}\partial_\alpha\partial_\beta = 0$$

  - In extremal limit, solutions scale with **exponents**

$$\gamma_\pm = \frac{d}{2}\left(-1 \pm \sqrt{1 + \frac{4m_{\text{eff}}^2 L^2}{d^2}}\right)$$

- Discussion around extremal limit and explicit example from multi-centred black branes follow through.

- Full reduction of gravitational perturbations doable (and checked).

  - **Tensor** modes and **EM vector** and **scalar** modes positive definite

  - **Gravitational vector** and **scalar** modes can be singular down to $\gamma = -d/2$.

Conclusion

Summary

- **UV sensitivity** of extremal black holes
  - Deformations to near-horizon geometry are UV sensitive, but no **breakdown of EFT**!

    → Generalisation to **extremal black branes**!

    - Implications for **Aretakis instability** of black branes. [Cvetic, Porfirio, Satz '20]

- **Constraints** on EFTs
  - **Near-horizon negativity** (speculative): Generalisation of bound from **WGC** for leading EFT.

    → Physical intuition – Holography, energy conditions?

Thanks for your attention!
Questions?